Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.1.2-d-sec-^n-a+b-sec-^m

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [879]. This is test number [118].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (879)	% 0. (0)
Mathematica	% 98.41 (865)	% 1.59 (14)
Maple	% 83.62 (735)	% 16.38 (144)
Maxima	% 35.15 (309)	% 64.85 (570)
Fricas	% 44.71 (393)	% 55.29 (486)
Sympy	% 3.41 (30)	% 96.59 (849)
Giac	% 32.88 (289)	% 67.12 (590)

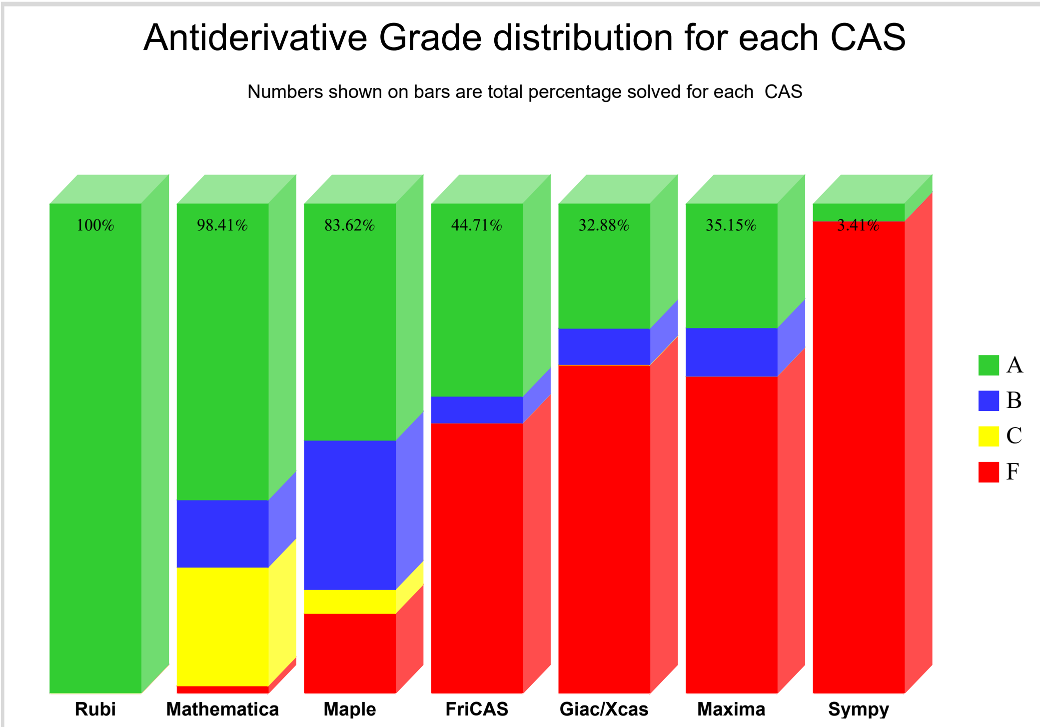
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

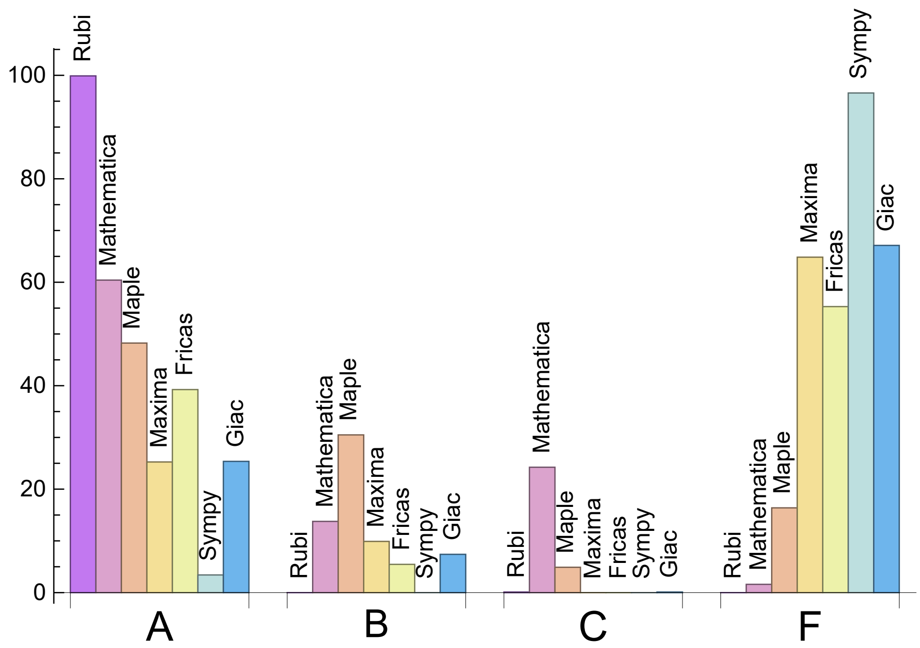
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.89	0.	0.11	0.
Mathematica	60.41	13.77	24.23	1.59
Maple	48.24	30.49	4.89	16.38
Maxima	25.26	9.9	0.	64.85
Fricas	39.25	5.46	0.	55.29
Sympy	3.41	0.	0.	96.59
Giac	25.37	7.39	0.11	67.12

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	152.93	0.91	128.	1.
Mathematica	6.55	1395.42	7.05	154.	1.05
Maple	0.8	516.9	2.47	218.	1.79
Maxima	1.66	678.71	4.96	131.	1.56
Fricas	1.7	555.54	4.46	309.	3.47
Sympy	0.6	6.2	0.34	0.	0.
Giac	1.74	172.81	1.55	130.	1.58

1.4 list of integrals that has no closed form antiderivative

{686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {286}

Mathematica {11, 21, 61, 71, 111, 112, 123, 131, 139, 146, 147, 151, 152, 157, 158, 163, 164, 186, 219, 227, 228, 235, 236, 237, 245, 249, 255, 256, 257, 264, 265, 266, 267, 269, 270, 271, 280, 281, 282, 283, 284, 285, 287, 294, 297, 298, 301, 302, 305, 306, 307, 310, 311, 314, 315, 318, 319, 320, 321, 322, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 388, 391, 392, 395, 396, 397, 408, 415, 416, 417, 425, 426, 427, 428, 433, 434, 436, 437, 440, 497, 531, 535, 536, 537, 538, 539, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 562, 563, 564, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 616, 617, 618, 621, 622, 623, 624, 625, 626, 627, 634, 635, 642, 643, 662, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 837, 838, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 859, 860, 861, 862, 863, 864, 866, 867, 868, 869, 870, 871, 872, 873, 874, 878, 879}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

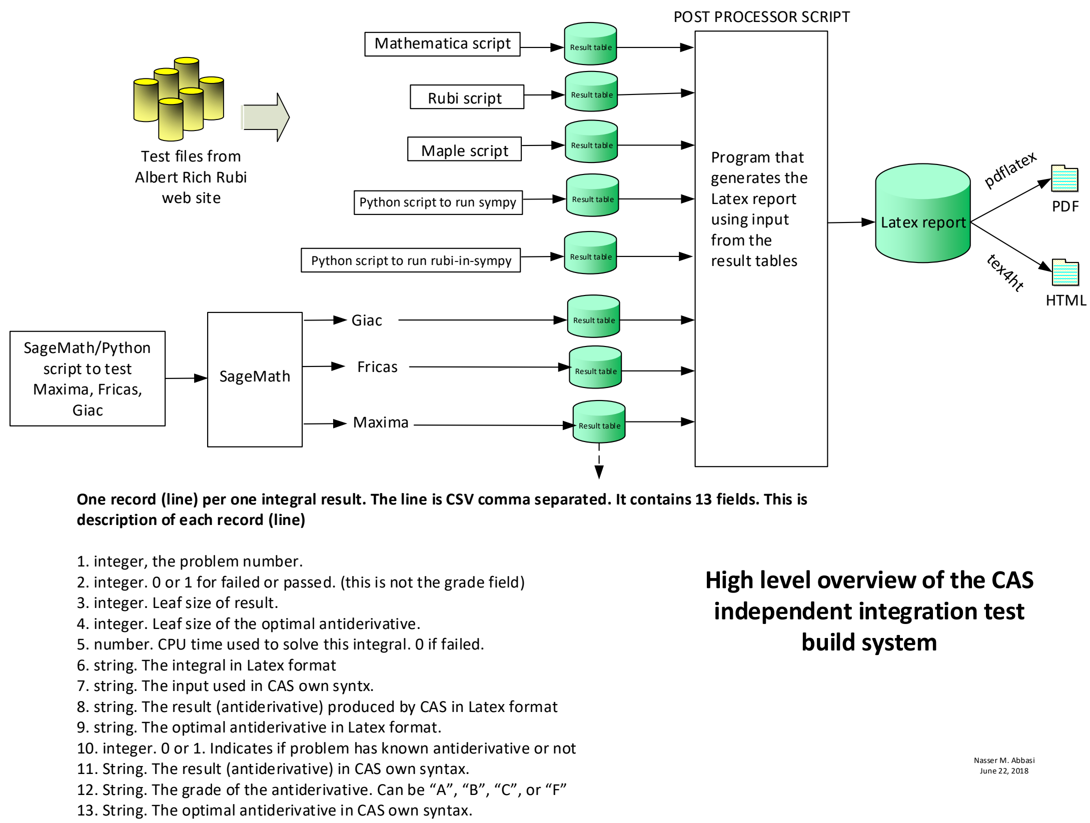
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

B grade: { }

C grade: { 286 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 36, 37, 38, 39, 40, 41, 46, 47, 49, 50, 51, 55, 56, 57, 59, 60, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 132, 134, 135, 136, 137, 140, 165, 166, 167, 168, 169, 170, 171, 175, 189, 204, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 242, 243, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 286, 291, 295, 296, 299, 300, 303, 304, 308, 309, 312, 313, 316, 317, 341, 342, 343, 344, 363, 385, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 439, 440, 443, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 531, 532, 533, 534, 537, 538, 539, 540, 544, 545, 546, 547, 554, 555, 556, 558, 559, 562, 563, 564, 565, 566, 570, 571, 572, 573, 574, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 627, 629, 630, 631, 632, 633, 636, 637, 638, 639, 644, 645, 646, 649, 650, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 782, 783, 784, 785, 786, 787, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 875, 876, 877 }

B grade: { 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 42, 43, 44, 45, 48, 52, 53, 54, 58, 61, 62, 70, 71, 77, 78, 80, 88, 146, 147, 151, 152, 157, 158, 163, 164, 253, 260, 280, 281, 282, 283, 284, 297, 298, 301, 302, 305, 306, 310, 311, 314, 315, 318, 319, 329, 330, 331, 333, 334, 335, 337, 338, 339, 345, 346, 347, 348, 431, 438, 480, 529, 530, 549, 551, 557, 568, 576, 616, 617, 622, 623, 624, 625, 626, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 780, 781, 788, 789, 790, 822, 878, 879 }

C grade: { 96, 97, 98, 111, 112, 113, 114, 115, 117, 118, 123, 131, 133, 138, 139, 141, 142, 143, 144, 145, 148, 149, 150, 153, 154, 155, 156, 159, 160, 161, 162, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 233, 234, 244, 272, 273, 274, 275, 276, 277, 278, 279, 285, 287, 294, 307, 320, 321, 322, 325, 326, 327, 328, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 418, 419, 505, 535, 536, 541, 542, 543, 548, 550, 552, 553, 560, 561, 567, 569, 575, 577, 578, 628, 634, 635, 640, 641, 642, 643, 647, 648, 654, 655, 661, 662, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874 }

F grade: { 288, 289, 290, 292, 293, 323, 324, 332, 336, 340, 441, 442, 444, 445}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 99, 100, 101, 102, 107, 108, 109, 110, 112, 116, 118, 121, 123, 142, 167, 168, 169, 170, 171, 175, 176, 177, 178, 182, 183, 184, 185, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 211, 212, 213, 214, 215, 216, 217, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 248, 249, 250, 251, 253, 254, 255, 256, 257, 260, 261, 262, 263, 264, 269, 270, 271, 351, 352, 354, 355, 359, 360, 361, 363, 367, 368, 369, 370, 374, 375, 376, 377, 378, 379, 381, 382, 383, 384, 386, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 404, 405, 406, 407, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 489, 490, 491, 492, 493, 497, 498, 500, 501, 503, 504, 508, 509, 510, 516, 517, 518, 519, 523, 524, 525, 526, 527, 528, 529, 530, 534, 558, 559, 581, 582, 583, 584, 585, 589, 590, 591, 592, 596, 597, 598, 599, 605, 606, 607, 608, 611, 612, 650, 678, 679, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793, 794, 795, 798, 799, 806, 813, 819, 820, 858 }

B grade: { 94, 95, 96, 97, 98, 103, 104, 105, 106, 111, 113, 114, 115, 117, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 165, 166, 172, 173, 174, 179, 180, 181, 186, 187, 188, 189, 194, 204, 210, 218, 219, 220, 226, 227, 228, 235, 236, 243, 244, 245, 246, 247, 252, 258, 259, 265, 266, 267, 268, 353, 356, 357, 358, 362, 364, 365, 366, 371, 372, 373, 380, 385, 387, 388, 396, 402, 403, 408, 409, 439, 440, 487, 488, 494, 495, 496, 499, 502, 505, 506, 507, 511, 512, 513, 514, 515, 520, 521, 522, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 586, 587, 588, 593, 594, 595, 600, 601, 602, 603, 604, 609, 610, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 630, 631, 632, 633, 637, 638, 639, 644, 645, 646, 651, 652, 653, 656, 657, 658, 659, 660, 663, 664, 665, 666, 667, 668, 670, 671, 796, 797, 800, 801, 802, 803, 804, 805, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 842, 843, 844, 848, 849, 850, 855, 856, 857, 862, 863, 864, 865, 866, 869, 870, 871, 872, 873 }

C grade: { 628, 629, 634, 635, 636, 640, 641, 642, 643, 647, 648, 649, 654, 655, 661, 662, 669, 672, 673, 674, 675, 676, 677, 680, 681, 682, 683, 684, 840, 841, 845, 846, 847, 851, 852, 853, 854, 859, 860, 861, 867, 868, 874 }

F grade: { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 242, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 441, 442, 443, 444, 445, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 875, 876, 877, 878, 879 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 221, 222, 229, 238, 247, 248, 269, 400, 401, 407, 414, 422, 423, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 523, 524, 525, 526, 527, 528, 529, 530, 686, 691, 693, 698, 703, 705, 707, 712, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793

B grade: { 30, 41, 42, 43, 44, 48, 94, 95, 96, 97, 98, 103, 104, 111, 112, 117, 118, 218, 219, 220, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 237, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 254, 255, 258, 259, 261, 262, 265, 266, 267, 268, 270, 271, 398, 399, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 415, 417, 418, 419, 420, 421, 424, 425, 426, 429, 430, 432, 433, 436, 437, 439, 440 }

C grade: { }

F grade: { 90, 91, 92, 93, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 236, 253, 256, 257, 260, 263, 264, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 416, 427, 428, 431, 434, 435, 438, 441, 442, 443, 444, 445, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687, 688, 689, 690, 692, 694, 695, 696, 697, 699, 700, 701, 702, 704, 706, 708, 709, 710, 711, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 135, 137, 140, 141, 142, 218, 221, 222, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 270, 271, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 446, 447, 448, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 490, 491, 492, 493, 494, 495, 496, 500, 501, 503, 504, 505, 508, 513, 523, 524, 525, 526, 527, 528, 529, 530, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 782, 783, 784, 785, 786, 787, 791, 792, 793 }

B grade: { 4, 5, 14, 117, 118, 129, 130, 131, 136, 138, 139, 219, 220, 227, 228, 243, 244, 252, 259, 265, 266, 267, 268, 269, 449, 450, 460, 488, 489, 497, 498, 499, 502, 506, 507, 509, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 521, 522 }

C grade: { }

F grade: { 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 441, 442, 443, 444, 445, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 777, 778, 779, 780, 781, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879 }

2.1.6 Sympy

A grade: { 4, 5, 6, 449, 450, 451, 686, 691, 703, 705, 707, 712, 720, 721, 722, 723, 724, 750, 751, 752, 753, 761, 762, 783, 784, 785, 786, 787, 791, 792 }

B grade: { }

C grade: { }

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2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	128	266	0	149
normalized size	1	1.	0.89	1.08	1.51	3.13	0.	1.75
time (sec)	N/A	0.059	0.163	0.031	1.18	1.675	0.	1.298

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	95	236	0	130
normalized size	1	1.	0.95	1.14	1.51	3.75	0.	2.06
time (sec)	N/A	0.046	0.133	0.029	1.048	1.636	0.	1.294

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	78	198	0	108
normalized size	1	1.	1.	1.09	1.66	4.21	0.	2.3
time (sec)	N/A	0.042	0.019	0.025	1.073	1.691	0.	1.346

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	39	162	37	85
normalized size	1	1.	1.	1.33	1.62	6.75	1.54	3.54
time (sec)	N/A	0.023	0.009	0.023	1.104	1.694	5.675	1.384

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	31	95	41	66
normalized size	1	1.	1.	1.5	1.94	5.94	2.56	4.12
time (sec)	N/A	0.007	0.002	0.005	1.057	1.767	1.735	1.352

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	21	27	38	15	53
normalized size	1	1.	1.73	1.4	1.8	2.53	1.	3.53
time (sec)	N/A	0.019	0.009	0.047	1.1	1.622	2.861	1.346

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	38	46	72	0	76
normalized size	1	1.	0.84	1.	1.21	1.89	0.	2.
time (sec)	N/A	0.035	0.05	0.056	1.081	1.674	0.	1.296

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	62	105	0	97
normalized size	1	1.	1.06	0.91	1.15	1.94	0.	1.8
time (sec)	N/A	0.04	0.067	0.095	1.083	1.692	0.	1.189

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	77	136	0	116
normalized size	1	1.	0.96	0.79	1.01	1.79	0.	1.53
time (sec)	N/A	0.055	0.097	0.103	1.085	1.722	0.	1.303

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	487	124	180	320	0	186
normalized size	1	1.	3.99	1.02	1.48	2.62	0.	1.52
time (sec)	N/A	0.095	1.513	0.036	0.993	1.726	0.	1.385

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	877	102	196	288	0	165
normalized size	1	1.	9.14	1.06	2.04	3.	0.	1.72
time (sec)	N/A	0.084	6.405	0.035	1.117	1.766	0.	1.469

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	318	78	115	246	0	143
normalized size	1	1.	4.3	1.05	1.55	3.32	0.	1.93
time (sec)	N/A	0.081	0.63	0.031	1.128	1.743	0.	1.436

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	219	58	109	215	0	122
normalized size	1	1.	4.06	1.07	2.02	3.98	0.	2.26
time (sec)	N/A	0.047	0.594	0.03	1.132	1.737	0.	1.484

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	171	50	55	193	0	107
normalized size	1	1.	5.03	1.47	1.62	5.68	0.	3.15
time (sec)	N/A	0.024	0.434	0.026	1.172	1.769	0.	1.485

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	47	51	70	131	0	107
normalized size	1	1.	1.38	1.5	2.06	3.85	0.	3.15
time (sec)	N/A	0.051	0.013	0.055	1.151	1.71	0.	1.396

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	65	82	0	86
normalized size	1	1.	0.76	1.16	1.44	1.82	0.	1.91
time (sec)	N/A	0.06	0.039	0.058	1.099	1.705	0.	1.327

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	41	64	82	113	0	108
normalized size	1	1.	0.72	1.12	1.44	1.98	0.	1.89
time (sec)	N/A	0.082	0.085	0.066	0.989	1.694	0.	1.357

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	53	90	112	154	0	130
normalized size	1	1.	0.61	1.03	1.29	1.77	0.	1.49
time (sec)	N/A	0.079	0.138	0.07	1.124	1.707	0.	1.375

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	61	96	128	186	0	151
normalized size	1	1.	0.59	0.93	1.24	1.81	0.	1.47
time (sec)	N/A	0.11	0.136	0.109	1.076	1.708	0.	1.373

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	487	124	242	329	0	186
normalized size	1	1.	4.27	1.09	2.12	2.89	0.	1.63
time (sec)	N/A	0.125	1.465	0.038	1.124	1.736	0.	1.396

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	877	101	211	286	0	165
normalized size	1	1.	9.43	1.09	2.27	3.08	0.	1.77
time (sec)	N/A	0.114	6.399	0.033	1.057	1.757	0.	1.354

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	154	80	140	254	0	143
normalized size	1	1.	2.14	1.11	1.94	3.53	0.	1.99
time (sec)	N/A	0.075	5.944	0.036	1.086	1.71	0.	1.37

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	235	71	123	251	0	135
normalized size	1	1.	3.56	1.08	1.86	3.8	0.	2.05
time (sec)	N/A	0.047	0.893	0.03	1.147	1.753	0.	1.353

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	211	65	86	238	0	108
normalized size	1	1.	4.4	1.35	1.79	4.96	0.	2.25
time (sec)	N/A	0.058	0.866	0.054	1.088	1.775	0.	1.383

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	72	100	159	0	135
normalized size	1	1.	1.37	1.22	1.69	2.69	0.	2.29
time (sec)	N/A	0.067	0.07	0.06	1.101	1.758	0.	1.39

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	96	119	0	108
normalized size	1	1.	0.7	1.17	1.52	1.89	0.	1.71
time (sec)	N/A	0.075	0.061	0.065	1.116	1.655	0.	1.354

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	51	100	127	151	0	130
normalized size	1	1.	0.6	1.18	1.49	1.78	0.	1.53
time (sec)	N/A	0.097	0.118	0.073	1.113	1.678	0.	1.412

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	63	121	158	194	0	151
normalized size	1	1.	0.6	1.15	1.5	1.85	0.	1.44
time (sec)	N/A	0.112	0.157	0.076	1.128	1.698	0.	1.373

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	73	143	193	230	0	173
normalized size	1	1.	0.57	1.11	1.5	1.78	0.	1.34
time (sec)	N/A	0.134	0.207	0.121	1.051	1.703	0.	1.325

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	211	146	365	366	0	208
normalized size	1	1.	1.55	1.07	2.68	2.69	0.	1.53
time (sec)	N/A	0.172	0.832	0.043	0.998	1.768	0.	1.423

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	498	123	257	325	0	186
normalized size	1	1.	4.49	1.11	2.32	2.93	0.	1.68
time (sec)	N/A	0.135	1.527	0.039	1.169	1.757	0.	1.366

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	877	102	236	292	0	165
normalized size	1	1.	9.14	1.06	2.46	3.04	0.	1.72
time (sec)	N/A	0.109	6.395	0.037	1.13	1.733	0.	1.39

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	773	93	157	281	0	157
normalized size	1	1.	8.49	1.02	1.73	3.09	0.	1.73
time (sec)	N/A	0.09	6.246	0.036	1.103	1.79	0.	1.302

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	272	86	149	286	0	174
normalized size	1	1.	3.73	1.18	2.04	3.92	0.	2.38
time (sec)	N/A	0.076	1.475	0.061	1.186	1.783	0.	1.41

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	241	86	115	270	0	174
normalized size	1	1.	3.3	1.18	1.58	3.7	0.	2.38
time (sec)	N/A	0.079	1.772	0.061	1.069	1.79	0.	1.413

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	91	94	131	201	0	157
normalized size	1	1.	1.25	1.29	1.79	2.75	0.	2.15
time (sec)	N/A	0.082	0.103	0.067	1.081	1.813	0.	1.398

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	56	111	140	157	0	130
normalized size	1	1.	0.64	1.28	1.61	1.8	0.	1.49
time (sec)	N/A	0.098	0.101	0.074	1.099	1.712	0.	1.416

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	63	133	173	190	0	151
normalized size	1	1.	0.62	1.3	1.7	1.86	0.	1.48
time (sec)	N/A	0.114	0.146	0.081	1.156	1.686	0.	1.387

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	73	169	223	231	0	173
normalized size	1	1.	0.57	1.33	1.76	1.82	0.	1.36
time (sec)	N/A	0.145	0.207	0.088	1.097	1.735	0.	1.389

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	83	185	252	267	0	194
normalized size	1	1.	0.56	1.26	1.71	1.82	0.	1.32
time (sec)	N/A	0.155	0.261	0.128	1.165	1.737	0.	1.277

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	229	168	424	413	0	230
normalized size	1	1.	1.47	1.08	2.72	2.65	0.	1.47
time (sec)	N/A	0.198	1.343	0.046	1.129	1.805	0.	1.473

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	374	183	277	331	0	154
normalized size	1	1.	3.63	1.78	2.69	3.21	0.	1.5
time (sec)	N/A	0.098	3.199	0.043	1.217	2.024	0.	1.368

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	250	143	219	301	0	136
normalized size	1	1.	2.94	1.68	2.58	3.54	0.	1.6
time (sec)	N/A	0.093	1.334	0.039	1.145	1.959	0.	1.427

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	194	99	161	266	0	113
normalized size	1	1.	3.8	1.94	3.16	5.22	0.	2.22
time (sec)	N/A	0.106	0.699	0.033	1.14	1.995	0.	1.393

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	109	58	101	181	0	73
normalized size	1	1.	2.87	1.53	2.66	4.76	0.	1.92
time (sec)	N/A	0.068	0.166	0.025	1.024	1.896	0.	1.343

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	31	53	0	22
normalized size	1	1.	0.77	0.77	1.41	2.41	0.	1.
time (sec)	N/A	0.024	0.026	0.027	1.086	1.528	0.	1.262

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	58	37	66	89	0	38
normalized size	1	1.	2.	1.28	2.28	3.07	0.	1.31
time (sec)	N/A	0.014	0.123	0.032	1.648	1.599	0.	1.3

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	89	68	124	116	0	78
normalized size	1	1.	2.02	1.55	2.82	2.64	0.	1.77
time (sec)	N/A	0.057	0.222	0.051	1.659	1.664	0.	1.222

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	117	103	180	149	0	99
normalized size	1	1.	1.58	1.39	2.43	2.01	0.	1.34
time (sec)	N/A	0.082	0.233	0.053	1.757	1.639	0.	1.284

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	143	136	238	180	0	119
normalized size	1	1.	1.52	1.45	2.53	1.91	0.	1.27
time (sec)	N/A	0.09	0.31	0.057	1.743	1.67	0.	1.322

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	173	171	293	213	0	136
normalized size	1	1.	1.47	1.45	2.48	1.81	0.	1.15
time (sec)	N/A	0.101	0.309	0.054	1.696	1.677	0.	1.308

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	300	162	257	427	0	165
normalized size	1	1.	2.44	1.32	2.09	3.47	0.	1.34
time (sec)	N/A	0.179	1.879	0.045	1.147	1.751	0.	1.421

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	247	120	196	387	0	143
normalized size	1	1.	2.78	1.35	2.2	4.35	0.	1.61
time (sec)	N/A	0.155	1.114	0.035	1.159	1.697	0.	1.417

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	160	77	132	305	0	104
normalized size	1	1.	2.42	1.17	2.	4.62	0.	1.58
time (sec)	N/A	0.118	0.354	0.031	1.191	1.708	0.	1.462

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	45	32	62	123	0	42
normalized size	1	1.	0.82	0.58	1.13	2.24	0.	0.76
time (sec)	N/A	0.066	0.067	0.028	1.125	1.522	0.	1.321

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	32	63	126	0	42
normalized size	1	1.	1.09	0.58	1.15	2.29	0.	0.76
time (sec)	N/A	0.049	0.13	0.031	1.183	1.561	0.	1.322

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	112	56	97	198	0	68
normalized size	1	1.	1.96	0.98	1.7	3.47	0.	1.19
time (sec)	N/A	0.069	0.264	0.039	1.638	1.636	0.	1.279

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	151	88	159	230	0	107
normalized size	1	1.	2.1	1.22	2.21	3.19	0.	1.49
time (sec)	N/A	0.129	0.477	0.054	1.677	1.621	0.	1.324

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	177	122	221	257	0	128
normalized size	1	1.	1.61	1.11	2.01	2.34	0.	1.16
time (sec)	N/A	0.175	0.372	0.061	1.734	1.641	0.	1.361

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	199	156	279	278	0	146
normalized size	1	1.	1.6	1.26	2.25	2.24	0.	1.18
time (sec)	N/A	0.19	0.427	0.059	1.747	1.683	0.	1.333

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	351	181	285	548	0	188
normalized size	1	1.	2.17	1.12	1.76	3.38	0.	1.16
time (sec)	N/A	0.292	0.981	0.047	1.191	1.774	0.	1.383

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	294	139	223	506	0	165
normalized size	1	1.	2.3	1.09	1.74	3.95	0.	1.29
time (sec)	N/A	0.265	1.255	0.039	1.191	1.773	0.	1.401

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	209	96	161	424	0	127
normalized size	1	1.	1.99	0.91	1.53	4.04	0.	1.21
time (sec)	N/A	0.222	0.486	0.036	1.115	1.718	0.	1.347

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	57	45	90	186	0	62
normalized size	1	1.	0.69	0.54	1.08	2.24	0.	0.75
time (sec)	N/A	0.123	0.114	0.033	1.188	1.558	0.	1.346

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	71	32	63	182	0	42
normalized size	1	1.	0.86	0.39	0.76	2.19	0.	0.51
time (sec)	N/A	0.097	0.16	0.031	1.166	1.599	0.	1.398

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	86	45	90	186	0	62
normalized size	1	1.	1.04	0.54	1.08	2.24	0.	0.75
time (sec)	N/A	0.081	0.219	0.039	1.12	1.61	0.	1.341

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	162	75	124	300	0	92
normalized size	1	1.	1.84	0.85	1.41	3.41	0.	1.05
time (sec)	N/A	0.112	0.271	0.04	1.712	1.66	0.	1.341

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	169	107	185	325	0	130
normalized size	1	1.	1.64	1.04	1.8	3.16	0.	1.26
time (sec)	N/A	0.221	0.562	0.06	1.718	1.902	0.	1.374

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	181	141	248	363	0	153
normalized size	1	1.	1.23	0.96	1.69	2.47	0.	1.04
time (sec)	N/A	0.29	0.57	0.064	1.761	1.913	0.	1.404

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	403	200	312	670	0	209
normalized size	1	1.	2.09	1.04	1.62	3.47	0.	1.08
time (sec)	N/A	0.393	1.57	0.047	1.126	2.088	0.	1.427

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	349	158	251	632	0	188
normalized size	1	1.	2.19	0.99	1.58	3.97	0.	1.18
time (sec)	N/A	0.369	1.22	0.04	1.212	2.049	0.	1.371

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	193	115	188	539	0	149
normalized size	1	1.	1.42	0.85	1.38	3.96	0.	1.1
time (sec)	N/A	0.323	0.908	0.039	1.186	1.752	0.	1.364

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	69	56	117	248	0	80
normalized size	1	1.	0.57	0.47	0.98	2.07	0.	0.67
time (sec)	N/A	0.169	0.206	0.039	1.205	1.584	0.	1.323

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	58	117	251	0	80
normalized size	1	1.	0.78	0.52	1.04	2.24	0.	0.71
time (sec)	N/A	0.154	0.226	0.035	1.146	1.592	0.	1.462

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	99	58	117	251	0	80
normalized size	1	1.	0.88	0.52	1.04	2.24	0.	0.71
time (sec)	N/A	0.125	0.259	0.035	1.322	1.586	0.	1.352

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	58	117	248	0	80
normalized size	1	1.	1.	0.52	1.04	2.21	0.	0.71
time (sec)	N/A	0.111	0.237	0.036	1.31	1.596	0.	1.368

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	224	94	151	397	0	112
normalized size	1	1.	2.02	0.85	1.36	3.58	0.	1.01
time (sec)	N/A	0.16	0.373	0.042	1.729	1.621	0.	1.442

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	263	126	213	444	0	151
normalized size	1	1.	2.09	1.	1.69	3.52	0.	1.2
time (sec)	N/A	0.304	0.457	0.063	1.52	1.684	0.	1.402

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	289	160	275	470	0	173
normalized size	1	1.	1.64	0.91	1.56	2.67	0.	0.98
time (sec)	N/A	0.393	0.552	0.063	1.658	1.687	0.	1.369

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	401	177	278	755	0	209
normalized size	1	1.	2.	0.88	1.39	3.78	0.	1.04
time (sec)	N/A	0.48	1.845	0.041	1.123	1.76	0.	1.472

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	219	134	215	671	0	170
normalized size	1	1.	1.24	0.76	1.21	3.79	0.	0.96
time (sec)	N/A	0.427	1.968	0.043	1.179	1.78	0.	1.392

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	97	71	144	316	0	97
normalized size	1	1.	0.61	0.45	0.91	1.99	0.	0.61
time (sec)	N/A	0.216	0.192	0.039	1.123	1.632	0.	1.354

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	97	58	117	312	0	80
normalized size	1	1.	0.61	0.36	0.74	1.96	0.	0.5
time (sec)	N/A	0.215	0.195	0.038	1.142	1.654	0.	1.443

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	110	45	90	312	0	62
normalized size	1	1.	0.79	0.32	0.65	2.24	0.	0.45
time (sec)	N/A	0.182	0.218	0.039	1.14	1.64	0.	1.536

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	125	58	117	312	0	80
normalized size	1	1.	0.87	0.41	0.82	2.18	0.	0.56
time (sec)	N/A	0.157	0.23	0.035	1.051	1.612	0.	1.471

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	138	71	144	316	0	97
normalized size	1	1.	0.97	0.5	1.01	2.21	0.	0.68
time (sec)	N/A	0.142	0.267	0.038	1.095	1.663	0.	1.504

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	280	113	178	514	0	135
normalized size	1	1.	1.94	0.78	1.24	3.57	0.	0.94
time (sec)	N/A	0.207	0.522	0.044	1.662	1.711	0.	1.375

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	319	145	240	541	0	174
normalized size	1	1.	2.01	0.91	1.51	3.4	0.	1.09
time (sec)	N/A	0.397	0.659	0.07	1.528	1.712	0.	1.564

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	345	179	302	591	0	196
normalized size	1	1.	1.6	0.83	1.4	2.75	0.	0.91
time (sec)	N/A	0.509	0.722	0.072	1.556	1.712	0.	1.41

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	58	82	0	212	0	162
normalized size	1	1.	0.48	0.67	0.	1.74	0.	1.33
time (sec)	N/A	0.207	0.14	0.211	0.	1.663	0.	4.851

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	48	72	0	185	0	136
normalized size	1	1.	0.56	0.84	0.	2.15	0.	1.58
time (sec)	N/A	0.152	0.102	0.169	0.	1.677	0.	4.858

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	36	62	0	155	0	111
normalized size	1	1.	0.64	1.11	0.	2.77	0.	1.98
time (sec)	N/A	0.083	0.095	0.141	0.	1.645	0.	4.852

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	42	0	104	0	84
normalized size	1	1.	1.12	1.62	0.	4.	0.	3.23
time (sec)	N/A	0.029	0.068	0.122	0.	1.652	0.	4.78

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	60	89	197	350	0	0
normalized size	1	1.	1.62	2.41	5.32	9.46	0.	0.
time (sec)	N/A	0.023	0.094	0.139	1.824	1.731	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	123	1068	647	0	0
normalized size	1	1.	1.	1.98	17.23	10.44	0.	0.
time (sec)	N/A	0.063	0.192	0.185	2.165	1.768	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	47	221	1430	720	0	0
normalized size	1	1.	0.46	2.17	14.02	7.06	0.	0.
time (sec)	N/A	0.118	0.098	0.22	2.268	1.795	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	47	310	2593	782	0	0
normalized size	1	1.	0.34	2.25	18.79	5.67	0.	0.
time (sec)	N/A	0.178	0.088	0.253	2.587	1.768	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	47	399	8961	846	0	0
normalized size	1	1.	0.27	2.29	51.5	4.86	0.	0.
time (sec)	N/A	0.235	0.087	0.294	3.68	1.868	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	70	93	0	262	0	243
normalized size	1	1.	0.43	0.57	0.	1.62	0.	1.5
time (sec)	N/A	0.275	0.523	0.166	0.	1.747	0.	5.212

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	60	83	0	230	0	204
normalized size	1	1.	0.52	0.72	0.	1.98	0.	1.76
time (sec)	N/A	0.194	0.179	0.148	0.	1.69	0.	4.836

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	48	73	0	189	0	163
normalized size	1	1.	0.56	0.85	0.	2.2	0.	1.9
time (sec)	N/A	0.12	0.13	0.135	0.	1.715	0.	4.932

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	38	63	0	158	0	126
normalized size	1	1.	0.64	1.07	0.	2.68	0.	2.14
time (sec)	N/A	0.061	0.088	0.125	0.	1.685	0.	4.816

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	181	1346	620	0	0
normalized size	1	1.	1.14	2.74	20.39	9.39	0.	0.
time (sec)	N/A	0.037	0.212	0.147	2.022	1.728	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	89	125	1084	662	0	0
normalized size	1	1.	1.37	1.92	16.68	10.18	0.	0.
time (sec)	N/A	0.118	0.193	0.162	2.049	1.734	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	108	222	0	736	0	554
normalized size	1	1.	1.02	2.09	0.	6.94	0.	5.23
time (sec)	N/A	0.127	0.362	0.198	0.	1.887	0.	7.48

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	120	311	0	803	0	702
normalized size	1	1.	0.83	2.16	0.	5.58	0.	4.88
time (sec)	N/A	0.187	0.517	0.227	0.	2.094	0.	7.364

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	80	105	0	312	0	282
normalized size	1	1.	0.39	0.52	0.	1.54	0.	1.39
time (sec)	N/A	0.374	0.2	0.169	0.	2.021	0.	5.508

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	70	95	0	277	0	243
normalized size	1	1.	0.48	0.65	0.	1.9	0.	1.66
time (sec)	N/A	0.23	0.502	0.157	0.	1.973	0.	5.224

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	60	85	0	236	0	204
normalized size	1	1.	0.52	0.73	0.	2.03	0.	1.76
time (sec)	N/A	0.155	0.166	0.135	0.	1.95	0.	5.426

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	50	75	0	204	0	165
normalized size	1	1.	0.56	0.84	0.	2.29	0.	1.85
time (sec)	N/A	0.095	0.091	0.128	0.	1.962	0.	4.919

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	360	214	1883	792	0	0
normalized size	1	1.	3.67	2.18	19.21	8.08	0.	0.
time (sec)	N/A	0.102	10.179	0.164	2.444	2.101	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	189	128	1867	706	0	493
normalized size	1	1.	2.01	1.36	19.86	7.51	0.	5.24
time (sec)	N/A	0.157	4.743	0.183	2.442	2.084	0.	7.194

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	150	224	0	763	0	0
normalized size	1	1.	1.42	2.11	0.	7.2	0.	0.
time (sec)	N/A	0.164	0.522	0.189	0.	2.049	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	151	313	0	830	0	707
normalized size	1	1.	1.05	2.17	0.	5.76	0.	4.91
time (sec)	N/A	0.235	0.771	0.214	0.	2.013	0.	8.195

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	161	402	0	911	0	856
normalized size	1	1.	0.88	2.21	0.	5.01	0.	4.7
time (sec)	N/A	0.294	0.769	0.261	0.	2.184	0.	7.982

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	30	42	0	107	0	77
normalized size	1	1.	1.11	1.56	0.	3.96	0.	2.85
time (sec)	N/A	0.03	0.111	0.15	0.	1.935	0.	1.508

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	188	91	197	464	0	88
normalized size	1	1.	4.95	2.39	5.18	12.21	0.	2.32
time (sec)	N/A	0.022	3.433	0.138	1.954	2.377	0.	1.518

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	260	103	1068	753	0	151
normalized size	1	1.	4.	1.58	16.43	11.58	0.	2.32
time (sec)	N/A	0.064	0.873	0.204	2.198	2.306	0.	1.409

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	106	314	0	914	0	277
normalized size	1	1.	0.76	2.24	0.	6.53	0.	1.98
time (sec)	N/A	0.276	0.202	0.19	0.	2.4	0.	10.076

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	86	221	0	846	0	184
normalized size	1	1.	0.83	2.12	0.	8.13	0.	1.77
time (sec)	N/A	0.158	0.147	0.173	0.	2.252	0.	9.511

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	83	121	0	701	0	178
normalized size	1	1.	1.14	1.66	0.	9.6	0.	2.44
time (sec)	N/A	0.088	0.077	0.132	0.	2.289	0.	9.723

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	64	95	0	428	0	88
normalized size	1	1.	1.39	2.07	0.	9.3	0.	1.91
time (sec)	N/A	0.036	0.047	0.127	0.	2.222	0.	9.774

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	5416	141	0	784	0	0
normalized size	1	1.	63.72	1.66	0.	9.22	0.	0.
time (sec)	N/A	0.066	23.561	0.134	0.	2.246	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	105	201	0	1123	0	0
normalized size	1	1.	0.97	1.86	0.	10.4	0.	0.
time (sec)	N/A	0.179	0.12	0.195	0.	2.389	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	118	380	0	1196	0	0
normalized size	1	1.	0.8	2.59	0.	8.14	0.	0.
time (sec)	N/A	0.248	0.24	0.226	0.	2.539	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	124	417	0	1095	0	331
normalized size	1	1.	0.68	2.28	0.	5.98	0.	1.81
time (sec)	N/A	0.425	0.434	0.188	0.	2.372	0.	10.149

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	114	322	0	1033	0	279
normalized size	1	1.	0.79	2.22	0.	7.12	0.	1.92
time (sec)	N/A	0.288	0.285	0.167	0.	2.201	0.	10.513

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	104	225	0	895	0	215
normalized size	1	1.	0.99	2.14	0.	8.52	0.	2.05
time (sec)	N/A	0.168	0.306	0.152	0.	2.326	0.	10.339

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	94	222	0	873	0	165
normalized size	1	1.	1.22	2.88	0.	11.34	0.	2.14
time (sec)	N/A	0.097	0.212	0.117	0.	2.641	0.	9.151

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	93	220	0	868	0	165
normalized size	1	1.	1.21	2.86	0.	11.27	0.	2.14
time (sec)	N/A	0.072	0.111	0.114	0.	2.594	0.	9.019

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	5534	370	0	1319	0	375
normalized size	1	1.	48.54	3.25	0.	11.57	0.	3.29
time (sec)	N/A	0.116	23.819	0.125	0.	3.114	0.	11.479

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	129	384	0	1384	0	0
normalized size	1	1.	0.9	2.67	0.	9.61	0.	0.
time (sec)	N/A	0.258	0.796	0.182	0.	2.434	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	197	560	0	1435	0	0
normalized size	1	1.	1.06	3.03	0.	7.76	0.	0.
time (sec)	N/A	0.391	2.838	0.216	0.	2.827	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	135	417	0	1220	0	343
normalized size	1	1.	0.74	2.28	0.	6.67	0.	1.87
time (sec)	N/A	0.424	1.259	0.179	0.	2.054	0.	10.284

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	125	316	0	1079	0	292
normalized size	1	1.	0.86	2.18	0.	7.44	0.	2.01
time (sec)	N/A	0.309	0.708	0.164	0.	2.039	0.	9.511

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	116	323	0	1062	0	216
normalized size	1	1.	1.08	3.02	0.	9.93	0.	2.02
time (sec)	N/A	0.176	0.627	0.164	0.	1.971	0.	10.562

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	115	315	0	1052	0	216
normalized size	1	1.	1.07	2.94	0.	9.83	0.	2.02
time (sec)	N/A	0.137	0.627	0.122	0.	1.988	0.	10.283

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	52	315	0	1057	0	216
normalized size	1	1.	0.49	2.94	0.	9.88	0.	2.02
time (sec)	N/A	0.113	0.057	0.113	0.	2.029	0.	10.15

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	5574	550	0	1565	0	427
normalized size	1	1.	38.71	3.82	0.	10.87	0.	2.97
time (sec)	N/A	0.176	23.793	0.156	0.	3.018	0.	12.477

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	169	552	0	1624	0	0
normalized size	1	1.	0.97	3.17	0.	9.33	0.	0.
time (sec)	N/A	0.372	1.766	0.202	0.	3.029	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	94	83	0	424	0	109
normalized size	1	1.	1.96	1.73	0.	8.83	0.	2.27
time (sec)	N/A	0.038	0.382	0.135	0.	1.95	0.	1.915

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	127	119	0	778	0	0
normalized size	1	1.	1.46	1.37	0.	8.94	0.	0.
time (sec)	N/A	0.071	0.456	0.143	0.	1.835	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	105	0	0	0	0	0
normalized size	1	1.	0.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.68	0.255	0.112	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	85	0	0	0	0	0
normalized size	1	1.	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.352	0.128	0.097	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	66	0	0	0	0	0
normalized size	1	1.	0.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	0.049	0.092	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	691	0	0	0	0	0
normalized size	1	1.	8.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	4.792	0.126	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	2700	0	0	0	0	0
normalized size	1	1.	35.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	16.045	0.103	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	96	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.508	0.302	0.116	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	106	0	0	0	0	0
normalized size	1	1.	0.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.384	0.419	0.099	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	66	0	0	0	0	0
normalized size	1	1.	0.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	0.078	0.093	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	2694	0	0	0	0	0
normalized size	1	1.	31.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	15.766	0.092	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	2700	0	0	0	0	0
normalized size	1	1.	31.4	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	15.993	0.109	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	155	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.548	0.35	0.131	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	95	0	0	0	0	0
normalized size	1	1.	0.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.394	0.161	0.119	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	85	0	0	0	0	0
normalized size	1	1.	0.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.302	0.108	0.102	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	65	0	0	0	0	0
normalized size	1	1.	0.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.228	0.067	0.119	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	718	0	0	0	0	0
normalized size	1	1.	9.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	4.51	0.112	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	240	0	0	0	0	0
normalized size	1	1.	3.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	1.973	0.109	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	766	766	111	0	0	0	0	0
normalized size	1	1.	0.14	0.	0.	0.	0.	0.
time (sec)	N/A	1.059	0.558	0.132	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	731	731	98	0	0	0	0	0
normalized size	1	1.	0.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.771	0.27	0.12	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	731	731	90	0	0	0	0	0
normalized size	1	1.	0.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.674	0.337	0.11	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	744	744	68	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.58	0.062	0.098	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	3007	0	0	0	0	0
normalized size	1	1.	33.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	16.344	0.099	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	3011	0	0	0	0	0
normalized size	1	1.	33.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	16.028	0.102	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	115	384	0	0	0	0
normalized size	1	1.	0.76	2.54	0.	0.	0.	0.
time (sec)	N/A	0.091	0.228	2.175	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	83	369	0	0	0	0
normalized size	1	1.	0.67	3.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.193	2.24	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	68	146	0	0	0	0
normalized size	1	1.	0.7	1.51	0.	0.	0.	0.
time (sec)	N/A	0.064	0.138	1.266	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	49	150	0	0	0	0
normalized size	1	1.	0.65	2.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.077	1.331	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	73	225	0	0	0	0
normalized size	1	1.	0.72	2.23	0.	0.	0.	0.
time (sec)	N/A	0.069	0.132	1.156	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	93	219	0	0	0	0
normalized size	1	1.	0.73	1.72	0.	0.	0.	0.
time (sec)	N/A	0.083	0.189	1.198	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	103	270	0	0	0	0
normalized size	1	1.	0.68	1.79	0.	0.	0.	0.
time (sec)	N/A	0.095	0.264	1.284	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	287	439	0	0	0	0
normalized size	1	1.	1.53	2.35	0.	0.	0.	0.
time (sec)	N/A	0.132	2.266	2.429	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	269	386	0	0	0	0
normalized size	1	1.	1.67	2.4	0.	0.	0.	0.
time (sec)	N/A	0.114	1.88	2.332	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	264	371	0	0	0	0
normalized size	1	1.	2.02	2.83	0.	0.	0.	0.
time (sec)	N/A	0.102	1.588	2.417	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	104	0	0	0	0
normalized size	1	1.	0.75	1.62	0.	0.	0.	0.
time (sec)	N/A	0.074	0.156	1.347	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	156	228	0	0	0	0
normalized size	1	1.	1.46	2.13	0.	0.	0.	0.
time (sec)	N/A	0.092	1.283	1.35	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	0	0	0
normalized size	1	1.	1.01	1.85	0.	0.	0.	0.
time (sec)	N/A	0.105	1.576	1.414	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	149	272	0	0	0	0
normalized size	1	1.	0.93	1.69	0.	0.	0.	0.
time (sec)	N/A	0.115	1.785	1.2	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	287	439	0	0	0	0
normalized size	1	1.	1.53	2.35	0.	0.	0.	0.
time (sec)	N/A	0.19	2.473	2.418	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	267	386	0	0	0	0
normalized size	1	1.	1.7	2.46	0.	0.	0.	0.
time (sec)	N/A	0.161	2.168	2.288	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	187	371	0	0	0	0
normalized size	1	1.	1.43	2.83	0.	0.	0.	0.
time (sec)	N/A	0.142	1.774	2.267	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	169	172	0	0	0	0
normalized size	1	1.	1.29	1.31	0.	0.	0.	0.
time (sec)	N/A	0.138	1.488	1.462	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	171	250	0	0	0	0
normalized size	1	1.	1.31	1.91	0.	0.	0.	0.
time (sec)	N/A	0.14	1.446	1.403	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	0	0	0
normalized size	1	1.	0.91	1.69	0.	0.	0.	0.
time (sec)	N/A	0.168	1.9	1.321	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	0	0	0
normalized size	1	1.	0.83	1.39	0.	0.	0.	0.
time (sec)	N/A	0.194	2.278	1.362	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	289	492	0	0	0	0
normalized size	1	1.	1.36	2.31	0.	0.	0.	0.
time (sec)	N/A	0.251	3.861	2.649	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	0	0	0
normalized size	1	1.	1.49	2.35	0.	0.	0.	0.
time (sec)	N/A	0.209	2.793	2.765	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	286	386	0	0	0	0
normalized size	1	1.	1.78	2.4	0.	0.	0.	0.
time (sec)	N/A	0.182	2.984	2.393	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	0	0	0
normalized size	1	1.	0.59	2.47	0.	0.	0.	0.
time (sec)	N/A	0.169	0.301	2.078	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	184	194	0	0	0	0
normalized size	1	1.	1.16	1.22	0.	0.	0.	0.
time (sec)	N/A	0.172	1.632	1.669	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	180	272	0	0	0	0
normalized size	1	1.	1.12	1.69	0.	0.	0.	0.
time (sec)	N/A	0.187	1.695	1.352	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	0	0	0
normalized size	1	1.	0.83	1.39	0.	0.	0.	0.
time (sec)	N/A	0.222	2.26	1.535	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	306	273	0	0	0	0
normalized size	1	1.	1.44	1.28	0.	0.	0.	0.
time (sec)	N/A	0.258	3.219	1.388	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	291	413	0	0	0	0
normalized size	1	1.	1.77	2.52	0.	0.	0.	0.
time (sec)	N/A	0.126	3.321	2.526	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	262	253	0	0	0	0
normalized size	1	1.	1.93	1.86	0.	0.	0.	0.
time (sec)	N/A	0.113	2.006	1.535	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	201	200	0	0	0	0
normalized size	1	1.	1.83	1.82	0.	0.	0.	0.
time (sec)	N/A	0.102	24.861	1.449	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	202	198	0	0	0	0
normalized size	1	1.	1.84	1.8	0.	0.	0.	0.
time (sec)	N/A	0.1	6.267	1.377	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	317	199	0	0	0	0
normalized size	1	1.	2.83	1.78	0.	0.	0.	0.
time (sec)	N/A	0.102	1.799	1.322	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	318	215	0	0	0	0
normalized size	1	1.	2.27	1.54	0.	0.	0.	0.
time (sec)	N/A	0.117	4.3	1.475	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	347	229	0	0	0	0
normalized size	1	1.	2.07	1.36	0.	0.	0.	0.
time (sec)	N/A	0.129	2.776	1.399	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	287	413	0	0	0	0
normalized size	1	1.	1.42	2.04	0.	0.	0.	0.
time (sec)	N/A	0.229	3.697	2.365	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	252	405	0	0	0	0
normalized size	1	1.	1.43	2.3	0.	0.	0.	0.
time (sec)	N/A	0.214	1.353	1.584	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	242	257	0	0	0	0
normalized size	1	1.	1.62	1.72	0.	0.	0.	0.
time (sec)	N/A	0.199	1.242	1.394	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	0	0	0
normalized size	1	1.	1.27	2.44	0.	0.	0.	0.
time (sec)	N/A	0.061	0.348	1.546	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	239	257	0	0	0	0
normalized size	1	1.	1.6	1.72	0.	0.	0.	0.
time (sec)	N/A	0.201	1.422	1.651	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	260	257	0	0	0	0
normalized size	1	1.	1.71	1.69	0.	0.	0.	0.
time (sec)	N/A	0.202	0.976	1.584	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	257	270	0	0	0	0
normalized size	1	1.	1.44	1.52	0.	0.	0.	0.
time (sec)	N/A	0.225	1.742	1.421	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	271	283	0	0	0	0
normalized size	1	1.	1.36	1.42	0.	0.	0.	0.
time (sec)	N/A	0.24	1.836	1.63	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	378	453	0	0	0	0
normalized size	1	1.	1.53	1.83	0.	0.	0.	0.
time (sec)	N/A	0.356	4.706	3.016	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	371	555	0	0	0	0
normalized size	1	1.	1.68	2.51	0.	0.	0.	0.
time (sec)	N/A	0.34	2.325	1.579	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	274	268	0	0	0	0
normalized size	1	1.	1.41	1.37	0.	0.	0.	0.
time (sec)	N/A	0.325	4.691	1.5	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	371	270	0	0	0	0
normalized size	1	1.	1.9	1.38	0.	0.	0.	0.
time (sec)	N/A	0.32	2.188	1.453	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	371	270	0	0	0	0
normalized size	1	1.	1.9	1.38	0.	0.	0.	0.
time (sec)	N/A	0.319	2.069	1.621	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	272	270	0	0	0	0
normalized size	1	1.	1.39	1.38	0.	0.	0.	0.
time (sec)	N/A	0.329	5.252	1.547	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	386	270	0	0	0	0
normalized size	1	1.	1.98	1.38	0.	0.	0.	0.
time (sec)	N/A	0.322	2.285	1.612	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	285	283	0	0	0	0
normalized size	1	1.	1.29	1.28	0.	0.	0.	0.
time (sec)	N/A	0.352	2.417	1.557	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	297	296	0	0	0	0
normalized size	1	1.	1.2	1.2	0.	0.	0.	0.
time (sec)	N/A	0.374	2.666	1.65	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	100	221	1706	953	0	0
normalized size	1	1.	0.86	1.91	14.71	8.22	0.	0.
time (sec)	N/A	0.167	0.496	0.26	2.8	1.797	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	75	186	894	814	0	0
normalized size	1	1.	1.04	2.58	12.42	11.31	0.	0.
time (sec)	N/A	0.113	0.22	0.215	2.835	1.811	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	54	147	325	495	0	0
normalized size	1	1.	1.46	3.97	8.78	13.38	0.	0.
time (sec)	N/A	0.058	0.102	0.214	2.986	1.777	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	52	27	130	0	0
normalized size	1	1.	1.08	1.44	0.75	3.61	0.	0.
time (sec)	N/A	0.054	0.086	0.188	2.594	1.623	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	49	68	153	181	0	0
normalized size	1	1.	0.64	0.88	1.99	2.35	0.	0.
time (sec)	N/A	0.109	0.154	0.21	2.545	1.656	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	61	80	274	211	0	0
normalized size	1	1.	0.53	0.7	2.38	1.83	0.	0.
time (sec)	N/A	0.164	0.184	0.217	3.078	1.556	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	71	90	396	238	0	0
normalized size	1	1.	0.46	0.59	2.59	1.56	0.	0.
time (sec)	N/A	0.216	0.24	0.213	2.694	1.607	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	112	246	3187	1052	0	0
normalized size	1	1.	0.7	1.54	19.92	6.58	0.	0.
time (sec)	N/A	0.234	0.556	0.233	3.3	1.841	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	99	214	3029	975	0	0
normalized size	1	1.	0.82	1.78	25.24	8.12	0.	0.
time (sec)	N/A	0.175	0.408	0.218	3.551	1.807	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	75	184	1543	830	0	0
normalized size	1	1.	1.	2.45	20.57	11.07	0.	0.
time (sec)	N/A	0.117	0.264	0.211	2.973	1.793	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	86	174	370	819	0	0
normalized size	1	1.	1.13	2.29	4.87	10.78	0.	0.
time (sec)	N/A	0.118	0.329	0.187	3.421	1.866	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	50	71	51	186	0	0
normalized size	1	1.	0.63	0.9	0.65	2.35	0.	0.
time (sec)	N/A	0.109	0.193	0.19	2.88	1.617	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	60	81	284	215	0	0
normalized size	1	1.	0.52	0.7	2.45	1.85	0.	0.
time (sec)	N/A	0.172	0.257	0.184	2.824	1.562	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	72	93	409	255	0	0
normalized size	1	1.	0.45	0.58	2.54	1.58	0.	0.
time (sec)	N/A	0.232	0.337	0.205	2.824	1.653	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	80	103	535	288	0	0
normalized size	1	1.	0.4	0.51	2.66	1.43	0.	0.
time (sec)	N/A	0.292	0.521	0.196	3.374	1.656	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	582	286	5211	1160	0	0
normalized size	1	1.	2.91	1.43	26.06	5.8	0.	0.
time (sec)	N/A	0.337	8.257	0.265	3.819	1.875	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	458	254	4683	1079	0	0
normalized size	1	1.	2.86	1.59	29.27	6.74	0.	0.
time (sec)	N/A	0.275	7.831	0.238	3.299	1.858	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	106	226	3815	1002	0	0
normalized size	1	1.	0.88	1.88	31.79	8.35	0.	0.
time (sec)	N/A	0.215	0.451	0.225	24.053	1.815	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	91	199	0	909	0	0
normalized size	1	1.	0.81	1.78	0.	8.12	0.	0.
time (sec)	N/A	0.217	0.656	0.226	0.	1.858	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	103	195	801	949	0	0
normalized size	1	1.	0.87	1.65	6.79	8.04	0.	0.
time (sec)	N/A	0.219	0.397	0.223	2.237	1.796	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	64	85	81	230	0	0
normalized size	1	1.	0.54	0.71	0.68	1.93	0.	0.
time (sec)	N/A	0.169	0.285	0.183	2.254	1.653	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	74	95	436	262	0	0
normalized size	1	1.	0.47	0.61	2.79	1.68	0.	0.
time (sec)	N/A	0.236	0.336	0.187	2.162	1.707	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	80	105	570	302	0	0
normalized size	1	1.	0.4	0.52	2.84	1.5	0.	0.
time (sec)	N/A	0.333	0.564	0.194	2.187	1.897	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	90	115	703	338	0	0
normalized size	1	1.	0.37	0.48	2.92	1.4	0.	0.
time (sec)	N/A	0.403	0.359	0.207	2.497	1.939	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	45	0	163	132	0	0
normalized size	1	1.	1.18	0.	4.29	3.47	0.	0.
time (sec)	N/A	0.056	0.081	0.162	1.916	2.083	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	54	147	325	495	0	0
normalized size	1	1.	1.46	3.97	8.78	13.38	0.	0.
time (sec)	N/A	0.058	0.133	0.222	2.249	2.004	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	299	127	477	545	0	124
normalized size	1	1.	7.87	3.34	12.55	14.34	0.	3.26
time (sec)	N/A	0.063	1.849	0.252	2.437	1.805	0.	2.417

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	125	222	1183	1310	0	0
normalized size	1	1.	0.98	1.73	9.24	10.23	0.	0.
time (sec)	N/A	0.267	0.236	0.224	2.359	1.913	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	89	181	643	936	0	0
normalized size	1	1.	0.94	1.91	6.77	9.85	0.	0.
time (sec)	N/A	0.164	0.093	0.197	2.267	2.162	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	99	122	439	0	0
normalized size	1	1.	1.34	1.77	2.18	7.84	0.	0.
time (sec)	N/A	0.059	0.065	0.185	2.222	2.043	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	102	100	140	764	0	0
normalized size	1	1.	1.1	1.08	1.51	8.22	0.	0.
time (sec)	N/A	0.112	0.209	0.168	2.267	2.001	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	120	120	381	863	0	0
normalized size	1	1.	0.92	0.92	2.91	6.59	0.	0.
time (sec)	N/A	0.219	0.249	0.21	2.261	2.138	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	117	130	482	926	0	0
normalized size	1	1.	0.69	0.77	2.85	5.48	0.	0.
time (sec)	N/A	0.345	1.082	0.212	2.282	2.059	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	252	281	6661	1555	0	0
normalized size	1	1.	1.45	1.61	38.28	8.94	0.	0.
time (sec)	N/A	0.419	0.589	0.215	3.631	2.288	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	220	237	2865	1497	0	0
normalized size	1	1.	1.64	1.77	21.38	11.17	0.	0.
time (sec)	N/A	0.283	0.541	0.195	3.251	2.285	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	220	146	0	903	0	0
normalized size	1	1.	2.27	1.51	0.	9.31	0.	0.
time (sec)	N/A	0.123	0.565	0.178	0.	2.003	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	120	146	1392	909	0	0
normalized size	1	1.	1.24	1.51	14.35	9.37	0.	0.
time (sec)	N/A	0.122	0.162	0.186	2.188	2.063	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	145	175	9688	1004	0	0
normalized size	1	1.	1.06	1.28	70.72	7.33	0.	0.
time (sec)	N/A	0.235	0.479	0.196	2.528	2.188	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	150	193	0	1068	0	0
normalized size	1	1.	0.85	1.09	0.	6.03	0.	0.
time (sec)	N/A	0.367	0.907	0.202	0.	2.309	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	163	203	0	1118	0	0
normalized size	1	1.	0.75	0.94	0.	5.15	0.	0.
time (sec)	N/A	0.516	1.255	0.211	0.	2.136	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	340	454	12215	1796	0	0
normalized size	1	1.	1.59	2.12	57.08	8.39	0.	0.
time (sec)	N/A	0.562	1.181	0.221	18.32	2.431	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	308	406	6734	1778	0	0
normalized size	1	1.	1.77	2.33	38.7	10.22	0.	0.
time (sec)	N/A	0.43	0.674	0.208	3.904	2.785	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	308	210	0	1127	0	0
normalized size	1	1.	2.25	1.53	0.	8.23	0.	0.
time (sec)	N/A	0.187	0.698	0.191	0.	2.089	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	266	210	3881	1122	0	0
normalized size	1	1.	1.94	1.53	28.33	8.19	0.	0.
time (sec)	N/A	0.188	1.74	0.196	4.65	2.014	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	146	208	4116	1133	0	0
normalized size	1	1.	1.07	1.52	30.04	8.27	0.	0.
time (sec)	N/A	0.242	0.861	0.194	5.667	2.105	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	186	236	0	1188	0	0
normalized size	1	1.	1.05	1.33	0.	6.71	0.	0.
time (sec)	N/A	0.38	1.232	0.199	0.	2.161	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	165	254	0	1256	0	0
normalized size	1	1.	0.76	1.17	0.	5.79	0.	0.
time (sec)	N/A	0.507	2.217	0.208	0.	2.211	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	140	255	2218	936	0	0
normalized size	1	1.	1.11	2.02	17.6	7.43	0.	0.
time (sec)	N/A	0.285	0.4	0.239	2.341	2.081	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	111	218	1179	826	0	0
normalized size	1	1.	1.31	2.56	13.87	9.72	0.	0.
time (sec)	N/A	0.185	0.286	0.201	2.217	2.128	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	76	180	639	614	0	0
normalized size	1	1.	1.41	3.33	11.83	11.37	0.	0.
time (sec)	N/A	0.109	0.082	0.193	2.185	2.108	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	40	95	117	240	0	0
normalized size	1	1.	1.48	3.52	4.33	8.89	0.	0.
time (sec)	N/A	0.039	0.027	0.182	1.941	2.014	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	90	96	136	404	0	0
normalized size	1	1.	1.45	1.55	2.19	6.52	0.	0.
time (sec)	N/A	0.078	0.208	0.155	2.202	1.934	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	118	116	377	448	0	0
normalized size	1	1.	1.2	1.18	3.85	4.57	0.	0.
time (sec)	N/A	0.151	0.226	0.194	1.893	1.974	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	122	126	478	482	0	0
normalized size	1	1.	0.91	0.94	3.57	3.6	0.	0.
time (sec)	N/A	0.235	0.29	0.21	1.99	1.976	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	71	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.383	0.23	0.194	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	71	0	0	0	0	0
normalized size	1	1.	0.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	0.137	0.337	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	71	0	0	0	0	0
normalized size	1	1.	0.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	0.148	0.24	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	716	716	71	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	0.23	0.197	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	673	673	71	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.478	0.221	0.192	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	624	71	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.435	0.139	0.188	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	662	662	71	0	0	0	0	0
normalized size	1	1.	0.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.483	0.147	0.25	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	715	715	71	0	0	0	0	0
normalized size	1	1.	0.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.54	0.175	0.174	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	760	0	0	0	0	0
normalized size	1	1.	9.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	6.748	0.171	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	749	0	0	0	0	0
normalized size	1	1.	9.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	7.514	0.165	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	3346	0	0	0	0	0
normalized size	1	1.	44.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.159	20.072	0.174	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	585	0	0	0	0	0
normalized size	1	1.	7.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	6.813	0.167	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	1982	0	0	0	0	0
normalized size	1	1.	25.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	14.654	0.112	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	2618	0	0	0	0	0
normalized size	1	1.	33.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	19.249	0.111	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F	F(-1)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	327	79	274	0	0	0	0	0
normalized size	1	0.24	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	7.396	0.121	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	2325	0	0	0	0	0
normalized size	1	1.	29.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	14.749	0.111	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.485	0.591	0.962	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	230	230	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.276	2.093	2.337	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	172	172	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.9	1.121	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	106	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.146	0.508	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	174	174	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.947	0.746	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.309	1.467	0.23	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	398	0	0	0	0	0
normalized size	1	1.	2.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	58.268	0.153	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	83	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.41	0.148	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.037	0.178	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	2938	0	0	0	0	0
normalized size	1	1.	49.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	15.691	0.158	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	2990	0	0	0	0	0
normalized size	1	1.	48.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	16.774	0.144	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.386	0.16	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.046	0.178	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	2951	0	0	0	0	0
normalized size	1	1.	40.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	6.212	0.167	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	3003	0	0	0	0	0
normalized size	1	1.	41.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	6.25	0.154	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.366	0.164	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	67	0	0	0	0	0
normalized size	1	1.	1.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.04	0.173	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	2951	0	0	0	0	0
normalized size	1	1.	40.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	6.193	0.177	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	3003	0	0	0	0	0
normalized size	1	1.	41.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	6.217	0.152	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	400	0	0	0	0	0
normalized size	1	1.	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.295	8.066	0.164	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	86	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	0.363	0.158	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0
normalized size	1	1.	1.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.091	0.193	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	2964	0	0	0	0	0
normalized size	1	1.	48.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	6.217	0.178	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	2992	0	0	0	0	0
normalized size	1	1.	44.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	6.233	0.155	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	0.344	0.167	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.12	0.189	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	2977	0	0	0	0	0
normalized size	1	1.	39.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	6.221	0.18	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	3005	0	0	0	0	0
normalized size	1	1.	38.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	6.206	0.158	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	88	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	0.315	0.168	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	71	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.12	0.191	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	2977	0	0	0	0	0
normalized size	1	1.	39.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	6.193	0.188	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	3005	0	0	0	0	0
normalized size	1	1.	38.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.145	6.195	0.162	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	429	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.334	24.799	0.183	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	346	0	0	0	0	0
normalized size	1	1.	3.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	13.498	0.172	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	213	0	0	0	0	0
normalized size	1	1.	4.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	70.318	0.199	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	1.246	0.179	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.167	1.886	0.171	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	332	0	0	0	0	0
normalized size	1	1.	2.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	2.068	0.172	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	185	0	0	0	0	0
normalized size	1	1.	2.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.385	0.191	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	346	0	0	0	0	0
normalized size	1	1.	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	1.141	0.176	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	69	213	0	0	0	0	0
normalized size	1	1.	3.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.531	0.198	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	72	2246	0	0	0	0	0
normalized size	1	1.	31.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	14.214	0.665	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	255	0	0	0	0	0
normalized size	1	1.	2.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	2.204	0.702	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	2248	0	0	0	0	0
normalized size	1	1.	25.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	6.242	0.721	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	1.003	0.753	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	2248	0	0	0	0	0
normalized size	1	1.	26.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	6.203	0.711	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	257	0	0	0	0	0
normalized size	1	1.	3.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.298	0.724	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	2250	0	0	0	0	0
normalized size	1	1.	25.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	6.208	0.751	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.224	0.79	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	2248	0	0	0	0	0
normalized size	1	1.	28.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	6.203	0.728	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	257	0	0	0	0	0
normalized size	1	1.	3.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.279	0.734	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	2250	0	0	0	0	0
normalized size	1	1.	23.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	6.209	0.762	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.206	0.786	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	154	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.353	1.278	0.31	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	123	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	0.621	0.279	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	95	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.209	0.242	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.103	0.267	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	711	0	0	0	0	0
normalized size	1	1.	8.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	6.577	0.201	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	3781	0	0	0	0	0
normalized size	1	1.	45.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	16.931	0.373	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	2529	0	0	0	0	0
normalized size	1	1.	25.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	14.941	0.195	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	2225	0	0	0	0	0
normalized size	1	1.	23.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	14.527	0.198	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	2424	0	0	0	0	0
normalized size	1	1.	25.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	14.804	0.2	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	3349	0	0	0	0	0
normalized size	1	1.	34.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	19.369	0.186	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	490	270	0	0	0	0
normalized size	1	1.	4.41	2.43	0.	0.	0.	0.
time (sec)	N/A	0.09	6.171	1.414	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	232	219	0	0	0	0
normalized size	1	1.	2.67	2.52	0.	0.	0.	0.
time (sec)	N/A	0.08	5.476	1.461	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	222	225	0	0	0	0
normalized size	1	1.	3.64	3.69	0.	0.	0.	0.
time (sec)	N/A	0.069	4.98	1.333	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	0	0	0
normalized size	1	1.	4.43	4.29	0.	0.	0.	0.
time (sec)	N/A	0.059	1.801	1.16	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	209	146	0	0	0	0
normalized size	1	1.	3.67	2.56	0.	0.	0.	0.
time (sec)	N/A	0.067	4.777	1.641	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	444	369	0	0	0	0
normalized size	1	1.	5.35	4.45	0.	0.	0.	0.
time (sec)	N/A	0.079	6.149	2.303	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	477	384	0	0	0	0
normalized size	1	1.	4.3	3.46	0.	0.	0.	0.
time (sec)	N/A	0.091	6.178	2.505	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	294	437	0	0	0	0
normalized size	1	1.	2.18	3.24	0.	0.	0.	0.
time (sec)	N/A	0.102	4.68	2.555	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	548	260	0	0	0	0
normalized size	1	1.	3.73	1.77	0.	0.	0.	0.
time (sec)	N/A	0.174	6.155	1.381	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	516	272	0	0	0	0
normalized size	1	1.	4.26	2.25	0.	0.	0.	0.
time (sec)	N/A	0.158	6.129	1.224	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	235	250	0	0	0	0
normalized size	1	1.	2.47	2.63	0.	0.	0.	0.
time (sec)	N/A	0.143	5.55	1.386	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	224	228	0	0	0	0
normalized size	1	1.	3.34	3.4	0.	0.	0.	0.
time (sec)	N/A	0.128	5.141	1.22	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	39	104	0	0	0	0
normalized size	1	1.	0.89	2.36	0.	0.	0.	0.
time (sec)	N/A	0.109	0.178	1.662	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	470	371	0	0	0	0
normalized size	1	1.	5.16	4.08	0.	0.	0.	0.
time (sec)	N/A	0.139	6.167	2.13	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	503	386	0	0	0	0
normalized size	1	1.	4.16	3.19	0.	0.	0.	0.
time (sec)	N/A	0.154	6.221	2.259	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	531	439	0	0	0	0
normalized size	1	1.	3.61	2.99	0.	0.	0.	0.
time (sec)	N/A	0.173	6.223	2.522	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	548	260	0	0	0	0
normalized size	1	1.	3.73	1.77	0.	0.	0.	0.
time (sec)	N/A	0.252	6.146	1.34	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	516	272	0	0	0	0
normalized size	1	1.	4.26	2.25	0.	0.	0.	0.
time (sec)	N/A	0.22	6.138	1.633	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	233	250	0	0	0	0
normalized size	1	1.	2.56	2.75	0.	0.	0.	0.
time (sec)	N/A	0.186	5.989	1.434	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	240	172	0	0	0	0
normalized size	1	1.	2.64	1.89	0.	0.	0.	0.
time (sec)	N/A	0.197	4.849	1.664	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	479	371	0	0	0	0
normalized size	1	1.	5.26	4.08	0.	0.	0.	0.
time (sec)	N/A	0.194	6.209	2.535	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	501	386	0	0	0	0
normalized size	1	1.	4.28	3.3	0.	0.	0.	0.
time (sec)	N/A	0.223	6.223	2.706	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	531	439	0	0	0	0
normalized size	1	1.	3.61	2.99	0.	0.	0.	0.
time (sec)	N/A	0.239	6.257	2.587	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	314	229	0	0	0	0
normalized size	1	1.	2.45	1.79	0.	0.	0.	0.
time (sec)	N/A	0.176	2.152	1.533	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	292	215	0	0	0	0
normalized size	1	1.	2.92	2.15	0.	0.	0.	0.
time (sec)	N/A	0.161	2.436	1.281	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	270	199	0	0	0	0
normalized size	1	1.	3.75	2.76	0.	0.	0.	0.
time (sec)	N/A	0.147	2.086	1.555	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	262	198	0	0	0	0
normalized size	1	1.	3.74	2.83	0.	0.	0.	0.
time (sec)	N/A	0.141	1.434	1.665	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	263	200	0	0	0	0
normalized size	1	1.	3.76	2.86	0.	0.	0.	0.
time (sec)	N/A	0.144	1.45	1.293	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	303	253	0	0	0	0
normalized size	1	1.	3.16	2.64	0.	0.	0.	0.
time (sec)	N/A	0.157	2.016	1.639	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	338	413	0	0	0	0
normalized size	1	1.	2.73	3.33	0.	0.	0.	0.
time (sec)	N/A	0.175	3.939	2.473	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	366	283	0	0	0	0
normalized size	1	1.	2.29	1.77	0.	0.	0.	0.
time (sec)	N/A	0.284	2.754	1.534	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	341	270	0	0	0	0
normalized size	1	1.	2.47	1.96	0.	0.	0.	0.
time (sec)	N/A	0.267	2.041	1.419	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	374	257	0	0	0	0
normalized size	1	1.	3.34	2.29	0.	0.	0.	0.
time (sec)	N/A	0.242	6.654	1.599	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	656	257	0	0	0	0
normalized size	1	1.	6.02	2.36	0.	0.	0.	0.
time (sec)	N/A	0.241	6.317	1.439	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	63	188	0	0	0	0
normalized size	1	1.	1.11	3.3	0.	0.	0.	0.
time (sec)	N/A	0.099	0.223	1.449	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	312	257	0	0	0	0
normalized size	1	1.	2.86	2.36	0.	0.	0.	0.
time (sec)	N/A	0.242	5.062	1.493	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	393	405	0	0	0	0
normalized size	1	1.	2.89	2.98	0.	0.	0.	0.
time (sec)	N/A	0.263	6.783	1.683	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	372	413	0	0	0	0
normalized size	1	1.	2.3	2.55	0.	0.	0.	0.
time (sec)	N/A	0.288	2.369	2.685	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	391	296	0	0	0	0
normalized size	1	1.	1.89	1.43	0.	0.	0.	0.
time (sec)	N/A	0.428	3.039	1.726	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	375	283	0	0	0	0
normalized size	1	1.	2.07	1.56	0.	0.	0.	0.
time (sec)	N/A	0.396	2.481	1.446	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	357	270	0	0	0	0
normalized size	1	1.	2.3	1.74	0.	0.	0.	0.
time (sec)	N/A	0.37	2.162	1.601	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	721	270	0	0	0	0
normalized size	1	1.	4.65	1.74	0.	0.	0.	0.
time (sec)	N/A	0.385	6.4	1.454	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	342	270	0	0	0	0
normalized size	1	1.	2.21	1.74	0.	0.	0.	0.
time (sec)	N/A	0.378	2.011	1.563	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	342	270	0	0	0	0
normalized size	1	1.	2.21	1.74	0.	0.	0.	0.
time (sec)	N/A	0.382	1.988	1.445	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	721	268	0	0	0	0
normalized size	1	1.	4.65	1.73	0.	0.	0.	0.
time (sec)	N/A	0.39	6.348	1.578	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	372	555	0	0	0	0
normalized size	1	1.	2.06	3.07	0.	0.	0.	0.
time (sec)	N/A	0.406	2.636	1.779	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	207	207	402	453	0	0	0	0
normalized size	1	1.	1.94	2.19	0.	0.	0.	0.
time (sec)	N/A	0.434	3.556	2.977	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	80	80	396	215	0	0
normalized size	1	1.	0.52	0.52	2.59	1.41	0.	0.
time (sec)	N/A	0.298	0.222	0.199	3.125	1.655	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	61	70	274	188	0	0
normalized size	1	1.	0.53	0.61	2.38	1.63	0.	0.
time (sec)	N/A	0.232	0.175	0.186	2.687	1.63	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	49	58	153	158	0	0
normalized size	1	1.	0.64	0.75	1.99	2.05	0.	0.
time (sec)	N/A	0.174	0.116	0.178	2.802	1.622	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	50	27	130	0	0
normalized size	1	1.	1.08	1.39	0.75	3.61	0.	0.
time (sec)	N/A	0.11	0.103	0.142	2.614	1.645	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	74	139	325	475	0	0
normalized size	1	1.	1.3	2.44	5.7	8.33	0.	0.
time (sec)	N/A	0.116	0.161	0.201	2.78	1.791	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	90	178	894	869	0	0
normalized size	1	1.	0.98	1.93	9.72	9.45	0.	0.
time (sec)	N/A	0.175	0.265	0.204	3.019	1.807	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	100	213	1706	944	0	0
normalized size	1	1.	0.74	1.57	12.54	6.94	0.	0.
time (sec)	N/A	0.23	0.483	0.211	3.175	1.848	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	72	83	409	232	0	0
normalized size	1	1.	0.45	0.52	2.54	1.44	0.	0.
time (sec)	N/A	0.311	0.289	0.169	2.678	1.676	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	60	71	284	192	0	0
normalized size	1	1.	0.52	0.61	2.45	1.66	0.	0.
time (sec)	N/A	0.242	0.236	0.168	2.855	1.622	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	50	61	51	163	0	0
normalized size	1	1.	0.63	0.77	0.65	2.06	0.	0.
time (sec)	N/A	0.176	0.165	0.15	2.863	1.636	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	81	172	370	799	0	0
normalized size	1	1.	0.84	1.79	3.85	8.32	0.	0.
time (sec)	N/A	0.183	0.138	0.138	2.906	1.736	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	92	182	1543	891	0	0
normalized size	1	1.	0.97	1.92	16.24	9.38	0.	0.
time (sec)	N/A	0.181	0.318	0.19	2.785	1.762	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	99	212	3029	965	0	0
normalized size	1	1.	0.71	1.51	21.64	6.89	0.	0.
time (sec)	N/A	0.243	0.431	0.2	3.478	1.87	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	112	244	3187	1031	0	0
normalized size	1	1.	0.62	1.36	17.71	5.73	0.	0.
time (sec)	N/A	0.302	0.538	0.214	3.468	1.84	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	90	95	570	279	0	0
normalized size	1	1.	0.45	0.47	2.84	1.39	0.	0.
time (sec)	N/A	0.406	0.24	0.183	2.892	1.665	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	74	85	436	239	0	0
normalized size	1	1.	0.47	0.54	2.79	1.53	0.	0.
time (sec)	N/A	0.299	0.256	0.168	2.803	1.62	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	64	75	81	207	0	0
normalized size	1	1.	0.54	0.63	0.68	1.74	0.	0.
time (sec)	N/A	0.231	0.236	0.149	2.511	1.646	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	93	185	801	888	0	0
normalized size	1	1.	0.67	1.34	5.8	6.43	0.	0.
time (sec)	N/A	0.282	0.236	0.194	3.125	1.825	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	90	197	0	969	0	0
normalized size	1	1.	0.68	1.49	0.	7.34	0.	0.
time (sec)	N/A	0.281	0.284	0.204	0.	1.858	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	95	216	3815	992	0	0
normalized size	1	1.	0.68	1.54	27.25	7.09	0.	0.
time (sec)	N/A	0.28	0.634	0.208	23.925	1.795	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	180	244	4683	1058	0	0
normalized size	1	1.	1.	1.36	26.02	5.88	0.	0.
time (sec)	N/A	0.343	5.475	0.224	3.027	1.868	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	190	276	5211	1139	0	0
normalized size	1	1.	0.86	1.25	23.69	5.18	0.	0.
time (sec)	N/A	0.403	5.566	0.235	3.748	1.965	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	136	120	482	886	0	0
normalized size	1	1.	0.72	0.63	2.55	4.69	0.	0.
time (sec)	N/A	0.414	0.303	0.193	2.227	1.771	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	116	110	381	828	0	0
normalized size	1	1.	0.77	0.73	2.52	5.48	0.	0.
time (sec)	N/A	0.283	0.196	0.182	2.217	1.752	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	100	98	140	764	0	0
normalized size	1	1.	0.88	0.87	1.24	6.76	0.	0.
time (sec)	N/A	0.17	0.091	0.138	2.178	1.697	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	76	95	91	122	439	0	0
normalized size	1	1.36	1.7	1.62	2.18	7.84	0.	0.
time (sec)	N/A	0.116	0.071	0.161	1.988	1.741	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	109	171	643	915	0	0
normalized size	1	1.	0.81	1.27	4.76	6.78	0.	0.
time (sec)	N/A	0.238	0.085	0.187	2.089	1.881	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	145	212	1183	1405	0	0
normalized size	1	1.	0.86	1.26	7.04	8.36	0.	0.
time (sec)	N/A	0.34	0.233	0.188	2.212	1.988	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	178	247	2222	1480	0	0
normalized size	1	1.	0.84	1.17	10.53	7.01	0.	0.
time (sec)	N/A	0.475	0.381	0.204	2.27	1.938	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	152	193	0	1077	0	0
normalized size	1	1.	0.64	0.81	0.	4.54	0.	0.
time (sec)	N/A	0.594	0.917	0.194	0.	1.885	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	133	183	0	1027	0	0
normalized size	1	1.	0.68	0.93	0.	5.21	0.	0.
time (sec)	N/A	0.446	0.612	0.184	0.	1.8	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	138	173	9688	964	0	0
normalized size	1	1.	0.88	1.1	61.71	6.14	0.	0.
time (sec)	N/A	0.307	0.881	0.175	3.356	1.79	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	131	138	1392	909	0	0
normalized size	1	1.	1.12	1.18	11.9	7.77	0.	0.
time (sec)	N/A	0.187	0.454	0.172	2.988	1.837	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	248	136	0	903	0	0
normalized size	1	1.	2.12	1.16	0.	7.72	0.	0.
time (sec)	N/A	0.188	0.861	0.17	0.	1.729	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	248	229	2865	1477	0	0
normalized size	1	1.	1.43	1.32	16.47	8.49	0.	0.
time (sec)	N/A	0.356	0.851	0.18	2.845	2.001	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	242	273	6661	1646	0	0
normalized size	1	1.	1.13	1.28	31.13	7.69	0.	0.
time (sec)	N/A	0.492	0.96	0.197	4.927	1.995	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	144	244	0	1215	0	0
normalized size	1	1.	0.61	1.03	0.	5.13	0.	0.
time (sec)	N/A	0.596	1.162	0.188	0.	1.834	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	141	234	0	1148	0	0
normalized size	1	1.	0.72	1.19	0.	5.83	0.	0.
time (sec)	N/A	0.458	0.93	0.18	0.	1.843	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	168	200	4116	1092	0	0
normalized size	1	1.	1.07	1.27	26.22	6.96	0.	0.
time (sec)	N/A	0.315	1.214	0.182	4.278	1.801	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	224	198	3881	1087	0	0
normalized size	1	1.	1.43	1.26	24.72	6.92	0.	0.
time (sec)	N/A	0.26	3.598	0.173	3.681	1.764	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	341	200	0	1087	0	0
normalized size	1	1.	2.17	1.27	0.	6.92	0.	0.
time (sec)	N/A	0.259	1.363	0.176	0.	1.833	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	328	396	6734	1717	0	0
normalized size	1	1.	1.53	1.85	31.47	8.02	0.	0.
time (sec)	N/A	0.503	1.223	0.199	4.765	1.977	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	348	444	12215	1886	0	0
normalized size	1	1.	1.37	1.75	48.09	7.43	0.	0.
time (sec)	N/A	0.646	1.502	0.212	23.526	2.051	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	244	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.4	2.511	2.663	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.974	1.253	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	105	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.152	0.628	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.244	0.969	0.848	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.412	1.673	0.299	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	128	266	0	221
normalized size	1	1.	0.89	1.08	1.51	3.13	0.	2.6
time (sec)	N/A	0.067	0.287	0.023	1.207	1.826	0.	1.328

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	95	236	0	165
normalized size	1	1.	0.95	1.14	1.51	3.75	0.	2.62
time (sec)	N/A	0.053	0.169	0.024	1.197	1.757	0.	1.287

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	78	198	0	144
normalized size	1	1.	1.	1.09	1.66	4.21	0.	3.06
time (sec)	N/A	0.049	0.021	0.019	1.053	1.939	0.	1.306

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	39	162	37	85
normalized size	1	1.	1.	1.33	1.62	6.75	1.54	3.54
time (sec)	N/A	0.026	0.012	0.018	1.062	1.977	3.752	1.255

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	31	95	41	66
normalized size	1	1.	1.	1.5	1.94	5.94	2.56	4.12
time (sec)	N/A	0.008	0.002	0.006	1.202	2.037	1.677	1.275

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	21	27	38	15	53
normalized size	1	1.	1.73	1.4	1.8	2.53	1.	3.53
time (sec)	N/A	0.023	0.009	0.034	1.167	1.747	2.442	1.237

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	38	46	72	0	111
normalized size	1	1.	0.92	1.	1.21	1.89	0.	2.92
time (sec)	N/A	0.038	0.066	0.045	1.077	1.601	0.	1.311

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	62	105	0	132
normalized size	1	1.	1.06	0.91	1.15	1.94	0.	2.44
time (sec)	N/A	0.046	0.072	0.049	1.224	1.719	0.	1.29

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	77	136	0	189
normalized size	1	1.	0.96	0.79	1.01	1.79	0.	2.49
time (sec)	N/A	0.056	0.125	0.048	1.033	1.685	0.	1.247

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	70	93	173	0	208
normalized size	1	1.	0.97	0.76	1.01	1.88	0.	2.26
time (sec)	N/A	0.058	0.117	0.047	1.28	1.704	0.	1.202

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	118	157	178	352	0	367
normalized size	1	1.	0.87	1.16	1.32	2.61	0.	2.72
time (sec)	N/A	0.106	0.599	0.031	1.207	1.823	0.	1.398

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	82	142	194	332	0	348
normalized size	1	1.	0.75	1.29	1.76	3.02	0.	3.16
time (sec)	N/A	0.094	0.277	0.028	1.1	1.764	0.	1.358

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	89	113	259	0	240
normalized size	1	1.	0.89	1.11	1.41	3.24	0.	3.
time (sec)	N/A	0.09	0.223	0.027	1.251	1.75	0.	1.375

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	45	78	108	236	0	174
normalized size	1	1.	0.76	1.32	1.83	4.	0.	2.95
time (sec)	N/A	0.054	0.104	0.025	1.162	1.682	0.	1.22

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	49	54	193	0	104
normalized size	1	1.	0.97	1.48	1.64	5.85	0.	3.15
time (sec)	N/A	0.026	0.075	0.023	1.168	1.737	0.	1.313

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	49	69	131	0	105
normalized size	1	1.	1.39	1.48	2.09	3.97	0.	3.18
time (sec)	N/A	0.055	0.016	0.043	1.178	1.682	0.	1.366

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	63	93	0	130
normalized size	1	1.	0.92	1.02	1.26	1.86	0.	2.6
time (sec)	N/A	0.066	0.072	0.046	1.152	1.605	0.	1.269

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	59	63	81	124	0	207
normalized size	1	1.	1.02	1.09	1.4	2.14	0.	3.57
time (sec)	N/A	0.089	0.168	0.051	1.089	1.622	0.	1.25

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	86	89	111	184	0	302
normalized size	1	1.	0.85	0.88	1.1	1.82	0.	2.99
time (sec)	N/A	0.088	0.17	0.056	1.047	1.646	0.	1.347

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	85	95	127	215	0	333
normalized size	1	1.	0.77	0.86	1.14	1.94	0.	3.
time (sec)	N/A	0.122	0.163	0.054	1.039	1.701	0.	1.303

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	120	206	244	421	0	495
normalized size	1	1.	0.63	1.09	1.29	2.23	0.	2.62
time (sec)	N/A	0.312	0.857	0.033	1.187	1.754	0.	1.333

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	90	160	213	348	0	446
normalized size	1	1.	0.69	1.23	1.64	2.68	0.	3.43
time (sec)	N/A	0.198	0.444	0.029	1.121	1.707	0.	1.349

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	70	118	143	309	0	277
normalized size	1	1.	0.71	1.19	1.44	3.12	0.	2.8
time (sec)	N/A	0.131	0.241	0.029	1.082	1.739	0.	1.381

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	95	126	281	0	196
normalized size	1	1.	0.75	1.3	1.73	3.85	0.	2.68
time (sec)	N/A	0.049	0.155	0.027	1.206	1.718	0.	1.265

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	88	68	89	246	0	177
normalized size	1	1.	1.31	1.01	1.33	3.67	0.	2.64
time (sec)	N/A	0.112	0.331	0.039	1.141	1.703	0.	1.362

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	105	90	103	176	0	185
normalized size	1	1.	1.33	1.14	1.3	2.23	0.	2.34
time (sec)	N/A	0.119	0.151	0.048	1.22	1.755	0.	1.33

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	80	76	99	153	0	230
normalized size	1	1.	0.8	0.76	0.99	1.53	0.	2.3
time (sec)	N/A	0.15	0.121	0.049	1.174	1.671	0.	1.288

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	100	102	128	197	0	401
normalized size	1	1.	0.81	0.83	1.04	1.6	0.	3.26
time (sec)	N/A	0.183	0.273	0.058	1.205	1.676	0.	1.366

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	130	123	161	265	0	448
normalized size	1	1.	0.81	0.77	1.01	1.66	0.	2.8
time (sec)	N/A	0.192	0.297	0.057	1.204	1.696	0.	1.296

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	159	145	196	324	0	582
normalized size	1	1.	0.86	0.78	1.06	1.75	0.	3.15
time (sec)	N/A	0.232	0.336	0.058	1.193	1.76	0.	1.286

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	154	302	371	528	0	799
normalized size	1	1.	0.63	1.24	1.52	2.16	0.	3.27
time (sec)	N/A	0.45	0.902	0.036	1.198	1.834	0.	1.357

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	125	225	263	443	0	622
normalized size	1	1.	0.7	1.26	1.47	2.47	0.	3.47
time (sec)	N/A	0.302	0.735	0.032	1.212	1.75	0.	1.394

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	101	188	243	394	0	486
normalized size	1	1.	0.69	1.29	1.66	2.7	0.	3.33
time (sec)	N/A	0.243	0.516	0.034	1.196	1.723	0.	1.334

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	77	135	163	339	0	298
normalized size	1	1.	0.72	1.26	1.52	3.17	0.	2.79
time (sec)	N/A	0.116	0.284	0.031	1.205	1.706	0.	1.297

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	280	114	155	324	0	242
normalized size	1	1.	2.69	1.1	1.49	3.12	0.	2.33
time (sec)	N/A	0.212	0.52	0.048	1.344	1.718	0.	1.282

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	119	109	122	294	0	230
normalized size	1	1.	1.1	1.01	1.13	2.72	0.	2.13
time (sec)	N/A	0.217	0.671	0.05	1.404	1.751	0.	1.321

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	128	131	138	239	0	286
normalized size	1	1.	1.11	1.14	1.2	2.08	0.	2.49
time (sec)	N/A	0.24	0.158	0.053	1.323	1.745	0.	1.332

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	104	116	147	224	0	429
normalized size	1	1.	0.72	0.8	1.01	1.54	0.	2.96
time (sec)	N/A	0.313	0.227	0.063	1.191	1.66	0.	1.319

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	133	138	180	285	0	574
normalized size	1	1.	0.77	0.8	1.04	1.65	0.	3.32
time (sec)	N/A	0.352	0.498	0.062	1.177	1.748	0.	1.377

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	156	174	230	358	0	743
normalized size	1	1.	0.73	0.82	1.08	1.68	0.	3.49
time (sec)	N/A	0.38	0.47	0.064	1.224	1.686	0.	1.29

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	114	205	267	444	0	513
normalized size	1	1.	0.72	1.3	1.69	2.81	0.	3.25
time (sec)	N/A	0.236	0.565	0.042	1.213	1.809	0.	1.184

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	258	400	0	1243	0	386
normalized size	1	1.	1.64	2.55	0.	7.92	0.	2.46
time (sec)	N/A	0.485	2.355	0.056	0.	2.923	0.	1.342

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	238	262	0	1076	0	285
normalized size	1	1.	2.	2.2	0.	9.04	0.	2.39
time (sec)	N/A	0.275	1.027	0.052	0.	2.851	0.	1.262

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	115	134	0	902	0	205
normalized size	1	1.	1.35	1.58	0.	10.61	0.	2.41
time (sec)	N/A	0.163	0.371	0.046	0.	2.183	0.	1.215

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	102	88	0	657	0	162
normalized size	1	1.	1.5	1.29	0.	9.66	0.	2.38
time (sec)	N/A	0.109	0.075	0.043	0.	2.082	0.	1.294

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	419	0	104
normalized size	1	1.	0.98	0.9	0.	8.55	0.	2.12
time (sec)	N/A	0.059	0.039	0.04	0.	1.697	0.	1.212

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	67	0	502	0	128
normalized size	1	1.	1.02	1.14	0.	8.51	0.	2.17
time (sec)	N/A	0.051	0.085	0.05	0.	1.758	0.	1.246

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	72	102	0	599	0	170
normalized size	1	1.	0.95	1.34	0.	7.88	0.	2.24
time (sec)	N/A	0.102	0.139	0.076	0.	1.794	0.	1.263

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	222	0	730	0	240
normalized size	1	1.	0.88	2.02	0.	6.64	0.	2.18
time (sec)	N/A	0.278	0.23	0.072	0.	1.839	0.	1.246

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	122	367	0	875	0	336
normalized size	1	1.	0.82	2.48	0.	5.91	0.	2.27
time (sec)	N/A	0.459	0.313	0.07	0.	1.877	0.	1.342

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	153	672	0	1057	0	531
normalized size	1	1.	0.79	3.48	0.	5.48	0.	2.75
time (sec)	N/A	0.687	0.564	0.083	0.	1.98	0.	1.285

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	357	405	0	2024	0	404
normalized size	1	1.	1.61	1.82	0.	9.12	0.	1.82
time (sec)	N/A	0.612	6.122	0.072	0.	6.954	0.	1.352

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	162	275	0	1704	0	447
normalized size	1	1.	0.99	1.68	0.	10.39	0.	2.73
time (sec)	N/A	0.346	1.419	0.058	0.	4.211	0.	1.329

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	146	225	0	1346	0	274
normalized size	1	1.	1.25	1.92	0.	11.5	0.	2.34
time (sec)	N/A	0.219	0.356	0.059	0.	4.103	0.	1.352

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	118	0	745	0	203
normalized size	1	1.	0.98	1.39	0.	8.76	0.	2.39
time (sec)	N/A	0.127	0.196	0.051	0.	1.758	0.	1.294

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	83	118	0	744	0	203
normalized size	1	1.	0.97	1.37	0.	8.65	0.	2.36
time (sec)	N/A	0.103	0.232	0.051	0.	1.804	0.	1.298

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	138	204	0	1053	0	242
normalized size	1	1.	1.27	1.87	0.	9.66	0.	2.22
time (sec)	N/A	0.169	0.442	0.061	0.	1.915	0.	1.164

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	172	242	0	1237	0	414
normalized size	1	1.	1.18	1.66	0.	8.47	0.	2.84
time (sec)	N/A	0.328	0.728	0.083	0.	1.939	0.	1.291

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	144	362	0	1445	0	356
normalized size	1	1.	0.69	1.74	0.	6.95	0.	1.71
time (sec)	N/A	0.585	0.725	0.083	0.	2.128	0.	1.29

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	176	508	0	1661	0	452
normalized size	1	1.	0.67	1.95	0.	6.36	0.	1.73
time (sec)	N/A	0.835	1.057	0.087	0.	2.238	0.	1.327

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	205	735	0	2921	0	517
normalized size	1	1.	0.89	3.2	0.	12.7	0.	2.25
time (sec)	N/A	0.707	4.805	0.064	0.	8.635	0.	1.332

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	194	685	0	2466	0	468
normalized size	1	1.	1.03	3.64	0.	13.12	0.	2.49
time (sec)	N/A	0.41	1.269	0.066	0.	8.568	0.	1.441

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	113	184	0	1291	0	342
normalized size	1	1.	0.76	1.23	0.	8.66	0.	2.3
time (sec)	N/A	0.232	0.418	0.056	0.	1.96	0.	1.405

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	115	195	0	1222	0	374
normalized size	1	1.	0.86	1.46	0.	9.12	0.	2.79
time (sec)	N/A	0.191	0.365	0.051	0.	1.894	0.	1.359

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	115	186	0	1296	0	343
normalized size	1	1.	0.86	1.4	0.	9.74	0.	2.58
time (sec)	N/A	0.175	0.454	0.061	0.	1.887	0.	1.371

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	205	664	0	1960	0	435
normalized size	1	1.	1.18	3.84	0.	11.33	0.	2.51
time (sec)	N/A	0.31	0.761	0.07	0.	2.135	0.	1.295

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	229	702	0	2225	0	482
normalized size	1	1.	1.03	3.15	0.	9.98	0.	2.16
time (sec)	N/A	0.623	0.881	0.086	0.	2.347	0.	1.352

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	199	827	0	2541	0	1037
normalized size	1	1.	0.67	2.79	0.	8.58	0.	3.5
time (sec)	N/A	0.998	2.042	0.092	0.	2.541	0.	1.416

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	416	1481	0	4581	0	799
normalized size	1	1.	1.32	4.69	0.	14.5	0.	2.53
time (sec)	N/A	1.106	6.233	0.07	0.	19.275	0.	1.434

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	250	1429	0	3977	0	755
normalized size	1	1.	0.97	5.52	0.	15.36	0.	2.92
time (sec)	N/A	0.752	4.184	0.069	0.	20.749	0.	1.536

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	158	285	0	1975	0	544
normalized size	1	1.	0.71	1.28	0.	8.9	0.	2.45
time (sec)	N/A	0.421	1.012	0.064	0.	2.191	0.	1.414

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	165	294	0	1976	0	582
normalized size	1	1.	0.8	1.43	0.	9.59	0.	2.83
time (sec)	N/A	0.353	1.086	0.064	0.	2.22	0.	1.443

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	164	297	0	1978	0	582
normalized size	1	1.	0.85	1.55	0.	10.3	0.	3.03
time (sec)	N/A	0.307	1.208	0.06	0.	2.192	0.	1.359

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	163	284	0	1974	0	544
normalized size	1	1.	0.89	1.54	0.	10.73	0.	2.96
time (sec)	N/A	0.307	1.109	0.061	0.	2.25	0.	1.35

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	268	1408	0	3174	0	718
normalized size	1	1.	1.11	5.82	0.	13.12	0.	2.97
time (sec)	N/A	0.535	1.518	0.073	0.	2.618	0.	1.247

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	293	1448	0	3573	0	761
normalized size	1	1.	0.98	4.84	0.	11.95	0.	2.55
time (sec)	N/A	1.037	1.674	0.097	0.	2.912	0.	1.473

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	326	1576	0	4038	0	830
normalized size	1	1.	0.84	4.07	0.	10.43	0.	2.14
time (sec)	N/A	1.456	6.359	0.101	0.	3.318	0.	1.389

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	34	63	89	0	41
normalized size	1	1.	0.97	1.1	2.03	2.87	0.	1.32
time (sec)	N/A	0.031	0.056	0.039	1.774	1.626	0.	1.248

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	73	63	119	211	0	80
normalized size	1	1.	1.3	1.12	2.12	3.77	0.	1.43
time (sec)	N/A	0.08	0.171	0.044	2.249	1.638	0.	1.12

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	108	94	177	348	0	101
normalized size	1	1.	1.33	1.16	2.19	4.3	0.	1.25
time (sec)	N/A	0.116	0.341	0.051	2.306	1.703	0.	1.252

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	141	125	231	504	0	119
normalized size	1	1.	1.33	1.18	2.18	4.75	0.	1.12
time (sec)	N/A	0.158	0.53	0.05	1.955	1.731	0.	1.211

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	69	51	95	154	0	58
normalized size	1	1.	0.99	0.73	1.36	2.2	0.	0.83
time (sec)	N/A	0.035	0.059	0.039	1.708	1.68	0.	1.274

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	162	87	150	309	0	93
normalized size	1	1.	1.71	0.92	1.58	3.25	0.	0.98
time (sec)	N/A	0.093	0.157	0.048	1.768	1.646	0.	1.278

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	241	123	209	482	0	115
normalized size	1	1.	2.01	1.02	1.74	4.02	0.	0.96
time (sec)	N/A	0.133	0.318	0.049	1.648	1.696	0.	1.256

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	344	159	262	672	0	132
normalized size	1	1.	2.37	1.1	1.81	4.63	0.	0.91
time (sec)	N/A	0.18	0.52	0.053	1.603	1.758	0.	1.283

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	401	1582	0	0	0	0
normalized size	1	1.	1.37	5.42	0.	0.	0.	0.
time (sec)	N/A	0.442	13.595	0.614	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	293	913	0	0	0	0
normalized size	1	1.	1.22	3.79	0.	0.	0.	0.
time (sec)	N/A	0.278	10.552	0.373	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	232	814	0	0	0	0
normalized size	1	1.	1.11	3.89	0.	0.	0.	0.
time (sec)	N/A	0.163	10.502	0.308	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	153	215	0	0	0	0
normalized size	1	1.	1.22	1.72	0.	0.	0.	0.
time (sec)	N/A	0.029	1.569	0.265	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	2713	829	0	0	0	0
normalized size	1	1.	8.22	2.51	0.	0.	0.	0.
time (sec)	N/A	0.323	18.818	0.287	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	1173	1257	0	0	0	0
normalized size	1	1.	2.96	3.17	0.	0.	0.	0.
time (sec)	N/A	0.599	18.559	0.286	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	550	2522	0	0	0	0
normalized size	1	1.	1.36	6.23	0.	0.	0.	0.
time (sec)	N/A	0.842	18.574	1.041	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	471	1852	0	0	0	0
normalized size	1	1.	1.38	5.42	0.	0.	0.	0.
time (sec)	N/A	0.599	14.116	0.658	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	282	282	408	1566	0	0	0	0
normalized size	1	1.	1.45	5.55	0.	0.	0.	0.
time (sec)	N/A	0.41	13.467	0.497	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	304	1106	0	0	0	0
normalized size	1	1.	1.22	4.44	0.	0.	0.	0.
time (sec)	N/A	0.292	9.936	0.331	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	882	1199	0	0	0	0
normalized size	1	1.	2.85	3.88	0.	0.	0.	0.
time (sec)	N/A	0.22	17.973	0.306	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	439	1029	0	0	0	0
normalized size	1	1.	1.31	3.08	0.	0.	0.	0.
time (sec)	N/A	0.334	11.579	0.289	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	390	390	1159	1440	0	0	0	0
normalized size	1	1.	2.97	3.69	0.	0.	0.	0.
time (sec)	N/A	0.544	18.341	0.257	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	463	463	615	2806	0	0	0	0
normalized size	1	1.	1.33	6.06	0.	0.	0.	0.
time (sec)	N/A	1.041	16.823	1.365	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	552	2523	0	0	0	0
normalized size	1	1.	1.38	6.32	0.	0.	0.	0.
time (sec)	N/A	0.78	16.165	1.017	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	474	1852	0	0	0	0
normalized size	1	1.	1.42	5.56	0.	0.	0.	0.
time (sec)	N/A	0.566	13.735	0.648	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	296	296	440	1775	0	0	0	0
normalized size	1	1.	1.49	6.	0.	0.	0.	0.
time (sec)	N/A	0.45	16.286	0.515	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	713	1514	0	0	0	0
normalized size	1	1.	2.03	4.3	0.	0.	0.	0.
time (sec)	N/A	0.334	17.65	0.347	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	784	1640	0	0	0	0
normalized size	1	1.	2.22	4.65	0.	0.	0.	0.
time (sec)	N/A	0.345	16.545	0.366	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	4588	1646	0	0	0	0
normalized size	1	1.	11.5	4.13	0.	0.	0.	0.
time (sec)	N/A	0.632	23.303	0.283	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	460	460	1026	1881	0	0	0	0
normalized size	1	1.	2.23	4.09	0.	0.	0.	0.
time (sec)	N/A	0.933	17.241	0.324	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	530	530	1688	2330	0	0	0	0
normalized size	1	1.	3.18	4.4	0.	0.	0.	0.
time (sec)	N/A	1.3	17.003	0.398	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	403	403	1150	2185	0	0	0	0
normalized size	1	1.	2.85	5.42	0.	0.	0.	0.
time (sec)	N/A	0.499	15.777	0.534	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	359	359	463	1852	0	0	0	0
normalized size	1	1.	1.29	5.16	0.	0.	0.	0.
time (sec)	N/A	0.673	14.495	0.683	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	301	301	365	1584	0	0	0	0
normalized size	1	1.	1.21	5.26	0.	0.	0.	0.
time (sec)	N/A	0.423	14.201	0.534	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	341	919	0	0	0	0
normalized size	1	1.	1.4	3.77	0.	0.	0.	0.
time (sec)	N/A	0.275	13.326	0.358	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	204	204	2189	639	0	0	0	0
normalized size	1	1.	10.73	3.13	0.	0.	0.	0.
time (sec)	N/A	0.156	18.649	0.311	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	93	143	0	0	0	0
normalized size	1	1.	0.94	1.44	0.	0.	0.	0.
time (sec)	N/A	0.04	1.247	0.245	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	140	178	0	0	0	0
normalized size	1	1.	1.32	1.68	0.	0.	0.	0.
time (sec)	N/A	0.022	1.248	0.245	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	5060	649	0	0	0	0
normalized size	1	1.	14.97	1.92	0.	0.	0.	0.
time (sec)	N/A	0.265	23.599	0.285	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	401	401	1195	1258	0	0	0	0
normalized size	1	1.	2.98	3.14	0.	0.	0.	0.
time (sec)	N/A	0.51	18.306	0.286	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	498	2477	0	0	0	0
normalized size	1	1.	1.25	6.21	0.	0.	0.	0.
time (sec)	N/A	0.775	15.09	0.851	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	470	1789	0	0	0	0
normalized size	1	1.	1.45	5.5	0.	0.	0.	0.
time (sec)	N/A	0.504	14.051	0.474	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	440	1451	0	0	0	0
normalized size	1	1.	1.71	5.65	0.	0.	0.	0.
time (sec)	N/A	0.319	13.242	0.366	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	249	837	0	0	0	0
normalized size	1	1.	1.05	3.53	0.	0.	0.	0.
time (sec)	N/A	0.293	8.557	0.272	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	244	817	0	0	0	0
normalized size	1	1.	1.03	3.46	0.	0.	0.	0.
time (sec)	N/A	0.229	9.369	0.251	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0
normalized size	1	1.	3.6	3.48	0.	0.	0.	0.
time (sec)	N/A	0.316	18.285	0.268	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	1077	1662	0	0	0	0
normalized size	1	1.	2.72	4.2	0.	0.	0.	0.
time (sec)	N/A	0.498	15.471	0.27	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	470	470	1745	2298	0	0	0	0
normalized size	1	1.	3.71	4.89	0.	0.	0.	0.
time (sec)	N/A	0.775	14.31	0.353	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	578	4176	0	0	0	0
normalized size	1	1.	1.35	9.78	0.	0.	0.	0.
time (sec)	N/A	0.933	17.34	0.926	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	556	3674	0	0	0	0
normalized size	1	1.	1.54	10.15	0.	0.	0.	0.
time (sec)	N/A	0.591	18.411	0.582	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	503	2731	0	0	0	0
normalized size	1	1.	1.49	8.1	0.	0.	0.	0.
time (sec)	N/A	0.502	14.018	0.329	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	486	2411	0	0	0	0
normalized size	1	1.	1.53	7.61	0.	0.	0.	0.
time (sec)	N/A	0.436	13.336	0.276	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	360	1781	0	0	0	0
normalized size	1	1.	1.18	5.86	0.	0.	0.	0.
time (sec)	N/A	0.408	7.813	0.263	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	448	448	1798	3889	0	0	0	0
normalized size	1	1.	4.01	8.68	0.	0.	0.	0.
time (sec)	N/A	0.564	14.144	0.31	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	510	510	1493	4580	0	0	0	0
normalized size	1	1.	2.93	8.98	0.	0.	0.	0.
time (sec)	N/A	0.814	18.598	0.45	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	562	562	2285	5638	0	0	0	0
normalized size	1	1.	4.07	10.03	0.	0.	0.	0.
time (sec)	N/A	1.167	15.412	0.579	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	535	535	2346	7838	0	0	0	0
normalized size	1	1.	4.39	14.65	0.	0.	0.	0.
time (sec)	N/A	0.86	15.563	0.5	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	97	502	0	0	0	0
normalized size	1	1.	0.64	3.32	0.	0.	0.	0.
time (sec)	N/A	0.099	0.32	3.964	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	85	397	0	0	0	0
normalized size	1	1.	0.69	3.23	0.	0.	0.	0.
time (sec)	N/A	0.091	0.226	3.412	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	148	0	0	0	0
normalized size	1	1.	0.73	1.53	0.	0.	0.	0.
time (sec)	N/A	0.075	0.108	1.346	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	0	0	0
normalized size	1	1.	0.69	2.03	0.	0.	0.	0.
time (sec)	N/A	0.063	0.073	1.273	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	76	228	0	0	0	0
normalized size	1	1.	0.75	2.26	0.	0.	0.	0.
time (sec)	N/A	0.077	0.131	1.557	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	88	262	0	0	0	0
normalized size	1	1.	0.69	2.06	0.	0.	0.	0.
time (sec)	N/A	0.089	0.335	1.356	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	99	290	0	0	0	0
normalized size	1	1.	0.66	1.92	0.	0.	0.	0.
time (sec)	N/A	0.098	0.536	1.26	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	139	689	0	0	0	0
normalized size	1	1.	0.7	3.44	0.	0.	0.	0.
time (sec)	N/A	0.148	0.864	4.871	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	126	660	0	0	0	0
normalized size	1	1.	0.72	3.77	0.	0.	0.	0.
time (sec)	N/A	0.129	1.216	4.376	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	93	514	0	0	0	0
normalized size	1	1.	0.69	3.81	0.	0.	0.	0.
time (sec)	N/A	0.11	0.329	3.414	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	82	202	0	0	0	0
normalized size	1	1.	0.76	1.87	0.	0.	0.	0.
time (sec)	N/A	0.107	0.183	1.535	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	0	0	0
normalized size	1	1.	0.78	2.53	0.	0.	0.	0.
time (sec)	N/A	0.109	0.18	1.434	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	100	321	0	0	0	0
normalized size	1	1.	0.71	2.28	0.	0.	0.	0.
time (sec)	N/A	0.122	0.424	1.712	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	120	362	0	0	0	0
normalized size	1	1.	0.69	2.07	0.	0.	0.	0.
time (sec)	N/A	0.138	0.712	1.43	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	177	847	0	0	0	0
normalized size	1	1.	0.76	3.62	0.	0.	0.	0.
time (sec)	N/A	0.241	3.274	5.777	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	134	738	0	0	0	0
normalized size	1	1.	0.71	3.9	0.	0.	0.	0.
time (sec)	N/A	0.202	1.428	4.544	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	106	631	0	0	0	0
normalized size	1	1.	0.67	3.99	0.	0.	0.	0.
time (sec)	N/A	0.189	0.463	3.526	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	108	303	0	0	0	0
normalized size	1	1.	0.65	1.83	0.	0.	0.	0.
time (sec)	N/A	0.194	0.552	1.615	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	106	376	0	0	0	0
normalized size	1	1.	0.68	2.41	0.	0.	0.	0.
time (sec)	N/A	0.193	0.443	1.528	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	132	421	0	0	0	0
normalized size	1	1.	0.66	2.12	0.	0.	0.	0.
time (sec)	N/A	0.228	0.91	1.568	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	159	470	0	0	0	0
normalized size	1	1.	0.68	2.01	0.	0.	0.	0.
time (sec)	N/A	0.241	1.25	1.5	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	256	1174	0	0	0	0
normalized size	1	1.	0.89	4.09	0.	0.	0.	0.
time (sec)	N/A	0.409	2.141	6.618	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	168	925	0	0	0	0
normalized size	1	1.	0.68	3.74	0.	0.	0.	0.
time (sec)	N/A	0.366	1.531	5.392	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	146	907	0	0	0	0
normalized size	1	1.	0.7	4.34	0.	0.	0.	0.
time (sec)	N/A	0.349	2.125	4.679	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	130	777	0	0	0	0
normalized size	1	1.	0.62	3.74	0.	0.	0.	0.
time (sec)	N/A	0.349	1.083	2.028	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	138	619	0	0	0	0
normalized size	1	1.	0.67	2.99	0.	0.	0.	0.
time (sec)	N/A	0.369	0.582	1.752	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	142	476	0	0	0	0
normalized size	1	1.	0.67	2.26	0.	0.	0.	0.
time (sec)	N/A	0.363	0.8	1.425	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	168	529	0	0	0	0
normalized size	1	1.	0.69	2.16	0.	0.	0.	0.
time (sec)	N/A	0.403	1.471	1.666	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	199	586	0	0	0	0
normalized size	1	1.	0.69	2.03	0.	0.	0.	0.
time (sec)	N/A	0.46	1.835	1.592	0.	0.	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	167	450	0	0	0	0
normalized size	1	1.	0.89	2.39	0.	0.	0.	0.
time (sec)	N/A	0.511	2.834	4.243	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	86	353	0	0	0	0
normalized size	1	1.	0.74	3.02	0.	0.	0.	0.
time (sec)	N/A	0.164	4.538	1.689	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0
normalized size	1	1.	1.29	3.06	0.	0.	0.	0.
time (sec)	N/A	0.092	0.316	1.154	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	49	187	0	0	0	0
normalized size	1	1.	0.53	2.01	0.	0.	0.	0.
time (sec)	N/A	0.15	0.217	1.492	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	178	226	0	0	0	0
normalized size	1	1.	1.32	1.67	0.	0.	0.	0.
time (sec)	N/A	0.207	6.232	1.415	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	196	516	0	0	0	0
normalized size	1	1.	1.14	3.	0.	0.	0.	0.
time (sec)	N/A	0.366	6.085	1.543	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	319	1002	0	0	0	0
normalized size	1	1.	0.93	2.93	0.	0.	0.	0.
time (sec)	N/A	0.944	6.397	6.195	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	355	868	0	0	0	0
normalized size	1	1.	1.27	3.11	0.	0.	0.	0.
time (sec)	N/A	0.643	5.818	4.619	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	587	608	0	0	0	0
normalized size	1	1.	2.74	2.84	0.	0.	0.	0.
time (sec)	N/A	0.402	6.656	2.751	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	633	707	0	0	0	0
normalized size	1	1.	3.04	3.4	0.	0.	0.	0.
time (sec)	N/A	0.358	6.651	3.605	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	255	788	0	0	0	0
normalized size	1	1.	1.12	3.47	0.	0.	0.	0.
time (sec)	N/A	0.369	4.385	3.836	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	323	809	0	0	0	0
normalized size	1	1.	1.32	3.32	0.	0.	0.	0.
time (sec)	N/A	0.44	6.406	4.456	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	281	1064	0	0	0	0
normalized size	1	1.	0.92	3.5	0.	0.	0.	0.
time (sec)	N/A	0.68	6.595	4.523	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	388	388	726	2014	0	0	0	0
normalized size	1	1.	1.87	5.19	0.	0.	0.	0.
time (sec)	N/A	0.961	6.783	7.598	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	315	315	697	1203	0	0	0	0
normalized size	1	1.	2.21	3.82	0.	0.	0.	0.
time (sec)	N/A	0.704	6.749	3.678	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	733	1760	0	0	0	0
normalized size	1	1.	2.34	5.62	0.	0.	0.	0.
time (sec)	N/A	0.683	6.72	5.928	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	306	306	724	1858	0	0	0	0
normalized size	1	1.	2.37	6.07	0.	0.	0.	0.
time (sec)	N/A	0.646	6.7	6.233	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	323	749	1936	0	0	0	0
normalized size	1	1.	2.32	5.99	0.	0.	0.	0.
time (sec)	N/A	0.667	6.729	6.461	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	712	1957	0	0	0	0
normalized size	1	1.	2.08	5.72	0.	0.	0.	0.
time (sec)	N/A	0.754	6.782	7.382	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	736	2216	0	0	0	0
normalized size	1	1.	1.81	5.46	0.	0.	0.	0.
time (sec)	N/A	1.021	6.879	7.682	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	321	788	0	0	0	0
normalized size	1	1.	1.35	3.32	0.	0.	0.	0.
time (sec)	N/A	0.653	5.133	0.353	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	96	283	0	0	0	0
normalized size	1	1.	0.7	2.05	0.	0.	0.	0.
time (sec)	N/A	0.354	2.271	0.266	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	925	0	0	0	0
normalized size	1	1.	1.	13.81	0.	0.	0.	0.
time (sec)	N/A	0.097	0.102	0.299	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	156	1021	0	0	0	0
normalized size	1	1.	0.81	5.32	0.	0.	0.	0.
time (sec)	N/A	0.377	0.576	0.336	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	203	1736	0	0	0	0
normalized size	1	1.	0.83	7.11	0.	0.	0.	0.
time (sec)	N/A	0.658	0.855	0.324	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	237	2050	0	0	0	0
normalized size	1	1.	0.78	6.72	0.	0.	0.	0.
time (sec)	N/A	0.847	1.188	0.396	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	411	1744	0	0	0	0
normalized size	1	1.	1.37	5.83	0.	0.	0.	0.
time (sec)	N/A	0.994	6.172	0.294	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	394	1207	0	0	0	0
normalized size	1	1.	1.58	4.85	0.	0.	0.	0.
time (sec)	N/A	0.727	7.708	0.342	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	129	1367	0	0	0	0
normalized size	1	1.	0.62	6.54	0.	0.	0.	0.
time (sec)	N/A	0.499	2.46	0.302	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	1219	0	0	0	0
normalized size	1	1.	0.83	6.52	0.	0.	0.	0.
time (sec)	N/A	0.406	0.653	0.311	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	197	1707	0	0	0	0
normalized size	1	1.	0.82	7.11	0.	0.	0.	0.
time (sec)	N/A	0.615	1.05	0.336	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	237	2050	0	0	0	0
normalized size	1	1.	0.78	6.77	0.	0.	0.	0.
time (sec)	N/A	0.871	1.688	0.389	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	602	2295	0	0	0	0
normalized size	1	1.	1.63	6.22	0.	0.	0.	0.
time (sec)	N/A	1.348	6.569	0.364	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	560	1982	0	0	0	0
normalized size	1	1.	1.78	6.31	0.	0.	0.	0.
time (sec)	N/A	1.071	6.504	0.275	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	538	1949	0	0	0	0
normalized size	1	1.	2.05	7.41	0.	0.	0.	0.
time (sec)	N/A	0.78	6.517	0.291	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	409	1661	0	0	0	0
normalized size	1	1.	1.56	6.34	0.	0.	0.	0.
time (sec)	N/A	0.78	5.694	0.262	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	200	1931	0	0	0	0
normalized size	1	1.	0.84	8.08	0.	0.	0.	0.
time (sec)	N/A	0.69	1.459	0.309	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	237	2050	0	0	0	0
normalized size	1	1.	0.78	6.77	0.	0.	0.	0.
time (sec)	N/A	0.936	2.245	0.352	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	286	2788	0	0	0	0
normalized size	1	1.	0.79	7.68	0.	0.	0.	0.
time (sec)	N/A	1.254	2.578	0.465	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	397	1755	0	0	0	0
normalized size	1	1.	1.27	5.62	0.	0.	0.	0.
time (sec)	N/A	0.888	5.837	0.321	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	329	996	0	0	0	0
normalized size	1	1.	1.34	4.05	0.	0.	0.	0.
time (sec)	N/A	0.617	8.634	0.313	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	216	0	0	0	0
normalized size	1	1.	1.	3.18	0.	0.	0.	0.
time (sec)	N/A	0.182	0.111	0.252	0.	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	171	0	0	0	0
normalized size	1	1.	1.	2.55	0.	0.	0.	0.
time (sec)	N/A	0.095	0.067	0.244	0.	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	96	736	0	0	0	0
normalized size	1	1.	0.68	5.18	0.	0.	0.	0.
time (sec)	N/A	0.242	2.474	0.28	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	147	1024	0	0	0	0
normalized size	1	1.	0.75	5.25	0.	0.	0.	0.
time (sec)	N/A	0.362	0.566	0.33	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	193	1736	0	0	0	0
normalized size	1	1.	0.78	6.97	0.	0.	0.	0.
time (sec)	N/A	0.567	0.713	0.303	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	478	1501	0	0	0	0
normalized size	1	1.	1.39	4.35	0.	0.	0.	0.
time (sec)	N/A	0.997	4.089	0.341	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	434	1144	0	0	0	0
normalized size	1	1.	2.11	5.55	0.	0.	0.	0.
time (sec)	N/A	0.52	6.	0.281	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	103	501	0	0	0	0
normalized size	1	1.	0.82	3.98	0.	0.	0.	0.
time (sec)	N/A	0.164	0.294	0.253	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	156	510	0	0	0	0
normalized size	1	1.	0.78	2.55	0.	0.	0.	0.
time (sec)	N/A	0.38	0.582	0.265	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	165	999	0	0	0	0
normalized size	1	1.	0.77	4.67	0.	0.	0.	0.
time (sec)	N/A	0.434	0.655	0.296	0.	0.	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	203	1315	0	0	0	0
normalized size	1	1.	0.7	4.55	0.	0.	0.	0.
time (sec)	N/A	0.672	0.877	0.319	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	250	1861	0	0	0	0
normalized size	1	1.	0.69	5.17	0.	0.	0.	0.
time (sec)	N/A	0.966	1.295	0.305	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	458	458	677	4591	0	0	0	0
normalized size	1	1.	1.48	10.02	0.	0.	0.	0.
time (sec)	N/A	1.413	6.664	0.384	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	487	3854	0	0	0	0
normalized size	1	1.	1.32	10.42	0.	0.	0.	0.
time (sec)	N/A	1.103	5.29	0.292	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	169	1343	0	0	0	0
normalized size	1	1.	0.61	4.85	0.	0.	0.	0.
time (sec)	N/A	0.665	1.026	0.31	0.	0.	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	178	1822	0	0	0	0
normalized size	1	1.	0.63	6.48	0.	0.	0.	0.
time (sec)	N/A	0.614	1.003	0.253	0.	0.	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	196	2070	0	0	0	0
normalized size	1	1.	0.65	6.85	0.	0.	0.	0.
time (sec)	N/A	0.645	1.184	0.277	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	208	3103	0	0	0	0
normalized size	1	1.	0.66	9.79	0.	0.	0.	0.
time (sec)	N/A	0.734	1.259	0.32	0.	0.	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	257	3614	0	0	0	0
normalized size	1	1.	0.66	9.24	0.	0.	0.	0.
time (sec)	N/A	1.022	1.64	0.337	0.	0.	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	292	4586	0	0	0	0
normalized size	1	1.	0.62	9.68	0.	0.	0.	0.
time (sec)	N/A	1.353	1.948	0.451	0.	0.	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	78	405	0	0	0	0
normalized size	1	1.	0.64	3.32	0.	0.	0.	0.
time (sec)	N/A	0.18	0.139	0.274	0.	0.	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	374	0	0	0	0
normalized size	1	1.	0.62	3.43	0.	0.	0.	0.
time (sec)	N/A	0.176	0.118	0.282	0.	0.	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	68	405	0	0	0	0
normalized size	1	1.	0.63	3.75	0.	0.	0.	0.
time (sec)	N/A	0.198	0.078	0.28	0.	0.	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	78	390	0	0	0	0
normalized size	1	1.	0.63	3.17	0.	0.	0.	0.
time (sec)	N/A	0.204	0.089	0.284	0.	0.	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	81	409	0	0	0	0
normalized size	1	1.	0.64	3.22	0.	0.	0.	0.
time (sec)	N/A	0.171	0.139	0.461	0.	0.	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	72	381	0	0	0	0
normalized size	1	1.	0.64	3.37	0.	0.	0.	0.
time (sec)	N/A	0.177	0.114	0.283	0.	0.	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	72	370	0	0	0	0
normalized size	1	1.	0.56	2.87	0.	0.	0.	0.
time (sec)	N/A	0.174	0.077	0.242	0.	0.	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	394	0	0	0	0
normalized size	1	1.	0.7	3.43	0.	0.	0.	0.
time (sec)	N/A	0.175	0.105	0.255	0.	0.	0.	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	142	0	0	0	0
normalized size	1	1.	1.	2.33	0.	0.	0.	0.
time (sec)	N/A	0.057	0.055	0.217	0.	0.	0.	0.

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	137	0	0	0	0
normalized size	1	1.	1.	2.54	0.	0.	0.	0.
time (sec)	N/A	0.057	0.056	0.234	0.	0.	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	144	0	0	0	0
normalized size	1	1.	1.	2.67	0.	0.	0.	0.
time (sec)	N/A	0.069	0.035	0.226	0.	0.	0.	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	139	0	0	0	0
normalized size	1	1.	1.	2.28	0.	0.	0.	0.
time (sec)	N/A	0.07	0.043	0.232	0.	0.	0.	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	61	145	0	0	0	0
normalized size	1	1.	1.	2.38	0.	0.	0.	0.
time (sec)	N/A	0.057	0.057	0.215	0.	0.	0.	0.

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	138	0	0	0	0
normalized size	1	1.	1.	2.56	0.	0.	0.	0.
time (sec)	N/A	0.057	0.056	0.241	0.	0.	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	136	0	0	0	0
normalized size	1	1.	0.87	2.19	0.	0.	0.	0.
time (sec)	N/A	0.057	0.044	0.241	0.	0.	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	61	142	0	0	0	0
normalized size	1	1.	1.11	2.58	0.	0.	0.	0.
time (sec)	N/A	0.058	0.047	0.235	0.	0.	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	7160	0	0	0	0	0
normalized size	1	1.	68.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	26.004	0.135	0.	0.	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	1.896	0.174	0.	0.	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	21877	0	0	0	0	0
normalized size	1	1.	60.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.668	26.681	0.12	0.	0.	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	18991	0	0	0	0	0
normalized size	1	1.	62.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.47	26.389	0.115	0.	0.	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	2505	0	0	0	0	0
normalized size	1	1.	9.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	18.301	0.099	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	7142	0	0	0	0	0
normalized size	1	1.	68.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	25.944	0.095	0.	0.	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.012	2.027	0.129	0.	0.	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	7313	0	0	0	0	0
normalized size	1	1.	67.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	26.451	0.096	0.	0.	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	18.632	0.133	0.	0.	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	412	412	28057	0	0	0	0	0
normalized size	1	1.	68.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.825	26.991	0.122	0.	0.	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	21890	0	0	0	0	0
normalized size	1	1.	61.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.653	26.562	0.112	0.	0.	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	19016	0	0	0	0	0
normalized size	1	1.	63.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.457	26.613	0.1	0.	0.	0.	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	7321	0	0	0	0	0
normalized size	1	1.	67.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	26.49	0.095	0.	0.	0.	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	24.271	0.088	0.	0.	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	19015	0	0	0	0	0
normalized size	1	1.	60.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.488	26.409	0.121	0.	0.	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	7195	0	0	0	0	0
normalized size	1	1.	27.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.332	25.971	0.119	0.	0.	0.	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	2759	0	0	0	0	0
normalized size	1	1.	12.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.229	19.014	0.109	0.	0.	0.	0.

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0
normalized size	1	1.	2.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	1.741	0.133	0.	0.	0.	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.912	0.114	0.	0.	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	310	0	0	0	0	0
normalized size	1	1.	2.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	1.705	0.126	0.	0.	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	0.905	0.114	0.	0.	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	10343	0	0	0	0	0
normalized size	1	1.	94.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	26.775	0.094	0.	0.	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	23.693	0.086	0.	0.	0.	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	21987	0	0	0	0	0
normalized size	1	1.	58.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	26.408	0.124	0.	0.	0.	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	19126	0	0	0	0	0
normalized size	1	1.	62.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.381	26.211	0.111	0.	0.	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	289	289	7325	0	0	0	0	0
normalized size	1	1.	25.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.359	26.245	0.101	0.	0.	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	10363	0	0	0	0	0
normalized size	1	1.	94.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	26.738	0.093	0.	0.	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	19.218	0.086	0.	0.	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	4543	0	0	0	0	0
normalized size	1	1.	26.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.257	21.323	0.097	0.	0.	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	4544	0	0	0	0	0
normalized size	1	1.	26.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.262	21.334	0.102	0.	0.	0.	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	7542	0	0	0	0	0
normalized size	1	1.	43.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	28.473	0.103	0.	0.	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	7588	0	0	0	0	0
normalized size	1	1.	43.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	28.271	0.096	0.	0.	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	32.23	0.225	0.	0.	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	42.929	0.251	0.	0.	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	27.375	0.219	0.	0.	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	33.285	0.254	0.	0.	0.	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	2.24	0.38	0.	0.	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	7.816	0.308	0.	0.	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	15.357	0.342	0.	0.	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	20.709	0.296	0.	0.	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	29.293	0.204	0.	0.	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	35.562	0.21	0.	0.	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	34.341	0.202	0.	0.	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	37.215	0.215	0.	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	31.319	0.207	0.	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	36.458	0.203	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	27.726	0.193	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	37.117	0.192	0.	0.	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	21.134	0.3	0.	0.	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	25.884	0.232	0.	0.	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	27.631	0.195	0.	0.	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	34.173	0.205	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	37.88	0.205	0.	0.	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	42.907	0.222	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	37.149	0.201	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	41.223	0.222	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	30.661	0.209	0.	0.	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	41.39	0.203	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	31.798	0.199	0.	0.	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	37.259	0.195	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	31.17	0.194	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	37.634	0.19	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	27.21	0.201	0.	0.	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	30.992	0.211	0.	0.	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	1.56	0.266	0.	0.	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	1.772	0.277	0.	0.	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	1.216	0.278	0.	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	2.803	0.244	0.	0.	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	21.95	0.316	0.	0.	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	27.059	0.255	0.	0.	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	30.265	0.204	0.	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	36.831	0.196	0.	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	29.577	0.203	0.	0.	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	35.975	0.191	0.	0.	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	30.631	0.207	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	37.615	0.193	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	31.363	0.204	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	37.272	0.194	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	35.02	0.26	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	41.64	0.195	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	15.451	0.212	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	43.26	0.198	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	34.618	0.202	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	41.927	0.23	0.	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	33.908	0.202	0.	0.	0.	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	38.819	0.219	0.	0.	0.	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	38.014	0.209	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	43.56	0.251	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	42.731	0.204	0.	0.	0.	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	50.09	0.195	0.	0.	0.	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	19.599	0.204	0.	0.	0.	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	56.813	0.206	0.	0.	0.	0.

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	231	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.349	0.833	3.113	0.	0.	0.	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	171	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	0.311	1.867	0.	0.	0.	0.

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	107	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.149	0.549	0.	0.	0.	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	5280	0	0	0	0	0
normalized size	1	1.	27.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.302	25.451	0.73	0.	0.	0.	0.

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	10428	0	0	0	0	0
normalized size	1	1.	34.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.444	32.737	0.244	0.	0.	0.	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	13.519	0.204	0.	0.	0.	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.42	0.212	0.	0.	0.	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	2.45	0.193	0.	0.	0.	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	2.229	0.181	0.	0.	0.	0.

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	2.178	0.816	0.	0.	0.	0.

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.496	0.835	0.	0.	0.	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	8908	0	0	0	0	0
normalized size	1	1.	32.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.349	26.226	0.275	0.	0.	0.	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	5564	0	0	0	0	0
normalized size	1	1.	25.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.221	22.753	0.229	0.	0.	0.	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	2828	0	0	0	0	0
normalized size	1	1.	27.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	14.571	0.238	0.	0.	0.	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.01	1.759	0.193	0.	0.	0.	0.

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	5.558	0.335	0.	0.	0.	0.

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	4.678	0.459	0.	0.	0.	0.

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	90	318	0	0	0	0
normalized size	1	1.	0.67	2.36	0.	0.	0.	0.
time (sec)	N/A	0.104	0.342	1.557	0.	0.	0.	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	77	290	0	0	0	0
normalized size	1	1.	0.69	2.61	0.	0.	0.	0.
time (sec)	N/A	0.089	0.49	1.383	0.	0.	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	66	262	0	0	0	0
normalized size	1	1.	0.76	3.01	0.	0.	0.	0.
time (sec)	N/A	0.076	0.224	1.607	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	228	0	0	0	0
normalized size	1	1.	0.87	3.74	0.	0.	0.	0.
time (sec)	N/A	0.067	0.111	1.472	0.	0.	0.	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	152	0	0	0	0
normalized size	1	1.	0.91	4.34	0.	0.	0.	0.
time (sec)	N/A	0.057	0.067	1.513	0.	0.	0.	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	148	0	0	0	0
normalized size	1	1.	0.89	2.6	0.	0.	0.	0.
time (sec)	N/A	0.067	0.14	1.556	0.	0.	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	397	0	0	0	0
normalized size	1	1.	0.78	4.78	0.	0.	0.	0.
time (sec)	N/A	0.075	0.403	3.502	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	502	0	0	0	0
normalized size	1	1.	0.86	4.52	0.	0.	0.	0.
time (sec)	N/A	0.087	0.298	3.946	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	113	398	0	0	0	0
normalized size	1	1.	0.71	2.49	0.	0.	0.	0.
time (sec)	N/A	0.187	0.767	1.828	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	98	362	0	0	0	0
normalized size	1	1.	0.73	2.68	0.	0.	0.	0.
time (sec)	N/A	0.172	0.584	1.443	0.	0.	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	321	0	0	0	0
normalized size	1	1.	0.78	3.18	0.	0.	0.	0.
time (sec)	N/A	0.153	0.295	1.498	0.	0.	0.	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	283	0	0	0	0
normalized size	1	1.	0.89	3.93	0.	0.	0.	0.
time (sec)	N/A	0.139	0.156	1.731	0.	0.	0.	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	202	0	0	0	0
normalized size	1	1.	0.91	2.97	0.	0.	0.	0.
time (sec)	N/A	0.136	0.291	1.653	0.	0.	0.	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	73	514	0	0	0	0
normalized size	1	1.	0.77	5.41	0.	0.	0.	0.
time (sec)	N/A	0.145	0.577	3.614	0.	0.	0.	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	660	0	0	0	0
normalized size	1	1.	0.92	4.89	0.	0.	0.	0.
time (sec)	N/A	0.164	0.355	4.947	0.	0.	0.	0.

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	142	689	0	0	0	0
normalized size	1	1.	0.89	4.31	0.	0.	0.	0.
time (sec)	N/A	0.183	0.535	5.016	0.	0.	0.	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	137	470	0	0	0	0
normalized size	1	1.	0.71	2.42	0.	0.	0.	0.
time (sec)	N/A	0.286	0.942	1.598	0.	0.	0.	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	110	421	0	0	0	0
normalized size	1	1.	0.69	2.65	0.	0.	0.	0.
time (sec)	N/A	0.26	0.714	1.764	0.	0.	0.	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	84	376	0	0	0	0
normalized size	1	1.	0.72	3.24	0.	0.	0.	0.
time (sec)	N/A	0.228	0.364	1.634	0.	0.	0.	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	87	303	0	0	0	0
normalized size	1	1.	0.69	2.4	0.	0.	0.	0.
time (sec)	N/A	0.231	0.522	1.954	0.	0.	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	84	631	0	0	0	0
normalized size	1	1.	0.71	5.35	0.	0.	0.	0.
time (sec)	N/A	0.23	1.113	3.619	0.	0.	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	738	0	0	0	0
normalized size	1	1.	0.84	4.95	0.	0.	0.	0.
time (sec)	N/A	0.246	0.876	4.534	0.	0.	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	177	847	0	0	0	0
normalized size	1	1.	0.91	4.37	0.	0.	0.	0.
time (sec)	N/A	0.285	0.764	5.513	0.	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	228	668	0	0	0	0
normalized size	1	1.	1.5	4.39	0.	0.	0.	0.
time (sec)	N/A	0.602	1.582	1.995	0.	0.	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	160	516	0	0	0	0
normalized size	1	1.	1.43	4.61	0.	0.	0.	0.
time (sec)	N/A	0.39	1.66	1.714	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	84	226	0	0	0	0
normalized size	1	1.	1.12	3.01	0.	0.	0.	0.
time (sec)	N/A	0.242	0.292	1.507	0.	0.	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	187	0	0	0	0
normalized size	1	1.	0.91	3.53	0.	0.	0.	0.
time (sec)	N/A	0.189	0.068	1.849	0.	0.	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0
normalized size	1	1.	1.	5.17	0.	0.	0.	0.
time (sec)	N/A	0.128	0.078	1.392	0.	0.	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	197	353	0	0	0	0
normalized size	1	1.	2.56	4.58	0.	0.	0.	0.
time (sec)	N/A	0.202	1.894	1.821	0.	0.	0.	0.

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	213	450	0	0	0	0
normalized size	1	1.	1.66	3.52	0.	0.	0.	0.
time (sec)	N/A	0.54	4.665	4.096	0.	0.	0.	0.

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	268	1064	0	0	0	0
normalized size	1	1.	1.1	4.36	0.	0.	0.	0.
time (sec)	N/A	0.727	1.812	4.984	0.	0.	0.	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	254	809	0	0	0	0
normalized size	1	1.	1.38	4.4	0.	0.	0.	0.
time (sec)	N/A	0.479	1.767	4.926	0.	0.	0.	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	196	788	0	0	0	0
normalized size	1	1.	1.17	4.72	0.	0.	0.	0.
time (sec)	N/A	0.419	3.456	3.556	0.	0.	0.	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	233	707	0	0	0	0
normalized size	1	1.	1.57	4.78	0.	0.	0.	0.
time (sec)	N/A	0.394	3.238	3.88	0.	0.	0.	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	242	608	0	0	0	0
normalized size	1	1.	1.57	3.95	0.	0.	0.	0.
time (sec)	N/A	0.445	3.266	3.063	0.	0.	0.	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	280	868	0	0	0	0
normalized size	1	1.	1.28	3.96	0.	0.	0.	0.
time (sec)	N/A	0.684	3.078	5.325	0.	0.	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	355	2216	0	0	0	0
normalized size	1	1.	1.03	6.4	0.	0.	0.	0.
time (sec)	N/A	1.057	3.599	7.931	0.	0.	0.	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	315	1957	0	0	0	0
normalized size	1	1.	1.12	6.94	0.	0.	0.	0.
time (sec)	N/A	0.807	2.751	7.339	0.	0.	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	288	1936	0	0	0	0
normalized size	1	1.	1.1	7.36	0.	0.	0.	0.
time (sec)	N/A	0.683	2.82	6.48	0.	0.	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	274	1858	0	0	0	0
normalized size	1	1.	1.11	7.55	0.	0.	0.	0.
time (sec)	N/A	0.658	1.988	6.548	0.	0.	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	291	1760	0	0	0	0
normalized size	1	1.	1.15	6.96	0.	0.	0.	0.
time (sec)	N/A	0.721	3.105	6.368	0.	0.	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	299	1203	0	0	0	0
normalized size	1	1.	1.17	4.72	0.	0.	0.	0.
time (sec)	N/A	0.746	2.775	3.999	0.	0.	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	336	2014	0	0	0	0
normalized size	1	1.	1.02	6.14	0.	0.	0.	0.
time (sec)	N/A	1.	3.327	7.894	0.	0.	0.	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	340	1726	0	0	0	0
normalized size	1	1.	1.39	7.07	0.	0.	0.	0.
time (sec)	N/A	0.659	8.764	0.327	0.	0.	0.	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	273	1011	0	0	0	0
normalized size	1	1.	1.42	5.27	0.	0.	0.	0.
time (sec)	N/A	0.432	8.123	0.309	0.	0.	0.	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	198	923	0	0	0	0
normalized size	1	1.	2.96	13.78	0.	0.	0.	0.
time (sec)	N/A	0.146	3.362	0.287	0.	0.	0.	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	14885	275	0	0	0	0
normalized size	1	1.	107.86	1.99	0.	0.	0.	0.
time (sec)	N/A	0.406	28.177	0.25	0.	0.	0.	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	23549	781	0	0	0	0
normalized size	1	1.	99.36	3.3	0.	0.	0.	0.
time (sec)	N/A	0.69	30.732	0.289	0.	0.	0.	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	383	2040	0	0	0	0
normalized size	1	1.	1.26	6.73	0.	0.	0.	0.
time (sec)	N/A	0.94	9.865	0.309	0.	0.	0.	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	344	1697	0	0	0	0
normalized size	1	1.	1.43	7.07	0.	0.	0.	0.
time (sec)	N/A	0.681	8.074	0.271	0.	0.	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	284	1209	0	0	0	0
normalized size	1	1.	1.52	6.47	0.	0.	0.	0.
time (sec)	N/A	0.469	6.346	0.28	0.	0.	0.	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	25369	1365	0	0	0	0
normalized size	1	1.	121.38	6.53	0.	0.	0.	0.
time (sec)	N/A	0.552	29.47	0.26	0.	0.	0.	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	24604	1204	0	0	0	0
normalized size	1	1.	98.81	4.84	0.	0.	0.	0.
time (sec)	N/A	0.775	31.274	0.293	0.	0.	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	51315	1742	0	0	0	0
normalized size	1	1.	171.62	5.83	0.	0.	0.	0.
time (sec)	N/A	1.024	32.087	0.253	0.	0.	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	363	363	477	2778	0	0	0	0
normalized size	1	1.	1.31	7.65	0.	0.	0.	0.
time (sec)	N/A	1.328	13.527	0.41	0.	0.	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	419	2040	0	0	0	0
normalized size	1	1.	1.38	6.73	0.	0.	0.	0.
time (sec)	N/A	1.006	11.873	0.304	0.	0.	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	391	1921	0	0	0	0
normalized size	1	1.	1.64	8.04	0.	0.	0.	0.
time (sec)	N/A	0.756	11.624	0.312	0.	0.	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	36372	1651	0	0	0	0
normalized size	1	1.	138.82	6.3	0.	0.	0.	0.
time (sec)	N/A	0.853	32.584	0.224	0.	0.	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	263	263	44191	1947	0	0	0	0
normalized size	1	1.	168.03	7.4	0.	0.	0.	0.
time (sec)	N/A	0.849	31.878	0.24	0.	0.	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	314	314	52888	1972	0	0	0	0
normalized size	1	1.	168.43	6.28	0.	0.	0.	0.
time (sec)	N/A	1.152	31.962	0.287	0.	0.	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	369	369	61979	2285	0	0	0	0
normalized size	1	1.	167.96	6.19	0.	0.	0.	0.
time (sec)	N/A	1.421	32.279	0.312	0.	0.	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	340	1726	0	0	0	0
normalized size	1	1.	1.37	6.93	0.	0.	0.	0.
time (sec)	N/A	0.651	8.94	0.263	0.	0.	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	195	195	265	1014	0	0	0	0
normalized size	1	1.	1.36	5.2	0.	0.	0.	0.
time (sec)	N/A	0.439	7.221	0.287	0.	0.	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	216	732	0	0	0	0
normalized size	1	1.	1.52	5.15	0.	0.	0.	0.
time (sec)	N/A	0.307	3.768	0.258	0.	0.	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	102	163	0	0	0	0
normalized size	1	1.	1.52	2.43	0.	0.	0.	0.
time (sec)	N/A	0.149	0.746	0.24	0.	0.	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	14986	208	0	0	0	0
normalized size	1	1.	220.38	3.06	0.	0.	0.	0.
time (sec)	N/A	0.235	28.447	0.234	0.	0.	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	21698	986	0	0	0	0
normalized size	1	1.	88.2	4.01	0.	0.	0.	0.
time (sec)	N/A	0.696	30.146	0.271	0.	0.	0.	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	51323	1745	0	0	0	0
normalized size	1	1.	164.5	5.59	0.	0.	0.	0.
time (sec)	N/A	0.952	32.066	0.266	0.	0.	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	360	419	1853	0	0	0	0
normalized size	1	1.	1.16	5.15	0.	0.	0.	0.
time (sec)	N/A	1.05	12.471	0.275	0.	0.	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	289	289	382	1305	0	0	0	0
normalized size	1	1.	1.32	4.52	0.	0.	0.	0.
time (sec)	N/A	0.752	8.659	0.3	0.	0.	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	330	997	0	0	0	0
normalized size	1	1.	1.54	4.66	0.	0.	0.	0.
time (sec)	N/A	0.526	8.958	0.27	0.	0.	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	245	502	0	0	0	0
normalized size	1	1.	1.22	2.51	0.	0.	0.	0.
time (sec)	N/A	0.462	7.386	0.269	0.	0.	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	260	491	0	0	0	0
normalized size	1	1.	2.06	3.9	0.	0.	0.	0.
time (sec)	N/A	0.233	7.181	0.245	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	47811	1134	0	0	0	0
normalized size	1	1.	232.09	5.5	0.	0.	0.	0.
time (sec)	N/A	0.598	32.06	0.267	0.	0.	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	345	345	51610	1491	0	0	0	0
normalized size	1	1.	149.59	4.32	0.	0.	0.	0.
time (sec)	N/A	1.089	32.208	0.317	0.	0.	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	391	391	527	3604	0	0	0	0
normalized size	1	1.	1.35	9.22	0.	0.	0.	0.
time (sec)	N/A	1.095	14.452	0.313	0.	0.	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	507	3101	0	0	0	0
normalized size	1	1.	1.6	9.78	0.	0.	0.	0.
time (sec)	N/A	0.822	13.62	0.275	0.	0.	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	398	2062	0	0	0	0
normalized size	1	1.	1.32	6.83	0.	0.	0.	0.
time (sec)	N/A	0.74	8.603	0.272	0.	0.	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	447	1812	0	0	0	0
normalized size	1	1.	1.59	6.45	0.	0.	0.	0.
time (sec)	N/A	0.688	11.299	0.244	0.	0.	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	311	1333	0	0	0	0
normalized size	1	1.	1.12	4.81	0.	0.	0.	0.
time (sec)	N/A	0.745	9.086	0.303	0.	0.	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	92128	3844	0	0	0	0
normalized size	1	1.	248.99	10.39	0.	0.	0.	0.
time (sec)	N/A	1.205	32.97	0.289	0.	0.	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	222	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.456	0.689	3.103	0.	0.	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	161	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.396	1.933	0.	0.	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	106	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.119	0.576	0.	0.	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	5216	0	0	0	0	0
normalized size	1	1.	26.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.365	25.647	0.782	0.	0.	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	10296	0	0	0	0	0
normalized size	1	1.	33.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.514	32.621	0.276	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [661] had the largest ratio of [0.56]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	4	1.	19	0.21
2	A	5	4	1.	19	0.21
3	A	5	5	1.	19	0.263
4	A	4	4	1.	17	0.235
5	A	2	1	1.	10	0.1
6	A	3	3	1.	17	0.176
7	A	4	4	1.	19	0.21
8	A	5	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	6	4	1.	19	0.21
10	A	7	5	1.	21	0.238
11	A	6	5	1.	21	0.238
12	A	6	6	1.	21	0.286
13	A	5	5	1.	19	0.263
14	A	4	4	1.	12	0.333
15	A	4	4	1.	19	0.21
16	A	4	4	1.	21	0.19
17	A	6	5	1.	21	0.238
18	A	6	5	1.	21	0.238
19	A	8	6	1.	21	0.286
20	A	11	4	1.	21	0.19
21	A	11	5	1.	21	0.238
22	A	9	5	1.	19	0.263
23	A	5	5	1.	12	0.417
24	A	6	5	1.	19	0.263
25	A	6	5	1.	21	0.238
26	A	7	5	1.	21	0.238
27	A	10	5	1.	21	0.238
28	A	11	4	1.	21	0.19
29	A	13	4	1.	21	0.19
30	A	15	4	1.	21	0.19
31	A	13	5	1.	21	0.238
32	A	12	5	1.	19	0.263
33	A	6	6	1.	12	0.5
34	A	8	6	1.	19	0.316
35	A	8	6	1.	21	0.286
36	A	8	6	1.	21	0.286
37	A	10	5	1.	21	0.238
38	A	12	5	1.	21	0.238
39	A	15	4	1.	21	0.19
40	A	15	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	17	4	1.	21	0.19
42	A	6	5	1.	21	0.238
43	A	6	6	1.	21	0.286
44	A	4	4	1.	21	0.19
45	A	3	3	1.	21	0.143
46	A	1	1	1.	19	0.053
47	A	2	2	1.	12	0.167
48	A	4	4	1.	19	0.21
49	A	5	5	1.	21	0.238
50	A	6	5	1.	21	0.238
51	A	7	5	1.	21	0.238
52	A	7	7	1.	21	0.333
53	A	6	6	1.	21	0.286
54	A	4	4	1.	21	0.19
55	A	2	2	1.	21	0.095
56	A	2	2	1.	19	0.105
57	A	3	3	1.	12	0.25
58	A	5	5	1.	19	0.263
59	A	6	6	1.	21	0.286
60	A	7	6	1.	21	0.286
61	A	8	7	1.	21	0.333
62	A	7	7	1.	21	0.333
63	A	5	5	1.	21	0.238
64	A	3	3	1.	21	0.143
65	A	3	3	1.	21	0.143
66	A	3	2	1.	19	0.105
67	A	4	4	1.	12	0.333
68	A	6	5	1.	19	0.263
69	A	7	6	1.	21	0.286
70	A	9	7	1.	21	0.333
71	A	8	7	1.	21	0.333
72	A	6	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	4	4	1.	21	0.19
74	A	4	4	1.	21	0.19
75	A	4	3	1.	21	0.143
76	A	4	2	1.	19	0.105
77	A	5	4	1.	12	0.333
78	A	7	5	1.	19	0.263
79	A	8	6	1.	21	0.286
80	A	9	7	1.	21	0.333
81	A	7	6	1.	21	0.286
82	A	5	4	1.	21	0.19
83	A	5	5	1.	21	0.238
84	A	5	4	1.	21	0.19
85	A	5	3	1.	21	0.143
86	A	5	2	1.	19	0.105
87	A	6	4	1.	12	0.333
88	A	8	5	1.	19	0.263
89	A	9	6	1.	21	0.286
90	A	4	4	1.	23	0.174
91	A	3	3	1.	23	0.13
92	A	2	2	1.	23	0.087
93	A	1	1	1.	21	0.048
94	A	2	2	1.	14	0.143
95	A	3	3	1.	21	0.143
96	A	4	3	1.	23	0.13
97	A	5	3	1.	23	0.13
98	A	6	3	1.	23	0.13
99	A	6	6	1.	23	0.261
100	A	4	4	1.	23	0.174
101	A	3	3	1.	23	0.13
102	A	2	2	1.	21	0.095
103	A	4	4	1.	14	0.286
104	A	5	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
105	A	5	5	1.	23	0.217
106	A	6	5	1.	23	0.217
107	A	6	6	1.	23	0.261
108	A	5	4	1.	23	0.174
109	A	4	3	1.	23	0.13
110	A	3	2	1.	21	0.095
111	A	5	5	1.	14	0.357
112	A	4	4	1.	21	0.19
113	A	4	4	1.	23	0.174
114	A	5	5	1.	23	0.217
115	A	6	5	1.	23	0.217
116	A	1	1	1.	22	0.045
117	A	2	2	1.	15	0.133
118	A	3	3	1.	22	0.136
119	A	5	5	1.	23	0.217
120	A	4	4	1.	23	0.174
121	A	3	3	1.	23	0.13
122	A	2	2	1.	21	0.095
123	A	5	4	1.	14	0.286
124	A	6	5	1.	21	0.238
125	A	7	6	1.	23	0.261
126	A	6	6	1.	23	0.261
127	A	5	5	1.	23	0.217
128	A	4	4	1.	23	0.174
129	A	3	3	1.	23	0.13
130	A	3	3	1.	21	0.143
131	A	6	5	1.	14	0.357
132	A	7	6	1.	21	0.286
133	A	8	6	1.	23	0.261
134	A	6	6	1.	23	0.261
135	A	5	5	1.	23	0.217
136	A	4	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	4	4	1.	23	0.174
138	A	4	3	1.	21	0.143
139	A	7	6	1.	14	0.429
140	A	8	7	1.	21	0.333
141	A	2	2	1.	22	0.091
142	A	5	4	1.	15	0.267
143	A	7	7	1.	23	0.304
144	A	6	6	1.	23	0.261
145	A	5	5	1.	21	0.238
146	A	3	3	1.	14	0.214
147	A	3	3	1.	21	0.143
148	A	8	7	1.	23	0.304
149	A	7	6	1.	23	0.261
150	A	6	5	1.	21	0.238
151	A	3	3	1.	14	0.214
152	A	3	3	1.	21	0.143
153	A	7	7	1.	23	0.304
154	A	6	6	1.	23	0.261
155	A	5	5	1.	23	0.217
156	A	4	4	1.	21	0.19
157	A	3	3	1.	14	0.214
158	A	3	3	1.	21	0.143
159	A	9	9	1.	23	0.391
160	A	8	8	1.	23	0.348
161	A	8	8	1.	23	0.348
162	A	8	7	1.	21	0.333
163	A	3	3	1.	14	0.214
164	A	3	3	1.	21	0.143
165	A	8	5	1.	21	0.238
166	A	7	5	1.	21	0.238
167	A	6	5	1.	21	0.238
168	A	5	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
169	A	6	5	1.	21	0.238
170	A	7	5	1.	21	0.238
171	A	8	5	1.	21	0.238
172	A	9	6	1.	23	0.261
173	A	8	6	1.	23	0.261
174	A	7	6	1.	23	0.261
175	A	4	4	1.	23	0.174
176	A	6	5	1.	23	0.217
177	A	7	6	1.	23	0.261
178	A	8	6	1.	23	0.261
179	A	16	5	1.	23	0.217
180	A	14	5	1.	23	0.217
181	A	12	5	1.	23	0.217
182	A	12	6	1.	23	0.261
183	A	12	5	1.	23	0.217
184	A	14	5	1.	23	0.217
185	A	16	5	1.	23	0.217
186	A	21	5	1.	23	0.217
187	A	18	5	1.	23	0.217
188	A	16	5	1.	23	0.217
189	A	15	6	1.	23	0.261
190	A	15	6	1.	23	0.261
191	A	16	5	1.	23	0.217
192	A	18	5	1.	23	0.217
193	A	21	5	1.	23	0.217
194	A	8	6	1.	23	0.261
195	A	7	6	1.	23	0.261
196	A	6	5	1.	23	0.217
197	A	6	5	1.	23	0.217
198	A	6	5	1.	23	0.217
199	A	7	6	1.	23	0.261
200	A	8	6	1.	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	9	7	1.	23	0.304
202	A	8	7	1.	23	0.304
203	A	7	6	1.	23	0.261
204	A	4	4	1.	23	0.174
205	A	7	6	1.	23	0.261
206	A	7	6	1.	23	0.261
207	A	8	7	1.	23	0.304
208	A	9	7	1.	23	0.304
209	A	10	7	1.	23	0.304
210	A	9	7	1.	23	0.304
211	A	8	6	1.	23	0.261
212	A	8	7	1.	23	0.304
213	A	8	7	1.	23	0.304
214	A	8	7	1.	23	0.304
215	A	8	6	1.	23	0.261
216	A	9	7	1.	23	0.304
217	A	10	7	1.	23	0.304
218	A	4	3	1.	25	0.12
219	A	3	3	1.	25	0.12
220	A	2	2	1.	25	0.08
221	A	1	1	1.	25	0.04
222	A	2	2	1.	25	0.08
223	A	3	2	1.	25	0.08
224	A	4	2	1.	25	0.08
225	A	6	5	1.	25	0.2
226	A	5	5	1.	25	0.2
227	A	4	4	1.	25	0.16
228	A	4	4	1.	25	0.16
229	A	2	2	1.	25	0.08
230	A	3	3	1.	25	0.12
231	A	5	4	1.	25	0.16
232	A	6	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
233	A	6	5	1.	25	0.2
234	A	5	5	1.	25	0.2
235	A	4	4	1.	25	0.16
236	A	4	4	1.	25	0.16
237	A	4	4	1.	25	0.16
238	A	3	2	1.	25	0.08
239	A	4	3	1.	25	0.12
240	A	5	4	1.	25	0.16
241	A	6	4	1.	25	0.16
242	A	2	2	1.	25	0.08
243	A	2	2	1.	25	0.08
244	A	2	2	1.	28	0.071
245	A	6	6	1.	25	0.24
246	A	5	5	1.	25	0.2
247	A	2	2	1.	25	0.08
248	A	3	3	1.	25	0.12
249	A	4	4	1.	25	0.16
250	A	5	5	1.	25	0.2
251	A	7	7	1.	25	0.28
252	A	6	6	1.	25	0.24
253	A	3	3	1.	25	0.12
254	A	3	3	1.	25	0.12
255	A	4	4	1.	25	0.16
256	A	5	5	1.	25	0.2
257	A	6	5	1.	25	0.2
258	A	8	8	1.	25	0.32
259	A	7	7	1.	25	0.28
260	A	4	3	1.	25	0.12
261	A	4	4	1.	25	0.16
262	A	4	4	1.	25	0.16
263	A	5	5	1.	25	0.2
264	A	6	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
265	A	7	6	1.	23	0.261
266	A	6	5	1.	23	0.217
267	A	5	4	1.	23	0.174
268	A	2	2	1.	23	0.087
269	A	3	3	1.	23	0.13
270	A	4	4	1.	23	0.174
271	A	5	5	1.	23	0.217
272	A	4	4	1.	27	0.148
273	A	3	3	1.	27	0.111
274	A	4	4	1.	27	0.148
275	A	7	6	1.	27	0.222
276	A	6	6	1.	27	0.222
277	A	5	5	1.	27	0.185
278	A	6	6	1.	27	0.222
279	A	7	6	1.	27	0.222
280	A	4	4	1.	27	0.148
281	A	4	4	1.	27	0.148
282	A	4	4	1.	27	0.148
283	A	4	4	1.	27	0.148
284	A	3	3	1.	25	0.12
285	A	3	3	1.	25	0.12
286	C	3	3	0.24	25	0.12
287	A	3	3	1.	25	0.12
288	A	8	6	1.	21	0.286
289	A	7	5	1.	21	0.238
290	A	6	4	1.	21	0.19
291	A	5	3	1.	19	0.158
292	A	6	4	1.	21	0.19
293	A	7	5	1.	21	0.238
294	A	4	4	1.	21	0.19
295	A	4	4	1.	21	0.19
296	A	2	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
297	A	3	3	1.	21	0.143
298	A	3	3	1.	21	0.143
299	A	4	4	1.	23	0.174
300	A	2	2	1.	23	0.087
301	A	2	2	1.	23	0.087
302	A	2	2	1.	23	0.087
303	A	4	4	1.	23	0.174
304	A	2	2	1.	23	0.087
305	A	2	2	1.	23	0.087
306	A	2	2	1.	23	0.087
307	A	4	4	1.	23	0.174
308	A	4	4	1.	23	0.174
309	A	2	2	1.	23	0.087
310	A	4	4	1.	23	0.174
311	A	4	4	1.	23	0.174
312	A	5	5	1.	25	0.2
313	A	3	3	1.	25	0.12
314	A	3	3	1.	25	0.12
315	A	3	3	1.	25	0.12
316	A	5	5	1.	25	0.2
317	A	3	3	1.	25	0.12
318	A	3	3	1.	25	0.12
319	A	3	3	1.	25	0.12
320	A	4	4	1.	26	0.154
321	A	4	4	1.	26	0.154
322	A	2	2	1.	26	0.077
323	A	4	4	1.	26	0.154
324	A	4	4	1.	26	0.154
325	A	5	5	1.	24	0.208
326	A	3	3	1.	24	0.125
327	A	5	5	1.	26	0.192
328	A	3	3	1.	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	2	2	1.	19	0.105
330	A	2	2	1.	21	0.095
331	A	3	3	1.	21	0.143
332	A	3	3	1.	22	0.136
333	A	2	2	1.	21	0.095
334	A	2	2	1.	23	0.087
335	A	3	3	1.	23	0.13
336	A	3	3	1.	24	0.125
337	A	2	2	1.	21	0.095
338	A	2	2	1.	23	0.087
339	A	3	3	1.	23	0.13
340	A	3	3	1.	24	0.125
341	A	6	6	1.	21	0.286
342	A	5	5	1.	21	0.238
343	A	4	4	1.	21	0.19
344	A	3	3	1.	19	0.158
345	A	3	3	1.	12	0.25
346	A	3	3	1.	19	0.158
347	A	3	3	1.	25	0.12
348	A	3	3	1.	25	0.12
349	A	3	3	1.	25	0.12
350	A	3	3	1.	25	0.12
351	A	7	5	1.	21	0.238
352	A	6	5	1.	21	0.238
353	A	5	5	1.	21	0.238
354	A	4	4	1.	21	0.19
355	A	5	5	1.	21	0.238
356	A	6	5	1.	21	0.238
357	A	7	5	1.	21	0.238
358	A	8	5	1.	21	0.238
359	A	10	7	1.	23	0.304
360	A	9	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
361	A	8	7	1.	23	0.304
362	A	7	6	1.	23	0.261
363	A	5	5	1.	23	0.217
364	A	8	7	1.	23	0.304
365	A	9	7	1.	23	0.304
366	A	10	7	1.	23	0.304
367	A	17	6	1.	23	0.261
368	A	15	6	1.	23	0.261
369	A	13	6	1.	23	0.261
370	A	13	7	1.	23	0.304
371	A	13	6	1.	23	0.261
372	A	15	6	1.	23	0.261
373	A	17	6	1.	23	0.261
374	A	9	7	1.	23	0.304
375	A	8	7	1.	23	0.304
376	A	7	6	1.	23	0.261
377	A	7	6	1.	23	0.261
378	A	7	6	1.	23	0.261
379	A	8	7	1.	23	0.304
380	A	9	7	1.	23	0.304
381	A	10	8	1.	23	0.348
382	A	9	8	1.	23	0.348
383	A	8	7	1.	23	0.304
384	A	8	7	1.	23	0.304
385	A	5	5	1.	23	0.217
386	A	8	7	1.	23	0.304
387	A	9	8	1.	23	0.348
388	A	10	8	1.	23	0.348
389	A	11	8	1.	23	0.348
390	A	10	8	1.	23	0.348
391	A	9	7	1.	23	0.304
392	A	9	8	1.	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
393	A	9	8	1.	23	0.348
394	A	9	8	1.	23	0.348
395	A	9	7	1.	23	0.304
396	A	10	8	1.	23	0.348
397	A	11	8	1.	23	0.348
398	A	5	3	1.	25	0.12
399	A	4	3	1.	25	0.12
400	A	3	3	1.	25	0.12
401	A	2	2	1.	25	0.08
402	A	3	3	1.	25	0.12
403	A	4	4	1.	25	0.16
404	A	5	4	1.	25	0.16
405	A	6	5	1.	25	0.2
406	A	4	4	1.	25	0.16
407	A	3	3	1.	25	0.12
408	A	5	5	1.	25	0.2
409	A	5	5	1.	25	0.2
410	A	6	6	1.	25	0.24
411	A	7	6	1.	25	0.24
412	A	6	5	1.	25	0.2
413	A	5	4	1.	25	0.16
414	A	4	3	1.	25	0.12
415	A	5	5	1.	25	0.2
416	A	5	5	1.	25	0.2
417	A	5	5	1.	25	0.2
418	A	6	6	1.	25	0.24
419	A	7	6	1.	25	0.24
420	A	6	6	1.	25	0.24
421	A	5	5	1.	25	0.2
422	A	4	4	1.	25	0.16
423	A	3	3	1.36	25	0.12
424	A	6	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
425	A	7	7	1.	25	0.28
426	A	8	8	1.	25	0.32
427	A	7	6	1.	25	0.24
428	A	6	6	1.	25	0.24
429	A	5	5	1.	25	0.2
430	A	4	4	1.	25	0.16
431	A	4	4	1.	25	0.16
432	A	7	7	1.	25	0.28
433	A	8	8	1.	25	0.32
434	A	7	7	1.	25	0.28
435	A	6	6	1.	25	0.24
436	A	5	5	1.	25	0.2
437	A	5	5	1.	25	0.2
438	A	5	4	1.	25	0.16
439	A	8	8	1.	25	0.32
440	A	9	9	1.	25	0.36
441	A	8	6	1.	23	0.261
442	A	7	5	1.	23	0.217
443	A	5	4	1.	21	0.19
444	A	7	5	1.	23	0.217
445	A	8	6	1.	23	0.261
446	A	6	4	1.	19	0.21
447	A	5	4	1.	19	0.21
448	A	5	5	1.	19	0.263
449	A	4	4	1.	17	0.235
450	A	2	1	1.	10	0.1
451	A	3	3	1.	17	0.176
452	A	4	4	1.	19	0.21
453	A	5	4	1.	19	0.21
454	A	6	4	1.	19	0.21
455	A	6	4	1.	19	0.21
456	A	7	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
457	A	6	5	1.	21	0.238
458	A	6	6	1.	21	0.286
459	A	5	5	1.	19	0.263
460	A	4	4	1.	12	0.333
461	A	4	4	1.	19	0.21
462	A	4	4	1.	21	0.19
463	A	6	5	1.	21	0.238
464	A	6	5	1.	21	0.238
465	A	8	6	1.	21	0.286
466	A	8	7	1.	21	0.333
467	A	7	7	1.	21	0.333
468	A	6	6	1.	19	0.316
469	A	5	4	1.	12	0.333
470	A	5	5	1.	19	0.263
471	A	5	5	1.	21	0.238
472	A	5	5	1.	21	0.238
473	A	7	6	1.	21	0.286
474	A	7	6	1.	21	0.286
475	A	9	7	1.	21	0.333
476	A	9	7	1.	21	0.333
477	A	8	7	1.	21	0.333
478	A	7	7	1.	19	0.368
479	A	6	5	1.	12	0.417
480	A	6	6	1.	19	0.316
481	A	6	6	1.	21	0.286
482	A	6	6	1.	21	0.286
483	A	6	6	1.	21	0.286
484	A	8	7	1.	21	0.333
485	A	8	7	1.	21	0.333
486	A	7	6	1.	12	0.5
487	A	8	8	1.	21	0.381
488	A	7	7	1.	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
489	A	6	6	1.	21	0.286
490	A	5	5	1.	21	0.238
491	A	3	3	1.	19	0.158
492	A	3	3	1.	12	0.25
493	A	5	5	1.	19	0.263
494	A	6	6	1.	21	0.286
495	A	7	6	1.	21	0.286
496	A	8	6	1.	21	0.286
497	A	8	8	1.	21	0.381
498	A	7	7	1.	21	0.333
499	A	6	6	1.	21	0.286
500	A	5	5	1.	21	0.238
501	A	5	5	1.	19	0.263
502	A	5	5	1.	12	0.417
503	A	6	6	1.	19	0.316
504	A	7	6	1.	21	0.286
505	A	8	6	1.	21	0.286
506	A	8	8	1.	21	0.381
507	A	7	7	1.	21	0.333
508	A	6	6	1.	21	0.286
509	A	6	6	1.	21	0.286
510	A	6	6	1.	19	0.316
511	A	6	6	1.	12	0.5
512	A	7	7	1.	19	0.368
513	A	8	7	1.	21	0.333
514	A	9	9	1.	21	0.429
515	A	8	8	1.	21	0.381
516	A	7	7	1.	21	0.333
517	A	7	6	1.	21	0.286
518	A	7	6	1.	21	0.286
519	A	7	6	1.	19	0.316
520	A	7	6	1.	12	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
521	A	8	7	1.	19	0.368
522	A	9	7	1.	21	0.333
523	A	2	2	1.	12	0.167
524	A	4	4	1.	12	0.333
525	A	5	5	1.	12	0.417
526	A	6	5	1.	12	0.417
527	A	3	3	1.	12	0.25
528	A	5	5	1.	12	0.417
529	A	6	6	1.	12	0.5
530	A	7	6	1.	12	0.5
531	A	5	5	1.	23	0.217
532	A	4	4	1.	23	0.174
533	A	3	3	1.	21	0.143
534	A	1	1	1.	14	0.071
535	A	6	6	1.	21	0.286
536	A	7	7	1.	23	0.304
537	A	7	6	1.	23	0.261
538	A	6	5	1.	23	0.217
539	A	5	5	1.	23	0.217
540	A	4	4	1.	21	0.19
541	A	5	5	1.	14	0.357
542	A	6	6	1.	21	0.286
543	A	7	7	1.	23	0.304
544	A	8	6	1.	23	0.261
545	A	7	5	1.	23	0.217
546	A	6	5	1.	23	0.217
547	A	5	5	1.	21	0.238
548	A	6	6	1.	14	0.429
549	A	6	6	1.	21	0.286
550	A	7	7	1.	23	0.304
551	A	8	7	1.	23	0.304
552	A	9	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
553	A	7	7	1.	14	0.5
554	A	6	6	1.	23	0.261
555	A	5	5	1.	23	0.217
556	A	4	4	1.	23	0.174
557	A	3	3	1.	23	0.13
558	A	1	1	1.	21	0.048
559	A	1	1	1.	14	0.071
560	A	6	6	1.	21	0.286
561	A	7	7	1.	23	0.304
562	A	6	6	1.	23	0.261
563	A	5	5	1.	23	0.217
564	A	4	4	1.	23	0.174
565	A	4	4	1.	23	0.174
566	A	5	5	1.	21	0.238
567	A	6	6	1.	14	0.429
568	A	7	7	1.	21	0.333
569	A	8	8	1.	23	0.348
570	A	6	6	1.	23	0.261
571	A	5	5	1.	23	0.217
572	A	5	5	1.	23	0.217
573	A	5	5	1.	23	0.217
574	A	5	5	1.	21	0.238
575	A	7	7	1.	14	0.5
576	A	8	8	1.	21	0.381
577	A	9	8	1.	23	0.348
578	A	8	7	1.	14	0.5
579	A	8	5	1.	21	0.238
580	A	7	5	1.	21	0.238
581	A	6	5	1.	21	0.238
582	A	5	4	1.	21	0.19
583	A	6	5	1.	21	0.238
584	A	7	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
585	A	8	5	1.	21	0.238
586	A	9	6	1.	23	0.261
587	A	8	6	1.	23	0.261
588	A	7	6	1.	23	0.261
589	A	6	5	1.	23	0.217
590	A	6	5	1.	23	0.217
591	A	7	6	1.	23	0.261
592	A	8	6	1.	23	0.261
593	A	9	7	1.	23	0.304
594	A	8	7	1.	23	0.304
595	A	7	6	1.	23	0.261
596	A	7	6	1.	23	0.261
597	A	7	6	1.	23	0.261
598	A	8	7	1.	23	0.304
599	A	9	7	1.	23	0.304
600	A	10	8	1.	23	0.348
601	A	9	8	1.	23	0.348
602	A	8	7	1.	23	0.304
603	A	8	7	1.	23	0.304
604	A	8	7	1.	23	0.304
605	A	8	7	1.	23	0.304
606	A	9	8	1.	23	0.348
607	A	10	8	1.	23	0.348
608	A	10	9	1.	23	0.391
609	A	6	6	1.	23	0.261
610	A	2	2	1.	23	0.087
611	A	4	4	1.	23	0.174
612	A	8	7	1.	23	0.304
613	A	9	8	1.	23	0.348
614	A	11	9	1.	23	0.391
615	A	10	9	1.	23	0.391
616	A	9	8	1.	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
617	A	9	8	1.	23	0.348
618	A	9	8	1.	23	0.348
619	A	9	8	1.	23	0.348
620	A	10	9	1.	23	0.391
621	A	11	10	1.	23	0.435
622	A	10	9	1.	23	0.391
623	A	10	9	1.	23	0.391
624	A	10	9	1.	23	0.391
625	A	10	9	1.	23	0.391
626	A	10	9	1.	23	0.391
627	A	11	10	1.	23	0.435
628	A	12	12	1.	25	0.48
629	A	7	7	1.	25	0.28
630	A	3	3	1.	25	0.12
631	A	8	8	1.	25	0.32
632	A	9	9	1.	25	0.36
633	A	10	9	1.	25	0.36
634	A	13	13	1.	25	0.52
635	A	12	12	1.	25	0.48
636	A	11	11	1.	25	0.44
637	A	8	8	1.	25	0.32
638	A	9	9	1.	25	0.36
639	A	10	9	1.	25	0.36
640	A	14	13	1.	25	0.52
641	A	13	13	1.	25	0.52
642	A	12	12	1.	25	0.48
643	A	12	12	1.	25	0.48
644	A	9	9	1.	25	0.36
645	A	10	9	1.	25	0.36
646	A	11	9	1.	25	0.36
647	A	13	13	1.	25	0.52
648	A	12	12	1.	25	0.48

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
649	A	3	3	1.	25	0.12
650	A	3	3	1.	25	0.12
651	A	7	7	1.	25	0.28
652	A	8	8	1.	25	0.32
653	A	9	9	1.	25	0.36
654	A	13	13	1.	25	0.52
655	A	9	9	1.	25	0.36
656	A	5	5	1.	25	0.2
657	A	8	8	1.	25	0.32
658	A	8	8	1.	25	0.32
659	A	9	9	1.	25	0.36
660	A	10	9	1.	25	0.36
661	A	14	14	1.	25	0.56
662	A	13	13	1.	25	0.52
663	A	9	9	1.	25	0.36
664	A	9	9	1.	25	0.36
665	A	9	9	1.	25	0.36
666	A	9	9	1.	25	0.36
667	A	10	10	1.	25	0.4
668	A	11	10	1.	25	0.4
669	A	5	5	1.	25	0.2
670	A	5	5	1.	25	0.2
671	A	7	7	1.	25	0.28
672	A	7	7	1.	25	0.28
673	A	5	5	1.	25	0.2
674	A	5	5	1.	25	0.2
675	A	5	5	1.	25	0.2
676	A	5	5	1.	25	0.2
677	A	2	2	1.	25	0.08
678	A	2	2	1.	25	0.08
679	A	3	3	1.	25	0.12
680	A	3	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
681	A	2	2	1.	25	0.08
682	A	2	2	1.	25	0.08
683	A	2	2	1.	25	0.08
684	A	2	2	1.	25	0.08
685	A	3	3	1.	21	0.143
686	A	0	0	0.	0	0.
687	A	10	7	1.	23	0.304
688	A	9	6	1.	23	0.261
689	A	8	5	1.	23	0.217
690	A	3	3	1.	21	0.143
691	A	0	0	0.	0	0.
692	A	3	3	1.	21	0.143
693	A	0	0	0.	0	0.
694	A	11	7	1.	23	0.304
695	A	10	6	1.	23	0.261
696	A	9	6	1.	23	0.261
697	A	3	3	1.	21	0.143
698	A	0	0	0.	0	0.
699	A	9	6	1.	23	0.261
700	A	8	5	1.	23	0.217
701	A	7	4	1.	23	0.174
702	A	3	3	1.	21	0.143
703	A	0	0	0.	0	0.
704	A	3	3	1.	21	0.143
705	A	0	0	0.	0	0.
706	A	3	3	1.	21	0.143
707	A	0	0	0.	0	0.
708	A	9	6	1.	23	0.261
709	A	8	5	1.	23	0.217
710	A	8	5	1.	23	0.217
711	A	3	3	1.	21	0.143
712	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
713	A	6	4	1.	23	0.174
714	A	6	4	1.	23	0.174
715	A	6	4	1.	23	0.174
716	A	6	4	1.	23	0.174
717	A	0	0	0.	0	0.
718	A	0	0	0.	0	0.
719	A	0	0	0.	0	0.
720	A	0	0	0.	0	0.
721	A	0	0	0.	0	0.
722	A	0	0	0.	0	0.
723	A	0	0	0.	0	0.
724	A	0	0	0.	0	0.
725	A	0	0	0.	0	0.
726	A	0	0	0.	0	0.
727	A	0	0	0.	0	0.
728	A	0	0	0.	0	0.
729	A	0	0	0.	0	0.
730	A	0	0	0.	0	0.
731	A	0	0	0.	0	0.
732	A	0	0	0.	0	0.
733	A	0	0	0.	0	0.
734	A	0	0	0.	0	0.
735	A	0	0	0.	0	0.
736	A	0	0	0.	0	0.
737	A	0	0	0.	0	0.
738	A	0	0	0.	0	0.
739	A	0	0	0.	0	0.
740	A	0	0	0.	0	0.
741	A	0	0	0.	0	0.
742	A	0	0	0.	0	0.
743	A	0	0	0.	0	0.
744	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
745	A	0	0	0.	0	0.
746	A	0	0	0.	0	0.
747	A	0	0	0.	0	0.
748	A	0	0	0.	0	0.
749	A	0	0	0.	0	0.
750	A	0	0	0.	0	0.
751	A	0	0	0.	0	0.
752	A	0	0	0.	0	0.
753	A	0	0	0.	0	0.
754	A	0	0	0.	0	0.
755	A	0	0	0.	0	0.
756	A	0	0	0.	0	0.
757	A	0	0	0.	0	0.
758	A	0	0	0.	0	0.
759	A	0	0	0.	0	0.
760	A	0	0	0.	0	0.
761	A	0	0	0.	0	0.
762	A	0	0	0.	0	0.
763	A	0	0	0.	0	0.
764	A	0	0	0.	0	0.
765	A	0	0	0.	0	0.
766	A	0	0	0.	0	0.
767	A	0	0	0.	0	0.
768	A	0	0	0.	0	0.
769	A	0	0	0.	0	0.
770	A	0	0	0.	0	0.
771	A	0	0	0.	0	0.
772	A	0	0	0.	0	0.
773	A	0	0	0.	0	0.
774	A	0	0	0.	0	0.
775	A	0	0	0.	0	0.
776	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
777	A	7	5	1.	23	0.217
778	A	6	4	1.	23	0.174
779	A	5	3	1.	21	0.143
780	A	6	4	1.	23	0.174
781	A	9	4	1.	23	0.174
782	A	0	0	0.	0	0.
783	A	0	0	0.	0	0.
784	A	0	0	0.	0	0.
785	A	0	0	0.	0	0.
786	A	0	0	0.	0	0.
787	A	0	0	0.	0	0.
788	A	8	5	1.	21	0.238
789	A	7	4	1.	21	0.19
790	A	3	3	1.	19	0.158
791	A	0	0	0.	0	0.
792	A	0	0	0.	0	0.
793	A	0	0	0.	0	0.
794	A	8	5	1.	21	0.238
795	A	7	5	1.	21	0.238
796	A	6	5	1.	21	0.238
797	A	5	5	1.	21	0.238
798	A	4	4	1.	21	0.19
799	A	5	5	1.	21	0.238
800	A	6	5	1.	21	0.238
801	A	7	5	1.	21	0.238
802	A	10	7	1.	23	0.304
803	A	9	7	1.	23	0.304
804	A	8	7	1.	23	0.304
805	A	7	6	1.	23	0.261
806	A	7	6	1.	23	0.261
807	A	8	7	1.	23	0.304
808	A	9	7	1.	23	0.304

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
809	A	10	7	1.	23	0.304
810	A	10	8	1.	23	0.348
811	A	9	8	1.	23	0.348
812	A	8	7	1.	23	0.304
813	A	8	7	1.	23	0.304
814	A	8	7	1.	23	0.304
815	A	9	8	1.	23	0.348
816	A	10	8	1.	23	0.348
817	A	11	10	1.	23	0.435
818	A	10	9	1.	23	0.391
819	A	9	8	1.	23	0.348
820	A	5	5	1.	23	0.217
821	A	3	3	1.	23	0.13
822	A	7	7	1.	23	0.304
823	A	11	10	1.	23	0.435
824	A	11	10	1.	23	0.435
825	A	10	9	1.	23	0.391
826	A	10	9	1.	23	0.391
827	A	10	9	1.	23	0.391
828	A	10	9	1.	23	0.391
829	A	11	10	1.	23	0.435
830	A	12	11	1.	23	0.478
831	A	11	10	1.	23	0.435
832	A	11	10	1.	23	0.435
833	A	11	10	1.	23	0.435
834	A	11	10	1.	23	0.435
835	A	11	10	1.	23	0.435
836	A	12	11	1.	23	0.478
837	A	10	10	1.	25	0.4
838	A	9	9	1.	25	0.36
839	A	4	4	1.	25	0.16
840	A	8	8	1.	25	0.32

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
841	A	13	13	1.	25	0.52
842	A	11	10	1.	25	0.4
843	A	10	10	1.	25	0.4
844	A	9	9	1.	25	0.36
845	A	12	12	1.	25	0.48
846	A	13	13	1.	25	0.52
847	A	14	14	1.	25	0.56
848	A	12	10	1.	25	0.4
849	A	11	10	1.	25	0.4
850	A	10	10	1.	25	0.4
851	A	13	13	1.	25	0.52
852	A	13	13	1.	25	0.52
853	A	14	14	1.	25	0.56
854	A	15	14	1.	25	0.56
855	A	10	10	1.	25	0.4
856	A	9	9	1.	25	0.36
857	A	8	8	1.	25	0.32
858	A	4	4	1.	25	0.16
859	A	4	4	1.	25	0.16
860	A	13	13	1.	25	0.52
861	A	14	14	1.	25	0.56
862	A	11	10	1.	25	0.4
863	A	10	10	1.	25	0.4
864	A	9	9	1.	25	0.36
865	A	9	9	1.	25	0.36
866	A	6	6	1.	25	0.24
867	A	10	10	1.	25	0.4
868	A	14	14	1.	25	0.56
869	A	11	11	1.	25	0.44
870	A	10	10	1.	25	0.4
871	A	10	10	1.	25	0.4
872	A	10	10	1.	25	0.4

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
873	A	10	10	1.	25	0.4
874	A	14	14	1.	25	0.56
875	A	8	6	1.	23	0.261
876	A	7	5	1.	23	0.217
877	A	5	4	1.	21	0.19
878	A	7	5	1.	23	0.217
879	A	10	5	1.	23	0.217

Chapter 3

Listing of integrals

3.1 $\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rubi [A] time = 0.058626, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 3767, 3768, 3770}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```


```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```


Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sec(c+dx))\,dx = a\int \sec^4(c+dx)\,dx + a\int \sec^5(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,\right.}{d}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a\tan}{}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c}{}$$

Mathematica [A] time = 0.163375, size = 76, normalized size = 0.89

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.031, size = 92, normalized size = 1.1

$$\frac{2\,a\tan\left(dx+c\right)}{3\,d}+\frac{a\tan\left(dx+c\right)\left(\sec\left(dx+c\right)\right)^{2}}{3\,d}+\frac{a\left(\sec\left(dx+c\right)\right)^{3}\tan\left(dx+c\right)}{4\,d}+\frac{3\,a\sec\left(dx+c\right)\tan\left(dx+c\right)}{8\,d}+\frac{3\,a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/4*a*sec(d*x+c)^3*tan(d
*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.17958, size = 128, normalized size = 1.51

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,a\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.67521, size = 266, normalized size = 3.13

$$\frac{9\,a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,a\cos\left(dx+c\right)^{3}+9\,a\cos\left(dx+c\right.\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

Giac [A] time = 1.29848, size = 149, normalized size = 1.75

$$\frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-39\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5 +
31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^4)/d
```

3.2 $\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0457791, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 3768, 3770, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))\,dx = a\int \sec^3(c+dx)\,dx + a\int \sec^4(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\text{t}\right.}{d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{a\tan^3(c+}{3d}$$

Mathematica [A] time = 0.133453, size = 60, normalized size = 0.95

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.029, size = 72, normalized size = 1.1

$$\frac{a\sec(dx+c)\tan(dx+c)}{2d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{2a\tan(dx+c)}{3d} + \frac{a\tan(dx+c)(\sec(dx+c))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*tan(d
*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.04754, size = 95, normalized size = 1.51

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.63606, size = 236, normalized size = 3.75

$$\frac{3\,a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sec^{3}\left(c+dx\right)dx+\int\sec^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

Giac [A] time = 1.29445, size = 130, normalized size = 2.06

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 + 9*
a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.3 $\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rubi [A] time = 0.0421372, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3787, 3767, 8, 3768, 3770}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))\,dx = a\int \sec^2(c+dx)\,dx + a\int \sec^3(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{a\,\text{Subst}(\int 1\,dx, x, -\tan(c+a$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.018978, size = 47, normalized size = 1.

$$\frac{a\tan(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Maple [A] time = 0.025, size = 51, normalized size = 1.1

$$\frac{a\tan(dx+c)}{d} + \frac{a\sec(dx+c)\tan(dx+c)}{2d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 1.07314, size = 78, normalized size = 1.66

$$-\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,a\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

Fricas [A] time = 1.69099, size = 198, normalized size = 4.21

$$\frac{a\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sec^2\left(c+dx\right)dx+\int\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sec(c + d*x)**2, x) + Integral(sec(c + d*x)**3, x))
```

Giac [A] time = 1.34646, size = 108, normalized size = 2.3

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^2)/d
```

3.4 $\int \sec(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0233012, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3787, 3770, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec(c+dx)\,dx + a\int \sec^2(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0090899, size = 24, normalized size = 1.

$$\frac{a\tan(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.023, size = 32, normalized size = 1.3

$$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*a*ln(sec(d*x+c)+tan(d*x+c))+a*tan(d*x+c)/d
```

Maxima [A] time = 1.10384, size = 39, normalized size = 1.62

$$\frac{a\log(\sec(dx+c)+\tan(dx+c))+a\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + a*tan(d*x + c))/d
```

Fricas [B] time = 1.69357, size = 162, normalized size = 6.75

$$\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,a\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [A] time = 5.67544, size = 37, normalized size = 1.54

$$\begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+a\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x\,(a\sec(c)+a)\sec(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + a*tan(c + d*x))/d, Ne(d, 0
)), (x*(a*sec(c) + a)*sec(c), True))
```

Giac [B] time = 1.38367, size = 85, normalized size = 3.54

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.5 $\int (a+a\sec(c+dx))\,dx$

Optimal. Leaf size=16

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0072979, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3770}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\,dx &= ax+a\int \sec(c+dx)\,dx\\ &= ax+\frac{a\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0015236, size = 16, normalized size = 1.

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] time = 0.005, size = 24, normalized size = 1.5

$$ax + \frac{a \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+a*sec(d*x+c),x)
```

```
[Out] a*x+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.05706, size = 31, normalized size = 1.94

$$ax + \frac{a \log(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + a*log(sec(d*x + c) + tan(d*x + c))/d
```

Fricas [B] time = 1.76666, size = 95, normalized size = 5.94

$$\frac{2\,adx + a \log(\sin(dx + c) + 1) - a \log(-\sin(dx + c) + 1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

Sympy [A] time = 1.73496, size = 41, normalized size = 2.56

$$ax + a\left(\begin{cases} \frac{\log{\left(\tan{\left(c+dx \right)}+\sec{\left(c+dx \right)} \right)}}{d} & \text{for}\: d \neq 0 \\ \frac{x\left(\tan{\left(c \right)}\sec{\left(c \right)}+\sec^{2}{\left(c \right)}\right)}{\tan{\left(c \right)}+\sec{\left(c \right)}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x)
```

```
[Out] a*x + a*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(c
)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

Giac [B] time = 1.35175, size = 66, normalized size = 4.12

$$ax + \frac{a\left(\log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx+c\right) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx+c\right) - 2\right|\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*sec(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/4*a*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*
x + c) + sin(d*x + c) - 2)))/d
```

3.6 $\int \cos(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=15

$$\frac{a\sin(c+dx)}{d}+ax$$

```
[Out] a*x + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0192023, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3787, 2637, 8}

$$\frac{a\sin(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x + (a*Sin[c + d*x])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))\,dx = a\int 1\,dx + a\int \cos(c+dx)\,dx$$
$$= ax + \frac{a\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0087643, size = 26, normalized size = 1.73

$$\frac{a\sin(c)\cos(dx)}{d} + \frac{a\cos(c)\sin(dx)}{d} + ax$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.047, size = 21, normalized size = 1.4

$$\frac{a\sin(dx+c) + a(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*sin(d*x+c)+a*(d*x+c))
```

Maxima [A] time = 1.09982, size = 27, normalized size = 1.8

$$\frac{(dx+c)a + a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a + a*sin(d*x + c))/d
```

Fricas [A] time = 1.62217, size = 38, normalized size = 2.53

$$\frac{adx + a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*d*x + a*sin(d*x + c))/d
```

Sympy [A] time = 2.86143, size = 15, normalized size = 1.

$$ax + a\left(\begin{cases}\sin(c) & \text{for}\ d = 0\\ \frac{\sin(c+dx)}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*x + a*Piecewise((sin(c), Eq(d, 0)), (sin(c + d*x)/d, True))
```

Giac [B] time = 1.34643, size = 53, normalized size = 3.53

$$\frac{(dx + c)a + \frac{2\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} + 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.7 $\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0346483, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 2635, 8, 2637}

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sec(c+dx))\,dx &= a\int \cos(c+dx)\,dx+a\int \cos^2(c+dx)\,dx\\ &= \frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0503867, size = 32, normalized size = 0.84

$$\frac{a(2(c+dx)+4\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.056, size = 38, normalized size = 1.

$$\frac{1}{d}\left(a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*sin(d*x+c))
```

Maxima [A] time = 1.0812, size = 46, normalized size = 1.21

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a+4\,a\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d
```

Fricas [A] time = 1.67417, size = 72, normalized size = 1.89

$$\frac{adx + (a\cos(dx+c) + 2a)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \cos^2(c+dx)\sec(c+dx)\,dx + \int \cos^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)**2, x)
)
```

Giac [A] time = 1.29553, size = 76, normalized size = 2.

$$\frac{(dx+c)a + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.8 $\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0404894, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 2633, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sec(c+dx))\,dx &= a\int \cos^2(c+dx)\,dx + a\int \cos^3(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx-\frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= \frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0665462, size = 57, normalized size = 1.06

$$\frac{a(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.095, size = 49, normalized size = 0.9

$$\frac{1}{d}\left(\frac{a\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*(cos(d*x+c)^2+2)*sin(d*x+c)+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.08348, size = 62, normalized size = 1.15

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a)/d
```

Fricas [A] time = 1.69214, size = 105, normalized size = 1.94

$$\frac{3\,adx+\left(2\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))/
d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cos^{3}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\cos^{3}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(cos(c + d*x)**3, x)
)
```

Giac [A] time = 1.18852, size = 97, normalized size = 1.8

$$\frac{3\,(dx+c)a+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 + 4*a*tan(1/2*d*x + 1/2*
c)^3 + 9*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.9 $\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0550533, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 2635, 8, 2633}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+a\sec(c+dx))\,dx &= a\int \cos^3(c+dx)\,dx + a\int \cos^4(c+dx)\,dx\\ &= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int\left(1-x^2\right)dx, x\right.}{d}\\ &= \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a\sin^3(}{3}\\ &= \frac{3ax}{8} + \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} -\end{aligned}$$

Mathematica [A] time = 0.0969441, size = 73, normalized size = 0.96

$$\frac{3a(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*
Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.103, size = 60, normalized size = 0.8

$$\frac{1}{d}\left(a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{3\,dx}{8} + \frac{3\,c}{8}\right) + \frac{a\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*(
cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A] time = 1.08461, size = 77, normalized size = 1.01

$$-\frac{32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.72212, size = 136, normalized size = 1.79

$$\frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^3+8\,a\cos\left(dx+c\right)^2+9\,a\cos\left(dx+c\right)+16\,a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c)
 + 16*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.30253, size = 116, normalized size = 1.53

$$\frac{9\,(dx+c)a+\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+39\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 + 49*a*tan(1/2*d*x + 1/
2*c)^5 + 31*a*tan(1/2*d*x + 1/2*c)^3 + 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^4)/d
```

3.10 $\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=122

$$\frac{3a^2\tan^3(c+dx)}{5d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^3(c+}{2d}$$

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) + (9*a^2*Tan[c + d*x])/(5*d) + (3*a^2*S
ec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) +
 (a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (3*a^2*Tan[c + d*x]^3)/(5*d)
```

Rubi [A] time = 0.0946708, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 3768, 3770, 4046, 3767}

$$\frac{3a^2\tan^3(c+dx)}{5d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^3(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(4*d) + (9*a^2*Tan[c + d*x])/(5*d) + (3*a^2*S
ec[c + d*x]*Tan[c + d*x])/(4*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) +
 (a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (3*a^2*Tan[c + d*x]^3)/(5*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \sec^5(c+dx)\,dx+\int \sec^4(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx\\ &= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{a^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{2}\left(3a^2\right)\int \sec^3(c+\\ &= \frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{a^2\sec^4(c+dx)\tan}{5d}\\ &= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{9a^2\tan(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(c+dx)}{4d}+\frac{a^2 s}{}\end{aligned}$$

Mathematica [B] time = 1.51316, size = 487, normalized size = 3.99

$$-\frac{a^2\sec(c)\sec^5(c+dx)\Big(80\sin(2c+dx)-140\sin(c+2dx)-140\sin(3c+2dx)-240\sin(2c+3dx)-30\sin(3c+4dx)-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*Sec[c]*Sec[c + d*x]^5*(75*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 75*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 15*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 15*Cos[
6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 150*Cos[d*x]*(Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
```

```
2]]) + 150*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 75*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 75*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 15*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
 - 15*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 400*Sin[d
*x] + 80*Sin[2*c + d*x] - 140*Sin[c + 2*d*x] - 140*Sin[3*c + 2*d*x] - 240*S
in[2*c + 3*d*x] - 30*Sin[3*c + 4*d*x] - 30*Sin[5*c + 4*d*x] - 48*Sin[4*c +
5*d*x]))/(640*d)
```

Maple [A] time = 0.036, size = 124, normalized size = 1.

$$\frac{6\,a^2\tan(dx+c)}{5\,d}+\frac{3\,a^2(\sec(dx+c))^2\tan(dx+c)}{5\,d}+\frac{a^2(\sec(dx+c))^3\tan(dx+c)}{2\,d}+\frac{3\,a^2\sec(dx+c)\tan(dx+c)}{4\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 6/5*a^2*tan(d*x+c)/d+3/5*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)^3
*tan(d*x+c)/d+3/4*a^2*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a^2*ln(sec(d*x+c)+tan(d
*x+c))+1/5*a^2*sec(d*x+c)^4*tan(d*x+c)/d
```

Maxima [A] time = 0.992602, size = 180, normalized size = 1.48

$$\frac{8\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^2+40\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2-15\,a^2\left(\frac{2\left(3\sin(dx+c)^3\right.}{\sin(dx+c)^4-2\,\mathrm{s}}\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 15*a^2*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.72649, size = 320, normalized size = 2.62

$$\frac{15\,a^2\cos(dx+c)^5\log(\sin(dx+c)+1) - 15\,a^2\cos(dx+c)^5\log(-\sin(dx+c)+1) + 2\left(24\,a^2\cos(dx+c)^4 + 15\,a^2\cos(\right.}{40\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/40*(15*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a^2*cos(d*x + c)^5*l
og(-sin(d*x + c) + 1) + 2*(24*a^2*cos(d*x + c)^4 + 15*a^2*cos(d*x + c)^3 +
12*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \sec^4(c+dx)\,dx + \int 2\sec^5(c+dx)\,dx + \int \sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**4, x) + Integral(2*sec(c + d*x)**5, x) + Integ
ral(sec(c + d*x)**6, x))
```

Giac [A] time = 1.38516, size = 186, normalized size = 1.52

$$\frac{15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 144\,a^2\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(15*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 - 70*a^2*tan(1/2*d*x +
```

```
1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 - 90*a^2*tan(1/2*d*x + 1/2*c)^3 +
 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.11 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=96

$$\frac{2a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{7a^2\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*Tan[c + d*x])/d + (7*a^2*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*
a^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.08384, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 3767, 4046, 3768, 3770}

$$\frac{2a^2\tan^3(c+dx)}{3d}+\frac{2a^2\tan(c+dx)}{d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{7a^2\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*Tan[c + d*x])/d + (7*a^2*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*
a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^4(c+dx)\,dx + \int \sec^3(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx$$
$$= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(7a^2\right)\int \sec^3(c+dx)\,dx - \frac{\left(2a^2\right)\text{Subst}\left(\int (1\right.}{}$$
$$= \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} +$$
$$= \frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2}{}$$

Mathematica [B] time = 6.40461, size = 877, normalized size = 9.14

$$-\frac{7\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(\sec(c+dx)a+a)^2\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d} + \frac{7\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-7*Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (
d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(32*d) + (7*Cos[c + d*x]^2*Log[Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)
/(32*d) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(64*
```

```
d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^2*Sec[c/2 +
(d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(12*d*(Cos[c/2] - Sin[c/2])
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^2*Sec[c/2 + (
d*x)/2]^4*(a + a*Sec[c + d*x])^2*(29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x
]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(3*d*(Cos[c/2
] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c + d*x]^2*
Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(64*d*(Cos[c/2 + (d*x)/2] + Si
n[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
d*x])^2*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin
[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d
*x])^2*(-29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2
+ (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*
(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(3*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.035, size = 102, normalized size = 1.1

$$\frac{7\,a^2\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{7\,a^2\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+\frac{4\,a^2\tan(dx+c)}{3\,d}+\frac{2\,a^2(\sec(dx+c))^2\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 7/8*a^2*sec(d*x+c)*tan(d*x+c)/d+7/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+4/3*a^2
*tan(d*x+c)/d+2/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*
x+c)/d
```

Maxima [A] time = 1.1169, size = 196, normalized size = 2.04

$$\frac{32\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^2-3\,a^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
```

```
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.76587, size = 288, normalized size = 3.

$$\frac{21\,a^2\cos(dx+c)^4\log(\sin(dx+c)+1) - 21\,a^2\cos(dx+c)^4\log(-\sin(dx+c)+1) + 2\left(32\,a^2\cos(dx+c)^3 + 21\,a^2\cos\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(21*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 21*a^2*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(32*a^2*cos(d*x + c)^3 + 21*a^2*cos(d*x + c)^2 +
16*a^2*cos(d*x + c) + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \sec^3(c+dx)\,dx + \int 2\sec^4(c+dx)\,dx + \int \sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**3, x) + Integral(2*sec(c + d*x)**4, x) + Integ
ral(sec(c + d*x)**5, x))
```

Giac [A] time = 1.46889, size = 165, normalized size = 1.72

$$\frac{21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 83\,a^2\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 21*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*a^2*tan(1/2*d*x +
1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.12 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=74

$$\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/(3*d) + (a^2*Sec[c + d
*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0808184, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3788, 3768, 3770, 4046, 3767, 8}

$$\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d + (5*a^2*Tan[c + d*x])/(3*d) + (a^2*Sec[c + d
*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \sec^3(c+dx)\,dx+\int \sec^2(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx\\ &= \frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d}+a^2\int \sec(c+dx)\,dx\\ &= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\sec^2(c+dx)\tan(c+\end{aligned}$$

$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^2\tan(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{d}+\frac{a^2\sec}{}$$

Mathematica [B] time = 0.630396, size = 318, normalized size = 4.3

$$-\frac{a^2\sec(c)\sec^3(c+dx)\left(6\sin(2c+dx)-6\sin(c+2dx)-6\sin(3c+2dx)-10\sin(2c+3dx)+3\cos(2c+3dx)\log\left(\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*Sec[c]*Sec[c + d*x]^3*(3*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[
(c + d*x)/2]] + 3*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 9*Cos[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]) + 9*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c
```

```
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Cos[2*c + 3*d*x]
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 3*Cos[4*c + 3*d*x]*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] - 24*Sin[d*x] + 6*Sin[2*c + d*x] - 6*Sin[c + 2
*d*x] - 6*Sin[3*c + 2*d*x] - 10*Sin[2*c + 3*d*x]))/(24*d)
```

Maple [A] time = 0.031, size = 78, normalized size = 1.1

$$\frac{5\,a^2\tan\left(dx+c\right)}{3\,d}+\frac{a^2\sec\left(dx+c\right)\tan\left(dx+c\right)}{d}+\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a^2\left(\sec\left(dx+c\right)\right)^2\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 5/3*a^2*tan(d*x+c)/d+a^2*sec(d*x+c)*tan(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(
d*x+c))+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.12788, size = 115, normalized size = 1.55

$$\frac{2\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2-3\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,a^2\tan\left(dx+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan
(d*x + c))/d
```

Fricas [A] time = 1.74291, size = 246, normalized size = 3.32

$$\frac{3\,a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(5\,a^2\cos\left(dx+c\right)^2+3\,a^2\cos\left(dx\right.\right.}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*si
n(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int \sec^{2}(c+dx)\,dx+\int 2\sec^{3}(c+dx)\,dx+\int \sec^{4}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**2, x) + Integral(2*sec(c + d*x)**3, x) + Integ
ral(sec(c + d*x)**4, x))
```

Giac [A] time = 1.4357, size = 143, normalized size = 1.93

$$\frac{3\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-8\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2*c
)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.13 $\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=54

$$\frac{2a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0470463, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3788, 3767, 8, 4046, 3770}

$$\frac{2a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^2(c+dx)\,dx + \int \sec(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(3a^2\right)\int \sec(c+dx)\,dx - \frac{\left(2a^2\right)\mathrm{Subst}(\int 1\,dx, x, -}{d}$$
$$= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [B] time = 0.594104, size = 219, normalized size = 4.06

$$a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{8\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{}{\left(\cos\left(\frac{1}{2}\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-6*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2)
+ (8*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(16*d)
```

Maple [A] time = 0.03, size = 58, normalized size = 1.1

$$\frac{3\,a^2\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+2\,\frac{a^2\tan(dx+c)}{d}+\frac{a^2\sec(dx+c)\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 3/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*tan(d*x+c)/d+1/2*a^2*sec(d*x+c)*t
an(d*x+c)/d
```

Maxima [A] time = 1.13197, size = 109, normalized size = 2.02

$$-\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 4\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right) - 8\,a^2\tan\left(dx+\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a^2*tan(d
*x + c))/d
```

Fricas [A] time = 1.73665, size = 215, normalized size = 3.98

$$\frac{3\,a^2\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - 3\,a^2\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(4\,a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log(
-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x
+ c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\sec\left(c+dx\right)dx + \int 2\sec^2\left(c+dx\right)dx + \int\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sec(c + d*x), x) + Integral(2*sec(c + d*x)**2, x) + Integral
(sec(c + d*x)**3, x))
```

Giac [A] time = 1.48383, size = 122, normalized size = 2.26

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.14 $\int (a + a\sec(c + dx))^2\, dx$

Optimal. Leaf size=34

$$\frac{a^2\tan(c+dx)}{d} + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} + a^2x$$

```
[Out] a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.0237475, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3773, 3770, 3767, 8}

$$\frac{a^2\tan(c+dx)}{d} + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} + a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\,dx &= a^2x + a^2\int \sec^2(c+dx)\,dx + \left(2a^2\right)\int \sec(c+dx)\,dx\\ &= a^2x + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^2\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}\\ &= a^2x + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.433958, size = 171, normalized size = 5.03

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}-2\log\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(d*x - 2*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]
/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(4*d)
```

Maple [A] time = 0.026, size = 50, normalized size = 1.5

$$a^2x + 2\,\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^2\tan(dx+c)}{d} + \frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*x+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*tan(d*x+c)/d+1/d*a^2*c
```

Maxima [A] time = 1.17209, size = 55, normalized size = 1.62

$$a^2x + \frac{2\,a^2\log(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a^2*log(sec(d*x + c) + tan(d*x + c))/d + a^2*tan(d*x + c)/d
```

Fricas [B] time = 1.769, size = 193, normalized size = 5.68

$$\frac{a^2dx\cos(dx+c)+a^2\cos(dx+c)\log(\sin(dx+c)+1)-a^2\cos(dx+c)\log(-\sin(dx+c)+1)+a^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d*
x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 1\,dx+\int 2\sec(c+dx)\,dx+\int\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(1, x) + Integral(2*sec(c + d*x), x) + Integral(sec(c + d*x)*
*2, x))
```

Giac [B] time = 1.4852, size = 107, normalized size = 3.15

$$\frac{(dx+c)a^2+2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

3.15 $\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=34

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.0508976, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3788, 8, 4045, 3770}

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int 1\,dx+\int \cos(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx\\ &= 2a^2x+\frac{a^2\sin(c+dx)}{d}+a^2\int \sec(c+dx)\,dx\\ &= 2a^2x+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0132871, size = 47, normalized size = 1.38

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c)\cos(dx)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Co
s[c]*Sin[d*x])/d
```

Maple [A] time = 0.055, size = 51, normalized size = 1.5

$$2\,a^2x+\frac{a^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a^2\sin\left(dx+c\right)}{d}+2\,\frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2*a^2*x+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*sin(d*x+c)/d+2/d*a^2*c
```

Maxima [A] time = 1.15053, size = 70, normalized size = 2.06

$$\frac{4\left(dx+c\right)a^2+a^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.70983, size = 131, normalized size = 3.85

$$\frac{4a^2dx + a^2\log(\sin(dx+c)+1) - a^2\log(-\sin(dx+c)+1) + 2a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) + 2
*a^2*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\cos(c+dx)\sec(c+dx)\,dx + \int \cos(c+dx)\sec^2(c+dx)\,dx + \int \cos(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)*sec(
c + d*x)**2, x) + Integral(cos(c + d*x), x))
```

Giac [B] time = 1.3963, size = 107, normalized size = 3.15

$$\frac{2(dx+c)a^2 + a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

3.16 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=45

$$\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

```
[Out] (3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rubi [A] time = 0.0602976, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3788, 2637, 4045, 8}

$$\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \cos(c+dx)\,dx + \int \cos^2(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx \\
&= \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(3a^2\right)\int 1\,dx \\
&= \frac{3a^2x}{2} + \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0393088, size = 34, normalized size = 0.76

$$\frac{a^2(6(c+dx)+8\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.058, size = 52, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,a^2\sin(dx+c)+a^2\,(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*sin(d*x+c)+a^2*(d*
x+c))
```

Maxima [A] time = 1.09854, size = 65, normalized size = 1.44

$$\frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})a^2+4\,(dx+c)a^2+8\,a^2\sin{(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*a^2 + 8*a^2*sin(d*x
 + c))/d
```

Fricas [A] time = 1.70518, size = 82, normalized size = 1.82

$$\frac{3\,a^2dx+\left(a^2\cos{(dx+c)}+4\,a^2\right)\sin{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\cos^2{(c+dx)}\sec{(c+dx)}\,dx+\int\cos^2{(c+dx)}\sec^2{(c+dx)}\,dx+\int\cos^2{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)**2*sec(c + d*x), x) + Integral(cos(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**2, x))
```

Giac [A] time = 1.32744, size = 86, normalized size = 1.91

$$\frac{3\,(dx+c)a^2+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x
+ 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.17 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=57

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+a^2x$$

```
[Out] a^2*x + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/d - (a^2*S
in[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0821145, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 2635, 8, 4044, 3013}

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] a^2*x + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/d - (a^2*S
in[c + d*x]^3)/(3*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \cos^2(c+dx)\,dx+\int \cos^3(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx\\ &= \frac{a^2\cos(c+dx)\sin(c+dx)}{d}+a^2\int 1\,dx+\int \cos(c+dx)\left(a^2+a^2\cos^2(c+dx)\right)d\\ &= a^2x+\frac{a^2\cos(c+dx)\sin(c+dx)}{d}-\frac{\mathrm{Subst}\left(\int\left(2a^2-a^2x^2\right)dx,x,-\sin(c+dx)\right)}{d}\\ &= a^2x+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0853281, size = 41, normalized size = 0.72

$$\frac{a^2(21\sin(c+dx)+6\sin(2(c+dx))+\sin(3(c+dx))+12dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(1
2*d)
```

Maple [A] time = 0.066, size = 64, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a^2\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}+2\,a^2\left(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2\right)+a^2\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*a^2*(cos(d*x+c)^2+2)*sin(d*x+c)+2*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+a^2*sin(d*x+c))
```

Maxima [A] time = 0.989147, size = 82, normalized size = 1.44

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{2}-6\,a^{2}\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^2 - 6*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.69427, size = 113, normalized size = 1.98

$$\frac{3\,a^{2}dx+\left(a^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}\cos\left(dx+c\right)+5\,a^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x + (a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x
+ c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35726, size = 108, normalized size = 1.89

$$\frac{3\,(dx+c)a^2+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 8*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.18 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=87

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

```
[Out] (7*a^2*x)/8 + (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a^2*Sin[c + d*x]^3)/(3*d
)
```

Rubi [A] time = 0.0793729, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 2633, 4045, 2635, 8}

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (7*a^2*x)/8 + (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a^2*Sin[c + d*x]^3)/(3*d
)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \cos^3(c+dx)\,dx + \int \cos^4(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\left(7a^2\right)\int \cos^2(c+dx)\,dx - \frac{\left(2a^2\right)\text{Subst}\left(\int\left(1-\right.\right.}{}$$

$$= \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{2}{}$$

$$= \frac{7a^2x}{8} + \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)\sin(c+}{4d}$$

Mathematica [A] time = 0.137689, size = 53, normalized size = 0.61

$$\frac{a^2(144\sin(c+dx)+48\sin(2(c+dx))+16\sin(3(c+dx))+3\sin(4(c+dx))+84dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)]
 + 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.07, size = 90, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^2\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}+a^2\left(\frac{\cos(d}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
^2*(cos(d*x+c)^2+2)*sin(d*x+c)+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.1238, size = 112, normalized size = 1.29

$$-\frac{64\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{2}-24\left(2\,dx+2\,c+\sin\left(2\right.\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
a^2)/d
```

Fricas [A] time = 1.70655, size = 154, normalized size = 1.77

$$\frac{21\,a^{2}dx+\left(6\,a^{2}\cos\left(dx+c\right)^{3}+16\,a^{2}\cos\left(dx+c\right)^{2}+21\,a^{2}\cos\left(dx+c\right)+32\,a^{2}\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*c
os(d*x + c) + 32*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37486, size = 130, normalized size = 1.49

$$\frac{21\,(dx+c)a^2+\frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+83\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)*a^2 + 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*a^2*tan(1/2*
d*x + 1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 + 75*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.19 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2x}{4}$$

```
[Out] (3*a^2*x)/4 + (2*a^2*Sin[c + d*x])/d + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - (a^2*Sin[c + d*x]^3)/d + (a
^2*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.109805, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3788, 2635, 8, 4044, 3013, 373}

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/4 + (2*a^2*Sin[c + d*x])/d + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - (a^2*Sin[c + d*x]^3)/d + (a
^2*Sin[c + d*x]^5)/(5*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)(a+a\sec(c+dx))^2\,dx &= \left(2a^2\right)\int \cos^4(c+dx)\,dx+\int \cos^5(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx\\
&= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^2\right)\int \cos^2(c+dx)\,dx+\int \cos^3(c+dx)\left(a^2+\right.\\
&= \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}+\frac{1}{4}\left(3a^2\right)\int 1\,dx-\frac{\text{Su}}{}\\
&= \frac{3a^2x}{4}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}-\frac{\text{Subst}\left(\int\left(2\right.\right.}{}\\
&= \frac{3a^2x}{4}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}+\frac{a^2\cos^3(c+dx)\sin(c+}{2d}
\end{aligned}$$

Mathematica [A] time = 0.136428, size = 61, normalized size = 0.59

$$\frac{a^2(110\sin(c+dx)+40\sin(2(c+dx))+15\sin(3(c+dx))+5\sin(4(c+dx))+\sin(5(c+dx))+60dx)}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(60*d*x + 110*Sin[c + d*x] + 40*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)]
 + 5*Sin[4*(c + d*x)] + Sin[5*(c + d*x)]))/(80*d)
```

Maple [A] time = 0.109, size = 96, normalized size = 0.9

$$\frac{1}{d}\left(\frac{a^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+2a^2\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*(cos(d*x+c)^2+2
)*sin(d*x+c))
```

Maxima [A] time = 1.07577, size = 128, normalized size = 1.24

$$\frac{16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2-80\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2+15(12dx+12c+\mathrm{si}}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 80
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A] time = 1.70839, size = 186, normalized size = 1.81

$$\frac{15a^2dx+\left(4a^2\cos(dx+c)^4+10a^2\cos(dx+c)^3+12a^2\cos(dx+c)^2+15a^2\cos(dx+c)+24a^2\right)\sin(dx+c)}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*c
os(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37274, size = 151, normalized size = 1.47

$$\frac{15\,(dx+c)a^2 + \frac{2\left(15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+144\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+65\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(15*(d*x + c)*a^2 + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^9 + 70*a^2*tan(1/2*
d*x + 1/2*c)^7 + 144*a^2*tan(1/2*d*x + 1/2*c)^5 + 90*a^2*tan(1/2*d*x + 1/2*
c)^3 + 65*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.20 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=114

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\ldots$$

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.125291, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3791, 3768, 3770, 3767}

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+3a^3\sec^5(c+dx)+a^3\sec^6(c+dx)\right)dx$$
$$= a^3\int \sec^3(c+dx)\,dx + a^3\int \sec^6(c+dx)\,dx + \left(3a^3\right)\int \sec^4(c+dx)\,dx + \left(3a^3\right)$$
$$= \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{2}a^3\int \sec(c+dx)$$
$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{3a^3}{}$$
$$= \frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{8d} +$$

Mathematica [B] time = 1.46479, size = 487, normalized size = 4.27

$$-\frac{a^3\sec(c)\sec^5(c+dx)\left(1440\sin(2c+dx)-1500\sin(c+2dx)-1500\sin(3c+2dx)-3040\sin(2c+3dx)-390\sin(3c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*Sec[c]*Sec[c + d*x]^5*(975*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 975*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 195*
Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1950*Cos[d*x]*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + 1950*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 975*Cos[2*c + 3*d*x]*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 975*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] - 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] - 195*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
```

```
4640*Sin[d*x] + 1440*Sin[2*c + d*x] - 1500*Sin[c + 2*d*x] - 1500*Sin[3*c +
 2*d*x] - 3040*Sin[2*c + 3*d*x] - 390*Sin[3*c + 4*d*x] - 390*Sin[5*c + 4*d*
x] - 608*Sin[4*c + 5*d*x]))/(3840*d)
```

Maple [A] time = 0.038, size = 124, normalized size = 1.1

$$\frac{13\,a^3\sec\left(dx+c\right)\tan\left(dx+c\right)}{8\,d}+\frac{13\,a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8\,d}+\frac{38\,a^3\tan\left(dx+c\right)}{15\,d}+\frac{19\,a^3\tan\left(dx+c\right)\left(\sec\left(dx\right.\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 13/8*a^3*sec(d*x+c)*tan(d*x+c)/d+13/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+38/15
*a^3*tan(d*x+c)/d+19/15/d*a^3*tan(d*x+c)*sec(d*x+c)^2+3/4*a^3*sec(d*x+c)^3*
tan(d*x+c)/d+1/5/d*a^3*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 1.12429, size = 242, normalized size = 2.12

$$16\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^3+240\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-45\,a^3\left(\frac{2\left(3\,\sin(dx+\right.}{\sin(dx+c)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.73609, size = 329, normalized size = 2.89

$$\frac{195\,a^3\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-195\,a^3\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(304\,a^3\cos\left(dx+c\right)^4+195\,a\right.}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(195*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 195*a^3*cos(d*x + c)^
5*log(-sin(d*x + c) + 1) + 2*(304*a^3*cos(d*x + c)^4 + 195*a^3*cos(d*x + c)
^3 + 152*a^3*cos(d*x + c)^2 + 90*a^3*cos(d*x + c) + 24*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3 \left(\int \sec^3(c+dx)\,dx + \int 3\sec^4(c+dx)\,dx + \int 3\sec^5(c+dx)\,dx + \int \sec^6(c+dx)\,dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**3, x) + Integral(3*sec(c + d*x)**4, x) + Integ
ral(3*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**6, x))
```

Giac [A] time = 1.39625, size = 186, normalized size = 1.63

$$\frac{195\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 195\,a^3 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(195\,a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^9 - 910\,a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^7 + 1664\,a^3 \tan\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2*d
*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x + 1/
2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.21 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=93

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\sec(c}{8d}$$

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
a^3*Tan[c + d*x]^3)/d
```

Rubi [A] time = 0.113988, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 3767, 8, 3768, 3770}

$$\frac{a^3\tan^3(c+dx)}{d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{15a^3\tan(c+dx)\sec(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
a^3*Tan[c + d*x]^3)/d
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+a^3\sec^5(c+dx)\right)dx\\ &= a^3\int \sec^2(c+dx)\,dx+a^3\int \sec^5(c+dx)\,dx+\left(3a^3\right)\int \sec^3(c+dx)\,dx+\left(3a^3\right)\\ &= \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\left(3a^3\right)\int \sec^3(c+\\ &= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}+\\ &= \frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a^3\tan(c+dx)}{d}+\frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}+\end{aligned}$$

Mathematica [B] time = 6.39876, size = 877, normalized size = 9.43

$$-\frac{15\cos^3(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(\sec(c+dx)a+a)^3\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d}+\frac{15\cos^3(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-15*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(64*d) + (15*Cos[c + d*x]^3*Log[Cos[c/2
+ (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^
3)/(64*d) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(1
28*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^3*Sec[c/2
 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(16*d*(Cos[c/2] - Sin[c/
```

```
2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^3*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (3*Cos[c
+ d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(8*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c + d*
x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(128*d*(Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Se
c[c + d*x])^3*Sin[(d*x)/2])/(16*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec
[c + d*x])^3*(-19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Cos[c/2] + Sin[c/2])*(Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (3*Cos[c + d*x]^3*Sec[c/2 + (d*
x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(8*d*(Cos[c/2] + Sin[c/2])*(Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.033, size = 101, normalized size = 1.1

$$3\,\frac{a^3\tan(dx+c)}{d}+\frac{15\,a^3\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{15\,a^3\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+\frac{a^3\tan(dx+c)(\sec(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 3*a^3*tan(d*x+c)/d+15/8*a^3*sec(d*x+c)*tan(d*x+c)/d+15/8/d*a^3*ln(sec(d*x+c
)+tan(d*x+c))+1/d*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4*a^3*sec(d*x+c)^3*tan(d*x+
c)/d
```

Maxima [A] time = 1.05683, size = 211, normalized size = 2.27

$$\frac{16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^3-a^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
```

```
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x + c
))/d
```

Fricas [A] time = 1.75651, size = 286, normalized size = 3.08

$$\frac{15\,a^3\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-15\,a^3\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,a^3\cos\left(dx+c\right)^3+15\,a^3\cos\left(\right.\right.}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2 +
8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int\sec^2\left(c+dx\right)dx+\int 3\sec^3\left(c+dx\right)dx+\int 3\sec^4\left(c+dx\right)dx+\int\sec^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**2, x) + Integral(3*sec(c + d*x)**3, x) + Integ
ral(3*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**5, x))
```

Giac [A] time = 1.35389, size = 165, normalized size = 1.77

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a^3\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x + 1
/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.22 $\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=72

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0752543, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3791, 3770, 3767, 8, 3768}

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\sec(c+dx)+3a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+a^3\sec^4(c+dx)\right)dx\\ &= a^3\int \sec(c+dx)\,dx+a^3\int \sec^4(c+dx)\,dx+\left(3a^3\right)\int \sec^2(c+dx)\,dx+\left(3a^3\right)\\ &= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int \sec(c+dx)\\ &= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{4a^3\tan(c+dx)}{d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^3}{}\end{aligned}$$

Mathematica [B] time = 5.94417, size = 154, normalized size = 2.14

$$-\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(-4\tan(c)\cos(c+dx)-\sec(c)(-20\sin(2c+dx)+9\sin(c+2dx)+9\sin(3c+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(60*Cos[c + d*x]^3*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 - Sec[c]*(50*Sin[d*x] - 20*Sin[2*c + d*x] + 9*Sin[c + 2*d*x] + 9*Sin[3*c +
 2*d*x] + 22*Sin[2*c + 3*d*x]) - 4*Cos[c + d*x]*Tan[c]))/(192*d)
```

Maple [A] time = 0.036, size = 80, normalized size = 1.1

$$\frac{5a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{11a^3\tan(dx+c)}{3d}+\frac{3a^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3\tan(dx+c)(\sec(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 5/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3*a^3*tan(d*x+c)/d+3/2*a^3*sec(d*x+c
)*tan(d*x+c)/d+1/3/d*a^3*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.08559, size = 140, normalized size = 1.94

$$\frac{4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3-9\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^3\log\left(\sec\left(a\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3*l
og(sec(d*x + c) + tan(d*x + c)) + 36*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.70999, size = 254, normalized size = 3.53

$$\frac{15\,a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-15\,a^3\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(22\,a^3\cos\left(dx+c\right)^2+9\,a^3\cos\left(d\right.\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*l
og(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2*a
^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int\sec\left(c+dx\right)dx+\int 3\sec^2\left(c+dx\right)dx+\int 3\sec^3\left(c+dx\right)dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sec(c + d*x), x) + Integral(3*sec(c + d*x)**2, x) + Integral
(3*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

Giac [A] time = 1.36951, size = 143, normalized size = 1.99

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a^3\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x + 1
/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.23 $\int (a + a\sec(c + dx))^3\,dx$

Optimal. Leaf size=66

$$\frac{5a^3\tan(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+a^3x$$

```
[Out] a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*Tan[c + d*x])/(2*d) +
((a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0473646, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3775, 3914, 3767, 8, 3770}

$$\frac{5a^3\tan(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*Tan[c + d*x])/(2*d) +
((a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\,dx &= \frac{\left(a^3+a^3\sec(c+dx)\right)\tan(c+dx)}{2d}+\frac{1}{2}a\int (a+a\sec(c+dx))(2a+5a\sec(c+dx))\,dx\\ &= a^3x+\frac{\left(a^3+a^3\sec(c+dx)\right)\tan(c+dx)}{2d}+\frac{1}{2}\left(5a^3\right)\int \sec^2(c+dx)\,dx+\frac{1}{2}\left(7a^3\right)\int \sec(c+a\\ &= a^3x+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\left(a^3+a^3\sec(c+dx)\right)\tan(c+dx)}{2d}-\frac{\left(5a^3\right)\mathrm{Subst}(\int 1\,dx,x}{2d}\\ &= a^3x+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^3\tan(c+dx)}{2d}+\frac{\left(a^3+a^3\sec(c+dx)\right)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [B] time = 0.892656, size = 235, normalized size = 3.56

$$\frac{1}{32}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{12\sin(dx)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(4*x - (14*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
+ 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - 1/(d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^2) + (12*Sin[d*x])/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + S
in[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c +
```

```
d*x)/2])))))/32
```

Maple [A] time = 0.03, size = 71, normalized size = 1.1

$$a^3x+\frac{a^3c}{d}+\frac{7\,a^3\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2\,d}+3\,\frac{a^3\tan\left(dx+c\right)}{d}+\frac{a^3\sec\left(dx+c\right)\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3,x)
```

```
[Out] a^3*x+1/d*a^3*c+7/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)/d+1/2*
a^3*sec(d*x+c)*tan(d*x+c)/d
```

Maxima [A] time = 1.14688, size = 123, normalized size = 1.86

$$a^3x-\frac{a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d}+\frac{3\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3\,a^3\tan\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/4*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1))/d + 3*a^3*log(sec(d*x + c) + tan(d*x + c))/d + 3
*a^3*tan(d*x + c)/d
```

Fricas [A] time = 1.75337, size = 251, normalized size = 3.8

$$\frac{4\,a^3dx\cos\left(dx+c\right)^2+7\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-7\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(6\,a^3\cos\left(dx\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
- 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a^3
)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 1\,dx+\int 3\sec\left(c+dx\right)dx+\int 3\sec^2\left(c+dx\right)dx+\int \sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(1, x) + Integral(3*sec(c + d*x), x) + Integral(3*sec(c + d*x
)**2, x) + Integral(sec(c + d*x)**3, x))
```

Giac [A] time = 1.35293, size = 135, normalized size = 2.05

$$\frac{2\left(dx+c\right)a^3+7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.24 $\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=48

$$\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+3a^3x$$

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Tan
[c + d*x])/d
```

Rubi [A] time = 0.0579294, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3791, 2637, 3770, 3767, 8}

$$\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Tan
[c + d*x])/d
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(3a^3+a^3\cos(c+dx)+3a^3\sec(c+dx)+a^3\sec^2(c+dx)\right)dx\\ &= 3a^3x+a^3\int \cos(c+dx)\,dx+a^3\int \sec^2(c+dx)\,dx+\left(3a^3\right)\int \sec(c+dx)\,dx\\ &= 3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\operatorname{Subst}(\int 1\,dx,x,-\tan(c}{d}\\ &= 3a^3x+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.866313, size = 211, normalized size = 4.4

$$\frac{1}{8}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(c)\cos(dx)}{d}+\frac{\cos(c)\sin(dx)}{d}+\frac{\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(3*x - (3*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]])/d + (3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d +
(Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d + Sin[(d*x)/2]/(d*(Cos[c/2] - Sin
[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(d*x)/2]/(d*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/8
```

Maple [A] time = 0.054, size = 65, normalized size = 1.4

$$3\,a^3x+3\,\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^3\tan(dx+c)}{d}+\frac{a^3\sin(dx+c)}{d}+3\,\frac{a^3c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 3*a^3*x+3/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*tan(d*x+c)/d+a^3*sin(d*x+c)/d
+3/d*a^3*c
```

Maxima [A] time = 1.0876, size = 86, normalized size = 1.79

$$\frac{6\,(dx+c)a^3 + 3\,a^3(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,a^3\sin(dx+c) + 2\,a^3\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.77482, size = 238, normalized size = 4.96

$$\frac{6\,a^3dx\cos(dx+c) + 3\,a^3\cos(dx+c)\log(\sin(dx+c)+1) - 3\,a^3\cos(dx+c)\log(-\sin(dx+c)+1) + 2\left(a^3\cos(dx+c)\right.}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*
a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(d*
x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\cos(c+dx)\sec(c+dx)\,dx + \int 3\cos(c+dx)\sec^2(c+dx)\,dx + \int \cos(c+dx)\sec^3(c+dx)\,dx + \int \cos(c+dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)*se
c(c + d*x)**2, x) + Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(co
s(c + d*x), x))
```

Giac [A] time = 1.38282, size = 108, normalized size = 2.25

$$\frac{3\,(dx+c)a^3+3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^4 - 1))/d
```

3.25 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=59

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

```
[Out] (7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.066875, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 3770}

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(3a^3+3a^3\cos(c+dx)+a^3\cos^2(c+dx)+a^3\sec(c+dx)\right)\,dx\\&=3a^3x+a^3\int \cos^2(c+dx)\,dx+a^3\int \sec(c+dx)\,dx+\left(3a^3\right)\int \cos(c+dx)\,dx\\&=3a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}\\&=\frac{7a^3x}{2}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0700981, size = 81, normalized size = 1.37

$$\frac{a^3\left(12\sin(c+dx)+\sin(2(c+dx))-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.06, size = 72, normalized size = 1.2

$$\frac{a^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{7a^3x}{2}+\frac{7a^3c}{2d}+3\frac{a^3\sin(dx+c)}{d}+\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+7/2*a^3*x+7/2/d*a^3*c+3*a^3*sin(d*x+c)/d+1/
d*a^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.10061, size = 100, normalized size = 1.69

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3+12\,(dx+c)a^3+2\,a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+12\,a^3\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 2*a^3*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.75841, size = 159, normalized size = 2.69

$$\frac{7\,a^3dx+a^3\log(\sin(dx+c)+1)-a^3\log(-\sin(dx+c)+1)+\left(a^3\cos(dx+c)+6\,a^3\right)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(7*a^3*d*x + a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (
a^3*cos(d*x + c) + 6*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

[Out] Timed out

Giac [A] time = 1.39002, size = 135, normalized size = 2.29

$$\frac{7\,(dx+c)a^3+2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(7*(d*x + c)*a^3 + 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.26 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=63

$$-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

```
[Out] (5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2
*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0746903, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 2633}

$$-\frac{a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2
*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3+3a^3\cos(c+dx)+3a^3\cos^2(c+dx)+a^3\cos^3(c+dx)\right)\,dx\\ &= a^3x+a^3\int \cos^3(c+dx)\,dx+\left(3a^3\right)\int \cos(c+dx)\,dx+\left(3a^3\right)\int \cos^2(c+dx)\,d\\ &= a^3x+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int 1\,dx-\frac{a^3\,\mathrm{Su}}{}\\ &= \frac{5a^3x}{2}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0614099, size = 44, normalized size = 0.7

$$\frac{a^3(45\sin(c+dx)+9\sin(2(c+dx))+\sin(3(c+dx))+30c+30dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(30*c + 30*d*x + 45*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + Sin[3*(c + d*x
)]))/(12*d)
```

Maple [A] time = 0.065, size = 74, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^3\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}+3\,a^3\,(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2)+3\,a^3\sin(dx+c)+a^3\,(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*a^3*(cos(d*x+c)^2+2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+3*a^3*sin(d*x+c)+a^3*(d*x+c))
```

Maxima [A] time = 1.1161, size = 96, normalized size = 1.52

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^{3}-12\left(dx+c\right)a^{3}-36\,a^{3}\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3 - 12*(d*x + c)*a^3 - 36*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.65539, size = 119, normalized size = 1.89

$$\frac{15\,a^{3}dx+\left(2\,a^{3}\cos\left(dx+c\right)^{2}+9\,a^{3}\cos\left(dx+c\right)+22\,a^{3}\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin(
d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35408, size = 108, normalized size = 1.71

$$\frac{15\,(dx+c)a^3+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*a^3*tan(1/2*d
*x + 1/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3
)/d
```

3.27 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=85

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

```
[Out] (15*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d
```

Rubi [A] time = 0.0972659, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 2633}

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (15*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cos(c+dx)+3a^3\cos^2(c+dx)+3a^3\cos^3(c+dx)+a^3\cos^4(c+dx)\right)dx$$
$$= a^3\int\cos(c+dx)\,dx + a^3\int\cos^4(c+dx)\,dx + \left(3a^3\right)\int\cos^2(c+dx)\,dx + \left(3a^3\right)$$
$$= \frac{a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}$$
$$= \frac{3a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{15a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^3\cos^3(c+dx)\sin(c}{4d}$$
$$= \frac{15a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{15a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^3\cos^3(c+dx)\sin(}{4d}$$

Mathematica [A] time = 0.1177, size = 51, normalized size = 0.6

$$\frac{a^3(104\sin(c+dx)+32\sin(2(c+dx))+8\sin(3(c+dx))+\sin(4(c+dx))+60dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]
+ Sin[4*(c + d*x)]))/(32*d)
```

Maple [A] time = 0.073, size = 100, normalized size = 1.2

$$\frac{1}{d}\left(a^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+a^3\left((\cos(dx+c))^2+2\right)\sin(dx+c)+3a^3(1/2\ \mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(
cos(d*x+c)^2+2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
a^3*sin(d*x+c))
```

Maxima [A] time = 1.11291, size = 127, normalized size = 1.49

$$-\frac{32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{3}-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{3}-24\left(2\,dx+2\,c+\sin\left(2\,dx\right.\right.}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^
3 - 32*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.67844, size = 151, normalized size = 1.78

$$\frac{15\,a^{3}dx+\left(2\,a^{3}\cos\left(dx+c\right)^{3}+8\,a^{3}\cos\left(dx+c\right)^{2}+15\,a^{3}\cos\left(dx+c\right)+24\,a^{3}\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(15*a^3*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 15*a^3*cos
(d*x + c) + 24*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41238, size = 130, normalized size = 1.53

$$\frac{15\,(dx+c)a^3 + \frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+73\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+49\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(15*(d*x + c)*a^3 + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 + 55*a^3*tan(1/2*d
*x + 1/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*a^3*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.28 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=105

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{13\ldots}{8\ldots}$$

```
[Out] (13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3)/
(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.111869, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3791, 2635, 8, 2633}

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{13\ldots}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3)/
(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\cos^2(c+dx)+3a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+a^3\cos^5(c+dx)\right)\,d$$
$$= a^3\int \cos^2(c+dx)\,dx + a^3\int \cos^5(c+dx)\,dx + \left(3a^3\right)\int \cos^3(c+dx)\,dx + \left(3a^3\right.$$
$$= \frac{a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{2}a^3\int 1\,dx + \frac{1}{4}\left(9\right.$$
$$= \frac{a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+dx)\sin(c}{4d}$$
$$= \frac{13a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+dx)\sin}{4d}$$

Mathematica [A] time = 0.156687, size = 63, normalized size = 0.6

$$\frac{a^3(1380\sin(c+dx)+480\sin(2(c+dx))+170\sin(3(c+dx))+45\sin(4(c+dx))+6\sin(5(c+dx))+780dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c + d
*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.076, size = 121, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)+3\,a^3\left(1/4\,\left((\cos(dx+c))^3+3/2\,\cos(dx+c)\right)\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(cos(d*x+c)^2+2)*si
n(d*x+c)+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.12754, size = 158, normalized size = 1.5

$$\frac{32\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+c\right)^{3}+15\sin\left(dx+c\right)\right)a^{3}-480\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)a^{3}+45\left(12\,dx+12\,c+\sin\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 48
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A] time = 1.69822, size = 194, normalized size = 1.85

$$\frac{195\,a^{3}dx+\left(24\,a^{3}\cos\left(dx+c\right)^{4}+90\,a^{3}\cos\left(dx+c\right)^{3}+152\,a^{3}\cos\left(dx+c\right)^{2}+195\,a^{3}\cos\left(dx+c\right)+304\,a^{3}\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*a
^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37286, size = 151, normalized size = 1.44

$$\frac{195\,(dx+c)a^3 + \frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1330\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+765\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*(d*x + c)*a^3 + 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*a^3*tan(
1/2*d*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*a^3*tan(1/2*d*x
 + 1/2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)
/d
```

3.29 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=129

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{2}{}$$

```
[Out] (23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^5
)/(5*d)
```

Rubi [A] time = 0.134471, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3791, 2633, 2635, 8}

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^5
)/(5*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sec(c+dx))^3\,dx &= \int \left(a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+3a^3\cos^5(c+dx)+a^3\cos^6(c+dx)\right) d\\ &= a^3\int \cos^3(c+dx)\,dx + a^3\int \cos^6(c+dx)\,dx + \left(3a^3\right)\int \cos^4(c+dx)\,dx + \left(3a^3\right.\\ &= \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}\left(5a^3\right)\int \cos^4(c\\ &= \frac{4a^3\sin(c+dx)}{d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{23a^3\cos^3(c+dx)\sin(c+dx)}{24d}\\ &= \frac{9a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{23a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{23a^3\cos^3(c+dx)\sin}{24d}\\ &= \frac{23a^3x}{16} + \frac{4a^3\sin(c+dx)}{d} + \frac{23a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{23a^3\cos^3(c+dx)\,s}{24d}\end{aligned}$$

Mathematica [A] time = 0.206919, size = 73, normalized size = 0.57

$$\frac{a^3(2520\sin(c+dx)+945\sin(2(c+dx))+380\sin(3(c+dx))+135\sin(4(c+dx))+36\sin(5(c+dx))+5\sin(6(c+dx}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c +
d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(
960*d)
```

Maple [A] time = 0.121, size = 143, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5\,(\cos(dx+c))^3}{4}+\frac{15\,\cos(dx+c)}{8}\right)+\frac{5\,dx}{16}+\frac{5\,c}{16}\right)+\frac{3\,a^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^3*(cos(
d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A] time = 1.05069, size = 193, normalized size = 1.5

$$\frac{192\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^3-5\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\right.}{960}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A] time = 1.70349, size = 230, normalized size = 1.78

$$\frac{345\,a^3dx+\left(40\,a^3\cos\left(dx+c\right)^5+144\,a^3\cos\left(dx+c\right)^4+230\,a^3\cos\left(dx+c\right)^3+272\,a^3\cos\left(dx+c\right)^2+345\,a^3\cos\left(dx+c\right)\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^3*cos(d*x + c)^4 + 230*
a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*a^
3)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32461, size = 173, normalized size = 1.34

$$\frac{345\,(dx+c)a^3 + \frac{2\left(345\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4554\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+5814\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3165\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+157}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(345*(d*x + c)*a^3 + 2*(345*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*a^3*ta
n(1/2*d*x + 1/2*c)^9 + 4554*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*a^3*tan(1/2*d
*x + 1/2*c)^5 + 3165*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*a^3*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.30 $\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=136

$$\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{41a$$

```
[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Se
c[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*
d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (
4*a^4*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.171728, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3791, 3768, 3770, 3767}

$$\frac{4a^4\tan^5(c+dx)}{5d}+\frac{4a^4\tan^3(c+dx)}{d}+\frac{8a^4\tan(c+dx)}{d}+\frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{41a$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Se
c[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*
d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (
4*a^4*Tan[c + d*x]^5)/(5*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec^3(c+dx)+4a^4\sec^4(c+dx)+6a^4\sec^5(c+dx)+4a^4\sec^6(c+dx)+\right.$$

$$= a^4\int \sec^3(c+dx)\,dx + a^4\int \sec^7(c+dx)\,dx + \left(4a^4\right)\int \sec^4(c+dx)\,dx + \left(4a^4\right.$$

$$= \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^4\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^4\sec^5(c+dx)\,\mathrm{ta}}{6d}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{11a^4\sec(c+dx)\tan(c+dx)}{4d} + \frac{41}{}$$

$$= \frac{11a^4\tanh^{-1}(\sin(c+dx))}{4d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+dx)}{16d} +$$

$$= \frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+dx)}{16d} +$$

Mathematica [A] time = 0.831869, size = 211, normalized size = 1.55

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(23520\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(23520*Cos[c +
 d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) - Sec[c]*(-11520*Sin[c] + 3750*Sin[d*x] + 3750*Sin[2*c +
 d*x] + 15360*Sin[c + 2*d*x] - 1920*Sin[3*c + 2*d*x] + 3845*Sin[2*c + 3*d*x
] + 3845*Sin[4*c + 3*d*x] + 6912*Sin[3*c + 4*d*x] + 735*Sin[4*c + 5*d*x] +
735*Sin[6*c + 5*d*x] + 1152*Sin[5*c + 6*d*x]))/(122880*d)
```

Maple [A] time = 0.043, size = 146, normalized size = 1.1

$$\frac{49\,a^{4}\sec\left(dx+c\right)\tan\left(dx+c\right)}{16\,d}+\frac{49\,a^{4}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{16\,d}+\frac{24\,a^{4}\tan\left(dx+c\right)}{5\,d}+\frac{12\,a^{4}\tan\left(dx+c\right)\left(\sec\left(dx+\right.\right.}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 49/16*a^4*sec(d*x+c)*tan(d*x+c)/d+49/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+24/
5*a^4*tan(d*x+c)/d+12/5/d*a^4*tan(d*x+c)*sec(d*x+c)^2+41/24*a^4*sec(d*x+c)^
3*tan(d*x+c)/d+4/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4+1/6*a^4*sec(d*x+c)^5*tan(d
*x+c)/d
```

Maxima [B] time = 0.998093, size = 365, normalized size = 2.68

$$128\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{4}+640\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{4}-5\,a^{4}\left(\frac{2\left(15\,\sin(dx+c\right.}{\sin(dx+c)^{6}-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 6
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 - 40
*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*s
in(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) -
 180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*a^4
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)))/d
```

Fricas [A] time = 1.76794, size = 366, normalized size = 2.69

$$\frac{735\,a^{4}\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-735\,a^{4}\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(1152\,a^{4}\cos\left(dx+c\right)^{5}+735\,a\right.}{480\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)^
6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x + c
)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x + c
) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4 \left(\int \sec^3(c+dx)\,dx + \int 4\sec^4(c+dx)\,dx + \int 6\sec^5(c+dx)\,dx + \int 4\sec^6(c+dx)\,dx + \int \sec^7(c+dx)\,dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Integ
ral(6*sec(c + d*x)**5, x) + Integral(4*sec(c + d*x)**6, x) + Integral(sec(c
 + d*x)**7, x))
```

Giac [A] time = 1.42317, size = 208, normalized size = 1.53

$$\frac{735\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 735\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(735\,a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{11} - 4165\,a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^9 + 9702\,a}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1/2
*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*x +
 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.31 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=111

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+7a^4\,\text{ta}$$

```
[Out] (7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*
Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.135141, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 3767, 8, 3768, 3770}

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+7a^4\,\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*
Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec^2(c+dx)+4a^4\sec^3(c+dx)+6a^4\sec^4(c+dx)+4a^4\sec^5(c+dx)+\right.$$

$$= a^4\int \sec^2(c+dx)\,dx + a^4\int \sec^6(c+dx)\,dx + \left(4a^4\right)\int \sec^3(c+dx)\,dx + \left(4a^4\right.$$

$$= \frac{2a^4\sec(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{d} + \left(2a^4\right)\int \sec(c+$$

$$= \frac{2a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx)}{2d} + $$

$$= \frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx)}{2d} + $$

Mathematica [B] time = 1.52693, size = 498, normalized size = 4.49

$$-\frac{a^4\sec(c)\sec^5(c+dx)\left(960\sin(2c+dx)-660\sin(c+2dx)-660\sin(3c+2dx)-1600\sin(2c+3dx)+60\sin(4c+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(a^4*Sec[c]*Sec[c + d*x]^5*(525*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*
Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1050*Cos[d*x]*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + 1050*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 525*Cos[2*c + 3*d*x]*Log[Cos[(
```

```
c + d*x)/2] + Sin[(c + d*x)/2]] - 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] - 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] - 105*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
 2360*Sin[d*x] + 960*Sin[2*c + d*x] - 660*Sin[c + 2*d*x] - 660*Sin[3*c + 2*
d*x] - 1600*Sin[2*c + 3*d*x] + 60*Sin[4*c + 3*d*x] - 210*Sin[3*c + 4*d*x] -
 210*Sin[5*c + 4*d*x] - 332*Sin[4*c + 5*d*x]))/(960*d)
```

Maple [A] time = 0.039, size = 123, normalized size = 1.1

$$\frac{83\,a^4\tan(dx+c)}{15\,d}+\frac{7\,a^4\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{7\,a^4\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+\frac{34\,a^4\tan(dx+c)(\sec(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 83/15*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*x+c)/d+7/2/d*a^4*ln(sec(d*x
+c)+tan(d*x+c))+34/15/d*a^4*tan(d*x+c)*sec(d*x+c)^2+a^4*sec(d*x+c)^3*tan(d*
x+c)/d+1/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 1.16907, size = 257, normalized size = 2.32

$$\frac{4\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^4+120\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4-15\,a^4\left(\frac{2\left(3\sin(dx+c)^3-\right.}{\sin(dx+c)^4-2\,\mathrm{s}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.75651, size = 325, normalized size = 2.93

$$\frac{105\,a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-105\,a^4\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(166\,a^4\cos(dx+c)^4+105\,a\right.}{60\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c)^
3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int\sec^2(c+dx)\,dx+\int 4\sec^3(c+dx)\,dx+\int 6\sec^4(c+dx)\,dx+\int 4\sec^5(c+dx)\,dx+\int\sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**2, x) + Integral(4*sec(c + d*x)**3, x) + Integ
ral(6*sec(c + d*x)**4, x) + Integral(4*sec(c + d*x)**5, x) + Integral(sec(c
 + d*x)**6, x))
```

Giac [A] time = 1.36585, size = 186, normalized size = 1.68

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+896\,a^4\,\mathrm{ta}\right.}{\left(\tan\left(\frac{1}{2}\,d\right.\right.}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*d*
```

```
x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/2*c
)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.32 $\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=96

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec(}{8d}$$

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
4*a^4*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.109265, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3791, 3770, 3767, 8, 3768}

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec(}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
4*a^4*Tan[c + d*x]^3)/(3*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sec(c+dx)+4a^4\sec^2(c+dx)+6a^4\sec^3(c+dx)+4a^4\sec^4(c+dx)+a^4\,\mathrm{s}\right.$$

$$= a^4\int \sec(c+dx)\,dx + a^4\int \sec^5(c+dx)\,dx + \left(4a^4\right)\int \sec^2(c+dx)\,dx + \left(4a^4\right)\int$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^4\sec(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^3(c+dx)\tan(c+}{4d}$$

$$= \frac{4a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^4}{}$$

$$= \frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^4}{}$$

Mathematica [B] time = 6.39469, size = 877, normalized size = 9.14

$$-\frac{35\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(\sec(c+dx)a+a)^4\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d}+\frac{35\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-35*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
(d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(128*d) + (35*Cos[c + d*x]^4*Log[Cos[c/2
 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])
^4)/(128*d) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/
(256*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^4*Sec[c
```

```
/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(24*d*(Cos[c/2] - Sin[
c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/
2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(
Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (5*Cos[
c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(12*d*
(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - (Cos[c +
 d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(256*d*(Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2])^4) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a
*Sec[c + d*x])^4*Sin[(d*x)/2])/(24*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*
Sec[c + d*x])^4*(-97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(Cos[c/2] + Sin[c/2])*
(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 +
(d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])
*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.037, size = 102, normalized size = 1.1

$$\frac{35\,a^4\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+\frac{20\,a^4\tan(dx+c)}{3\,d}+\frac{27\,a^4\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{4\,a^4\tan(dx+c)(\sec(dx}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 35/8/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x
+c)*tan(d*x+c)/d+4/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2+1/4*a^4*sec(d*x+c)^3*tan
(d*x+c)/d
```

Maxima [A] time = 1.13003, size = 236, normalized size = 2.46

$$64\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^4-3\,a^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
```

```
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4*log(sec(d
*x + c) + tan(d*x + c)) + 192*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.73283, size = 292, normalized size = 3.04

$$\frac{105\,a^4\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-105\,a^4\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(160\,a^4\cos\left(dx+c\right)^3+81\,a^4\cos\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^4
*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)^2
 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int\sec\left(c+dx\right)dx+\int 4\sec^2\left(c+dx\right)dx+\int 6\sec^3\left(c+dx\right)dx+\int 4\sec^4\left(c+dx\right)dx+\int\sec^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sec(c + d*x), x) + Integral(4*sec(c + d*x)**2, x) + Integral
(6*sec(c + d*x)**3, x) + Integral(4*sec(c + d*x)**4, x) + Integral(sec(c +
d*x)**5, x))
```

Giac [A] time = 1.3898, size = 165, normalized size = 1.72

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,a^4\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*d*
x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.33 $\int (a + a\sec(c + dx))^4 \, dx$

Optimal. Leaf size=91

$$\frac{5a^4\tan(c+dx)}{d} + \frac{6a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d} + \frac{4\tan(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{3d} +$$

```
[Out] a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (5*a^4*Tan[c + d*x])/d + ((a^2 +
a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + (4*(a^4 + a^4*Sec[c + d*x])*Tan[c
 + d*x])/(3*d)
```

Rubi [A] time = 0.0898229, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3775, 3917, 3914, 3767, 8, 3770}

$$\frac{5a^4\tan(c+dx)}{d} + \frac{6a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{3d} + \frac{4\tan(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{3d} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4,x]
```


```
[Out] a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (5*a^4*Tan[c + d*x])/d + ((a^2 +
a^2*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + (4*(a^4 + a^4*Sec[c + d*x])*Tan[c
 + d*x])/(3*d)
```


Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3917

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
 1))/(f*m), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m + (b
*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3914

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^4\,dx = \frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{1}{3}a\int (a+a\sec(c+dx))^2(3a+8a\sec(c+dx))\,dx$$

$$= \frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4\left(a^4+a^4\sec(c+dx)\right)\tan(c+dx)}{3d}+\frac{1}{6}a\int (a+a\sec$$

$$= a^4x+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4\left(a^4+a^4\sec(c+dx)\right)\tan(c+dx)}{3d}+\left(5a^4\right)\int \mathrm{s}$$

$$= a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4\left(a^4+a^4\sec(c+dx\right.}{3d}$$

$$= a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4\tan(c+dx)}{d}+\frac{\left(a^2+a^2\sec(c+dx)\right)^2\tan(c+dx)}{3d}+\frac{4}{-}$$

Mathematica [B] time = 6.24585, size = 773, normalized size = 8.49

$$\frac{1}{16}x\cos^4(c+dx)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^4+\frac{5\sin\left(\frac{dx}{2}\right)\cos^4(c+dx)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^4}{12d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{5\sin}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (x*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/16 - (3*Cos[
c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^
8*(a + a*Sec[c + d*x])^4)/(8*d) + (3*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2]
+ Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4)/(8*d) +
(Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Sin[(d*x)/2])/(
96*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (
Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(13*Cos[c/2] - 1
1*Sin[c/2]))/(192*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (
d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4
*Sin[(d*x)/2])/(12*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
(d*x)/2])) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*Si
n[(d*x)/2])/(96*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*
x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(-1
3*Cos[c/2] - 11*Sin[c/2]))/(192*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^2) + (5*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^8*(a + a*S
ec[c + d*x])^4*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.036, size = 93, normalized size = 1.

$$a^4x+\frac{a^4c}{d}+6\,\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{20\,a^4\tan(dx+c)}{3\,d}+2\,\frac{a^4\sec(dx+c)\tan(dx+c)}{d}+\frac{a^4\tan(dx+c)\,(s}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4,x)
```

```
[Out] a^4*x+1/d*a^4*c+6/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3*a^4*tan(d*x+c)/d+2*a
^4*sec(d*x+c)*tan(d*x+c)/d+1/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.10326, size = 157, normalized size = 1.73

$$a^4x+\frac{\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^4}{3\,d}-\frac{a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{d}+\frac{4\,a^4\log(\mathrm{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4/d - a^4*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d + 4*
a^4*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^4*tan(d*x + c)/d
```

Fricas [A] time = 1.79029, size = 281, normalized size = 3.09

$$\frac{3\,a^4dx\cos\left(dx+c\right)^3+9\,a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-9\,a^4\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(20\,a^4\cos\left(d\right.\right.}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1)
- 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 + 6*
a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int 1\,dx+\int 4\sec\left(c+dx\right)dx+\int 6\sec^2\left(c+dx\right)dx+\int 4\sec^3\left(c+dx\right)dx+\int\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(1, x) + Integral(4*sec(c + d*x), x) + Integral(6*sec(c + d*x
)**2, x) + Integral(4*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**4, x))
```

Giac [A] time = 1.3023, size = 157, normalized size = 1.73

$$\frac{3\left(dx+c\right)a^4+18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-38\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 38*a
^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^3)/d
```

3.34 $\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

```
[Out] 4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4*
a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0757867, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3791, 2637, 3770, 3767, 8, 3768}

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] 4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4*
a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(4a^4+a^4\cos(c+dx)+6a^4\sec(c+dx)+4a^4\sec^2(c+dx)+a^4\sec^3(c+dx)\right) d$$

$$= 4a^4x + a^4\int \cos(c+dx)\,dx + a^4\int \sec^3(c+dx)\,dx + \left(4a^4\right)\int \sec^2(c+dx)\,dx + ($$

$$= 4a^4x + \frac{6a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^4\sin(c+dx)}{d} + \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} +$$

$$= 4a^4x + \frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^4\sin(c+dx)}{d} + \frac{4a^4\tan(c+dx)}{d} + \frac{a^4\sec(c}{}$$

Mathematica [B] time = 1.47475, size = 272, normalized size = 3.73

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin(c)\cos(dx)}{d}+\frac{4\cos(c)\sin(dx)}{d}+\frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-s\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
```

```
^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))))/64
```

Maple [A] time = 0.061, size = 86, normalized size = 1.2

$$\frac{a^4 \sin(dx+c)}{d} + 4a^4x + 4\frac{a^4c}{d} + \frac{13a^4 \ln(\sec(dx+c) + \tan(dx+c))}{2d} + 4\frac{a^4 \tan(dx+c)}{d} + \frac{a^4 \sec(dx+c) \tan(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^4,x)
```

```
[Out] a^4*sin(d*x+c)/d+4*a^4*x+4/d*a^4*c+13/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a
^4*tan(d*x+c)/d+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

Maxima [A] time = 1.18637, size = 149, normalized size = 2.04

$$\frac{16(dx+c)a^4 - a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 12a^4(\log(\sin(dx+c)+1) - \log(\sin(}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.78313, size = 286, normalized size = 3.92

$$\frac{16a^4dx\cos(dx+c)^2 + 13a^4\cos(dx+c)^2\log(\sin(dx+c)+1) - 13a^4\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\left(2a^4\text{co}\right.}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) + 1
) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^2
+ 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40973, size = 174, normalized size = 2.38

$$\frac{8\,(dx+c)a^4+13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{2\left(7\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 \cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx \cdots\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.35 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

```
[Out] (13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (
a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

Rubi [A] time = 0.078554, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3791, 2637, 2635, 8, 3770, 3767}

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (
a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(6a^4+4a^4\cos(c+dx)+a^4\cos^2(c+dx)+4a^4\sec(c+dx)+a^4\sec^2(c+dx)\right)$$

$$= 6a^4x + a^4\int \cos^2(c+dx)\,dx + a^4\int \sec^2(c+dx)\,dx + \left(4a^4\right)\int \cos(c+dx)\,dx +$$

$$= 6a^4x + \frac{4a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{4a^4\sin(c+dx)}{d} + \frac{a^4\cos(c+dx)\sin(c+dx)}{2d}$$

$$= \frac{13a^4x}{2} + \frac{4a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{4a^4\sin(c+dx)}{d} + \frac{a^4\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [B] time = 1.77213, size = 241, normalized size = 3.3

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{16\sin(c)\cos(dx)}{d}+\frac{\sin(2c)\cos(2dx)}{d}+\frac{16\cos(c)\sin(dx)}{d}+\frac{\cos(2c)\sin(2dx)}{d}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x])/
```

```
d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

Maple [A] time = 0.061, size = 86, normalized size = 1.2

$$\frac{a^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{13a^4x}{2}+\frac{13a^4c}{2d}+4\frac{a^4\sin(dx+c)}{d}+4\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^4\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*cos(d*x+c)*sin(d*x+c)/d+13/2*a^4*x+13/2/d*a^4*c+4*a^4*sin(d*x+c)/d+
4/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*tan(d*x+c)/d
```

Maxima [A] time = 1.06882, size = 115, normalized size = 1.58

$$\frac{(2dx+2c+\sin(2dx+2c))a^4+24(dx+c)a^4+8a^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+16a^4\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*tan
(d*x + c))/d
```

Fricas [A] time = 1.7898, size = 270, normalized size = 3.7

$$\frac{13a^4dx\cos(dx+c)+4a^4\cos(dx+c)\log(\sin(dx+c)+1)-4a^4\cos(dx+c)\log(-\sin(dx+c)+1)+\left(a^4\cos(dx+c\right.}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 4
*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*cos(
d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41339, size = 174, normalized size = 2.38

$$\frac{13\,(dx+c)a^4 + 8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\left(7\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\cdots\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
+ 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.36 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=73

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

```
[Out] 6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*C
os[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0815262, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3791, 2637, 2635, 8, 2633, 3770}

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] 6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*C
os[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sec(c+dx))^4\,dx &= \int \left(4a^4+6a^4\cos(c+dx)+4a^4\cos^2(c+dx)+a^4\cos^3(c+dx)+a^4\sec(c+dx)\right)\\ &= 4a^4x+a^4\int \cos^3(c+dx)\,dx+a^4\int \sec(c+dx)\,dx+\left(4a^4\right)\int \cos^2(c+dx)\,dx+\\ &= 4a^4x+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{6a^4\sin(c+dx)}{d}+\frac{2a^4\cos(c+dx)\sin(c+dx)}{d}\\ &= 6a^4x+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{7a^4\sin(c+dx)}{d}+\frac{2a^4\cos(c+dx)\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.103329, size = 91, normalized size = 1.25

$$\frac{a^4\left(81\sin(c+dx)+12\sin(2(c+dx))+\sin(3(c+dx))-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(\sin\left(\frac{1}{2}(c+d\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + Sin
[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.067, size = 94, normalized size = 1.3

$$\frac{\sin(dx+c)(\cos(dx+c))^2a^4}{3d}+\frac{20a^4\sin(dx+c)}{3d}+2\frac{a^4\cos(dx+c)\sin(dx+c)}{d}+6a^4x+6\frac{a^4c}{d}+\frac{a^4\ln(\sec(dx+c)+\ldots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*si
n(d*x+c)/d+6*a^4*x+6/d*a^4*c+1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.08054, size = 131, normalized size = 1.79

$$-\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4-6(2dx+2c+\sin(2dx+2c))a^4-24(dx+c)a^4-3a^4(\log(\sin(dx+c)+1)-\ldots}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^4 - 24*(d*x + c)*a^4 - 3*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x
 + c) - 1)) - 36*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.81348, size = 201, normalized size = 2.75

$$\frac{36a^4dx+3a^4\log(\sin(dx+c)+1)-3a^4\log(-\sin(dx+c)+1)+2\left(a^4\cos(dx+c)^2+6a^4\cos(dx+c)+20a^4\right)\sin(\ldots}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1
) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39801, size = 157, normalized size = 2.15

$$\frac{18\,(dx+c)a^4+3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+38\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(18*(d*x + c)*a^4 + 3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 + 38*a^
4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^3)/d
```

3.37 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=87

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

```
[Out] (35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3
*d)
```

Rubi [A] time = 0.098184, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 2633}

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3
*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4+4a^4\cos(c+dx)+6a^4\cos^2(c+dx)+4a^4\cos^3(c+dx)+a^4\cos^4(c+dx)\right.$$

$$= a^4x+a^4\int \cos^4(c+dx)\,dx+\left(4a^4\right)\int \cos(c+dx)\,dx+\left(4a^4\right)\int \cos^3(c+dx)\,dx$$

$$= a^4x+\frac{4a^4\sin(c+dx)}{d}+\frac{3a^4\cos(c+dx)\sin(c+dx)}{d}+\frac{a^4\cos^3(c+dx)\sin(c+d}{4d}$$

$$= 4a^4x+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c+dx)\sin(c+}{4d}$$

$$= \frac{35a^4x}{8}+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c+dx)\sin(c}{4d}$$

Mathematica [A] time = 0.100717, size = 56, normalized size = 0.64

$$\frac{a^4(672\sin(c+dx)+168\sin(2(c+dx))+32\sin(3(c+dx))+3\sin(4(c+dx))+420c+420dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3*
(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.074, size = 111, normalized size = 1.3

$$\frac{1}{d}\left(a^{4}\left(\frac{\sin\left(dx+c\right)}{4}\left(\left(\cos\left(dx+c\right)\right)^{3}+\frac{3\cos\left(dx+c\right)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^{4}\left(\left(\cos\left(dx+c\right)\right)^{2}+2\right)\sin\left(dx+c\right)}{3}+6a^{4}(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a
^4*(cos(d*x+c)^2+2)*sin(d*x+c)+6*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+4*a^4*sin(d*x+c)+a^4*(d*x+c))
```

Maxima [A] time = 1.09911, size = 140, normalized size = 1.61

$$-\frac{128\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)a^{4}-3\left(12dx+12c+\sin\left(4dx+4c\right)+8\sin\left(2dx+2c\right)\right)a^{4}-144\left(2dx+2c+\sin\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 3*(12*d*x + 12*c + sin(4
*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 144*(2*d*x + 2*c + sin(2*d*x + 2*c)
)*a^4 - 96*(d*x + c)*a^4 - 384*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.71224, size = 157, normalized size = 1.8

$$\frac{105a^{4}dx+\left(6a^{4}\cos\left(dx+c\right)^{3}+32a^{4}\cos\left(dx+c\right)^{2}+81a^{4}\cos\left(dx+c\right)+160a^{4}\right)\sin\left(dx+c\right)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4*
cos(d*x + c) + 160*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41584, size = 130, normalized size = 1.49

$$\frac{105\,(dx+c)a^4+\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+279\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*a^4*tan(1
/2*d*x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*a^4*tan(1/2*d*x +
1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.38 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=102

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^4}{2}$$

```
[Out] (7*a^4*x)/2 + (8*a^4*Sin[c + d*x])/d + (7*a^4*Cos[c + d*x]*Sin[c + d*x])/(2
*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d - (8*a^4*Sin[c + d*x]^3)/(3*d) +
(a^4*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.11388, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3791, 2637, 2635, 8, 2633}

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^4}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (7*a^4*x)/2 + (8*a^4*Sin[c + d*x])/d + (7*a^4*Cos[c + d*x]*Sin[c + d*x])/(2
*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d - (8*a^4*Sin[c + d*x]^3)/(3*d) +
(a^4*Sin[c + d*x]^5)/(5*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos(c+dx)+4a^4\cos^2(c+dx)+6a^4\cos^3(c+dx)+4a^4\cos^4(c+dx)+a^4\right.$$

$$= a^4\int \cos(c+dx)\,dx + a^4\int \cos^5(c+dx)\,dx + \left(4a^4\right)\int \cos^2(c+dx)\,dx + \left(4a^4\right)$$

$$= \frac{a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\sin(c+dx)}{d} + \frac{a^4\cos^3(c+dx)\sin(c+dx)}{d} + (2a$$

$$= 2a^4x + \frac{8a^4\sin(c+dx)}{d} + \frac{7a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^3(c+dx)\sin(c+}{d}$$

$$= \frac{7a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{7a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^3(c+dx)\sin(c+}{d}$$

Mathematica [A] time = 0.146386, size = 63, normalized size = 0.62

$$\frac{a^4(1470\sin(c+dx)+480\sin(2(c+dx))+145\sin(3(c+dx))+30\sin(4(c+dx))+3\sin(5(c+dx))+840dx)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c + d
*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.081, size = 133, normalized size = 1.3

$$\frac{1}{d}\left(\frac{a^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)+4\,a^4\left(1/4\,\left((\cos(dx+c))^3+3/2\,\cos(dx+c)\right)\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^4*(cos(d*x+c)^2+2)*
sin(d*x+c)+4*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*sin(d*x+c))
```

Maxima [A] time = 1.15616, size = 173, normalized size = 1.7

$$\frac{8\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)a^4-240\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^4+15\,(12\,dx+12\,c+\mathrm{si}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 240
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 1
20*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.6863, size = 190, normalized size = 1.86

$$\frac{105\,a^4dx+\left(6\,a^4\cos(dx+c)^4+30\,a^4\cos(dx+c)^3+68\,a^4\cos(dx+c)^2+105\,a^4\cos(dx+c)+166\,a^4\right)\sin(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4*
cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38698, size = 151, normalized size = 1.48

$$\frac{105\,(dx+c)a^4+\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+896\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+790\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+375\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(105*(d*x + c)*a^4 + 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 + 490*a^4*tan(1
/2*d*x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 + 790*a^4*tan(1/2*d*x +
1/2*c)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.39 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=127

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}+$$

```
[Out] (49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)/(5
*d)
```

Rubi [A] time = 0.1453, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3791, 2635, 8, 2633}

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)/(5
*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos^2(c+dx)+4a^4\cos^3(c+dx)+6a^4\cos^4(c+dx)+4a^4\cos^5(c+dx)+a\right.$$
$$= a^4\int\cos^2(c+dx)\,dx + a^4\int\cos^6(c+dx)\,dx + \left(4a^4\right)\int\cos^3(c+dx)\,dx + \left(4a^4\right)$$
$$= \frac{a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^4\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^5(c+dx)\sin(}{6d}$$
$$= \frac{a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{41a^4\cos^3(c+dx)\sin(c}{24d}$$
$$= \frac{11a^4x}{4} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\cos^3(c+dx)\sin}{24d}$$
$$= \frac{49a^4x}{16} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\cos^3(c+dx)\sin}{24d}$$

Mathematica [A] time = 0.207286, size = 73, normalized size = 0.57

$$\frac{a^4(5280\sin(c+dx)+1905\sin(2(c+dx))+720\sin(3(c+dx))+225\sin(4(c+dx))+48\sin(5(c+dx))+5\sin(6(c+dx}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c +
 d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/
(960*d)
```

Maple [A] time = 0.088, size = 169, normalized size = 1.3

$$\frac{1}{d}\left(a^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a^4
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a^4*(cos(
d*x+c)^2+2)*sin(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.09748, size = 223, normalized size = 1.76

$$\frac{256\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^4-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + sin(2*
d*x + 2*c))*a^4)/d
```

Fricas [A] time = 1.73505, size = 231, normalized size = 1.82

$$\frac{735a^4dx+\left(40a^4\cos(dx+c)^5+192a^4\cos(dx+c)^4+410a^4\cos(dx+c)^3+576a^4\cos(dx+c)^2+735a^4\cos(dx+c\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(735*a^4*d*x + (40*a^4*cos(d*x + c)^5 + 192*a^4*cos(d*x + c)^4 + 410*
a^4*cos(d*x + c)^3 + 576*a^4*cos(d*x + c)^2 + 735*a^4*cos(d*x + c) + 1152*a
^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38852, size = 173, normalized size = 1.36

$$\frac{735\,(dx+c)a^4 + \frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+9702\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+11802\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7355\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3105\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(735*(d*x + c)*a^4 + 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 + 4165*a^4*ta
n(1/2*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 + 11802*a^4*tan(1/2*
d*x + 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 + 3105*a^4*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.40 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=147

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{9a^4\sin^5(c+dx)}{5d}-\frac{16a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{11a^4\sin(c$$

```
[Out] (11*a^4*x)/4 + (8*a^4*Sin[c + d*x])/d + (11*a^4*Cos[c + d*x]*Sin[c + d*x])/
(4*d) + (11*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (2*a^4*Cos[c + d*x]^5*
Sin[c + d*x])/(3*d) - (16*a^4*Sin[c + d*x]^3)/(3*d) + (9*a^4*Sin[c + d*x]^5
)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.154827, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3791, 2633, 2635, 8}

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{9a^4\sin^5(c+dx)}{5d}-\frac{16a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{2a^4\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{11a^4\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```



```
[Out] (11*a^4*x)/4 + (8*a^4*Sin[c + d*x])/d + (11*a^4*Cos[c + d*x]*Sin[c + d*x])/
(4*d) + (11*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (2*a^4*Cos[c + d*x]^5*
Sin[c + d*x])/(3*d) - (16*a^4*Sin[c + d*x]^3)/(3*d) + (9*a^4*Sin[c + d*x]^5
)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4\,dx = \int \left(a^4\cos^3(c+dx)+4a^4\cos^4(c+dx)+6a^4\cos^5(c+dx)+4a^4\cos^6(c+dx)+a$$

$$= a^4\int \cos^3(c+dx)\,dx + a^4\int \cos^7(c+dx)\,dx + \left(4a^4\right)\int \cos^4(c+dx)\,dx + \left(4a^4\right)$$

$$= \frac{a^4\cos^3(c+dx)\sin(c+dx)}{d} + \frac{2a^4\cos^5(c+dx)\sin(c+dx)}{3d} + \left(3a^4\right)\int \cos^2(c+d$$

$$= \frac{8a^4\sin(c+dx)}{d} + \frac{3a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{11a^4\cos^3(c+dx)\sin(c+dx)}{6d} +$$

$$= \frac{3a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{11a^4\cos^3(c+dx)\sin(c$$

$$= \frac{11a^4x}{4} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{11a^4\cos^3(c+dx)\sin($$

Mathematica [A] time = 0.26099, size = 83, normalized size = 0.56

$$\frac{a^4(33915\sin(c+dx)+13020\sin(2(c+dx))+5495\sin(3(c+dx))+2100\sin(4(c+dx))+651\sin(5(c+dx))+140\sin(6}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*(18480*d*x + 33915*Sin[c + d*x] + 13020*Sin[2*(c + d*x)] + 5495*Sin[3*
(c + d*x)] + 2100*Sin[4*(c + d*x)] + 651*Sin[5*(c + d*x)] + 140*Sin[6*(c +
d*x)] + 15*Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.128, size = 185, normalized size = 1.3

$$\frac{1}{d}\left(\frac{a^4\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6\,(\cos(dx+c))^4}{5}+\frac{8\,(\cos(dx+c))^2}{5}\right)+4\,a^4\left(1/6\,\left((\cos(dx+c))^5+5/4\,(\cos(d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(1/7*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+
c)+4*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+6/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^4*(cos(
d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A] time = 1.16521, size = 252, normalized size = 1.71

$$-\frac{48\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)a^4-672\left(3\sin(dx+c)^5-10\sin(dx+c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1680*(48*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*
sin(d*x + c))*a^4 - 672*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x
+ c))*a^4 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) -
 48*sin(2*d*x + 2*c))*a^4 + 560*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 210
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4)/d
```

Fricas [A] time = 1.73657, size = 267, normalized size = 1.82

$$\frac{1155\,a^4dx+\left(60\,a^4\cos(dx+c)^6+280\,a^4\cos(dx+c)^5+576\,a^4\cos(dx+c)^4+770\,a^4\cos(dx+c)^3+908\,a^4\cos(dx+\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/420*(1155*a^4*d*x + (60*a^4*cos(d*x + c)^6 + 280*a^4*cos(d*x + c)^5 + 576
*a^4*cos(d*x + c)^4 + 770*a^4*cos(d*x + c)^3 + 908*a^4*cos(d*x + c)^2 + 115
5*a^4*cos(d*x + c) + 1816*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2765, size = 194, normalized size = 1.32

$$\frac{1155\,(dx+c)a^4 + \frac{2\left(1155\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+7700\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+21791\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+33792\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+31521\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/420*(1155*(d*x + c)*a^4 + 2*(1155*a^4*tan(1/2*d*x + 1/2*c)^13 + 7700*a^4*
tan(1/2*d*x + 1/2*c)^11 + 21791*a^4*tan(1/2*d*x + 1/2*c)^9 + 33792*a^4*tan(
1/2*d*x + 1/2*c)^7 + 31521*a^4*tan(1/2*d*x + 1/2*c)^5 + 14700*a^4*tan(1/2*d
*x + 1/2*c)^3 + 5565*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^7)/d
```

3.41 $\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx$

Optimal. Leaf size=156

$$\frac{a^5\tan^7(c+dx)}{7d}+\frac{13a^5\tan^5(c+dx)}{5d}+\frac{28a^5\tan^3(c+dx)}{3d}+\frac{16a^5\tan(c+dx)}{d}+\frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5a^5\tan(c$$

```
[Out] (93*a^5*ArcTanh[Sin[c + d*x]])/(16*d) + (16*a^5*Tan[c + d*x])/d + (93*a^5*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (85*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24
*d) + (5*a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (28*a^5*Tan[c + d*x]^3)/(
3*d) + (13*a^5*Tan[c + d*x]^5)/(5*d) + (a^5*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.197865, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3791, 3768, 3770, 3767}

$$\frac{a^5\tan^7(c+dx)}{7d}+\frac{13a^5\tan^5(c+dx)}{5d}+\frac{28a^5\tan^3(c+dx)}{3d}+\frac{16a^5\tan(c+dx)}{d}+\frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5a^5\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (93*a^5*ArcTanh[Sin[c + d*x]])/(16*d) + (16*a^5*Tan[c + d*x])/d + (93*a^5*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (85*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24
*d) + (5*a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (28*a^5*Tan[c + d*x]^3)/(
3*d) + (13*a^5*Tan[c + d*x]^5)/(5*d) + (a^5*Tan[c + d*x]^7)/(7*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^5\,dx = \int \left(a^5\sec^3(c+dx)+5a^5\sec^4(c+dx)+10a^5\sec^5(c+dx)+10a^5\sec^6(c+dx)+\right.$$

$$= a^5\int \sec^3(c+dx)\,dx + a^5\int \sec^8(c+dx)\,dx + \left(5a^5\right)\int \sec^4(c+dx)\,dx + \left(5a^5\right)$$

$$= \frac{a^5\sec(c+dx)\tan(c+dx)}{2d} + \frac{5a^5\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{5a^5\sec^5(c+dx)\,\mathrm{ta}}{6d}$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d} + \frac{16a^5\tan(c+dx)}{d} + \frac{17a^5\sec(c+dx)\tan(c+dx)}{4d} + \frac{85}{}$$

$$= \frac{17a^5\tanh^{-1}(\sin(c+dx))}{4d} + \frac{16a^5\tan(c+dx)}{d} + \frac{93a^5\sec(c+dx)\tan(c+dx)}{16d} +$$

$$= \frac{93a^5\tanh^{-1}(\sin(c+dx))}{16d} + \frac{16a^5\tan(c+dx)}{d} + \frac{93a^5\sec(c+dx)\tan(c+dx)}{16d} +$$

Mathematica [A] time = 1.34252, size = 229, normalized size = 1.47

$$-\frac{a^5(\cos(c+dx)+1)^5\sec^{10}\left(\frac{1}{2}(c+dx)\right)\sec^7(c+dx)\left(624960\cos^7(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -(a^5*(1 + Cos[c + d*x])^5*Sec[(c + d*x)/2]^10*Sec[c + d*x]^7*(624960*Cos[c
 + d*x]^7*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) - Sec[c]*(374080*Sin[d*x] - 162400*Sin[2*c + d*x] + 11
8825*Sin[c + 2*d*x] + 118825*Sin[3*c + 2*d*x] + 305088*Sin[2*c + 3*d*x] - 1
6800*Sin[4*c + 3*d*x] + 62860*Sin[3*c + 4*d*x] + 62860*Sin[5*c + 4*d*x] + 1
07296*Sin[4*c + 5*d*x] + 9765*Sin[5*c + 6*d*x] + 9765*Sin[7*c + 6*d*x] + 15
```

```
328*Sin[6*c + 7*d*x])))/(3440640*d)
```

Maple [A] time = 0.046, size = 168, normalized size = 1.1

$$\frac{93\,a^5\sec(dx+c)\tan(dx+c)}{16\,d}+\frac{93\,a^5\ln(\sec(dx+c)+\tan(dx+c))}{16\,d}+\frac{958\,a^5\tan(dx+c)}{105\,d}+\frac{479\,a^5\tan(dx+c)(\sec}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x)
```

```
[Out] 93/16*a^5*sec(d*x+c)*tan(d*x+c)/d+93/16/d*a^5*ln(sec(d*x+c)+tan(d*x+c))+958
/105*a^5*tan(d*x+c)/d+479/105/d*a^5*tan(d*x+c)*sec(d*x+c)^2+85/24*a^5*sec(d
*x+c)^3*tan(d*x+c)/d+76/35/d*a^5*tan(d*x+c)*sec(d*x+c)^4+5/6*a^5*sec(d*x+c)
^5*tan(d*x+c)/d+1/7/d*a^5*tan(d*x+c)*sec(d*x+c)^6
```

Maxima [B] time = 1.12852, size = 424, normalized size = 2.72

$$96\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)^3+35\tan(dx+c)\right)a^5+2240\left(3\tan(dx+c)^5+10\tan(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/3360*(96*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*a^5 + 2240*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x
+ c))*a^5 + 5600*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 - 175*a^5*(2*(15*sin
(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(
d*x + c) - 1)) - 2100*a^5*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.80474, size = 413, normalized size = 2.65

$$\frac{9765\,a^5\cos(dx+c)^7\log(\sin(dx+c)+1) - 9765\,a^5\cos(dx+c)^7\log(-\sin(dx+c)+1) + 2\left(15328\,a^5\cos(dx+c)^6 + 9\right.}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3360*(9765*a^5*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 9765*a^5*cos(d*x +
c)^7*log(-sin(d*x + c) + 1) + 2*(15328*a^5*cos(d*x + c)^6 + 9765*a^5*cos(d*
x + c)^5 + 7664*a^5*cos(d*x + c)^4 + 5950*a^5*cos(d*x + c)^3 + 3648*a^5*cos
(d*x + c)^2 + 1400*a^5*cos(d*x + c) + 240*a^5)*sin(d*x + c))/(d*cos(d*x + c
)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^5\left(\int \sec^3(c+dx)\,dx + \int 5\sec^4(c+dx)\,dx + \int 10\sec^5(c+dx)\,dx + \int 10\sec^6(c+dx)\,dx + \int 5\sec^7(c+dx)\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**5,x)
```

```
[Out] a**5*(Integral(sec(c + d*x)**3, x) + Integral(5*sec(c + d*x)**4, x) + Integ
ral(10*sec(c + d*x)**5, x) + Integral(10*sec(c + d*x)**6, x) + Integral(5*s
ec(c + d*x)**7, x) + Integral(sec(c + d*x)**8, x))
```

Giac [A] time = 1.47336, size = 230, normalized size = 1.47

$$\frac{9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 9765\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(9765\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 65100\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 1\right.}{}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1680*(9765*a^5*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9765*a^5*log(abs(tan(
1/2*d*x + 1/2*c) - 1)) - 2*(9765*a^5*tan(1/2*d*x + 1/2*c)^13 - 65100*a^5*ta
n(1/2*d*x + 1/2*c)^11 + 184233*a^5*tan(1/2*d*x + 1/2*c)^9 - 285696*a^5*tan(
1/2*d*x + 1/2*c)^7 + 260183*a^5*tan(1/2*d*x + 1/2*c)^5 - 132020*a^5*tan(1/2
*d*x + 1/2*c)^3 + 43995*a^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 -
 1)^7)/d
```

3.42 $\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=103

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(2*a*d) + (4*Tan[c + d*x])/(a*d) - (3*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c +
d*x])) + (4*Tan[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.098244, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3818, 3787, 3768, 3770, 3767}

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```


```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(2*a*d) + (4*Tan[c + d*x])/(a*d) - (3*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c +
d*x])) + (4*Tan[c + d*x]^3)/(3*a*d)
```


Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^3(c+dx)(3a-4a\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec^3(c+dx)\,dx}{a} + \frac{4\int \sec^4(c+dx)\,dx}{a}$$
$$= -\frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec(c+dx)\,dx}{2a} - \frac{4\,\mathrm{Subst}\left(\int \left(1\right.\right.}{}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{4\tan(c+dx)}{ad} - \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^3(c+dx)\tan(c+}{d(a+a\sec(c+dx)}$$

Mathematica [B] time = 3.19851, size = 374, normalized size = 3.63

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(6\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{8}\sec(c)\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-12\sin(2c+dx)-6\sin(c+2dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*(6*Sec[c/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2]
*Sec[c]*Sec[c + d*x]^3*(9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 27
```

```
*Cos[d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]) + 27*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 9*Cos[2*c + 3*d*x]*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + 48*Sin[d*x] - 12*Sin[2*c + d*x] - 6*Sin[c + 2*d
*x] - 6*Sin[3*c + 2*d*x] + 20*Sin[2*c + 3*d*x]))/8))/(3*a*d*(1 + Sec[c + d*
x]))
```

Maple [A] time = 0.043, size = 183, normalized size = 1.8

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{5}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{3}{2da}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)-1/3/a/d/(tan(1/2*d*x+1/2*c)+1)^3+1/a/d/(tan(1/2*d*
x+1/2*c)+1)^2-5/2/a/d/(tan(1/2*d*x+1/2*c)+1)-3/2/a/d*ln(tan(1/2*d*x+1/2*c)+
1)-1/3/a/d/(tan(1/2*d*x+1/2*c)-1)^3-1/a/d/(tan(1/2*d*x+1/2*c)-1)^2-5/2/a/d/
(tan(1/2*d*x+1/2*c)-1)+3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.21708, size = 277, normalized size = 2.69

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
+ 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(d*x
 + c) + 1)))/d
```

Fricas [A] time = 2.02354, size = 331, normalized size = 3.21

$$-\frac{9\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx+c\right)+1\right)-9\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(1\right.}{12\left(ad\cos\left(dx+c\right)^{4}+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(9*(cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 9*(cos(d
*x + c)^4 + cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*cos(d*x + c)^3 +
 7*cos(d*x + c)^2 - cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a
*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^{5}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.36813, size = 154, normalized size = 1.5

$$-\frac{\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{3}a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a - 6*tan(1/2*d*x + 1/2*c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^5 - 16*t
an(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 -
1)^3*a))/d
```

3.43 $\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*Tan[c + d*x])/(a*d) + (3*Sec[c + d*x
]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d
*x]))
```

Rubi [A] time = 0.0927384, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3818, 3787, 3767, 8, 3768, 3770}

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*Tan[c + d*x])/(a*d) + (3*Sec[c + d*x
]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d
*x]))
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \sec^2(c+dx)(2a-3a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{2\int \sec^2(c+dx)\,dx}{a}+\frac{3\int \sec^3(c+dx)\,dx}{a}\\ &= \frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{3\int \sec(c+dx)\,dx}{2a}+\frac{2\,\mathrm{Subst}(\int 1\,dx, x\ldots}{a\ldots}\\ &= \frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{2\tan(c+dx)}{ad}+\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [B] time = 1.33355, size = 250, normalized size = 2.94

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\ldots}\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]*(-4*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]
*(-6*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^(-2) - (4*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])))))/(2*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.039, size = 143, normalized size = 1.7

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{3}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{3}{2da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{2da}\left(t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2+3/2/a/d/(tan(1/2
*d*x+1/2*c)+1)+3/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)+1/2/a/d/(tan(1/2*d*x+1/2*c)
-1)^2+3/2/a/d/(tan(1/2*d*x+1/2*c)-1)-3/2/a/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.14508, size = 219, normalized size = 2.58

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) + 1)
))/d
```

Fricas [A] time = 1.95933, size = 301, normalized size = 3.54

$$\frac{3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(4\cos\right.}{4\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x
 + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c)^2 + co
s(d*x + c) - 1)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^4\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.42713, size = 136, normalized size = 1.6

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/a - 2*tan(1/2*d*x + 1/2*c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.44 $\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=51

$$\frac{\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a*d)) + Tan[c + d*x]/(a*d) + Tan[c + d*x]/(d*(a +
a*Sec[c + d*x]))
```

Rubi [A] time = 0.105528, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3790, 3789, 3770, 3794}

$$\frac{\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}+\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a*d)) + Tan[c + d*x]/(a*d) + Tan[c + d*x]/(d*(a +
a*Sec[c + d*x]))
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{\tan(c+dx)}{ad} - \int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= \frac{\tan(c+dx)}{ad} - \frac{\int \sec(c+dx)\,dx}{a} + \int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= -\frac{\tanh^{-1}(\sin(c+dx))}{ad} + \frac{\tan(c+dx)}{ad} + \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}
\end{aligned}
$$

Mathematica [B] time = 0.699149, size = 194, normalized size = 3.8

$$
\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}\right.\right.}\right.\right.}{ad(\sec(c+dx)+}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]*(Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(a*d
*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.033, size = 99, normalized size = 1.9

$$
\frac{1}{ad}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{1}{ad}\ln\left(\tan\left(\frac{dx}{2}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)-1/a/d/(tan(1/2*d*x+1/2*c)+1)-1/a/d*ln(tan(1/2*d*x+
1/2*c)+1)-1/a/d/(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.14049, size = 161, normalized size = 3.16

$$-\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.99497, size = 266, normalized size = 5.22

$$-\frac{\left(\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left(\cos(dx+c)^2+\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2\,(2\,\cos}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c)
^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*x
+ c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.3932, size = 113, normalized size = 2.22

$$-\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a))/d
```

3.45 $\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad} - \frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.067653, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3789, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad} - \frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\int \sec(c+dx)\,dx}{a} - \int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{ad} - \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 0.165871, size = 109, normalized size = 2.87

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(\right.\right.\right.\right.}{ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]*(Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*S
in[(d*x)/2]))/(a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.025, size = 58, normalized size = 1.5

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)+1/a/d*ln(tan(1/2*d
*x+1/2*c)+1)
```

Maxima [A] time = 1.02375, size = 101, normalized size = 2.66

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.89611, size = 181, normalized size = 4.76

$$\frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1)-(\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2\sin(dx+c)}{2\left(ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.34261, size = 73, normalized size = 1.92

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/
a - tan(1/2*d*x + 1/2*c)/a)/d
```

3.46 $\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.0238029, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3794}

$$\frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx = \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [A] time = 0.0257344, size = 17, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] Tan[(c + d*x)/2]/(a*d)
```

Maple [A] time = 0.027, size = 17, normalized size = 0.8

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.08566, size = 31, normalized size = 1.41

$$\frac{\sin\left(dx+c\right)}{ad(\cos\left(dx+c\right)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(a*d*(cos(d*x + c) + 1))
```

Fricas [A] time = 1.52776, size = 53, normalized size = 2.41

$$\frac{\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.26239, size = 22, normalized size = 1.

$$\frac{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] tan(1/2*d*x + 1/2*c)/(a*d)
```

3.47 $\int \frac{1}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{a} - \frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

```
[Out] x/a - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.0135317, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3777, 8}

$$\frac{x}{a} - \frac{\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-1),x]
```

```
[Out] x/a - Tan[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+a\sec(c+dx)}\,dx &= -\frac{\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int a\,dx}{a^2}\\ &= \frac{x}{a} - \frac{\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.122824, size = 58, normalized size = 2.

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(dx\cos\left(c+\frac{dx}{2}\right)-2\sin\left(\frac{dx}{2}\right)+dx\cos\left(\frac{dx}{2}\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-1),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(d*x*Cos[(d*x)/2] + d*x*Cos[c + (d*x)/2] - 2*Sin
[(d*x)/2]))/(2*a*d)
```

Maple [A] time = 0.032, size = 37, normalized size = 1.3

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.6481, size = 66, normalized size = 2.28

$$\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c
) + 1)))/d
```

Fricas [A] time = 1.59883, size = 89, normalized size = 3.07

$$\frac{dx\cos(dx+c)+dx-\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.29995, size = 38, normalized size = 1.31

$$\frac{\frac{dx+c}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d
```

3.48 $\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x}{a}$$

```
[Out] -(x/a) + (2*Sin[c + d*x])/(a*d) - Sin[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.0571631, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3819, 3787, 2637, 8}

$$\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(x/a) + (2*Sin[c + d*x])/(a*d) - Sin[c + d*x]/(d*(a + a*Sec[c + d*x]))
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos(c+dx)(-2a+a\sec(c+dx))\,dx}{a^2}$$
$$= -\frac{\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int 1\,dx}{a} + \frac{2\int \cos(c+dx)\,dx}{a}$$
$$= -\frac{x}{a} + \frac{2\sin(c+dx)}{ad} - \frac{\sin(c+dx)}{d(a+a\sec(c+dx))}$$

Mathematica [B] time = 0.222399, size = 89, normalized size = 2.02

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{3dx}{2}\right)-2dx\cos\left(c+\frac{dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)-2dx\cos\left(\frac{dx}{2}\right)\right)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x)/
2]))/(4*a*d)
```

Maple [A] time = 0.051, size = 68, normalized size = 1.6

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\tan(1/2\,dx+c/2)}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)+2/d/a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-
2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.65903, size = 124, normalized size = 2.82

$$-\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos
(d*x + c) + 1)))/d
```

Fricas [A] time = 1.66372, size = 116, normalized size = 2.64

$$-\frac{dx\cos(dx+c)+dx-(\cos(dx+c)+2)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.22235, size = 78, normalized size = 1.77

$$-\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d
*x + 1/2*c)^2 + 1)*a))/d
```

3.49 $\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sin(c+dx)}{ad}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3x}{2a}$$

```
[Out] (3*x)/(2*a) - (2*Sin[c + d*x])/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) - (Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.0816637, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3819, 3787, 2635, 8, 2637}

$$-\frac{2\sin(c+dx)}{ad}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sin[c + d*x])/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) - (Cos[c + d*x]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \cos^2(c+dx)(-3a+2a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{2\int \cos(c+dx)\,dx}{a}+\frac{3\int \cos^2(c+dx)\,dx}{a}\\ &= -\frac{2\sin(c+dx)}{ad}+\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{3\int 1\,dx}{2a}\\ &= \frac{3x}{2a}-\frac{2\sin(c+dx)}{ad}+\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.232865, size = 117, normalized size = 1.58

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-4\sin\left(c+\frac{dx}{2}\right)-3\sin\left(c+\frac{3dx}{2}\right)-3\sin\left(2c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{5dx}{2}\right)+12dx\cos\right.}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2] -
 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c +
(3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2]))/(16*a*d)
```

Maple [A] time = 0.053, size = 103, normalized size = 1.4

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{3}}{da\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{2}}-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}+3\,\frac{\arctan\left(\tan\left(1/\right.\right.}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*
c)^3-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+3/a/d*arctan(tan(1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.75684, size = 180, normalized size = 2.43

$$-\frac{\frac{\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{3\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}}{a+\frac{2\,a\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{a\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}}-\frac{3\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a}+\frac{\sin\left(dx+c\right)}{a\left(\cos\left(dx+c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.63877, size = 149, normalized size = 2.01

$$\frac{3\,dx\cos\left(dx+c\right)+3\,dx+\left(\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*d*x*cos(d*x + c) + 3*d*x + (cos(d*x + c)^2 - cos(d*x + c) - 4)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.28448, size = 99, normalized size = 1.34

$$\frac{\frac{3(dx+c)}{a} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} - \frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.50 $\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3x}{2a}$$

```
[Out] (-3*x)/(2*a) + (4*Sin[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*
d) - (Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) - (4*Sin[c + d*
x]^3)/(3*a*d)
```

Rubi [A] time = 0.0903989, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3819, 3787, 2633, 2635, 8}

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sin[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*
d) - (Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Sec[c + d*x])) - (4*Sin[c + d*
x]^3)/(3*a*d)
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \cos^3(c+dx)(-4a+3a\sec(c+dx))\,dx}{a^2}\\ &= -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{3\int \cos^2(c+dx)\,dx}{a}+\frac{4\int \cos^3(c+dx)\,dx}{a}\\ &= -\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{3\int 1\,dx}{2a}-\frac{4\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cdot\right.}{ad}\\ &= -\frac{3x}{2a}+\frac{4\sin(c+dx)}{ad}-\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{4\sin^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.310296, size = 143, normalized size = 1.52

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(21\sin\left(c+\frac{dx}{2}\right)+18\sin\left(c+\frac{3dx}{2}\right)+18\sin\left(2c+\frac{3dx}{2}\right)-2\sin\left(2c+\frac{5dx}{2}\right)-2\sin\left(3c+\frac{5dx}{2}\right)+\sin\left(\right.\right.}{48ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
+ 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[2*
c + (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3*c
+ (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

Maple [A] time = 0.057, size = 136, normalized size = 1.5

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\,\frac{(\tan(1/2\,dx+c/2))^5}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+\frac{16}{3\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3}+3\,\frac{\tan(}{da\left(1+(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a/d*tan(1/2*d*x+1/2*c)+5/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c
)^5+16/3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+3/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-3/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.74326, size = 238, normalized size = 2.53

$$\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3
*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.66955, size = 180, normalized size = 1.91

$$-\frac{9\,dx\cos(dx+c)+9\,dx-\left(2\,\cos(dx+c)^3-\cos(dx+c)^2+7\,\cos(dx+c)+16\right)\sin(dx+c)}{6\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*c
os(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32232, size = 119, normalized size = 1.27

$$-\frac{\frac{9\,(dx+c)}{a}-\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c)
^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/
2*c)^2 + 1)^3*a))/d
```

3.51 $\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}+1$$

```
[Out] (15*x)/(8*a) - (4*Sin[c + d*x])/(a*d) + (15*Cos[c + d*x]*Sin[c + d*x])/(8*a
*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (Cos[c + d*x]^3*Sin[c + d*x
])/(d*(a + a*Sec[c + d*x])) + (4*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.101267, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3819, 3787, 2635, 8, 2633}

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\sec(c+dx)+a)}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (15*x)/(8*a) - (4*Sin[c + d*x])/(a*d) + (15*Cos[c + d*x]*Sin[c + d*x])/(8*a
*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (Cos[c + d*x]^3*Sin[c + d*x
])/(d*(a + a*Sec[c + d*x])) + (4*Sin[c + d*x]^3)/(3*a*d)
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \cos^4(c+dx)(-5a+4a\sec(c+dx))\,dx}{a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{4\int \cos^3(c+dx)\,dx}{a} + \frac{5\int \cos^4(c+dx)\,dx}{a}$$

$$= \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{15\int \cos^2(c+dx)\,dx}{4a} + \frac{4\,\mathrm{Subst}\left(\int \left(1\right.\right.}{}$$

$$= -\frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3(c+dx)\sin(c}{d(a+a\sec(c+d}$$

$$= \frac{15x}{8a} - \frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^3(c+dx)\,\mathrm{s}}{d(a+a\sec(}$$

Mathematica [A] time = 0.309493, size = 173, normalized size = 1.47

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-168\sin\left(c+\frac{dx}{2}\right)-120\sin\left(c+\frac{3dx}{2}\right)-120\sin\left(2c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{5dx}{2}\right)+40\sin\left(3c+\frac{5dx}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2]
 - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120*S
```

```
in[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2] - 5
*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2] + 3
*Sin[5*c + (9*d*x)/2]))/(384*a*d)
```

Maple [A] time = 0.054, size = 171, normalized size = 1.5

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{25}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-4}-\frac{115}{12\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/a/d*tan(1/2*d*x+1/2*c)-25/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^7-115/12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-109/12/a
/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-7/4/a/d/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)+15/4/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.69611, size = 293, normalized size = 2.48

$$-\frac{\frac{\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{115\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{75\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a+\frac{4\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}-\frac{45\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.67651, size = 213, normalized size = 1.81

$$\frac{45\,dx\cos(dx+c)+45\,dx+\left(6\cos(dx+c)^4-2\cos(dx+c)^3+13\cos(dx+c)^2-19\cos(dx+c)-64\right)\sin(dx+c)}{24\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(45*d*x*cos(d*x + c) + 45*d*x + (6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 +
 13*cos(d*x + c)^2 - 19*cos(d*x + c) - 64)*sin(d*x + c))/(a*d*cos(d*x + c)
+ a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.30793, size = 136, normalized size = 1.15

$$\frac{\frac{45\,(dx+c)}{a}-\frac{24\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(75\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+115\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+109\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2*
c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.52 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{8\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{\tan(c+dx)\sec^3(}{3d(a\sec(c+dx)}$$

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (16*Tan[c + d*x])/(3*a^2*d) + (7*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*d) - (8*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d
*(1 + Sec[c + d*x])) - (Sec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*
x])^2)
```

Rubi [A] time = 0.179415, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{8\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{\tan(c+dx)\sec^3(}{3d(a\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (16*Tan[c + d*x])/(3*a^2*d) + (7*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*d) - (8*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d
*(1 + Sec[c + d*x])) - (Sec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*
x])^2)
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
```

```
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^3(c+dx)(3a-5a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec^2(c+dx)\left(16a^2-21a^2\sec\right.}{3a^4}$$

$$= -\frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{16\int \sec^2(c+dx)\,dx}{3a^2} + \frac{7\int \sec^3}{}$$

$$= \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{7\int s}{}$$

$$= \frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{16\tan(c+dx)}{3a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec^2(c+dx)\tan(}{3a^2d(1+\sec(c+}$$

Mathematica [B] time = 1.87855, size = 300, normalized size = 2.44

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-2\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)-2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+3\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-\frac{}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right.\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^2*(-2*Sec[c/2]*Sin[(d*x)/2] - 40*Cos[(c + d*
x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]^3*(-14*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + 14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^(-2) - (8*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) -
2*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.045, size = 162, normalized size = 1.3

$$-\frac{1}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{7}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{2da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{5}{2da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + \frac{7}{2da^2}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*tan(1/2*d*x+1/2*c)^3-7/2/a^2/d*tan(1/2*d*x+1/2*c)-1/2/a^2/d/(tan
(1/2*d*x+1/2*c)+1)^2+5/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)+7/2/a^2/d*ln(tan(1/2*
d*x+1/2*c)+1)+1/2/a^2/d/(tan(1/2*d*x+1/2*c)-1)^2+5/2/a^2/d/(tan(1/2*d*x+1/2
*c)-1)-7/2/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.14704, size = 257, normalized size = 2.09

$$-\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A] time = 1.75098, size = 427, normalized size = 3.47

$$\frac{21\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-21\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx\right.}{12\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x +
 c) + 1) - 21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin
(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 43*cos(d*x + c)^2 + 6*cos(d*x + c)
- 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*c
os(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.42132, size = 165, normalized size = 1.34

$$\frac{\frac{21\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{21\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+21a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4*ta
n(1/2*d*x + 1/2*c))/a^6)/d
```

3.53 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{4\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (-2*ArcTanh[Sin[c + d*x]])/(a^2*d) + (4*Tan[c + d*x])/(3*a^2*d) + (2*Tan[c
+ d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a
+ a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.155168, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3816, 4008, 3787, 3770, 3767, 8}

$$\frac{4\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2\tan(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*ArcTanh[Sin[c + d*x]])/(a^2*d) + (4*Tan[c + d*x])/(3*a^2*d) + (2*Tan[c
+ d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a
+ a*Sec[c + d*x])^2)
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
```

```
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(2a-4a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sec(c+dx)\left(-6a^2+4a^2\sec(c+dx)\right)}{3a^4}$$

$$= \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{4\int \sec^2(c+dx)\,dx}{3a^2} - \frac{2\int \sec(c+dx)\,d}{a^2}$$

$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{4\,\mathrm{Subst}(\int 1}{}$$

$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{4\tan(c+dx)}{3a^2d} + \frac{2\tan(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

Mathematica [B] time = 1.11365, size = 247, normalized size = 2.78

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\ }{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right.\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(Sec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x
)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*(2*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x
]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/
2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.035, size = 120, normalized size = 1.4

$$\frac{1}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-2\frac{\ln\left(\tan\left(1/2\,dx+c/2\right)+1\right)}{a^2d}-\frac{1}{a^2d}\left(\tan\left(\frac{d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*tan(1/2*d*x+1/2*c)^3+5/2/a^2/d*tan(1/2*d*x+1/2*c)-1/a^2/d/(tan(1/
2*d*x+1/2*c)+1)-2/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)-1/a^2/d/(tan(1/2*d*x+1/2*c
)-1)+2/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.15948, size = 196, normalized size = 2.2

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.69682, size = 387, normalized size = 4.35

$$-\frac{3\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+\right.\right.}{3\left(a^{2}d\cos\left(dx+c\right)^{3}+2\,a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c)
 + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x +
 c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^{4}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.41666, size = 143, normalized size = 1.61

$$-\frac{\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{2}}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{2}}+\frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{2}}-\frac{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^
2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.54 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^2*d) - (5*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x]
)) + Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.118135, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3799, 3998, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^2*d) - (5*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x]
)) + Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\sec(c+dx)(-2a+3a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}\\ &= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \sec(c+dx)\,dx}{a^2}-\frac{5\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{5\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [B] time = 0.353587, size = 160, normalized size = 2.42

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-s\right.\right.\right.}{3a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]^2*(6*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[
c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d
*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.031, size = 77, normalized size = 1.2

$$-\frac{1}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{1}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*tan(1/2*d*x+1/2*c)^3-3/2/a^2/d*tan(1/2*d*x+1/2*c)-1/a^2/d*ln(tan
(1/2*d*x+1/2*c)-1)+1/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.19073, size = 132, normalized size = 2.

$$-\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A] time = 1.70834, size = 305, normalized size = 4.62

$$\frac{3\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(-\sin\left(dx+c\right)+\right.}{6\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c
) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.46181, size = 104, normalized size = 1.58

$$\frac{\frac{6 \log\left(\left|\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)+1\right|\right)}{a^2}-\frac{6 \log\left(\left|\tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)-1\right|\right)}{a^2}-\frac{a^4 \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^3+9\, a^4 \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)}{a^6}}{6\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.55 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{2\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] -Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(3*d*(a^2 + a
^2*Sec[c + d*x]))
```

Rubi [A] time = 0.0656275, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3797, 3794}

$$\frac{2\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(3*d*(a^2 + a
^2*Sec[c + d*x]))
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{2\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}$$
$$= -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{2\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

Mathematica [A] time = 0.0666604, size = 45, normalized size = 0.82

$$\frac{\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[(c + d*x)/2]^3*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(12*a^2*d)
```

Maple [A] time = 0.028, size = 32, normalized size = 0.6

$$\frac{1}{2\,da^2}\left(\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.12465, size = 62, normalized size = 1.13

$$\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

Fricas [A] time = 1.52225, size = 123, normalized size = 2.24

$$\frac{(\cos(dx+c)+2)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.32052, size = 42, normalized size = 0.76

$$\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.56 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*d*(a^2 + a^2*Se
c[c + d*x]))
```

Rubi [A] time = 0.0493647, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3796, 3794}

$$\frac{\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}+\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Tan[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2) + Tan[c + d*x]/(3*d*(a^2 + a^2*Se
c[c + d*x]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}$$
$$= \frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

Mathematica [A] time = 0.129731, size = 60, normalized size = 1.09

$$\frac{\sec\left(\frac{c}{2}\right)\left(-3\sin\left(c+\frac{dx}{2}\right)+2\sin\left(c+\frac{3dx}{2}\right)+3\sin\left(\frac{dx}{2}\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(3*Sin[(d*x)/2] - 3*Sin[c + (d*x)/2] + 2*Sin[c
 + (3*d*x)/2]))/(12*a^2*d)
```

Maple [A] time = 0.031, size = 32, normalized size = 0.6

$$\frac{1}{2\,da^2}\left(-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.18325, size = 63, normalized size = 1.15

$$\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

Fricas [A] time = 1.56105, size = 126, normalized size = 2.29

$$\frac{(2\,\cos(dx+c)+1)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.32207, size = 42, normalized size = 0.76

$$-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.57 $\int \frac{1}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$-\frac{4\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] x/a^2 - (4*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - Tan[c + d*x]/(3*d*(
a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.0694143, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3777, 3919, 3794}

$$-\frac{4\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (4*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - Tan[c + d*x]/(3*d*(
a + a*Sec[c + d*x])^2)
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{-3a+a\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}\\ &= \frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{4\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{x}{a^2}-\frac{\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{4\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.264481, size = 112, normalized size = 1.96

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(12\sin\left(c+\frac{dx}{2}\right)-10\sin\left(c+\frac{3dx}{2}\right)+9dx\cos\left(c+\frac{dx}{2}\right)+3dx\cos\left(c+\frac{3dx}{2}\right)+3dx\cos\left(2c+\frac{3dx}{2}\right)-\right.}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-2),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*d*x*Cos[(d*x)/2] + 9*d*x*Cos[c + (d*x)/2] +
 3*d*x*Cos[c + (3*d*x)/2] + 3*d*x*Cos[2*c + (3*d*x)/2] - 18*Sin[(d*x)/2] +
12*Sin[c + (d*x)/2] - 10*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A] time = 0.039, size = 56, normalized size = 1.

$$\frac{1}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*tan(1/2*d*x+1/2*c)^3-3/2/a^2/d*tan(1/2*d*x+1/2*c)+2/a^2/d*arctan(
tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.6379, size = 97, normalized size = 1.7

$$-\frac{\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.63589, size = 198, normalized size = 3.47

$$\frac{3 dx \cos(dx+c)^2 + 6 dx \cos(dx+c) + 3 dx - (5 \cos(dx+c) + 4) \sin(dx+c)}{3 \left(a^2 d \cos(dx+c)^2 + 2 a^2 d \cos(dx+c) + a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) +
4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^2(c+dx)+2\sec(c+dx)+1}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.27906, size = 68, normalized size = 1.19

$$\frac{\frac{6\,(dx+c)}{a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/a^6)/d
```

3.58 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=72

$$\frac{10\sin(c+dx)}{3a^2d} - \frac{2\sin(c+dx)}{a^2d(\sec(c+dx)+1)} - \frac{2x}{a^2} - \frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (-2*x)/a^2 + (10*Sin[c + d*x])/(3*a^2*d) - (2*Sin[c + d*x])/(a^2*d*(1 + Sec
[c + d*x])) - Sin[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.129373, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3817, 4020, 3787, 2637, 8}

$$\frac{10\sin(c+dx)}{3a^2d} - \frac{2\sin(c+dx)}{a^2d(\sec(c+dx)+1)} - \frac{2x}{a^2} - \frac{\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 + (10*Sin[c + d*x])/(3*a^2*d) - (2*Sin[c + d*x])/(a^2*d*(1 + Sec
[c + d*x])) - Sin[c + d*x]/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)(-4a+2a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2} \\
&= -\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos(c+dx)\left(-10a^2+6a^2\sec(c+dx)\right)\,d}{3a^4} \\
&= -\frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{2\int 1\,dx}{a^2} + \frac{10\int \cos(c+dx)\,dx}{3a^2} \\
&= -\frac{2x}{a^2} + \frac{10\sin(c+dx)}{3a^2d} - \frac{2\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\sec(c+dx))^2}
\end{aligned}$$

Mathematica [B] time = 0.477439, size = 151, normalized size = 2.1

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-30\sin\left(c+\frac{dx}{2}\right)+41\sin\left(c+\frac{3dx}{2}\right)+9\sin\left(2c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{5dx}{2}\right)+3\sin\left(3c+\frac{5dx}{2}\right)-36\right.}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2
] - 12*d*x*Cos[c + (3*d*x)/2] - 12*d*x*Cos[2*c + (3*d*x)/2] + 66*Sin[(d*x)/
```

```
2] - 30*Sin[c + (d*x)/2] + 41*Sin[c + (3*d*x)/2] + 9*Sin[2*c + (3*d*x)/2] +
 3*Sin[2*c + (5*d*x)/2] + 3*Sin[3*c + (5*d*x)/2]))/(48*a^2*d)
```

Maple [A] time = 0.054, size = 88, normalized size = 1.2

$$-\frac{1}{6\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\tan(1/2\,dx+c/2)}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)}-4\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*tan(1/2*d*x+1/2*c)^3+5/2/a^2/d*tan(1/2*d*x+1/2*c)+2/a^2/d*tan(1/
2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.67684, size = 159, normalized size = 2.21

$$\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\,\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x + c)
/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.62132, size = 230, normalized size = 3.19

$$-\frac{6\,dx\cos(dx+c)^2+12\,dx\cos(dx+c)+6\,dx-\left(3\,\cos(dx+c)^2+14\,\cos(dx+c)+10\right)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*d*x*cos(d*x + c)^2 + 12*d*x*cos(d*x + c) + 6*d*x - (3*cos(d*x + c)^
2 + 14*cos(d*x + c) + 10)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.32418, size = 107, normalized size = 1.49

$$-\frac{\frac{12(dx+c)}{a^2} - \frac{12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^2} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 +
 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/
d
```

3.59 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=110

$$-\frac{16\sin(c+dx)}{3a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{8\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (7*x)/(2*a^2) - (16*Sin[c + d*x])/(3*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*d) - (8*Cos[c + d*x]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (
Cos[c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.175024, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3817, 4020, 3787, 2635, 8, 2637}

$$-\frac{16\sin(c+dx)}{3a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{8\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*x)/(2*a^2) - (16*Sin[c + d*x])/(3*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*d) - (8*Cos[c + d*x]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (
Cos[c + d*x]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
```

```
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(-5a+3a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos^2(c+dx)\left(-21a^2+16a^2\sec(c\right.}{3a^4}$$

$$= -\frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{16\int\cos(c+dx)\,dx}{3a^2} + \frac{7\int\cos^2(c+}{a^2}$$

$$= -\frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)\sin(c}{3d(a+a\sec(c+}$$

$$= \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos(c+dx)}{3d(a+a\sec}$$

Mathematica [A] time = 0.371987, size = 177, normalized size = 1.61

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(147\sin\left(c+\frac{dx}{2}\right)-239\sin\left(c+\frac{3dx}{2}\right)-63\sin\left(2c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{5dx}{2}\right)-15\sin\left(3c+\frac{5dx}{2}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(252*d*x*Cos[(d*x)/2] + 252*d*x*Cos[c + (d*x)/
2] + 84*d*x*Cos[c + (3*d*x)/2] + 84*d*x*Cos[2*c + (3*d*x)/2] - 381*Sin[(d*x
)/2] + 147*Sin[c + (d*x)/2] - 239*Sin[c + (3*d*x)/2] - 63*Sin[2*c + (3*d*x)
/2] - 15*Sin[2*c + (5*d*x)/2] - 15*Sin[3*c + (5*d*x)/2] + 3*Sin[3*c + (7*d*
x)/2] + 3*Sin[4*c + (7*d*x)/2]))/(192*a^2*d)
```

Maple [A] time = 0.061, size = 122, normalized size = 1.1

$$\frac{1}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-5\frac{(\tan(1/2dx+c/2))^3}{da^2\left(1+(\tan(1/2dx+c/2))^2\right)^2}-3\frac{\tan(1/2dx+c/2)}{da^2\left(1+(\tan(1/2dx+c/2))^2\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2/d*tan(1/2*d*x+1/2*c)^3-7/2/a^2/d*tan(1/2*d*x+1/2*c)-5/a^2/d/(1+tan(
1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-3/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^2
*tan(1/2*d*x+1/2*c)+7/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.73415, size = 221, normalized size = 2.01

$$-\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^2)/d
```

Fricas [A] time = 1.64118, size = 257, normalized size = 2.34

$$\frac{21\,dx\cos\left(dx+c\right)^{2}+42\,dx\cos\left(dx+c\right)+21\,dx+\left(3\,\cos\left(dx+c\right)^{3}-6\,\cos\left(dx+c\right)^{2}-43\,\cos\left(dx+c\right)-32\right)\sin\left(dx+c\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c)
^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cos^{2}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.361, size = 128, normalized size = 1.16

$$\frac{\frac{21\,(dx+c)}{a^{2}}-\frac{6\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{2}}+\frac{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-21\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 21*
a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.60 $\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^2(c+a}{3d(a\sec(c+dx)+a}$$

```
[Out] (-5*x)/a^2 + (12*Sin[c + d*x])/(a^2*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(a^2
*d) - (10*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Cos[
c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*Sin[c + d*x]^3)/
(a^2*d)
```

Rubi [A] time = 0.190435, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3817, 4020, 3787, 2633, 2635, 8}

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^2(c+a}{3d(a\sec(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-5*x)/a^2 + (12*Sin[c + d*x])/(a^2*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(a^2
*d) - (10*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Cos[
c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*Sin[c + d*x]^3)/
(a^2*d)
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^3(c+dx)(-6a+4a\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \cos^3(c+dx)\left(-36a^2+30a^2\sec\right.}{3a^4}$$

$$= -\frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{10\int \cos^2(c+dx)\,dx}{a^2} + \frac{12\int \cos^3}{}$$

$$= -\frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int 1}{a^2}$$

$$= -\frac{5x}{a^2} + \frac{12\sin(c+dx)}{a^2d} - \frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\cos^2(c+}{3d(a+a}$$

Mathematica [A] time = 0.427158, size = 199, normalized size = 1.6

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-156\sin\left(c+\frac{dx}{2}\right)+342\sin\left(c+\frac{3dx}{2}\right)+118\sin\left(2c+\frac{3dx}{2}\right)+30\sin\left(2c+\frac{5dx}{2}\right)+30\sin\left(3c+\frac{5d}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-360*d*x*Cos[(d*x)/2] - 360*d*x*Cos[c + (d*x)
/2] - 120*d*x*Cos[c + (3*d*x)/2] - 120*d*x*Cos[2*c + (3*d*x)/2] + 516*Sin[(
d*x)/2] - 156*Sin[c + (d*x)/2] + 342*Sin[c + (3*d*x)/2] + 118*Sin[2*c + (3*
d*x)/2] + 30*Sin[2*c + (5*d*x)/2] + 30*Sin[3*c + (5*d*x)/2] - 3*Sin[3*c + (
7*d*x)/2] - 3*Sin[4*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2] + Sin[5*c + (9*d*
x)/2]))/(192*a^2*d)
```

Maple [A] time = 0.059, size = 156, normalized size = 1.3

$$-\frac{1}{6da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{9}{2da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+10\frac{(\tan(1/2\,dx+c/2))^5}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+\frac{40}{3da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2/d*tan(1/2*d*x+1/2*c)^3+9/2/a^2/d*tan(1/2*d*x+1/2*c)+10/a^2/d/(1+ta
n(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+40/3/a^2/d/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)^3+6/a^2/d/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)-10/a^2/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.74651, size = 279, normalized size = 2.25

$$\frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1) -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^2)/d
```

Fricas [A] time = 1.68341, size = 278, normalized size = 2.24

$$-\frac{15\,dx\cos(dx+c)^2+30\,dx\cos(dx+c)+15\,dx-\left(\cos(dx+c)^4-\cos(dx+c)^3+6\,\cos(dx+c)^2+33\,\cos(dx+c)+\right.}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(15*d*x*cos(d*x + c)^2 + 30*d*x*cos(d*x + c) + 15*d*x - (cos(d*x + c)^
4 - cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 33*cos(d*x + c) + 24)*sin(d*x + c))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33315, size = 146, normalized size = 1.18

$$-\frac{\frac{30\,(dx+c)}{a^2}-\frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(30*(d*x + c)/a^2 - 4*(15*tan(1/2*d*x + 1/2*c)^5 + 20*tan(1/2*d*x + 1/
2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (a^
4*tan(1/2*d*x + 1/2*c)^3 - 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.61 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=162

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{76\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{\tan(c+dx)\,s}{5d(a\sec(c}$$

```
[Out] (13*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (152*Tan[c + d*x])/(15*a^3*d) + (13*
Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - (Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) - (11*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Se
c[c + d*x])^2) - (76*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c +
d*x]))
```

Rubi [A] time = 0.292155, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{76\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{\tan(c+dx)\,s}{5d(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (13*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (152*Tan[c + d*x])/(15*a^3*d) + (13*
Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - (Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) - (11*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Se
c[c + d*x])^2) - (76*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c +
d*x]))
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^4(c+dx)(4a-7a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^3(c+dx)\left(33a^2-43a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\sec^2(c+dx)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)} - \cdots$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\sec^2(c+dx)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)} - \cdots$$

$$= \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\sec^3(c+dx)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76 \cdots}{15 \cdots}$$

$$= \frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{152\tan(c+dx)}{15a^3d} + \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec^4(c+dx)\tan \cdots}{5d(a+a\sec(c \cdots}$$

Mathematica [B] time = 0.98114, size = 351, normalized size = 2.17

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(-4329\sin\left(c-\frac{dx}{2}\right)+1989\sin\left(c+\frac{dx}{2}\right)-3575\sin\left(2c+\frac{dx}{2}\right)-475\sin\left(c \cdots\right.\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^3*(24960*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Se
c[c/2]*Sec[c]*Sec[c + d*x]^2*(-1235*Sin[(d*x)/2] + 3805*Sin[(3*d*x)/2] - 43
29*Sin[c - (d*x)/2] + 1989*Sin[c + (d*x)/2] - 3575*Sin[2*c + (d*x)/2] - 475
*Sin[c + (3*d*x)/2] + 2005*Sin[2*c + (3*d*x)/2] - 2275*Sin[3*c + (3*d*x)/2]
 + 2673*Sin[c + (5*d*x)/2] + 105*Sin[2*c + (5*d*x)/2] + 1593*Sin[3*c + (5*d
*x)/2] - 975*Sin[4*c + (5*d*x)/2] + 1325*Sin[2*c + (7*d*x)/2] + 255*Sin[3*c
 + (7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2] - 195*Sin[5*c + (7*d*x)/2] + 304*S
in[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*x)/2] + 214*Sin[5*c + (9*d*x)/2]))
/(480*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.047, size = 181, normalized size = 1.1

$$-\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{7}{2\,da^3}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-2/3/d/a^3*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*
tan(1/2*d*x+1/2*c)-1/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+7/2/d/a^3/(tan(1/2*d*
x+1/2*c)+1)+13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)+1/2/d/a^3/(tan(1/2*d*x+1/2*
c)-1)^2+7/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)-13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1
)
```

Maxima [A] time = 1.19141, size = 285, normalized size = 1.76

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [A] time = 1.77443, size = 548, normalized size = 3.38

$$\frac{195\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-195\left(\cos(dx+c)^5+3\cos\right.}{60\left(a^3d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x +
 c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*c
os(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x + c
)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x +
 c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.38272, size = 188, normalized size = 1.16

$$\frac{\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{60\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{\cdots}}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3 + 60*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 40
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.62 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=128

$$\frac{9\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{3\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{3\tan(c+dx)\sec^2(c+dx)}{5ad(a\sec(c+dx)+a)^2}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + (9*Tan[c + d*x])/(5*a^3*d) - (Sec[c +
d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (3*Sec[c + d*x]^2*Tan[c
 + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (3*Tan[c + d*x])/(d*(a^3 + a^3*Se
c[c + d*x]))
```

Rubi [A] time = 0.264708, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{9\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{3\tan(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{3\tan(c+dx)\sec^2(c+dx)}{5ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + (9*Tan[c + d*x])/(5*a^3*d) - (Sec[c +
d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (3*Sec[c + d*x]^2*Tan[c
 + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (3*Tan[c + d*x])/(d*(a^3 + a^3*Se
c[c + d*x]))
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
```

```
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)(3a-6a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)\left(18a^2-27a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{3\tan(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)} + \frac{\int \sec(c$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{3\tan(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)} + \frac{9\int \sec$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{3\,\mathrm{ta}}{d\left(a^3+a\right.}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{9\tan(c+dx)}{5a^3d} - \frac{\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sec^2(c+dx)\tan(c}{5ad(a+a\sec(c+d}$$

Mathematica [B] time = 1.25505, size = 294, normalized size = 2.3

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(8\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+20\cos^5\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(Sec[c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)
/2]^2*Sec[c/2]*Sin[(d*x)/2] + 76*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] +
 20*Cos[(c + d*x)/2]^5*(3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos
[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/2] + 8*Cos[(c + d*x)/2]^3*Tan
[c/2]))/(5*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.039, size = 139, normalized size = 1.1

$$\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{1}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-3\,\frac{\ln\left(\tan\left(1/2\,dx\right.\right.}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+1/2/d/a^3*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*t
an(1/2*d*x+1/2*c)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)-3/d/a^3*ln(tan(1/2*d*x+1/2
*c)+1)-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.19071, size = 223, normalized size = 1.74

$$\frac{\frac{40\,\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)/a^3)/d
```

Fricas [A] time = 1.7734, size = 506, normalized size = 3.95

$$-\frac{15\left(\cos(dx+c)^4+3\,\cos(dx+c)^3+3\,\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^4+3\,\cos\right.}{10\left(a^3d\cos(dx+c)^4+3\,a^3d\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x +
 c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3 +
57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
```

```
4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.4012, size = 165, normalized size = 1.29

$$-\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3}-\frac{a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+10a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+85a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a
^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85*a^
12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.63 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{29\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^3*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*S
ec[c + d*x])^3) + (7*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (29*Ta
n[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.221938, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3816, 4008, 3998, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{29\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^3*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(5*d*(a + a*S
ec[c + d*x])^3) + (7*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (29*Ta
n[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)(2a-5a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2} \\
&= -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(-14a^2+15a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{15a^4} \\
&= -\frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \sec(c+dx)\,dx}{a^3} - \frac{29\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}}{15a^2} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{29\tan(}{15d\left(a^3+a^3\right.}
\end{aligned}$$

Mathematica [A] time = 0.485943, size = 209, normalized size = 1.99

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(14\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+3\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+60\cos^5\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(60*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*S
ec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 88*Cos
[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]*Tan[c/2] + 14*Co
s[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.036, size = 96, normalized size = 0.9

$$-\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{1}{da^3}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*t
an(1/2*d*x+1/2*c)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tan(1/2*d*x+1
/2*c)+1)
```

Maxima [A] time = 1.11534, size = 161, normalized size = 1.53

$$-\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/a^3)/d
```

Fricas [A] time = 1.71835, size = 424, normalized size = 4.04

$$\frac{15\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\mathrm{c}}{30\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(sin(d
*x + c) + 1) - 15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
log(-sin(d*x + c) + 1) - 2*(22*cos(d*x + c)^2 + 51*cos(d*x + c) + 32)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^{4}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.34659, size = 127, normalized size = 1.21

$$\frac{\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{3}}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*a^12*tan(1/2*d*x + 1
```

```
/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.64 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{7\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{8\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(15*a*d*(a + a
*Sec[c + d*x])^2) + (7*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.123383, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3799, 4000, 3794}

$$\frac{7\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{8\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(15*a*d*(a + a
*Sec[c + d*x])^2) + (7*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)(-3a+5a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$
$$= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{7\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}$$
$$= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{7\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

Mathematica [A] time = 0.113871, size = 57, normalized size = 0.69

$$\frac{\left(10\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{120a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[(c + d*x)/2]^5*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*
(c + d*x))/2]))/(120*a^3*d)
```

Maple [A] time = 0.033, size = 45, normalized size = 0.5

$$\frac{1}{4da^3}\left(\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

Maxima [A] time = 1.18824, size = 90, normalized size = 1.08

$$\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

Fricas [A] time = 1.55799, size = 186, normalized size = 2.24

$$\frac{\left(2\,\cos\left(dx+c\right)^2 + 6\,\cos\left(dx+c\right) + 7\right)\sin\left(dx+c\right)}{15\left(a^3 d\cos\left(dx+c\right)^3 + 3\,a^3 d\cos\left(dx+c\right)^2 + 3\,a^3 d\cos\left(dx+c\right) + a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(d*x + c)^2 + 6*cos(d*x + c) + 7)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.3465, size = 62, normalized size = 0.75

$$\frac{3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{5} + 10\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} + 15\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

3.65 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{\tan(c+dx)}{5d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\tan(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] -Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + Tan[c + d*x]/(5*a*d*(a + a*Sec
[c + d*x])^2) + Tan[c + d*x]/(5*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.0967388, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3797, 3796, 3794}

$$\frac{\tan(c+dx)}{5d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\tan(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + Tan[c + d*x]/(5*a*d*(a + a*Sec
[c + d*x])^2) + Tan[c + d*x]/(5*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a}\\ &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{5a^2}\\ &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\tan(c+dx)}{5ad(a+a\sec(c+dx))^2}+\frac{\tan(c+dx)}{5d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.159927, size = 71, normalized size = 0.86

$$\frac{\sec\left(\frac{c}{2}\right)\left(-5\sin\left(c+\frac{dx}{2}\right)+5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{80a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(5*Sin[(d*x)/2] - 5*Sin[c + (d*x)/2] + 5*Sin[c
 + (3*d*x)/2] + Sin[2*c + (5*d*x)/2]))/(80*a^3*d)
```

Maple [A] time = 0.031, size = 32, normalized size = 0.4

$$\frac{1}{4\,da^3}\left(-\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.16613, size = 63, normalized size = 0.76

$$\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{20\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)
^5)/(a^3*d)
```

Fricas [A] time = 1.59893, size = 182, normalized size = 2.19

$$\frac{\left(\cos\left(dx+c\right)^2 + 3\,\cos\left(dx+c\right) + 1\right)\sin\left(dx+c\right)}{5\left(a^3 d\cos\left(dx+c\right)^3 + 3\,a^3 d\cos\left(dx+c\right)^2 + 3\,a^3 d\cos\left(dx+c\right) + a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.39828, size = 42, normalized size = 0.51

$$-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{20\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

3.66 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{2\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{2\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*Tan[c + d*x])/(15*a*d*(a + a
*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.080896, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3796, 3794}

$$\frac{2\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{2\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) + (2*Tan[c + d*x])/(15*a*d*(a + a
*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a}$$
$$= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{2\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}$$
$$= \frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{2\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

Mathematica [A] time = 0.2189, size = 86, normalized size = 1.04

$$\frac{\sec\left(\frac{c}{2}\right)\left(-30\sin\left(c+\frac{dx}{2}\right)+20\sin\left(c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+40\sin\left(\frac{dx}{2}\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(40*Sin[(d*x)/2] - 30*Sin[c + (d*x)/2] + 20*Si
n[c + (3*d*x)/2] - 15*Sin[2*c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2]))/(240*
a^3*d)
```

Maple [A] time = 0.039, size = 45, normalized size = 0.5

$$\frac{1}{4da^3}\left(\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

Maxima [A] time = 1.11988, size = 90, normalized size = 1.08

$$\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

Fricas [A] time = 1.60977, size = 186, normalized size = 2.24

$$\frac{\left(7\,\cos\left(dx+c\right)^{2}+6\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 1.34133, size = 62, normalized size = 0.75

$$\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

3.67 $\int \frac{1}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{22\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x}{a^3}-\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] x/a^3 - Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - (7*Tan[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - (22*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]
))
```

Rubi [A] time = 0.112323, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3777, 3922, 3919, 3794}

$$-\frac{22\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x}{a^3}-\frac{7\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - Tan[c + d*x]/(5*d*(a + a*Sec[c + d*x])^3) - (7*Tan[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - (22*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]
))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
```

```
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\sec(c+dx))^3}\,dx &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\int \frac{-5a+2a\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}\\ &= -\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\int \frac{15a^2-7a^2\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^4}\\ &= \frac{x}{a^3}-\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{22\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{15a^2}\\ &= \frac{x}{a^3}-\frac{\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{7\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{22\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.271052, size = 162, normalized size = 1.84

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(60dx\cos^5\left(\frac{1}{2}(c+dx)\right)+26\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)-3\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)-3\sec}{15a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-3),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(60*d*x*Cos[(c + d*x)/2]^5 - 3*Sec[c/2]*
Sin[(d*x)/2] + 26*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 128*Cos[(c + d
*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] - 3*Cos[(c + d*x)/2]*Tan[c/2] + 26*Cos[(c +
```

```
d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.04, size = 75, normalized size = 0.9

$$-\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{1}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*t
an(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.71245, size = 124, normalized size = 1.41

$$-\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.66019, size = 300, normalized size = 3.41

$$\frac{15\,dx\cos\left(dx+c\right)^3+45\,dx\cos\left(dx+c\right)^2+45\,dx\cos\left(dx+c\right)+15\,dx-\left(32\,\cos\left(dx+c\right)^2+51\,\cos\left(dx+c\right)+22\right)\sin\left(dx+\right.}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (32*cos(d*x + c)^2 + 51*cos(d*x + c) + 22)*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)/a
**3
```

Giac [A] time = 1.34064, size = 92, normalized size = 1.05

$$\frac{\frac{60\,(dx+c)}{a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.68 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{24\sin(c+dx)}{5a^3d}-\frac{3\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{3\sin(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

```
[Out] (-3*x)/a^3 + (24*Sin[c + d*x])/(5*a^3*d) - Sin[c + d*x]/(5*d*(a + a*Sec[c +
 d*x])^3) - (3*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (3*Sin[c + d*
x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.220573, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3817, 4020, 3787, 2637, 8}

$$\frac{24\sin(c+dx)}{5a^3d}-\frac{3\sin(c+dx)}{d\left(a^3\sec(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{3\sin(c+dx)}{5ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 + (24*Sin[c + d*x])/(5*a^3*d) - Sin[c + d*x]/(5*d*(a + a*Sec[c +
 d*x])^3) - (3*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (3*Sin[c + d*
x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
```

```
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(-6a+3a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-27a^2+18a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)} - \int\cos(c+d$$

$$= -\frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{3\sin(c+dx)}{d\left(a^3+a^3\sec(c+dx)\right)} - \frac{3\int 1\,dx}{a^3} +$$

$$= -\frac{3x}{a^3} + \frac{24\sin(c+dx)}{5a^3d} - \frac{\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{3\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{3\sin(c+}{d\left(a^3+a^3\sec\right.}$$

Mathematica [A] time = 0.561546, size = 169, normalized size = 1.64

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(20(\sin(c+dx)-3dx)\cos^5\left(\frac{1}{2}(c+dx)\right)-12\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c\right.\right.}{5a^3d(\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(Sec[c/2]*Sin[(d*x)/2] - 12*Cos[(c + d*x
)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 96*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2]
+ 20*Cos[(c + d*x)/2]^5*(-3*d*x + Sin[c + d*x]) + Cos[(c + d*x)/2]*Tan[c/2]
 - 12*Cos[(c + d*x)/2]^3*Tan[c/2]))/(5*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.06, size = 107, normalized size = 1.

$$\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\tan(1/2\,dx+c/2)}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)}-6\,\frac{\arctan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-1/2/d/a^3*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*t
an(1/2*d*x+1/2*c)+2/d/a^3*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a
^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.71797, size = 185, normalized size = 1.8

$$\frac{\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.90152, size = 325, normalized size = 3.16

$$-\frac{15\,dx\cos\left(dx+c\right)^{3}+45\,dx\cos\left(dx+c\right)^{2}+45\,dx\cos\left(dx+c\right)+15\,dx-\left(5\,\cos\left(dx+c\right)^{3}+39\,\cos\left(dx+c\right)^{2}+57\,\cos\left(c\right)\right.}{5\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*sin
(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 1.37433, size = 130, normalized size = 1.26

$$-\frac{\frac{60\left(dx+c\right)}{a^{3}}-\frac{40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{3}}-\frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+85\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
+ 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3 +
 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.69 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=147

$$-\frac{152\sin(c+dx)}{15a^3d}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{76\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{13x}{2a^3}-\frac{11\sin(c+dx)\cos(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+}{5d(a\,\text{se}}$$

```
[Out] (13*x)/(2*a^3) - (152*Sin[c + d*x])/(15*a^3*d) + (13*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) -
 (11*Cos[c + d*x]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (76*Cos[c
 + d*x]*Sin[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.290027, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3817, 4020, 3787, 2635, 8, 2637}

$$-\frac{152\sin(c+dx)}{15a^3d}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{76\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{13x}{2a^3}-\frac{11\sin(c+dx)\cos(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{\sin(c+}{5d(a\,\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*x)/(2*a^3) - (152*Sin[c + d*x])/(15*a^3*d) + (13*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) -
 (11*Cos[c + d*x]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (76*Cos[c
 + d*x]*Sin[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(-7a+4a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-43a^2+33a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\cos(c+dx)\sin(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)} - \frac{\int \cos^2 \cdots}{\cdots}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{76\cos(c+dx)\sin(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)} - \frac{152\int \cdots}{\cdots}$$

$$= -\frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+dx)\sin \cdots}{15ad(a+a\sec(\cdots}$$

$$= \frac{13x}{2a^3} - \frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{11\cos(c+ \cdots}{15ad(a+ \cdots}$$

Mathematica [A] time = 0.569939, size = 181, normalized size = 1.23

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(15(-12\sin(c+dx)+\sin(2(c+dx))+26dx)\cos^5\left(\frac{1}{2}(c+dx)\right)+46\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx \cdots\right.}{15a^3d(\mathrm{se}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^3*(-3*Sec[c/2]*Sin[(d*x)/2] + 46*Cos[(c +
d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 508*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)
/2] + 15*Cos[(c + d*x)/2]^5*(26*d*x - 12*Sin[c + d*x] + Sin[2*(c + d*x)]) -
 3*Cos[(c + d*x)/2]*Tan[c/2] + 46*Cos[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(
1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.064, size = 141, normalized size = 1.

$$-\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{2}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{31}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 7\,\frac{(\tan(1/2\,dx+c/2))^3}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - 5\,\frac{\cdots}{da^3\left(1 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+2/3/d/a^3*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*
tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-
5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+13/d/a^3*arctan(tan(1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.76111, size = 248, normalized size = 1.69

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.91309, size = 363, normalized size = 2.47

$$\frac{195\,dx\cos(dx+c)^3+585\,dx\cos(dx+c)^2+585\,dx\cos(dx+c)+195\,dx+\left(15\cos(dx+c)^4-45\cos(dx+c)^3-479\right.}{30\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x + c
) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2 -
 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.4042, size = 153, normalized size = 1.04

$$\frac{\frac{390\,(dx+c)}{a^3}-\frac{60\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.70 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=193

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{43\tan(c+dx)\sec^3(c+dx)}{35a^4d(\sec(c+dx)+1)^2}-\frac{288\tan(c+dx)\sec^2(c+dx)}{35a^4d(\sec(c+dx)+1)}+\frac{21\tan(c+$$

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (576*Tan[c + d*x])/(35*a^4*d) + (21*
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - (43*Sec[c + d*x]^3*Tan[c + d*x])/(35
*a^4*d*(1 + Sec[c + d*x])^2) - (288*Sec[c + d*x]^2*Tan[c + d*x])/(35*a^4*d*
(1 + Sec[c + d*x])) - (Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x
])^4) - (2*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.393318, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 3787, 3767, 8, 3768, 3770}

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{43\tan(c+dx)\sec^3(c+dx)}{35a^4d(\sec(c+dx)+1)^2}-\frac{288\tan(c+dx)\sec^2(c+dx)}{35a^4d(\sec(c+dx)+1)}+\frac{21\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (21*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (576*Tan[c + d*x])/(35*a^4*d) + (21*
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - (43*Sec[c + d*x]^3*Tan[c + d*x])/(35
*a^4*d*(1 + Sec[c + d*x])^2) - (288*Sec[c + d*x]^2*Tan[c + d*x])/(35*a^4*d*
(1 + Sec[c + d*x])) - (Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x
])^4) - (2*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^5(c+dx)(5a-9a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^4(c+dx)\left(56a^2-73a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} - \int$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} - 2$$

$$= -\frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^4(c+dx)\tan(c+dx)}{5ad(a+a\sec(c+dx))^3} - 2$$

$$= \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec^3(c+dx)\tan(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\,s}{5}$$

$$= \frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{576\tan(c+dx)}{35a^4d} + \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec^3(c+dx)}{35a^4d(1+\sec}$$

Mathematica [B] time = 1.5704, size = 403, normalized size = 2.09

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(-61054\sin\left(c-\frac{dx}{2}\right)+33614\sin\left(c+\frac{dx}{2}\right)-51842\sin\left(2c+\frac{dx}{2}\right)-12460\,s\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(376320*Cos[(c + d*x)/2]^7*(Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + S
ec[c/2]*Sec[c]*Sec[c + d*x]^2*(-24402*Sin[(d*x)/2] + 55556*Sin[(3*d*x)/2] -
 61054*Sin[c - (d*x)/2] + 33614*Sin[c + (d*x)/2] - 51842*Sin[2*c + (d*x)/2]
 - 12460*Sin[c + (3*d*x)/2] + 33716*Sin[2*c + (3*d*x)/2] - 34300*Sin[3*c +
(3*d*x)/2] + 39788*Sin[c + (5*d*x)/2] - 2940*Sin[2*c + (5*d*x)/2] + 26068*S
in[3*c + (5*d*x)/2] - 16660*Sin[4*c + (5*d*x)/2] + 21351*Sin[2*c + (7*d*x)/
2] + 1295*Sin[3*c + (7*d*x)/2] + 14911*Sin[4*c + (7*d*x)/2] - 5145*Sin[5*c
+ (7*d*x)/2] + 7329*Sin[3*c + (9*d*x)/2] + 1225*Sin[4*c + (9*d*x)/2] + 5369
*Sin[5*c + (9*d*x)/2] - 735*Sin[6*c + (9*d*x)/2] + 1152*Sin[4*c + (11*d*x)/
2] + 280*Sin[5*c + (11*d*x)/2] + 872*Sin[6*c + (11*d*x)/2])))/(2240*a^4*d*(
```

```
1 + Sec[c + d*x])^4)
```

Maple [A] time = 0.047, size = 200, normalized size = 1.

$$-\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{9}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{13}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{111}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\,da^4}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4
*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*tan(1/2*d*x+1/2*c)-1/2/d/a^4/(tan(1/2*d*x
+1/2*c)+1)^2+9/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)+21/2/d/a^4*ln(tan(1/2*d*x+1/2
*c)+1)+1/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)^2+9/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)-
21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.12619, size = 312, normalized size = 1.62

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right.}{a^4}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) + 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c)/(co
s(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^
4)/d
```

Fricas [A] time = 2.08836, size = 670, normalized size = 3.47

$$\frac{735\left(\cos\left(dx+c\right)^{6}+4\cos\left(dx+c\right)^{5}+6\cos\left(dx+c\right)^{4}+4\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-735\left(\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*
x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 + 4*
cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*log(
-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 4548*co
s(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^{7}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**7/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.4268, size = 209, normalized size = 1.08

$$\frac{\frac{2940\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{4}}-\frac{2940\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^{4}}+\frac{280\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}a^{4}}-\frac{5a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+63a^{24}\tan\left(\frac{1}{2}dx+\right.}{}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)^7
+ 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 3885*a
^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.71 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4} dx$

Optimal. Leaf size=159

$$\frac{244\tan(c+dx)}{105a^4d} - \frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{88\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2} + \frac{4\tan(c+dx)}{a^4d(\sec(c+dx)+1)} - \frac{\tan(c+dx)\sec^4(c+}{7d(a\sec(c+dx)+a}$$

```
[Out] (-4*ArcTanh[Sin[c + d*x]])/(a^4*d) + (244*Tan[c + d*x])/(105*a^4*d) - (88*S
ec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + (4*Tan[c + d
*x])/(a^4*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a
*Sec[c + d*x])^4) - (12*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c +
 d*x])^3)
```

Rubi [A] time = 0.368714, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{244\tan(c+dx)}{105a^4d} - \frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{88\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2} + \frac{4\tan(c+dx)}{a^4d(\sec(c+dx)+1)} - \frac{\tan(c+dx)\sec^4(c+}{7d(a\sec(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-4*ArcTanh[Sin[c + d*x]])/(a^4*d) + (244*Tan[c + d*x])/(105*a^4*d) - (88*S
ec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) + (4*Tan[c + d
*x])/(a^4*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a
*Sec[c + d*x])^4) - (12*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c +
 d*x])^3)
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^4(c+dx)(4a-8a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)\left(36a^2-52a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} -$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} +$$

$$= -\frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec^3(c+dx)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} +$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sec}{35a}$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{244\tan(c+dx)}{105a^4d} - \frac{88\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sec^4(c+dx)\,\mathrm{t}}{7d(a+a\sec}$$

Mathematica [B] time = 1.22005, size = 349, normalized size = 2.19

$$\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(-20524\sin\left(c-\frac{dx}{2}\right)+14644\sin\left(c+\frac{dx}{2}\right)-16660\sin\left(2c+\frac{dx}{2}\right)-4690\sin\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^4*(107520*Cos[(c + d*x)/2]^7*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Se
c[c/2]*Sec[c]*Sec[c + d*x]*(-10780*Sin[(d*x)/2] + 18788*Sin[(3*d*x)/2] - 20
524*Sin[c - (d*x)/2] + 14644*Sin[c + (d*x)/2] - 16660*Sin[2*c + (d*x)/2] -
4690*Sin[c + (3*d*x)/2] + 14378*Sin[2*c + (3*d*x)/2] - 9100*Sin[3*c + (3*d*
x)/2] + 11668*Sin[c + (5*d*x)/2] - 630*Sin[2*c + (5*d*x)/2] + 9358*Sin[3*c
+ (5*d*x)/2] - 2940*Sin[4*c + (5*d*x)/2] + 4228*Sin[2*c + (7*d*x)/2] + 315*
Sin[3*c + (7*d*x)/2] + 3493*Sin[4*c + (7*d*x)/2] - 420*Sin[5*c + (7*d*x)/2]
 + 664*Sin[3*c + (9*d*x)/2] + 105*Sin[4*c + (9*d*x)/2] + 559*Sin[5*c + (9*d
*x)/2])))/(1680*a^4*d*(1 + Sec[c + d*x])^4)
```

Maple [A] time = 0.04, size = 158, normalized size = 1.

$$\frac{1}{56\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7}{40\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{23}{24\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{49}{8\,d a^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{d a^4}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5+23/24/d/a^4
*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*tan(1/2*d*x+1/2*c)-1/d/a^4/(tan(1/2*d*x+1/
2*c)+1)-4/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)-1/d/a^4/(tan(1/2*d*x+1/2*c)-1)+4/d
/a^4*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.21242, size = 251, normalized size = 1.58

$$\frac{\frac{1680\,\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Fricas [A] time = 2.04913, size = 632, normalized size = 3.97

$$-\frac{210\left(\cos(dx+c)^5+4\,\cos(dx+c)^4+6\,\cos(dx+c)^3+4\,\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-210\left(\cos\right.}{105\left(a^4 d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d
*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4*c
os(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(-si
n(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(d*x
+ c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a
^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4
*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.37133, size = 188, normalized size = 1.18

$$-\frac{\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4}-\frac{15a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+147a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+805a^{24}\tan\cdots}{a^{28}}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/2*c
)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))/a^2
8)/d
```

3.72 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=136

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{43\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}+\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{2\tan(c+dx)\sec}{7ad(a\sec(c+d}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) + (11*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*
x])^2) - (43*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^3*
Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*Sec[c + d*x]^2*Tan[c + d*x]
)/(7*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.323221, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3816, 4019, 4008, 3998, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{43\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}+\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}-\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}-\frac{2\tan(c+dx)\sec}{7ad(a\sec(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) + (11*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*
x])^2) - (43*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^3*
Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*Sec[c + d*x]^2*Tan[c + d*x]
)/(7*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
```

```
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^3(c+dx)(3a-7a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)\left(20a^2-35a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}$$

$$= \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec(c+}{}}{}$$

$$= \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} + \frac{\int \sec(c}{a}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\sec^2(c+dx)}{7ad(a+a\sec}$$

Mathematica [A] time = 0.907558, size = 193, normalized size = 1.42

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(\sec\left(\frac{c}{2}\right)\left(-434\sin\left(c+\frac{dx}{2}\right)+525\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+203\sin\left(2c+\frac{5dx}{2}\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^4*(1344*Cos[(c + d*x)/2]^7*(Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec
[c/2]*(686*Sin[(d*x)/2] - 434*Sin[c + (d*x)/2] + 525*Sin[c + (3*d*x)/2] - 1
47*Sin[2*c + (3*d*x)/2] + 203*Sin[2*c + (5*d*x)/2] - 21*Sin[3*c + (5*d*x)/2
] + 32*Sin[3*c + (7*d*x)/2])))/(84*a^4*d*(1 + Sec[c + d*x])^4)
```

Maple [A] time = 0.039, size = 115, normalized size = 0.9

$$-\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{1}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{11}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{15}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{da^4}\ln\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5-11/24/d/a^4
*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*tan(1/2*d*x+1/2*c)-1/d/a^4*ln(tan(1/2*d*x+
1/2*c)-1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.18645, size = 188, normalized size = 1.38

$$-\frac{\frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1}+\frac{77 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{168\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 +
168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Fricas [A] time = 1.75158, size = 539, normalized size = 3.96

$$\frac{21 \left(\cos(dx+c)^4 + 4 \cos(dx+c)^3 + 6 \cos(dx+c)^2 + 4 \cos(dx+c) + 1\right) \log(\sin(dx+c)+1) - 21 \left(\cos(dx+c)^4 + 4 \right.}{42 \left(a^4 d \cos(dx+c)^4 + 4 a^4 \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x
+ c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6
*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos(d*
x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a^4*d
*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d
*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^5(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.36403, size = 149, normalized size = 1.1

$$\frac{\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{168\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}-\frac{3a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+77a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+315a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^{28}}}{168d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*x
+ 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2*c)
)/a^28)/d
```

3.73 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{\tan(c+dx)}{5d\left(a^4\sec(c+dx)+a^4\right)}-\frac{8\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

```
[Out] (Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*Tan[c + d*x
])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(35*d*(a^2 + a^2*Sec[
c + d*x])^2) + Tan[c + d*x]/(5*d*(a^4 + a^4*Sec[c + d*x]))
```

Rubi [A] time = 0.168655, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3810, 3799, 4000, 3794}

$$\frac{\tan(c+dx)}{5d\left(a^4\sec(c+dx)+a^4\right)}-\frac{8\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + (3*Tan[c + d*x
])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(35*d*(a^2 + a^2*Sec[
c + d*x])^2) + Tan[c + d*x]/(5*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
```

```
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$

$$= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{3\int \frac{\sec(c+dx)(-3a+5a\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{35a^3}$$

$$= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{\int \frac{\sec(\ldots}{a+a\,\mathrm{se}\ldots}}{5\ldots}$$

$$= \frac{\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{8\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{\mathrm{t}\ldots}{5d\left(a^4\ldots\right.}$$

Mathematica [A] time = 0.206315, size = 69, normalized size = 0.57

$$\frac{\left(35\sin\left(\frac{1}{2}(c+dx)\right)+21\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec^7\left(\frac{1}{2}(c+dx)\right)}{1120a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[(c + d*x)/2]^7*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin[
(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(1120*a^4*d)
```

Maple [A] time = 0.039, size = 56, normalized size = 0.5

$$\frac{1}{8\,da^4}\left(\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/
2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.2047, size = 117, normalized size = 0.98

$$\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

Fricas [A] time = 1.58351, size = 248, normalized size = 2.07

$$\frac{\left(2\,\cos\left(dx+c\right)^3+8\,\cos\left(dx+c\right)^2+13\,\cos\left(dx+c\right)+12\right)\sin\left(dx+c\right)}{35\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^4(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.32315, size = 80, normalized size = 0.67

$$\frac{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d*
x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.74 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{13\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{13\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}-\frac{11\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - (11*Tan[c + d*x])/(35*a*d*(a +
a*Sec[c + d*x])^3) + (13*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) +
 (13*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rubi [A] time = 0.154121, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3799, 4000, 3796, 3794}

$$\frac{13\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{13\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}-\frac{11\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - (11*Tan[c + d*x])/(35*a*d*(a +
a*Sec[c + d*x])^3) + (13*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) +
 (13*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
```

```
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec(c+dx)(-4a+7a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}$$

$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{13\int \frac{\mathrm{se}}{a+a}}{10}$$

$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{11\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{13\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{13}{105d\left(a^4\right.}$$

Mathematica [A] time = 0.226075, size = 87, normalized size = 0.78

$$\frac{\sec\left(\frac{c}{2}\right)\left(-35\sin\left(c+\frac{dx}{2}\right)+2\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)+35\sin\left(\frac{dx}{2}\right)\right)\sec^7\left(\frac{1}{2}(c+dx)\right)}{1680a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(35*Sin[(d*x)/2] - 35*Sin[c + (d*x)/2] + 2*(21
*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*c + (7*d*x)/2])))/(168
0*a^4*d)
```

Maple [A] time = 0.035, size = 58, normalized size = 0.5

$$\frac{1}{8\,da^{4}}\left(-\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}-\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}+\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5+1/3*tan(1/2*d
*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.14586, size = 117, normalized size = 1.04

$$\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{21\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

Fricas [A] time = 1.59153, size = 251, normalized size = 2.24

$$\frac{\left(8\,\cos\left(dx+c\right)^{3}+32\,\cos\left(dx+c\right)^{2}+52\,\cos\left(dx+c\right)+13\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.46162, size = 80, normalized size = 0.71

$$-\frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*
d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.75 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{8\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{8\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] -Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(35*a*d*(a +
a*Sec[c + d*x])^3) + (8*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) +
(8*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rubi [A] time = 0.125312, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3797, 3796, 3794}

$$\frac{8\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{8\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(35*a*d*(a +
a*Sec[c + d*x])^3) + (8*Tan[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) +
(8*Tan[c + d*x])/(105*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$
$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}$$
$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{8\int \frac{\text{s}}{a+}}{}$$
$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{8\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{8}{105d(}$$

Mathematica [A] time = 0.258605, size = 99, normalized size = 0.88

$$\frac{\sec\left(\frac{c}{2}\right)\left(-175\sin\left(c+\frac{dx}{2}\right)+168\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+91\sin\left(2c+\frac{5dx}{2}\right)+13\sin\left(3c+\frac{7dx}{2}\right)+280\sin\left(\frac{dx}{2}\right)}{6720a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(280*Sin[(d*x)/2] - 175*Sin[c + (d*x)/2] + 168
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 91*Sin[2*c + (5*d*x)/2] +
13*Sin[3*c + (7*d*x)/2]))/(6720*a^4*d)
```

Maple [A] time = 0.035, size = 58, normalized size = 0.5

$$\frac{1}{8\,da^4}\left(\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*
x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.32243, size = 117, normalized size = 1.04

$$\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\, a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

Fricas [A] time = 1.5864, size = 251, normalized size = 2.24

$$\frac{\left(13 \cos(dx+c)^3 + 52 \cos(dx+c)^2 + 32 \cos(dx+c) + 8\right) \sin(dx+c)}{105 \left(a^4 d \cos(dx+c)^4 + 4 a^4 d \cos(dx+c)^3 + 6 a^4 d \cos(dx+c)^2 + 4 a^4 d \cos(dx+c) + a^4 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\, dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.35183, size = 80, normalized size = 0.71

$$\frac{15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 21 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 105 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{840 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*d
*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.76 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4} dx$

Optimal. Leaf size=112

$$\frac{2\tan(c+dx)}{35d\left(a^4\sec(c+dx)+a^4\right)}+\frac{2\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (3*Tan[c + d*x])/(35*a*d*(a + a
*Sec[c + d*x])^3) + (2*Tan[c + d*x])/(35*d*(a^2 + a^2*Sec[c + d*x])^2) + (2
*Tan[c + d*x])/(35*d*(a^4 + a^4*Sec[c + d*x]))
```

Rubi [A] time = 0.110848, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3796, 3794}

$$\frac{2\tan(c+dx)}{35d\left(a^4\sec(c+dx)+a^4\right)}+\frac{2\tan(c+dx)}{35d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{3\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4) + (3*Tan[c + d*x])/(35*a*d*(a + a
*Sec[c + d*x])^3) + (2*Tan[c + d*x])/(35*d*(a^2 + a^2*Sec[c + d*x])^2) + (2
*Tan[c + d*x])/(35*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx = \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$

$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{6\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^2}$$

$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{2\int \frac{\sec(}{a+a\sec}}{35}$$

$$= \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{3\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2\tan(c+dx)}{35d\left(a^2+a^2\sec(c+dx)\right)^2} + \frac{2\,\mathrm{ta}}{35d\left(a^4+\right.}$$

Mathematica [A] time = 0.237308, size = 112, normalized size = 1.

$$\frac{\sec\left(\frac{c}{2}\right)\left(-210\sin\left(c+\frac{dx}{2}\right)+147\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+49\sin\left(2c+\frac{5dx}{2}\right)-35\sin\left(3c+\frac{5dx}{2}\right)+12\sin\left(3c+\right.\right.}{2240a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(210*Sin[(d*x)/2] - 210*Sin[c + (d*x)/2] + 147
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 49*Sin[2*c + (5*d*x)/2] -
35*Sin[3*c + (5*d*x)/2] + 12*Sin[3*c + (7*d*x)/2]))/(2240*a^4*d)
```

Maple [A] time = 0.036, size = 58, normalized size = 0.5

$$\frac{1}{8da^4}\left(-\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1
/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.30959, size = 117, normalized size = 1.04

$$\frac{\frac{35 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

Fricas [A] time = 1.59648, size = 248, normalized size = 2.21

$$\frac{\left(12 \cos(dx+c)^3 + 13 \cos(dx+c)^2 + 8 \cos(dx+c) + 2\right) \sin(dx+c)}{35 \left(a^4 d \cos(dx+c)^4 + 4 a^4 d \cos(dx+c)^3 + 6 a^4 d \cos(dx+c)^2 + 4 a^4 d \cos(dx+c) + a^4 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(12*cos(d*x + c)^3 + 13*cos(d*x + c)^2 + 8*cos(d*x + c) + 2)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(c+dx)}{\sec^4(c+dx) + 4\sec^3(c+dx) + 6\sec^2(c+dx) + 4\sec(c+dx) + 1}\, dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)
**2 + 4*sec(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.36765, size = 80, normalized size = 0.71

$$-\frac{5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{280\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d
*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.77 $\int \frac{1}{(a+a\sec(c+dx))^4} dx$

Optimal. Leaf size=111

$$-\frac{32\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}-\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}+\frac{x}{a^4}-\frac{2\tan(c+dx)}{7ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

```
[Out] x/a^4 - (11*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*x])^2) - (32*Tan[c + d*x
])/(21*a^4*d*(1 + Sec[c + d*x])) - Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4
) - (2*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.160296, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3777, 3922, 3919, 3794}

$$-\frac{32\tan(c+dx)}{21a^4d(\sec(c+dx)+1)}-\frac{11\tan(c+dx)}{21a^4d(\sec(c+dx)+1)^2}+\frac{x}{a^4}-\frac{2\tan(c+dx)}{7ad(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-4),x]
```

```
[Out] x/a^4 - (11*Tan[c + d*x])/(21*a^4*d*(1 + Sec[c + d*x])^2) - (32*Tan[c + d*x
])/(21*a^4*d*(1 + Sec[c + d*x])) - Tan[c + d*x]/(7*d*(a + a*Sec[c + d*x])^4
) - (2*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
```

```
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{-7a+3a\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} + \frac{\int \frac{35a^2-20a^2\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} - \frac{\int \frac{-105a^3+55a}{a+a\sec}}{105}$$

$$= \frac{x}{a^4} - \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} - \frac{32\int \frac{\mathrm{se}}{a+a}}{2}$$

$$= \frac{x}{a^4} - \frac{11\tan(c+dx)}{21a^4d(1+\sec(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\tan(c+dx)}{7ad(a+a\sec(c+dx))^3} - \frac{32}{21d\left(a^4\right.}$$

Mathematica [B] time = 0.372984, size = 224, normalized size = 2.02

$$\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(1652\sin\left(c+\frac{dx}{2}\right) - 1428\sin\left(c+\frac{3dx}{2}\right) + 756\sin\left(2c+\frac{3dx}{2}\right) - 560\sin\left(2c+\frac{5dx}{2}\right) + 168\sin\left(3c+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-4),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(735*d*x*Cos[(d*x)/2] + 735*d*x*Cos[c + (d*x)/
2] + 441*d*x*Cos[c + (3*d*x)/2] + 441*d*x*Cos[2*c + (3*d*x)/2] + 147*d*x*Co
s[2*c + (5*d*x)/2] + 147*d*x*Cos[3*c + (5*d*x)/2] + 21*d*x*Cos[3*c + (7*d*x
)/2] + 21*d*x*Cos[4*c + (7*d*x)/2] - 1988*Sin[(d*x)/2] + 1652*Sin[c + (d*x)
/2] - 1428*Sin[c + (3*d*x)/2] + 756*Sin[2*c + (3*d*x)/2] - 560*Sin[2*c + (5
*d*x)/2] + 168*Sin[3*c + (5*d*x)/2] - 104*Sin[3*c + (7*d*x)/2]))/(2688*a^4*
d)
```

Maple [A] time = 0.042, size = 94, normalized size = 0.9

$$\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{1}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{11}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{15}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan(\tan(1}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*
tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d
*x+1/2*c))
```

Maxima [A] time = 1.72866, size = 151, normalized size = 1.36

$$-\frac{\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d
```

Fricas [A] time = 1.62136, size = 397, normalized size = 3.58

$$\frac{21\,dx\cos(dx+c)^4+84\,dx\cos(dx+c)^3+126\,dx\cos(dx+c)^2+84\,dx\cos(dx+c)+21\,dx-\left(52\,\cos(dx+c)^3+124\,\mathrm{c}\right.}{21\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(21*d*x*cos(d*x + c)^4 + 84*d*x*cos(d*x + c)^3 + 126*d*x*cos(d*x + c)^
2 + 84*d*x*cos(d*x + c) + 21*d*x - (52*cos(d*x + c)^3 + 124*cos(d*x + c)^2
+ 107*cos(d*x + c) + 32)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(
d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Integral(1/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec
(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.44155, size = 112, normalized size = 1.01

$$\frac{\frac{168\,(dx+c)}{a^4}+\frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-315\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/2
*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x + 1
/2*c))/a^28)/d
```

3.78 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

<u>**Optimal**</u>. Leaf size=126

$$\frac{664\sin(c+dx)}{105a^4d}-\frac{4\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{88\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4x}{a^4}-\frac{12\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\sin(c+dx}{7d(a\sec(c+dx}$$

```
[Out] (-4*x)/a^4 + (664*Sin[c + d*x])/(105*a^4*d) - (88*Sin[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) - (4*Sin[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) - Sin[c
 + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - (12*Sin[c + d*x])/(35*a*d*(a + a*Sec
[c + d*x])^3)
```

<u>**Rubi**</u> **[A]** time = 0.304229, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3817, 4020, 3787, 2637, 8}

$$\frac{664\sin(c+dx)}{105a^4d}-\frac{4\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{88\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{4x}{a^4}-\frac{12\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{\sin(c+dx}{7d(a\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (-4*x)/a^4 + (664*Sin[c + d*x])/(105*a^4*d) - (88*Sin[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) - (4*Sin[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) - Sin[c
 + d*x]/(7*d*(a + a*Sec[c + d*x])^4) - (12*Sin[c + d*x])/(35*a*d*(a + a*Sec
[c + d*x])^3)
```


<u>Rule 3817</u>

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

<u>Rule 4020</u>

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
```

```
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(-8a+4a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-52a^2+36a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx}{}}{}$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{4\sin}{d\left(a^4+a^4\right.}$$

$$= -\frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} - \frac{4\sin}{d\left(a^4+a^4\right.}$$

$$= -\frac{4x}{a^4} + \frac{664\sin(c+dx)}{105a^4d} - \frac{88\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{12\sin(c}{35ad(a+a\,\mathrm{se}}$$

Mathematica [B] time = 0.457358, size = 263, normalized size = 2.09

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(46130\sin\left(c+\frac{dx}{2}\right)-46116\sin\left(c+\frac{3dx}{2}\right)+18060\sin\left(2c+\frac{3dx}{2}\right)-19292\sin\left(2c+\frac{5dx}{2}\right)+21\dots\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^7*(29400*d*x*Cos[(d*x)/2] + 29400*d*x*Cos[c + (
d*x)/2] + 17640*d*x*Cos[c + (3*d*x)/2] + 17640*d*x*Cos[2*c + (3*d*x)/2] + 5
880*d*x*Cos[2*c + (5*d*x)/2] + 5880*d*x*Cos[3*c + (5*d*x)/2] + 840*d*x*Cos[
3*c + (7*d*x)/2] + 840*d*x*Cos[4*c + (7*d*x)/2] - 60830*Sin[(d*x)/2] + 4613
0*Sin[c + (d*x)/2] - 46116*Sin[c + (3*d*x)/2] + 18060*Sin[2*c + (3*d*x)/2]
- 19292*Sin[2*c + (5*d*x)/2] + 2100*Sin[3*c + (5*d*x)/2] - 3791*Sin[3*c + (
7*d*x)/2] - 735*Sin[4*c + (7*d*x)/2] - 105*Sin[4*c + (9*d*x)/2] - 105*Sin[5
*c + (9*d*x)/2]))/(26880*a^4*d)
```

Maple [A] time = 0.063, size = 126, normalized size = 1.

$$-\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{23}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{49}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\tan(\dots}{da^4\left(1+(\text{ta}\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5-23/24/d/a^
4*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*tan(1/2*d*x+1/2*c)+2/d/a^4*tan(1/2*d*x+1/
2*c)/(1+tan(1/2*d*x+1/2*c)^2)-8/d/a^4*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.52047, size = 213, normalized size = 1.69

$$\frac{\frac{1680\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4)/d
```

Fricas [A] time = 1.68441, size = 444, normalized size = 3.52

$$-\frac{420\,dx\cos(dx+c)^4+1680\,dx\cos(dx+c)^3+2520\,dx\cos(dx+c)^2+1680\,dx\cos(dx+c)+420\,dx-\left(105\,\cos(dx+c\right.}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(420*d*x*cos(d*x + c)^4 + 1680*d*x*cos(d*x + c)^3 + 2520*d*x*cos(d*x
 + c)^2 + 1680*d*x*cos(d*x + c) + 420*d*x - (105*cos(d*x + c)^4 + 1184*cos(
d*x + c)^3 + 2636*cos(d*x + c)^2 + 2236*cos(d*x + c) + 664)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40201, size = 151, normalized size = 1.2

$$-\frac{\frac{3360\,(dx+c)}{a^4}-\frac{1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5145\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x + 1
/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

3.79 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=176

$$-\frac{576\sin(c+dx)}{35a^4d}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{288\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)}-\frac{43\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)^2}+\frac{21x}{2a^4}-\frac{2\sin(}{5ad(}$$

```
[Out] (21*x)/(2*a^4) - (576*Sin[c + d*x])/(35*a^4*d) + (21*Cos[c + d*x]*Sin[c + d
*x])/(2*a^4*d) - (43*Cos[c + d*x]*Sin[c + d*x])/(35*a^4*d*(1 + Sec[c + d*x]
)^2) - (288*Cos[c + d*x]*Sin[c + d*x])/(35*a^4*d*(1 + Sec[c + d*x])) - (Cos
[c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*Cos[c + d*x]*Sin[
c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rubi [A] time = 0.392953, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3817, 4020, 3787, 2635, 8, 2637}

$$-\frac{576\sin(c+dx)}{35a^4d}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{288\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)}-\frac{43\sin(c+dx)\cos(c+dx)}{35a^4d(\sec(c+dx)+1)^2}+\frac{21x}{2a^4}-\frac{2\sin(}{5ad(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (21*x)/(2*a^4) - (576*Sin[c + d*x])/(35*a^4*d) + (21*Cos[c + d*x]*Sin[c + d
*x])/(2*a^4*d) - (43*Cos[c + d*x]*Sin[c + d*x])/(35*a^4*d*(1 + Sec[c + d*x]
)^2) - (288*Cos[c + d*x]*Sin[c + d*x])/(35*a^4*d*(1 + Sec[c + d*x])) - (Cos
[c + d*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - (2*Cos[c + d*x]*Sin[
c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^4}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(-9a+5a\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(-73a^2+56a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} - \int \frac{\cos^2(c}{}$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{288\cos}{35d\left(a\right.}$$

$$= -\frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{2\cos(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^3} - \frac{288\cos}{35d\left(a\right.}$$

$$= -\frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+dx)\sin}{7d(a+a\sec(}$$

$$= \frac{21x}{2a^4} - \frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos(c+dx)\sin(c+dx)}{35a^4d(1+\sec(c+dx))^2} - \frac{\cos(c+}{7d(a+}$$

Mathematica [A] time = 0.551801, size = 289, normalized size = 1.64

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(128730\sin\left(c+\frac{dx}{2}\right)-140826\sin\left(c+\frac{3dx}{2}\right)+44310\sin\left(2c+\frac{3dx}{2}\right)-60487\sin\left(2c+\frac{5dx}{2}\right)+122\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(102900*d*x*Cos[(d*x)/2] + 102900*d*x*Cos[c +
(d*x)/2] + 61740*d*x*Cos[c + (3*d*x)/2] + 61740*d*x*Cos[2*c + (3*d*x)/2] +
20580*d*x*Cos[2*c + (5*d*x)/2] + 20580*d*x*Cos[3*c + (5*d*x)/2] + 2940*d*x*
Cos[3*c + (7*d*x)/2] + 2940*d*x*Cos[4*c + (7*d*x)/2] - 179830*Sin[(d*x)/2]
+ 128730*Sin[c + (d*x)/2] - 140826*Sin[c + (3*d*x)/2] + 44310*Sin[2*c + (3*
d*x)/2] - 60487*Sin[2*c + (5*d*x)/2] + 1225*Sin[3*c + (5*d*x)/2] - 12001*Si
n[3*c + (7*d*x)/2] - 3185*Sin[4*c + (7*d*x)/2] - 315*Sin[4*c + (9*d*x)/2] -
 315*Sin[5*c + (9*d*x)/2] + 35*Sin[5*c + (11*d*x)/2] + 35*Sin[6*c + (11*d*x
)/2]))/(35840*a^4*d)
```

Maple [A] time = 0.063, size = 160, normalized size = 0.9

$$\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{9}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{13}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{111}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-9\,\frac{(\tan(1/2}{da^4\left(1+(\tan(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5+13/8/d/a^4*
tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*tan(1/2*d*x+1/2*c)-9/d/a^4/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-7/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2
*d*x+1/2*c)+21/d/a^4*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.6581, size = 275, normalized size = 1.56

$$-\frac{\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) - 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a^4)/d
```

Fricas [A] time = 1.68707, size = 470, normalized size = 2.67

$$\frac{735\,dx\cos(dx+c)^4+2940\,dx\cos(dx+c)^3+4410\,dx\cos(dx+c)^2+2940\,dx\cos(dx+c)+735\,dx+\left(35\,\cos(dx+c)\right.}{70\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x +
 c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*x
+ c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) - 11
52)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*
cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36911, size = 173, normalized size = 0.98

$$\frac{\frac{2940\,(dx+c)}{a^4} - \frac{280\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^4} + \frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+455\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3885\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2*c
)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 - 38
85*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.80 $\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=200

$$\frac{181\tan(c+dx)}{63a^5d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{29\tan(c+dx)\sec^3(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{67\tan(c+dx)\sec^2(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}+\frac{5\tan(c+}{d\left(a^5\sec(c+a\right.}$$

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^5*d) + (181*Tan[c + d*x])/(63*a^5*d) - (Sec[c
 + d*x]^5*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) - (5*Sec[c + d*x]^4*Ta
n[c + d*x])/(21*a*d*(a + a*Sec[c + d*x])^4) - (29*Sec[c + d*x]^3*Tan[c + d*
x])/(63*a^2*d*(a + a*Sec[c + d*x])^3) - (67*Sec[c + d*x]^2*Tan[c + d*x])/(6
3*a^3*d*(a + a*Sec[c + d*x])^2) + (5*Tan[c + d*x])/(d*(a^5 + a^5*Sec[c + d*
x]))
```

Rubi [A] time = 0.480232, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3816, 4019, 4008, 3787, 3770, 3767, 8}

$$\frac{181\tan(c+dx)}{63a^5d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{29\tan(c+dx)\sec^3(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{67\tan(c+dx)\sec^2(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}+\frac{5\tan(c+}{d\left(a^5\sec(c+a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```


```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^5*d) + (181*Tan[c + d*x])/(63*a^5*d) - (Sec[c
 + d*x]^5*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) - (5*Sec[c + d*x]^4*Ta
n[c + d*x])/(21*a*d*(a + a*Sec[c + d*x])^4) - (29*Sec[c + d*x]^3*Tan[c + d*
x])/(63*a^2*d*(a + a*Sec[c + d*x])^3) - (67*Sec[c + d*x]^2*Tan[c + d*x])/(6
3*a^3*d*(a + a*Sec[c + d*x])^2) + (5*Tan[c + d*x])/(d*(a^5 + a^5*Sec[c + d*
x]))
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\sec^5(c+dx)(5a-10a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^4(c+dx)\left(60a^2-85a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - \int$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - 6$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - 6$$

$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3(c+dx)\tan(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - 6$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sec^3}{63a^2d}$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{181\tan(c+dx)}{63a^5d} - \frac{\sec^5(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sec^4(c+dx)\tan}{21ad(a+a\sec(}$$

Mathematica [B] time = 1.84508, size = 401, normalized size = 2.

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\sec\left(\frac{c}{2}\right)\sec(c)\left(-56952\sin\left(c-\frac{dx}{2}\right)+43722\sin\left(c+\frac{dx}{2}\right)-47208\sin\left(2c+\frac{dx}{2}\right)-18144\,\mathrm{si}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^5*(322560*Cos[(c + d*x)/2]^9*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Se
c[c/2]*Sec[c]*Sec[c + d*x]*(-33978*Sin[(d*x)/2] + 52002*Sin[(3*d*x)/2] - 56
952*Sin[c - (d*x)/2] + 43722*Sin[c + (d*x)/2] - 47208*Sin[2*c + (d*x)/2] -
18144*Sin[c + (3*d*x)/2] + 41796*Sin[2*c + (3*d*x)/2] - 28350*Sin[3*c + (3*
d*x)/2] + 34578*Sin[c + (5*d*x)/2] - 5691*Sin[2*c + (5*d*x)/2] + 28719*Sin[
3*c + (5*d*x)/2] - 11550*Sin[4*c + (5*d*x)/2] + 15517*Sin[2*c + (7*d*x)/2]
- 504*Sin[3*c + (7*d*x)/2] + 13186*Sin[4*c + (7*d*x)/2] - 2835*Sin[5*c + (7
```

```
*d*x)/2] + 4149*Sin[3*c + (9*d*x)/2] + 252*Sin[4*c + (9*d*x)/2] + 3582*Sin[
5*c + (9*d*x)/2] - 315*Sin[6*c + (9*d*x)/2] + 496*Sin[4*c + (11*d*x)/2] + 6
3*Sin[5*c + (11*d*x)/2] + 433*Sin[6*c + (11*d*x)/2])))/(2016*a^5*d*(1 + Sec
[c + d*x])^5)
```

Maple [A] time = 0.041, size = 177, normalized size = 0.9

$$\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{1}{14\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{8\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3}{2\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{129}{16\,da^5}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+1/14/d/a^5*tan(1/2*d*x+1/2*c)^7+3/8/d/a^5*
tan(1/2*d*x+1/2*c)^5+3/2/d/a^5*tan(1/2*d*x+1/2*c)^3+129/16/d/a^5*tan(1/2*d*
x+1/2*c)-1/d/a^5/(tan(1/2*d*x+1/2*c)+1)-5/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)-1/
d/a^5/(tan(1/2*d*x+1/2*c)-1)+5/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.12283, size = 278, normalized size = 1.39

$$\frac{\frac{2016\,\sin(dx+c)}{\left(a^5-\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1512\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{72\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040\,\log}{}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

Fricas [A] time = 1.76029, size = 755, normalized size = 3.78

$$-\frac{315\left(\cos\left(dx+c\right)^{6}+5\,\cos\left(dx+c\right)^{5}+10\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+5\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\log\left(\sin\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/126*(315*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos
(d*x + c)^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 315*
(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 5*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(496*cos(d*x
+ c)^5 + 2165*cos(d*x + c)^4 + 3633*cos(d*x + c)^3 + 2840*cos(d*x + c)^2 +
946*cos(d*x + c) + 63)*sin(d*x + c))/(a^5*d*cos(d*x + c)^6 + 5*a^5*d*cos(d*
x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 5*a^5*d*cos(
d*x + c)^2 + a^5*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^{7}\left(c+dx\right)}{\sec^{5}\left(c+dx\right)+5\sec^{4}\left(c+dx\right)+10\sec^{3}\left(c+dx\right)+10\sec^{2}\left(c+dx\right)+5\sec\left(c+dx\right)+1}\,dx}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**7/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A] time = 1.47177, size = 209, normalized size = 1.04

$$-\frac{\frac{5040\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{5}}-\frac{5040\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{5}}+\frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a^{5}}-\frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+378\,a^{40}\tan}{}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^5 + 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 - 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 + 72*a^40*tan(1/2*d*x + 1/2*c)
^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 + 1512*a^40*tan(1/2*d*x + 1/2*c)^3 + 8
127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.81 $\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=177

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{34\tan(c+dx)\sec^2(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}-\frac{661\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{\tan(c}{9d(a}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^5*d) - (Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*S
ec[c + d*x])^5) - (13*Sec[c + d*x]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d
*x])^4) - (34*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*x])^
3) + (173*Tan[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (661*Tan[c + d
*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.427265, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3816, 4019, 4008, 3998, 3770, 3794}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{34\tan(c+dx)\sec^2(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}-\frac{661\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{\tan(c}{9d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(a^5*d) - (Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*S
ec[c + d*x])^5) - (13*Sec[c + d*x]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d
*x])^4) - (34*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*x])^
3) + (173*Tan[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (661*Tan[c + d
*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\sec^4(c+dx)(4a-9a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec^3(c+dx)\left(39a^2-63a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} -$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} +$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c+dx)\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} +$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\sec^2(c}{105a^2d(}$$

Mathematica [A] time = 1.96751, size = 219, normalized size = 1.24

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(\sec\left(\frac{c}{2}\right)\left(-25515\sin\left(c+\frac{dx}{2}\right)+29757\sin\left(c+\frac{3dx}{2}\right)-11235\sin\left(2c+\frac{3dx}{2}\right)+14733\sin\left(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^5*(80640*Cos[(c + d*x)/2]^9*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Se
c[c/2]*(35973*Sin[(d*x)/2] - 25515*Sin[c + (d*x)/2] + 29757*Sin[c + (3*d*x)
/2] - 11235*Sin[2*c + (3*d*x)/2] + 14733*Sin[2*c + (5*d*x)/2] - 2835*Sin[3*
c + (5*d*x)/2] + 4077*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c + (7*d*x)/2] + 488
*Sin[4*c + (9*d*x)/2])))/(2520*a^5*d*(1 + Sec[c + d*x])^5)
```

Maple [A] time = 0.043, size = 134, normalized size = 0.8

$$-\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9 - \frac{3}{56\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{1}{5\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{13}{24\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{31}{16\,da^5}\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-3/56/d/a^5*tan(1/2*d*x+1/2*c)^7-1/5/d/a^5
*tan(1/2*d*x+1/2*c)^5-13/24/d/a^5*tan(1/2*d*x+1/2*c)^3-31/16/d/a^5*tan(1/2*
d*x+1/2*c)-1/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^5*ln(tan(1/2*d*x+1/2*c)+1
)
```

Maxima [A] time = 1.179, size = 215, normalized size = 1.21

$$-\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1}+\frac{2730\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 50
40*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)/(co
s(d*x + c) + 1) - 1)/a^5)/d
```

Fricas [A] time = 1.78017, size = 671, normalized size = 3.79

$$\frac{315\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\log(\sin(dx+c)+1)-31}{630\,(a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(
d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^
5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x +
c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x + c)^
3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*d*cos
```

```
(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*c
os(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^6(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**6/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A] time = 1.39204, size = 170, normalized size = 0.96

$$\frac{\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^5}-\frac{5040\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^5}-\frac{35a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+270a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+1008a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+2730a^{40}\tan}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1/2
*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d*x
+ 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.82 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=159

$$\frac{4\tan(c+dx)}{45d\left(a^5\sec(c+dx)+a^5\right)}-\frac{32\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{4\,\mathrm{ta}}{63}$$

```
[Out] (Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (4*Sec[c + d*x
]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(105*a
^2*d*(a + a*Sec[c + d*x])^3) - (32*Tan[c + d*x])/(315*a*d*(a^2 + a^2*Sec[c
+ d*x])^2) + (4*Tan[c + d*x])/(45*d*(a^5 + a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.215953, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3810, 3799, 4000, 3794}

$$\frac{4\tan(c+dx)}{45d\left(a^5\sec(c+dx)+a^5\right)}-\frac{32\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{4\,\mathrm{ta}}{63}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (4*Sec[c + d*x
]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(105*a
^2*d*(a + a*Sec[c + d*x])^3) - (32*Tan[c + d*x])/(315*a*d*(a^2 + a^2*Sec[c
+ d*x])^2) + (4*Tan[c + d*x])/(45*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
```

```
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}$$

$$= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}$$

$$= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} + \frac{4\int}{}$$

$$= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{}{315}$$

$$= \frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{}{315}$$

Mathematica [A] time = 0.191781, size = 97, normalized size = 0.61

$$\frac{\left(126\sin\left(\frac{1}{2}(c+dx)\right)+84\sin\left(\frac{3}{2}(c+dx)\right)+36\sin\left(\frac{5}{2}(c+dx)\right)+9\sin\left(\frac{7}{2}(c+dx)\right)+\sin\left(\frac{9}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)\mathrm{se}}{315a^5d(\sec(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^5*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*x
))/2] + 36*Sin[(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*x)
)/2]))/(315*a^5*d*(1 + Sec[c + d*x])^5)
```

Maple [A] time = 0.039, size = 71, normalized size = 0.5

$$\frac{1}{16\,da^5}\left(\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{4}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{6}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{4}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5+4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.12254, size = 144, normalized size = 0.91

$$\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{420\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{180\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.63197, size = 316, normalized size = 1.99

$$\frac{\left(8\,\cos\left(dx+c\right)^4+40\,\cos\left(dx+c\right)^3+84\,\cos\left(dx+c\right)^2+100\,\cos\left(dx+c\right)+83\right)\sin\left(dx+c\right)}{315\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx+c\right)+a^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(d
*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^5(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**5/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A] time = 1.35392, size = 97, normalized size = 0.61

$$\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+180\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+378\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{5040\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

3.83 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=159

$$\frac{\tan(c+dx)}{9d\left(a^5\sec(c+dx)+a^5\right)}-\frac{8\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{5\tan(c}{63ad(}$$

```
[Out] -(Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (5*Sec[c + d*
x]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) + Tan[c + d*x]/(21*a^2*d
*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(63*a*d*(a^2 + a^2*Sec[c + d*x]
)^2) + Tan[c + d*x]/(9*d*(a^5 + a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.214803, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3811, 3810, 3799, 4000, 3794}

$$\frac{\tan(c+dx)}{9d\left(a^5\sec(c+dx)+a^5\right)}-\frac{8\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}-\frac{\tan(c+dx)\sec^4(c+dx)}{9d(a\sec(c+dx)+a)^5}+\frac{5\tan(c}{63ad(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```


```
[Out] -(Sec[c + d*x]^4*Tan[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5) + (5*Sec[c + d*
x]^3*Tan[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) + Tan[c + d*x]/(21*a^2*d
*(a + a*Sec[c + d*x])^3) - (8*Tan[c + d*x])/(63*a*d*(a^2 + a^2*Sec[c + d*x]
)^2) + Tan[c + d*x]/(9*d*(a^5 + a^5*Sec[c + d*x]))
```


Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
```

```
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{5\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3} + \frac{\int \sec \ldots}{\ldots}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3} - \frac{\ldots}{63a^3 \ldots}$$

$$= -\frac{\sec^4(c+dx)\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\sec^3(c+dx)\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3} - \frac{\ldots}{63a^3 \ldots}$$

Mathematica [A] time = 0.19492, size = 97, normalized size = 0.61

$$\frac{\sec\left(\frac{c}{2}\right)\left(-63\sin\left(c+\frac{dx}{2}\right)+84\sin\left(c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)+63\sin\left(\frac{dx}{2}\right)\right)\sec^9\left(\frac{1}{2}\ldots\right)}{8064a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(63*Sin[(d*x)/2] - 63*Sin[c + (d*x)/2] + 84*Si
n[c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Sin[3*c + (7*d*x)/2] + Sin[4
*c + (9*d*x)/2]))/(8064*a^5*d)
```

Maple [A] time = 0.038, size = 58, normalized size = 0.4

$$\frac{1}{16\,da^5}\left(-\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{2}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-2/7*tan(1/2*d*x+1/2*c)^7+2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.14216, size = 117, normalized size = 0.74

$$\frac{\frac{63\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{18\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.65416, size = 312, normalized size = 1.96

$$\frac{\left(2\,\cos\left(dx+c\right)^4+10\,\cos\left(dx+c\right)^3+21\,\cos\left(dx+c\right)^2+25\,\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)}{63\left(a^5d\cos\left(dx+c\right)^5+5\,a^5d\cos\left(dx+c\right)^4+10\,a^5d\cos\left(dx+c\right)^3+10\,a^5d\cos\left(dx+c\right)^2+5\,a^5d\cos\left(dx+c\right)+a^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*x
 + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^4(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**4/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A] time = 1.44324, size = 80, normalized size = 0.5

$$-\frac{7 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{9} + 18 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{7} - 42 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} - 63 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{1008\,a^{5} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

3.84 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=139

$$\frac{2\tan(c+dx)}{45d\left(a^5\sec(c+dx)+a^5\right)}+\frac{2\tan(c+dx)}{45a^3d(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{15a^2d(a\sec(c+dx)+a)^3}-\frac{2\tan(c+dx)}{9ad(a\sec(c+dx)+a)^4}+\frac{\text{ta}}{9d(a\,\text{se}}$$

```
[Out] Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - (2*Tan[c + d*x])/(9*a*d*(a + a*
Sec[c + d*x])^4) + Tan[c + d*x]/(15*a^2*d*(a + a*Sec[c + d*x])^3) + (2*Tan[
c + d*x])/(45*a^3*d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(45*d*(a^5 +
 a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.182088, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3799, 4000, 3796, 3794}

$$\frac{2\tan(c+dx)}{45d\left(a^5\sec(c+dx)+a^5\right)}+\frac{2\tan(c+dx)}{45a^3d(a\sec(c+dx)+a)^2}+\frac{\tan(c+dx)}{15a^2d(a\sec(c+dx)+a)^3}-\frac{2\tan(c+dx)}{9ad(a\sec(c+dx)+a)^4}+\frac{\text{ta}}{9d(a\,\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - (2*Tan[c + d*x])/(9*a*d*(a + a*
Sec[c + d*x])^4) + Tan[c + d*x]/(15*a^2*d*(a + a*Sec[c + d*x])^3) + (2*Tan[
c + d*x])/(45*a^3*d*(a + a*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(45*d*(a^5 +
 a^5*Sec[c + d*x]))
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
```

```
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{\int \frac{\sec(c+dx)(-5a+9a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{3a^2}$$

$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3} + \frac{2\int \frac{\sec(c+}{(a+a\sec(}}{15a}$$

$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3} + \frac{2\tan}{45a^3d(a+a}$$

$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{2\tan(c+dx)}{9ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{15a^2d(a+a\sec(c+dx))^3} + \frac{2\tan}{45a^3d(a+a}$$

Mathematica [A] time = 0.218125, size = 110, normalized size = 0.79

$$\frac{\sec\left(\frac{c}{2}\right)\left(-45\sin\left(c+\frac{dx}{2}\right)+54\sin\left(c+\frac{3dx}{2}\right)-30\sin\left(2c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)+}{5760a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(81*Sin[(d*x)/2] - 45*Sin[c + (d*x)/2] + 54*Si
n[c + (3*d*x)/2] - 30*Sin[2*c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Si
n[3*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2]))/(5760*a^5*d)
```

Maple [A] time = 0.039, size = 45, normalized size = 0.3

$$\frac{1}{16\,d a^{5}}\left(\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}-\frac{2}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1
/2*c))
```

Maxima [A] time = 1.13959, size = 90, normalized size = 0.65

$$\frac{\frac{45\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{18\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{5\,\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{720\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.64016, size = 312, normalized size = 2.24

$$\frac{\left(2\,\cos\left(dx+c\right)^{4}+10\,\cos\left(dx+c\right)^{3}+21\,\cos\left(dx+c\right)^{2}+10\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{45\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+c\right)+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**3/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A] time = 1.53563, size = 62, normalized size = 0.45

$$\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{720\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d*
x + 1/2*c))/(a^5*d)
```

3.85 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$\frac{2\tan(c+dx)}{63d\left(a^5\sec(c+dx)+a^5\right)}+\frac{2\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}+\frac{5\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}-\frac{\cdots}{9d(\cdots}$$

```
[Out] -Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (5*Tan[c + d*x])/(63*a*d*(a +
a*Sec[c + d*x])^4) + Tan[c + d*x]/(21*a^2*d*(a + a*Sec[c + d*x])^3) + (2*Ta
n[c + d*x])/(63*a*d*(a^2 + a^2*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(63*d*(a
^5 + a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.157324, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3797, 3796, 3794}

$$\frac{2\tan(c+dx)}{63d\left(a^5\sec(c+dx)+a^5\right)}+\frac{2\tan(c+dx)}{63ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{\tan(c+dx)}{21a^2d(a\sec(c+dx)+a)^3}+\frac{5\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}-\frac{\cdots}{9d(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (5*Tan[c + d*x])/(63*a*d*(a +
a*Sec[c + d*x])^4) + Tan[c + d*x]/(21*a^2*d*(a + a*Sec[c + d*x])^3) + (2*Ta
n[c + d*x])/(63*a*d*(a^2 + a^2*Sec[c + d*x])^2) + (2*Tan[c + d*x])/(63*d*(a
^5 + a^5*Sec[c + d*x]))
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3} + \frac{2\int \frac{\text{se}}{(a+a\,\text{s}}}{2}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3} + \frac{2\,\text{t}}{63a^3d(a}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{5\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\tan(c+dx)}{21a^2d(a+a\sec(c+dx))^3} + \frac{2\,\text{t}}{63a^3d(a}$$

Mathematica [A] time = 0.22972, size = 125, normalized size = 0.87

$$\frac{\sec\left(\frac{c}{2}\right)\left(-315\sin\left(c+\frac{dx}{2}\right)+273\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+117\sin\left(2c+\frac{5dx}{2}\right)-63\sin\left(3c+\frac{5dx}{2}\right)+45\sin\left(3c\right.\right.}{16128a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(315*Sin[(d*x)/2] - 315*Sin[c + (d*x)/2] + 273
*Sin[c + (3*d*x)/2] - 147*Sin[2*c + (3*d*x)/2] + 117*Sin[2*c + (5*d*x)/2] -
 63*Sin[3*c + (5*d*x)/2] + 45*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)/2]
))/(16128*a^5*d)
```

Maple [A] time = 0.035, size = 58, normalized size = 0.4

$$\frac{\frac{1}{16}\left(-\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{2}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/2*d*x+1/2*c)^7-2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.05075, size = 117, normalized size = 0.82

$$\frac{\frac{63\sin(dx+c)}{\cos(dx+c)+1}-\frac{42\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{18\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.61194, size = 312, normalized size = 2.18

$$\frac{\left(5\cos(dx+c)^4+25\cos(dx+c)^3+21\cos(dx+c)^2+10\cos(dx+c)+2\right)\sin(dx+c)}{63\left(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10\,a^5d\cos(dx+c)^3+10\,a^5d\cos(dx+c)^2+5\,a^5d\cos(dx+c)+a^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
```

```
5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\,dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)**2/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c +
d*x)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A] time = 1.4709, size = 80, normalized size = 0.56

$$-\frac{7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+42\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{1008\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

3.86 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$\frac{8\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{8\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{4\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}+$$

```
[Out] Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (4*Tan[c + d*x])/(63*a*d*(a + a
*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*x])^3) + (
8*Tan[c + d*x])/(315*a*d*(a^2 + a^2*Sec[c + d*x])^2) + (8*Tan[c + d*x])/(31
5*d*(a^5 + a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.14238, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3796, 3794}

$$\frac{8\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}+\frac{8\tan(c+dx)}{315ad\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{4\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{4\tan(c+dx)}{63ad(a\sec(c+dx)+a)^4}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```


```
[Out] Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) + (4*Tan[c + d*x])/(63*a*d*(a + a
*Sec[c + d*x])^4) + (4*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*x])^3) + (
8*Tan[c + d*x])/(315*a*d*(a^2 + a^2*Sec[c + d*x])^2) + (8*Tan[c + d*x])/(31
5*d*(a^5 + a^5*Sec[c + d*x]))
```


Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
```

```
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^5}\,dx = \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a}$$
$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{21a^2}$$
$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} + \frac{8\int \frac{\mathrm{sec}}{(a+a\,\mathrm{s}}}{10}$$
$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} + \frac{8\,\mathrm{t}}{315a^3d(a}$$
$$= \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} + \frac{4\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{4\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} + \frac{8\,\mathrm{t}}{315a^3d(a}$$

Mathematica [A] time = 0.266656, size = 138, normalized size = 0.97

$$\frac{\sec\left(\frac{c}{2}\right)\left(-5040\sin\left(c+\frac{dx}{2}\right)+3612\sin\left(c+\frac{3dx}{2}\right)-3360\sin\left(2c+\frac{3dx}{2}\right)+1728\sin\left(2c+\frac{5dx}{2}\right)-1260\sin\left(3c+\frac{5dx}{2}\right)+432}{80640a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(5418*Sin[(d*x)/2] - 5040*Sin[c + (d*x)/2] + 3
612*Sin[c + (3*d*x)/2] - 3360*Sin[2*c + (3*d*x)/2] + 1728*Sin[2*c + (5*d*x)
/2] - 1260*Sin[3*c + (5*d*x)/2] + 432*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c +
(7*d*x)/2] + 83*Sin[4*c + (9*d*x)/2]))/(80640*a^5*d)
```

Maple [A] time = 0.038, size = 71, normalized size = 0.5

$$\frac{1}{16\,da^5}\left(\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{4}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{6}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{4}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.0955, size = 144, normalized size = 1.01

$$\frac{\frac{315 \sin(dx+c)}{\cos(dx+c)+1} - \frac{420 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{378 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{180 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} + \frac{35 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\, a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.66341, size = 316, normalized size = 2.21

$$\frac{\left(83 \cos(dx+c)^4 + 100 \cos(dx+c)^3 + 84 \cos(dx+c)^2 + 40 \cos(dx+c) + 8\right) \sin(dx+c)}{315 \left(a^5 d \cos(dx+c)^5 + 5\, a^5 d \cos(dx+c)^4 + 10\, a^5 d \cos(dx+c)^3 + 10\, a^5 d \cos(dx+c)^2 + 5\, a^5 d \cos(dx+c) + a^5 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos(
d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(c+dx)}{\sec^5(c+dx)+5\sec^4(c+dx)+10\sec^3(c+dx)+10\sec^2(c+dx)+5\sec(c+dx)+1}\, dx}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(sec(c + d*x)/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x
)**3 + 10*sec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A] time = 1.50382, size = 97, normalized size = 0.68

$$\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-180\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+378\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{5040\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

3.87 $\int \frac{1}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=144

$$-\frac{488\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}-\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{34\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{x}{a^5}-\frac{13\tan(c+dx)}{63ad(a\sec(c+dx)+a)}$$

```
[Out] x/a^5 - Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - (13*Tan[c + d*x])/(63*a
*d*(a + a*Sec[c + d*x])^4) - (34*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*
x])^3) - (173*Tan[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (488*Tan[c
 + d*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.207306, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3777, 3922, 3919, 3794}

$$-\frac{488\tan(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}-\frac{173\tan(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{34\tan(c+dx)}{105a^2d(a\sec(c+dx)+a)^3}+\frac{x}{a^5}-\frac{13\tan(c+dx)}{63ad(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5),x]
```

```
[Out] x/a^5 - Tan[c + d*x]/(9*d*(a + a*Sec[c + d*x])^5) - (13*Tan[c + d*x])/(63*a
*d*(a + a*Sec[c + d*x])^4) - (34*Tan[c + d*x])/(105*a^2*d*(a + a*Sec[c + d*
x])^3) - (173*Tan[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (488*Tan[c
 + d*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
```

```
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^5}\,dx = -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{-9a+4a\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} + \frac{\int \frac{63a^2-39a^2\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{\int \frac{-315a^3 \ldots}{(a+ \ldots}}{\ldots}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{17 \ldots}{315a^3d(\ldots}$$

$$= \frac{x}{a^5} - \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{\ldots}{315 \ldots}$$

$$= \frac{x}{a^5} - \frac{\tan(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{13\tan(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{34\tan(c+dx)}{105a^2d(a+a\sec(c+dx))^3} - \frac{\ldots}{315 \ldots}$$

Mathematica [A] time = 0.522156, size = 280, normalized size = 1.94

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(100800\sin\left(c+\frac{dx}{2}\right) - 88284\sin\left(c+\frac{3dx}{2}\right) + 56700\sin\left(2c+\frac{3dx}{2}\right) - 43236\sin\left(2c+\frac{5dx}{2}\right) + 189 \ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(39690*d*x*Cos[(d*x)/2] + 39690*d*x*Cos[c + (d
*x)/2] + 26460*d*x*Cos[c + (3*d*x)/2] + 26460*d*x*Cos[2*c + (3*d*x)/2] + 11
340*d*x*Cos[2*c + (5*d*x)/2] + 11340*d*x*Cos[3*c + (5*d*x)/2] + 2835*d*x*Co
s[3*c + (7*d*x)/2] + 2835*d*x*Cos[4*c + (7*d*x)/2] + 315*d*x*Cos[4*c + (9*d
*x)/2] + 315*d*x*Cos[5*c + (9*d*x)/2] - 116676*Sin[(d*x)/2] + 100800*Sin[c
+ (d*x)/2] - 88284*Sin[c + (3*d*x)/2] + 56700*Sin[2*c + (3*d*x)/2] - 43236*
Sin[2*c + (5*d*x)/2] + 18900*Sin[3*c + (5*d*x)/2] - 12384*Sin[3*c + (7*d*x)
/2] + 3150*Sin[4*c + (7*d*x)/2] - 1726*Sin[4*c + (9*d*x)/2]))/(161280*a^5*d
)
```

Maple [A] time = 0.044, size = 113, normalized size = 0.8

$$-\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{3}{56\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{1}{5\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{13}{24\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31}{16\,da^5}\,\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+3/56/d/a^5*tan(1/2*d*x+1/2*c)^7-1/5/d/a^5
*tan(1/2*d*x+1/2*c)^5+13/24/d/a^5*tan(1/2*d*x+1/2*c)^3-31/16/d/a^5*tan(1/2*
d*x+1/2*c)+2/d/a^5*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.66243, size = 178, normalized size = 1.24

$$-\frac{\frac{\frac{9765\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{2730\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{270\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 10
```

```
080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

Fricas [A] time = 1.71136, size = 514, normalized size = 3.57

$$\frac{315\,dx\cos\left(dx+c\right)^{5}+1575\,dx\cos\left(dx+c\right)^{4}+3150\,dx\cos\left(dx+c\right)^{3}+3150\,dx\cos\left(dx+c\right)^{2}+1575\,dx\cos\left(dx+c\right)+315}{315\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863*c
os(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x +
c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^{5}\left(c+dx\right)+5\sec^{4}\left(c+dx\right)+10\sec^{3}\left(c+dx\right)+10\sec^{2}\left(c+dx\right)+5\sec\left(c+dx\right)+1}\,dx}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Integral(1/(sec(c + d*x)**5 + 5*sec(c + d*x)**4 + 10*sec(c + d*x)**3 + 10*s
ec(c + d*x)**2 + 5*sec(c + d*x) + 1), x)/a**5
```

Giac [A] time = 1.3747, size = 135, normalized size = 0.94

$$\frac{\frac{5040\,(dx+c)}{a^{5}}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*tan
(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/2*
d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.88 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=159

$$\frac{496\sin(c+dx)}{63a^5d}-\frac{5\sin(c+dx)}{d\left(a^5\sec(c+dx)+a^5\right)}-\frac{67\sin(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}-\frac{29\sin(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{5x}{a^5}-\frac{5\sin(c}{21ad(a\sec(c}$$

```
[Out] (-5*x)/a^5 + (496*Sin[c + d*x])/(63*a^5*d) - Sin[c + d*x]/(9*d*(a + a*Sec[c
 + d*x])^5) - (5*Sin[c + d*x])/(21*a*d*(a + a*Sec[c + d*x])^4) - (29*Sin[c
+ d*x])/(63*a^2*d*(a + a*Sec[c + d*x])^3) - (67*Sin[c + d*x])/(63*a^3*d*(a
+ a*Sec[c + d*x])^2) - (5*Sin[c + d*x])/(d*(a^5 + a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.397208, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3817, 4020, 3787, 2637, 8}

$$\frac{496\sin(c+dx)}{63a^5d}-\frac{5\sin(c+dx)}{d\left(a^5\sec(c+dx)+a^5\right)}-\frac{67\sin(c+dx)}{63a^3d(a\sec(c+dx)+a)^2}-\frac{29\sin(c+dx)}{63a^2d(a\sec(c+dx)+a)^3}-\frac{5x}{a^5}-\frac{5\sin(c}{21ad(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```



```
[Out] (-5*x)/a^5 + (496*Sin[c + d*x])/(63*a^5*d) - Sin[c + d*x]/(9*d*(a + a*Sec[c
 + d*x])^5) - (5*Sin[c + d*x])/(21*a*d*(a + a*Sec[c + d*x])^4) - (29*Sin[c
+ d*x])/(63*a^2*d*(a + a*Sec[c + d*x])^3) - (67*Sin[c + d*x])/(63*a^3*d*(a
+ a*Sec[c + d*x])^2) - (5*Sin[c + d*x])/(d*(a^5 + a^5*Sec[c + d*x]))
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\cos(c+dx)(-10a+5a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos(c+dx)\left(-85a^2+60a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - \frac{\int \frac{\cos(c+}{}}{}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - \frac{6}{63a^3d(}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - \frac{6}{63a^3d(}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin(c+dx)}{63a^2d(a+a\sec(c+dx))^3} - \frac{6}{63a^3d(}$$

$$= -\frac{5x}{a^5} + \frac{496\sin(c+dx)}{63a^5d} - \frac{\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{5\sin(c+dx)}{21ad(a+a\sec(c+dx))^4} - \frac{29\sin}{63a^2d(a+}$$

Mathematica [B] time = 0.659211, size = 319, normalized size = 2.01

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(143010\sin\left(c+\frac{dx}{2}\right)-138726\sin\left(c+\frac{3dx}{2}\right)+73290\sin\left(2c+\frac{3dx}{2}\right)-70389\sin\left(2c+\frac{5dx}{2}\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^9*(79380*d*x*Cos[(d*x)/2] + 79380*d*x*Cos[c + (
d*x)/2] + 52920*d*x*Cos[c + (3*d*x)/2] + 52920*d*x*Cos[2*c + (3*d*x)/2] + 2
2680*d*x*Cos[2*c + (5*d*x)/2] + 22680*d*x*Cos[3*c + (5*d*x)/2] + 5670*d*x*C
os[3*c + (7*d*x)/2] + 5670*d*x*Cos[4*c + (7*d*x)/2] + 630*d*x*Cos[4*c + (9*
d*x)/2] + 630*d*x*Cos[5*c + (9*d*x)/2] - 175014*Sin[(d*x)/2] + 143010*Sin[c
 + (d*x)/2] - 138726*Sin[c + (3*d*x)/2] + 73290*Sin[2*c + (3*d*x)/2] - 7038
9*Sin[2*c + (5*d*x)/2] + 20475*Sin[3*c + (5*d*x)/2] - 21141*Sin[3*c + (7*d*
x)/2] + 1575*Sin[4*c + (7*d*x)/2] - 3091*Sin[4*c + (9*d*x)/2] - 567*Sin[5*c
 + (9*d*x)/2] - 63*Sin[5*c + (11*d*x)/2] - 63*Sin[6*c + (11*d*x)/2]))/(6451
2*a^5*d)
```

Maple [A] time = 0.07, size = 145, normalized size = 0.9

$$\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{1}{14\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{8\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{3}{2\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{129}{16\,da^5}\tan\left(\frac{d}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^5,x)
```

```
[Out] 1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-1/14/d/a^5*tan(1/2*d*x+1/2*c)^7+3/8/d/a^5*
tan(1/2*d*x+1/2*c)^5-3/2/d/a^5*tan(1/2*d*x+1/2*c)^3+129/16/d/a^5*tan(1/2*d*
x+1/2*c)+2/d/a^5*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-10/d/a^5*arcta
n(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.52837, size = 240, normalized size = 1.51

$$\frac{\frac{2016\sin(dx+c)}{\left(a^5+\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\sin(dx+c)}{\cos(dx+c)+1}-\frac{1512\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{72\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

Fricas [A] time = 1.71159, size = 541, normalized size = 3.4

$$-\frac{315\,dx\cos\left(dx+c\right)^{5}+1575\,dx\cos\left(dx+c\right)^{4}+3150\,dx\cos\left(dx+c\right)^{3}+3150\,dx\cos\left(dx+c\right)^{2}+1575\,dx\cos\left(dx+c\right)+3}{63\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*co
s(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x + c)
^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d
*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5
*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.56401, size = 174, normalized size = 1.09

$$-\frac{\frac{5040\,(dx+c)}{a^5}-\frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^5}-\frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+378\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1512\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8127\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x + 1/
2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c)^3
 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.89 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx$

Optimal. Leaf size=215

$$-\frac{7664\sin(c+dx)}{315a^5d}+\frac{31\sin(c+dx)\cos(c+dx)}{2a^5d}-\frac{3832\sin(c+dx)\cos(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}-\frac{577\sin(c+dx)\cos(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{28\sin(}{45a^2d(}$$

```
[Out] (31*x)/(2*a^5) - (7664*Sin[c + d*x])/(315*a^5*d) + (31*Cos[c + d*x]*Sin[c +
 d*x])/(2*a^5*d) - (Cos[c + d*x]*Sin[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5)
 - (17*Cos[c + d*x]*Sin[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) - (28*Cos
[c + d*x]*Sin[c + d*x])/(45*a^2*d*(a + a*Sec[c + d*x])^3) - (577*Cos[c + d*
x]*Sin[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (3832*Cos[c + d*x]*Si
n[c + d*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rubi [A] time = 0.509367, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3817, 4020, 3787, 2635, 8, 2637}

$$-\frac{7664\sin(c+dx)}{315a^5d}+\frac{31\sin(c+dx)\cos(c+dx)}{2a^5d}-\frac{3832\sin(c+dx)\cos(c+dx)}{315d\left(a^5\sec(c+dx)+a^5\right)}-\frac{577\sin(c+dx)\cos(c+dx)}{315a^3d(a\sec(c+dx)+a)^2}-\frac{28\sin(}{45a^2d(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (31*x)/(2*a^5) - (7664*Sin[c + d*x])/(315*a^5*d) + (31*Cos[c + d*x]*Sin[c +
 d*x])/(2*a^5*d) - (Cos[c + d*x]*Sin[c + d*x])/(9*d*(a + a*Sec[c + d*x])^5)
 - (17*Cos[c + d*x]*Sin[c + d*x])/(63*a*d*(a + a*Sec[c + d*x])^4) - (28*Cos
[c + d*x]*Sin[c + d*x])/(45*a^2*d*(a + a*Sec[c + d*x])^3) - (577*Cos[c + d*
x]*Sin[c + d*x])/(315*a^3*d*(a + a*Sec[c + d*x])^2) - (3832*Cos[c + d*x]*Si
n[c + d*x])/(315*d*(a^5 + a^5*Sec[c + d*x]))
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^5}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)(-11a+6a\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)\left(-111a^2+85a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3} - \frac{\int \text{co}}{}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3} - \frac{577}{315}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3} - \frac{577}{315}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)\sin(c+dx)}{63ad(a+a\sec(c+dx))^4} - \frac{28\cos(c+dx)\sin(c+dx)}{45a^2d(a+a\sec(c+dx))^3} - \frac{577}{315}$$

$$= -\frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c+dx)}{63ad(a+a\,\text{se}}$$

$$= \frac{31x}{2a^5} - \frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos(c+dx)\sin(c+dx)}{9d(a+a\sec(c+dx))^5} - \frac{17\cos(c}{63ad(a}$$

Mathematica [A] time = 0.721878, size = 345, normalized size = 1.6

$$\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(7194600\sin\left(c+\frac{dx}{2}\right) - 7472241\sin\left(c+\frac{3dx}{2}\right) + 3432975\sin\left(2c+\frac{3dx}{2}\right) - 3871989\sin\left(2c+\frac{5}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(4921560*d*x*Cos[(d*x)/2] + 4921560*d*x*Cos[c
+ (d*x)/2] + 3281040*d*x*Cos[c + (3*d*x)/2] + 3281040*d*x*Cos[2*c + (3*d*x)
/2] + 1406160*d*x*Cos[2*c + (5*d*x)/2] + 1406160*d*x*Cos[3*c + (5*d*x)/2] +
 351540*d*x*Cos[3*c + (7*d*x)/2] + 351540*d*x*Cos[4*c + (7*d*x)/2] + 39060*
d*x*Cos[4*c + (9*d*x)/2] + 39060*d*x*Cos[5*c + (9*d*x)/2] - 9163224*Sin[(d*
x)/2] + 7194600*Sin[c + (d*x)/2] - 7472241*Sin[c + (3*d*x)/2] + 3432975*Sin
[2*c + (3*d*x)/2] - 3871989*Sin[2*c + (5*d*x)/2] + 801675*Sin[3*c + (5*d*x)
/2] - 1186056*Sin[3*c + (7*d*x)/2] - 17640*Sin[4*c + (7*d*x)/2] - 175184*Si
n[4*c + (9*d*x)/2] - 45360*Sin[5*c + (9*d*x)/2] - 3465*Sin[5*c + (11*d*x)/2
```

```
] - 3465*Sin[6*c + (11*d*x)/2] + 315*Sin[6*c + (13*d*x)/2] + 315*Sin[7*c +
(13*d*x)/2]))/(1290240*a^5*d)
```

Maple [A] time = 0.072, size = 179, normalized size = 0.8

$$-\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{5}{56\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{3}{5\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{25}{8\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{351}{16\,da^5}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+5/56/d/a^5*tan(1/2*d*x+1/2*c)^7-3/5/d/a^5
*tan(1/2*d*x+1/2*c)^5+25/8/d/a^5*tan(1/2*d*x+1/2*c)^3-351/16/d/a^5*tan(1/2*
d*x+1/2*c)-11/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-9/d/a^5
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+31/d/a^5*arctan(tan(1/2*d*x+
1/2*c))
```

Maxima [A] time = 1.55647, size = 302, normalized size = 1.4

$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^5+\frac{2a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}-\frac{15750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3024\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{156240\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/(a^5 + 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) + 1)
- 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9)/a^5 - 156240*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^5)/d
```

Fricas [A] time = 1.7123, size = 591, normalized size = 2.75

$$\frac{9765\,dx\cos\left(dx+c\right)^{5}+48825\,dx\cos\left(dx+c\right)^{4}+97650\,dx\cos\left(dx+c\right)^{3}+97650\,dx\cos\left(dx+c\right)^{2}+48825\,dx\cos\left(dx+\right.}{630\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(d
*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x +
(315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440*co
s(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(d*x
+ c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c
)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4101, size = 196, normalized size = 0.91

$$\frac{\frac{78120\,(dx+c)}{a^{5}}-\frac{5040\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{5}}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-450\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+3024\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15750\,a^{40}\tan\left(\right.}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(78120*(d*x + c)/a^5 - 5040*(11*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d
*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*tan(1/2*d*x +
```

```
1/2*c)^9 - 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2*c)
^5 - 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*c))/
a^45)/d
```

3.90 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{8\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{35d}+\frac{4a\tan(c+}{5d\sqrt{a\sec(c+}}$$

```
[Out] (4*a*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sec[c + d*x]^3*Tan
[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (8*Sqrt[a + a*Sec[c + d*x]]*Tan
[c + d*x])/(35*d) + (12*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.206857, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3803, 3800, 4001, 3792}

$$\frac{2a\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{8\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{35d}+\frac{4a\tan(c+}{5d\sqrt{a\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*a*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sec[c + d*x]^3*Tan
[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (8*Sqrt[a + a*Sec[c + d*x]]*Tan
[c + d*x])/(35*d) + (12*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{6}{7}\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{12(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35ad} + \frac{12\int \sec(c\cdots}{\cdots}$$

$$= \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{8\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{35d} + \frac{12(a+a\sec(c\cdots}{\cdots}$$

$$= \frac{4a\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{8\sqrt{a+a\sec(c+dx)}\tan(\cdots}{35d}$$

Mathematica [A] time = 0.139788, size = 58, normalized size = 0.48

$$\frac{2a\tan(c+dx)\left(5\sec^3(c+dx)+6\sec^2(c+dx)+8\sec(c+dx)+16\right)}{35d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*(16 + 8*Sec[c + d*x] + 6*Sec[c + d*x]^2 + 5*Sec[c + d*x]^3)*Tan[c + d*
x])/(35*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.211, size = 82, normalized size = 0.7

$$-\frac{32\left(\cos\left(dx+c\right)\right)^{4}-16\left(\cos\left(dx+c\right)\right)^{3}-4\left(\cos\left(dx+c\right)\right)^{2}-2\cos\left(dx+c\right)-10}{35\,d\left(\cos\left(dx+c\right)\right)^{3}\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/35/d*(16*cos(d*x+c)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.66307, size = 212, normalized size = 1.74

$$\frac{2\left(16\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+6\cos\left(dx+c\right)+5\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x +
c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a (\sec (c + dx) + 1)} \sec^4 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**4, x)
```

Giac [A] time = 4.85081, size = 162, normalized size = 1.33

$$-\frac{2\sqrt{2}\left(35\,a^4 - \left(35\,a^4 + \left(9\,a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 49\,a^4\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2\right)\mathrm{sgn}\left(\cos\left(dx + c\right)\right)\tan\left(\frac{1}{2}\,dx\right.}{35\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*sqrt(2)*(35*a^4 - (35*a^4 + (9*a^4*tan(1/2*d*x + 1/2*c)^2 - 49*a^4)*t
an(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*d*
x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)*d)
```

3.91 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}-\frac{4\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{14a\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (14*a*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (4*Sqrt[a + a*Sec[c +
 d*x]]*Tan[c + d*x])/(15*d) + (2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(
5*a*d)
```

Rubi [A] time = 0.152013, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3800, 4001, 3792}

$$\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}-\frac{4\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{14a\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (14*a*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) - (4*Sqrt[a + a*Sec[c +
 d*x]]*Tan[c + d*x])/(15*d) + (2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(
5*a*d)
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
+ 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad} + \frac{2\int \sec(c+dx)\left(\frac{3a}{2}-a\sec(c+dx)\right)\sqrt{a+a}}{5a}$$

$$= -\frac{4\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad} + \frac{7}{15}\int$$

$$= \frac{14a\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} - \frac{4\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2(a+a\sec(c+dx))}{5ad}$$

Mathematica [A] time = 0.102349, size = 48, normalized size = 0.56

$$\frac{2a\tan(c+dx)\left(3\sec^2(c+dx)+4\sec(c+dx)+8\right)}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*(8 + 4*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[a*(1
+ Sec[c + d*x])])
```

Maple [A] time = 0.169, size = 72, normalized size = 0.8

$$-\frac{16\,(\cos(dx+c))^3-8\,(\cos(dx+c))^2-2\,\cos(dx+c)-6}{15\,d\,(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(8*cos(d*x+c)^3-4*cos(d*x+c)^2-cos(d*x+c)-3)*(a*(cos(d*x+c)+1)/cos(
d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.67688, size = 185, normalized size = 2.15

$$\frac{2 \left(8 \cos \left(dx+c\right)^{2}+4 \cos \left(dx+c\right)+3\right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sin \left(dx+c\right)}{15 \left(d \cos \left(dx+c\right)^{3}+d \cos \left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \left(\sec \left(c+dx\right)+1\right)} \sec^{3}\left(c+dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**3, x)
```

Giac [A] time = 4.85842, size = 136, normalized size = 1.58

$$\frac{2\sqrt{2}\left(15\,a^3 + \left(7\,a^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 10\,a^3\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2\right)\mathrm{sgn}\left(\cos\left(dx + c\right)\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{15\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - a\right)^2\sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(2)*(15*a^3 + (7*a^3*tan(1/2*d*x + 1/2*c)^2 - 10*a^3)*tan(1/2*d*x
+ 1/2*c)^2)*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)
^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.92 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a + a*Sec[c + d
*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0826257, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3798, 3792}

$$\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a + a*Sec[c + d
*x]]*Tan[c + d*x])/(3*d)
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3}\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{2a\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

Mathematica [A] time = 0.0946142, size = 36, normalized size = 0.64

$$\frac{2a\tan(c+dx)(\sec(c+dx)+2)}{3d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*(2 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.141, size = 62, normalized size = 1.1

$$-\frac{4\,(\cos(dx+c))^2 - 2\,\cos(dx+c) - 2}{3\,d\sin(dx+c)\cos(dx+c)}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(2*cos(d*x+c)^2-cos(d*x+c)-1)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/si
n(d*x+c)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
3/4)*sqrt(a)*d*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x
 + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x
+ 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x
+ 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x +
 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x
+ 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x +
 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x +
6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2
*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(2*cos
(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 4
*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c
)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*
x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + s
in(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
^2 + (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*
x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + c
os(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)), x) + sqrt(a)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(3/4)*d)
```

Fricas [A] time = 1.64511, size = 155, normalized size = 2.77

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(2\cos(dx+c)+1)\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(2*cos(d*x + c) + 1)*sin(d*x +
c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**2, x)
```

Giac [A] time = 4.85203, size = 111, normalized size = 1.98

$$\frac{2\sqrt{2}\left(a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3a^{2}\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(2)*(a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2)*sgn(cos(d*x + c))*tan(1/2*
d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)*d)
```

3.93 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0294322, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3792}

$$\frac{2a\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{2a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.0682461, size = 29, normalized size = 1.12

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [A] time = 0.122, size = 42, normalized size = 1.6

$$-2\,{\frac {-1+\cos \left(dx+c \right) }{d\sin \left(dx+c \right) }\sqrt {{\frac {a \left(\cos \left(dx+c \right) +1 \right) }{\cos \left(dx+c \right) }}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec \left(d x + c\right) + a} \sec \left(d x + c\right)\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [A] time = 1.65249, size = 104, normalized size = 4.

$$\frac{2 \, \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sin\left(d x + c\right)}{d \cos\left(d x + c\right) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x), x)
```

Giac [B] time = 4.77986, size = 84, normalized size = 3.23

$$-\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}a\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a*sgn(cos(d*x + c))*tan(1/2*
d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

3.94 $\int \sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

Rubi [A] time = 0.0226566, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,dx = -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

Mathematica [A] time = 0.0943218, size = 60, normalized size = 1.62

$$\frac{\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/
2]*Sqrt[a*(1 + Sec[c + d*x])])/d
```

Maple [B] time = 0.139, size = 89, normalized size = 2.4

$$-\frac{\sqrt{2}}{d}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)/cos(d*x+c))
```

Maxima [B] time = 1.82355, size = 197, normalized size = 5.32

$$\sqrt{a}\arctan\left(\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

Fricas [A] time = 1.73095, size = 350, normalized size = 9.46

$$\left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d}, -\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) +
 1))/d, -2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sec(c + d*x) + a), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.95 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Si
n[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0628327, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3805, 3774, 203}

$$\frac{a\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*Si
n[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx &= \frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}\int \sqrt{a+a\sec(c+dx)}\,dx\\ &= \frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.191779, size = 62, normalized size = 1.

$$\frac{a\tan(c+dx)\left(\cos(c+dx)+\frac{\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(Cos[c + d*x] + ArcTanh[Sqrt[1 - Sec[c + d*x]]]/Sqrt[1 - Sec[c + d*x]])*
Tan[c + d*x])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.185, size = 123, normalized size = 2.

$$-\frac{1}{2\,d\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\,\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)+2\,(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+2*cos(
d*x+c)^2-2*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.16503, size = 1068, normalized size = 17.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

Fricas [A] time = 1.76771, size = 647, normalized size = 10.44

$$\left[\frac{\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin\right.}{2\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x +
 c) - a)/(cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c))/(d*cos(d*x + c) + d), -(sqrt(a)*(cos(d*x + c) + 1)
*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d
*x + c))) - sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c))/(d*cos(d*x + c) + d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\,(\sec(c+dx)+1)}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.96 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=102

$$\frac{3a\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (3*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) +
 (3*a*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]*Sin[c
+ d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.118228, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3805, 3774, 203}

$$\frac{3a\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (3*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) +
 (3*a*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]*Sin[c
+ d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{3}{4}\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{3}{8}\int \sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\right.}{4d}$$

$$= \frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.0976047, size = 47, normalized size = 0.46

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

Maple [B] time = 0.22, size = 221, normalized size = 2.2

$$\frac{1}{16\,d\sin(dx+c)\cos(dx+c)}\left(3\sqrt{2}\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\text{Artanh}\left(1/2\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(3*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(
d*x+c)+3*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-8*cos
(d*x+c)^4-4*cos(d*x+c)^3+12*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [B] time = 2.26818, size = 1430, normalized size = 14.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
```

```
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 1)))/d
```

Fricas [A] time = 1.79466, size = 720, normalized size = 7.06

$$\left[\frac{3\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\cos(dx+c)^2+3\cos(dx\right.}{8\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
 + c) - a)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/
4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 + 3*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) +
 d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec(c+dx)+1\right)}\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.97 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{5a\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{5\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{5a\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (5*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d) +
 (5*a*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (5*a*Cos[c + d*x]*Sin[
c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]^2*Sin[c + d*x])
/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.177855, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3805, 3774, 203}

$$\frac{5a\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{5\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{5a\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (5*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d) +
 (5*a*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (5*a*Cos[c + d*x]*Sin[
c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]^2*Sin[c + d*x])
/(3*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{5}{6}\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{5}{8}\int \cos(c+dx)\sqrt{a+}$$
$$= \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$
$$= \frac{5\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{5a\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{5a\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.0884142, size = 47, normalized size = 0.34

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

Maple [B] time = 0.253, size = 310, normalized size = 2.3

$$-\frac{1}{192\,d\sin(dx+c)(\cos(dx+c))^2}\left(15\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{5/2}\sqrt{2}\text{Artanh}\left(1/2\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/192/d*(15*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*c
os(d*x+c)^2+30*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)
*cos(d*x+c)+15*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)
+64*cos(d*x+c)^6+16*cos(d*x+c)^5+40*cos(d*x+c)^4-120*cos(d*x+c)^3)*(a*(cos(
d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [B] time = 2.58687, size = 2593, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arct
an2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(
```

```
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))/d
```

Fricas [A] time = 1.7675, size = 782, normalized size = 5.67

$$\left[\frac{15\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\cos(dx+c)^3+10\cos(\right.}{48\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*cos(d*x + c)^3 + 10*cos(d*x + c)^2
+ 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c) + d), -1/24*(15*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*cos(
d*x + c)^3 + 10*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.98 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{35a\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{35a\ldots}{9\ldots}$$

```
[Out] (35*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d)
 + (35*a*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (35*a*Cos[c + d*x]
*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (7*a*Cos[c + d*x]^2*Sin[c
+ d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(
4*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.235099, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3805, 3774, 203}

$$\frac{35a\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{35a\ldots}{9\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (35*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d)
 + (35*a*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (35*a*Cos[c + d*x]
*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (7*a*Cos[c + d*x]^2*Sin[c
+ d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(
4*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{7}{8}\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{7a\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{35}{48}\int \cos^2(c+dx)\sqrt{a}$$

$$= \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a\cos^3(c+dx)\sin(c}{4d\sqrt{a+a\sec(c+}$$

$$= \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+dx)\sin(c+d}{24d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{7a\cos^2(c+dx)\sin(c+d}{24d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{35a\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{35a\cos(c+dx)\sin(c+d}{96d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.0869189, size = 47, normalized size = 0.27

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\text{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x
])]*Tan[(c + d*x)/2])/d
```

Maple [B] time = 0.294, size = 399, normalized size = 2.3

$$\frac{1}{3072\,d\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{3}}\left(105\sqrt{2}\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{3}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3072/d*(105*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(7/2)+315*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(7/2)+315*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)+105*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(
d*x+c)-768*cos(d*x+c)^8-128*cos(d*x+c)^7-224*cos(d*x+c)^6-560*cos(d*x+c)^5+
1680*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c
)^3
```

Maxima [B] time = 3.6796, size = 8961, normalized size = 51.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((36*(sin(4*d*x + 4*c)^3 + (cos(4*
d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 9*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c
) + 9*sin(4*d*x + 4*c)^3 + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 + 2
*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 9*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x +
```

4*c), cos(4*d*x + 4*c))) + 36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) - 9)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 9*cos(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 36*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (9*cos(4*d*x + 4*c)^3 + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 - 10*cos(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) + 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 + 26*cos(4*d*x + 4*c)^2 + 25*cos(4*d*x + 4*c) + 8)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 - (32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*cos(4*d*x + 4*c)^2 + 2*(16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 7*cos(4*d*x + 4*c) - 9)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8*sin(4*d*x + 4*c)^2 - 2*(64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 7*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 9*cos(4*d*x + 4*c))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 4*(9*cos(4*d*x + 4*c)^3 + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c)^2 - 8*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 9*(2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 2*(cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(9*cos(4*d*x + 4*c) + 8)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (9*cos(4*d*x + 4*c) + 8)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((64*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 20*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(4*d*x + 4*c) + 8*(cos(4*d*x + 4*c)

```
^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 5*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 5*sin(4*d*x + 4*c)^3 + 4
*(5*sin(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c)^2 + 10*cos(4*d*x + 4*c) - 11)*
sin(4*d*x + 4*c) - 64*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*
sin(4*d*x + 4*c) + 40*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*
x + 4*c) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 10*(2*sin(4*d*x + 4*c)^3
+ 2*(cos(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(4*d*x + 4*c) + cos(1/4*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (16*cos(4*d*x +
 4*c)^2 + 16*sin(4*d*x + 4*c)^2 - 17*cos(4*d*x + 4*c) + 1)*sin(1/4*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 5*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*si
n(4*d*x + 4*c) + 2*(32*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d
*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*c
os(4*d*x + 4*c)^2 + 8*(4*cos(4*d*x + 4*c)^2 - sin(4*d*x + 4*c)^2 - 40*sin(4
*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*cos(4*
d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*(cos(4
*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*s
in(4*d*x + 4*c)^2 - 85*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 5
*(8*cos(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*sin(1/4*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)) - (64*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*
cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
3 + 5*cos(4*d*x + 4*c)^3 + 4*(5*cos(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c) -
8)*sin(4*d*x + 4*c)^2 - 18*cos(4*d*x + 4*c)^2 + 8*(cos(4*d*x + 4*c)^2 + sin
(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/4*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 37*cos(4*d*x + 4*c) - 24)*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + (5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c)^2
+ 4*(5*cos(4*d*x + 4*c)^3 + (5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c)^2 -
14*cos(4*d*x + 4*c)^2 + 16*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos
(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 8
*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*cos(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 43*cos(4*d*x + 4*c) - 24)*s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - 24*cos(4*d*x + 4*c)
^2 + 2*(10*cos(4*d*x + 4*c)^3 + 10*(cos(4*d*x + 4*c) - 4)*sin(4*d*x + 4*c)^
2 - 50*cos(4*d*x + 4*c)^2 + (16*cos(4*d*x + 4*c)^2 + 16*sin(4*d*x + 4*c)^2
- 21*cos(4*d*x + 4*c) + 5)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) - 5*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 48*cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + (8*cos(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)^2 - 5*cos(4*d*x + 4*c))*c
os(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(128*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2*sin(4*d*x + 4*c) + 8*(5*(cos(4*d*x
```

```
 + 4*c) - 4)*sin(4*d*x + 4*c) + 8*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 2*(5*cos(4*d*x + 4*c) - 24)*sin(4*d*x + 4*c) + 21*cos(1/4*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - 5*(cos(4*d*x + 4*c
) + 1)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*sin(4*d*x + 4*c)*sin(1/4*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)))*sqrt(a) - 105*((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)
^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c)
+ 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4
*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(
4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + s
in(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arct
an2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(
1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*s
in(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (4*(cos(4*d*x
+ 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c) + 1)*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4
*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)
^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(
```

```
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1))) - 1) - (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*
c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d
*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x
+ 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + 1) + (4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x + 4*c)
 + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(cos(4*d*x
 + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4*d*x +
4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
) - 1))*sqrt(a))/((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*d*x
 + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(cos(4
*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(sin(
```

```
4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*c))
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*d)
```

Fricas [A] time = 1.86806, size = 846, normalized size = 4.86

$$\left[\frac{105\sqrt{-a}(\cos(dx+c)+1)\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48\cos(dx+c)^4+56\cos\right.}{384\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*sqrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*cos(d*x + c)^4 + 56*cos(d*x + c)
^3 + 70*cos(d*x + c)^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(105*sqrt(a)*(cos(d*x +
c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) - (48*cos(d*x + c)^4 + 56*cos(d*x + c)^3 + 70*cos(d*x + c)
^2 + 105*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.99 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{68a^2\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{68\tan(c+dx)(a\sec(c+dx)+}{105d}$$

```
[Out] (68*a^2*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (34*a^2*Sec[c + d*x
]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sec[c + d*x]^4*T
an[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (136*a*Sqrt[a + a*Sec[c + d*x
]]*Tan[c + d*x])/(315*d) + (68*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(10
5*d)
```

Rubi [A] time = 0.275272, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3814, 21, 3803, 3800, 4001, 3792}

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{68a^2\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{68\tan(c+dx)(a\sec(c+dx)+}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (68*a^2*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (34*a^2*Sec[c + d*x
]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sec[c + d*x]^4*T
an[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (136*a*Sqrt[a + a*Sec[c + d*x
]]*Tan[c + d*x])/(315*d) + (68*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(10
5*d)
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(2a)\int \frac{\sec^4(c+dx)\left(\frac{17a}{2}+\frac{17}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d$$

$$= \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(17a)\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{21}(34a)\int \mathrm{sec}$$

$$= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{68(a+a\sec(c}{}$$

$$= \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} - \frac{136a\sqrt{a+a\,\mathrm{se}}}{}$$

$$= \frac{68a^2\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\tan}{9d\sqrt{a+a\sec(c}}$$

Mathematica [A] time = 0.522539, size = 70, normalized size = 0.43

$$\frac{2a^2\tan(c+dx)\left(35\sec^4(c+dx)+85\sec^3(c+dx)+102\sec^2(c+dx)+136\sec(c+dx)+272\right)}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(272 + 136*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 85*Sec[c + d*x]^3 + 3
5*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.166, size = 93, normalized size = 0.6

$$-\frac{2a\left(272(\cos(dx+c))^5-136(\cos(dx+c))^4-34(\cos(dx+c))^3-17(\cos(dx+c))^2-50\cos(dx+c)-35\right)}{315d(\cos(dx+c))^4\sin(dx+c)}\sqrt{\frac{a(\cos}{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/315/d*a*(272*cos(d*x+c)^5-136*cos(d*x+c)^4-34*cos(d*x+c)^3-17*cos(d*x+c)
^2-50*cos(d*x+c)-35)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d
```

```
*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [A] time = 1.74658, size = 262, normalized size = 1.62

$$\frac{2 \left(272 \, a \cos\left(dx+c\right)^{4}+136 \, a \cos\left(dx+c\right)^{3}+102 \, a \cos\left(dx+c\right)^{2}+85 \, a \cos\left(dx+c\right)+35 \, a\right) \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \sin\left(dx+c\right)}{315 \left(d \cos\left(dx+c\right)^{5}+d \cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 2/315*(272*a*cos(d*x + c)^4 + 136*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2 +
 85*a*cos(d*x + c) + 35*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(3/2),x)

[Out] Timed out
```

Giac [A] time = 5.21234, size = 243, normalized size = 1.5

$$\frac{4\left(315\sqrt{2}a^6\text{sgn}\left(\cos\left(dx+c\right)\right)-\left(525\sqrt{2}a^6\text{sgn}\left(\cos\left(dx+c\right)\right)-\left(819\sqrt{2}a^6\text{sgn}\left(\cos\left(dx+c\right)\right)+47\left(2\sqrt{2}a^6\text{sgn}\left(\cos\left(dx+\right.\right.\right.\right.\right.}{315\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 4/315*(315*sqrt(2)*a^6*sgn(cos(d*x + c)) - (525*sqrt(2)*a^6*sgn(cos(d*x + c
)) - (819*sqrt(2)*a^6*sgn(cos(d*x + c)) + 47*(2*sqrt(2)*a^6*sgn(cos(d*x + c
))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x +
1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2
*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d
)
```

3.100 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{152a^2\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}-\frac{4\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{38a\tan(c+dx)\sqrt{\cdots}}{10\cdots}$$

```
[Out] (152*a^2*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (38*a*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(105*d) - (4*(a + a*Sec[c + d*x])^(3/2)*Tan[c +
 d*x])/(35*d) + (2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.193941, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3800, 4001, 3793, 3792}

$$\frac{152a^2\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}-\frac{4\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{38a\tan(c+dx)\sqrt{\cdots}}{10\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (152*a^2*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (38*a*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(105*d) - (4*(a + a*Sec[c + d*x])^(3/2)*Tan[c +
 d*x])/(35*d) + (2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
```

```
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad} + \frac{2\int \sec(c+dx)\left(\frac{5a}{2}-a\sec(c+dx)\right)(a+\\ &= -\frac{4(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad} + \frac{19\\ &= \frac{38a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d} - \frac{4(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d} + \frac{2(a\\ &= \frac{152a^2\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{105d} - \frac{4(a+a\sec(c\end{aligned}$$

Mathematica [A] time = 0.178844, size = 60, normalized size = 0.52

$$\frac{2a^2\tan(c+dx)\left(15\sec^3(c+dx)+39\sec^2(c+dx)+52\sec(c+dx)+104\right)}{105d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(104 + 52*Sec[c + d*x] + 39*Sec[c + d*x]^2 + 15*Sec[c + d*x]^3)*Tan[
c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.148, size = 83, normalized size = 0.7

$$-\frac{2a\left(104\left(\cos\left(dx+c\right)\right)^{4}-52\left(\cos\left(dx+c\right)\right)^{3}-13\left(\cos\left(dx+c\right)\right)^{2}-24\cos\left(dx+c\right)-15\right)}{105d\left(\cos\left(dx+c\right)\right)^{3}\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/105/d*a*(104*cos(d*x+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-
15)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.69004, size = 230, normalized size = 1.98

$$\frac{2\left(104a\cos\left(dx+c\right)^{3}+52a\cos\left(dx+c\right)^{2}+39a\cos\left(dx+c\right)+15a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 +
 d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.83551, size = 204, normalized size = 1.76

$$-\frac{4\left(105\sqrt{2}a^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(140\sqrt{2}a^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+19\left(2\sqrt{2}a^5\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7\sqrt{2}a^5\right.\right.\right.}{105\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -4/105*(105*sqrt(2)*a^5*sgn(cos(d*x + c)) - (140*sqrt(2)*a^5*sgn(cos(d*x +
c)) + 19*(2*sqrt(2)*a^5*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2
)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*ta
n(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a)*d)
```

3.101 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=86

$$\frac{8a^2\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (8*a^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(5*d) + (2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/
(5*d)
```

Rubi [A] time = 0.119957, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3798, 3793, 3792}

$$\frac{8a^2\tan(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{5d}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(5*d) + (2*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/
(5*d)
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{3}{5}\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$$
$$= \frac{2a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{5d}+\frac{2(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{1}{5}(4a$$
$$= \frac{8a^2\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\frac{2a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{5d}+\frac{2(a+a\sec(c+d}{5}$$

Mathematica [A] time = 0.129918, size = 48, normalized size = 0.56

$$\frac{2a^2\tan(c+dx)\left(\sec^2(c+dx)+3\sec(c+dx)+6\right)}{5d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(6 + 3*Sec[c + d*x] + Sec[c + d*x]^2)*Tan[c + d*x])/(5*d*Sqrt[a*(1 +
 Sec[c + d*x])])
```

Maple [A] time = 0.135, size = 73, normalized size = 0.9

$$-\frac{2a\left(6(\cos(dx+c))^3-3(\cos(dx+c))^2-2\cos(dx+c)-1\right)}{5d(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d*a*(6*cos(d*x+c)^3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*(a*(cos(d*x+c)+1)/c
os(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.71473, size = 189, normalized size = 2.2

$$\frac{2\left(6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sec^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**2, x)
```

Giac [A] time = 4.93236, size = 163, normalized size = 1.9

$$\frac{4\left(5\sqrt{2}a^{4}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+\left(2\sqrt{2}a^{4}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-5\sqrt{2}a^{4}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)}{5\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 4/5*(5*sqrt(2)*a^4*sgn(cos(d*x + c)) + (2*sqrt(2)*a^4*sgn(cos(d*x + c))*tan
(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)
^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)*d)
```

3.102 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (8*a^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0614329, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3793, 3792}

$$\frac{8a^2\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{2a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3}(4a)\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{8a^2\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

Mathematica [A] time = 0.0875366, size = 38, normalized size = 0.64

$$\frac{2a^2\tan(c+dx)(\sec(c+dx)+5)}{3d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(5 + Sec[c + d*x])*Tan[c + d*x])/(3*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.125, size = 63, normalized size = 1.1

$$-\frac{2a\left(5(\cos(dx+c))^2-4\cos(dx+c)-1\right)}{3d\sin(dx+c)\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*a*(5*cos(d*x+c)^2-4*cos(d*x+c)-1)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2
)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [A] time = 1.68546, size = 158, normalized size = 2.68

$$\frac{2\,(5\,a\cos\left(dx+c\right)+a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*cos(d*x + c) + a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\,(\sec\left(c+dx\right)+1))^{\frac{3}{2}}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x), x)
```

Giac [A] time = 4.81613, size = 126, normalized size = 2.14

$$\frac{4\left(2\sqrt{2}a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\sqrt{2}a^{3}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 4/3*(2*sqrt(2)*a^3*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 3*sqrt(2)*a^3
*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.103 $\int (a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0368724, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3775, 21, 3774, 203}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2*
a^2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^{3/2}\,dx &= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(2a)\int\frac{\frac{a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx\\ &= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+a\int\sqrt{a+a\sec(c+dx)}\,dx\\ &= \frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{\left(2a^2\right)\text{Subst}\left(\int\frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.212093, size = 75, normalized size = 1.14

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/d
```

Maple [B] time = 0.147, size = 181, normalized size = 2.7

$$\frac{a}{d\left(\cos(dx+c)+1\right)}\left(-\cos(dx+c)\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}-\sqrt{2}\text{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*a*(-cos(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2^(1/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*sin(d*x+c))*(a*(cos(d*x+c)+1)
/cos(d*x+c))^(1/2)/(cos(d*x+c)+1)
```

Maxima [B] time = 2.022, size = 1346, normalized size = 20.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
```

```
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*d)
```

Fricas [A] time = 1.72774, size = 620, normalized size = 9.39

$$\left[\frac{(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [((a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) + 2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c) + d), -2*((a*cos(d*x + c) + a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(c+dx)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.104 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (3*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a^
2*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.118184, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3814, 21, 3805, 3774, 203}

$$\frac{a^2\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a^
2*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= -\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-(2a)\int \frac{\cos(c+dx)\left(-\frac{3a}{2}-\frac{3}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx\\
&= -\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(3a)\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx\\
&= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}(3a)\int \sqrt{a+a\sec(c+dx)}\,dx\\
&= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.192966, size = 89, normalized size = 1.37

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))
)/(2*d)
```

Maple [B] time = 0.162, size = 125, normalized size = 1.9

$$-\frac{a}{2\,d\sin(dx+c)}\left(3\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)+2\,(\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*a*(3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+2*
cos(d*x+c)^2-2*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.04867, size = 1084, normalized size = 16.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
```

```
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.73447, size = 662, normalized size = 10.18

$$\left[\frac{2\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(a\cos\left(dx+c\right)+a\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c}{\cos(dx+c)+1}\right.}{2\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + 3*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x
+ c) - a)/(cos(d*x + c) + 1)))/(d*cos(d*x + c) + d), (a*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*(a*cos(d*x + c) + a)*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.105 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=106

$$\frac{7a^2\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (7*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) +
 (7*a^2*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]*Si
n[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.126602, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3813, 21, 3805, 3774, 203}

$$\frac{7a^2\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d) +
 (7*a^2*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]*Si
n[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\,dx &= \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}a\int \frac{\cos(c+dx)\left(\frac{7a}{2}+\frac{7}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx \\
&= \frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{4}(7a)\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,dx \\
&= \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{8}(7a)\int \sqrt{a+a\sec(c+ \\
&= \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}-\frac{\left(7a^2\right)\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx,x\right.}{4d} \\
&= \frac{7a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos(c+dx)\sin(c+d
\end{aligned}
$$

Mathematica [A] time = 0.362199, size = 108, normalized size = 1.02

$$
\frac{a\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left((7\sin(c+dx)+\sin(2(c+dx)))\sqrt{1-\sec(c+dx)}+7\tan(c+dx)\tanh^{-1}\left(\sqrt{1-\sec(c+\right.\right.}{4d(\cos(c+dx)+1)\sqrt{1-\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(7*Sin[c
 + d*x] + Sin[2*(c + d*x)]) + 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x
]))/(4*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.198, size = 222, normalized size = 2.1

$$\frac{a}{16\,d\cos(dx+c)\sin(dx+c)}\left(7\sqrt{2}\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/16/d*a*(7*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*co
s(d*x+c)+7*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-8*c
os(d*x+c)^4-20*cos(d*x+c)^3+28*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.88736, size = 736, normalized size = 6.94

$$\left[\frac{7\left(a\cos\left(dx+c\right)+a\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\,a\cos\left(dx+c\right)^2+7\,a\cos\right.}{8\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d
*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a*cos(d*x + c)^2 + 7*a*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d
), -1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^2 + 7*
a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.48015, size = 554, normalized size = 5.23

$$-\frac{7\sqrt{-a}a\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-7\sqrt{-a}a\log\left(\left|\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(7*sqrt(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 7*sqrt(-a
)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*sqrt(2)*(7*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a^2*s
gn(cos(d*x + c)) - 95*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^4*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 53*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^4*sgn(cos(d*
x + c)) - 5*sqrt(-a)*a^5*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.106 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=144

$$\frac{11a^2\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (11*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
+ (11*a^2*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Cos[c + d*
x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^2*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.186515, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3813, 21, 3805, 3774, 203}

$$\frac{11a^2\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (11*a^(3/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
+ (11*a^2*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Cos[c + d*
x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^2*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}a\int \frac{\cos^2(c+dx)\left(\frac{11a}{2}+\frac{11}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(11a)\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(11a)\int \cos(c$$

$$= \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+dx)\sin(c}{3d\sqrt{a+a\sec(c+}$$

$$= \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^2(c+dx)\sin(c}{3d\sqrt{a+a\sec(c+}$$

$$= \frac{11a^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{11a^2\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\cos(c+dx)\sin(c}{12d\sqrt{a+a\sec(c+}$$

Mathematica [A] time = 0.517337, size = 120, normalized size = 0.83

$$\frac{a\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left((35\sin(c+dx)+11\sin(2(c+dx))+2\sin(3(c+dx)))\sqrt{1-\sec(c+dx)}+33\tan(c+d\right.}{24d(\cos(c+dx)+1)\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(35*Sin[
c + d*x] + 11*Sin[2*(c + d*x)] + 2*Sin[3*(c + d*x)]) + 33*ArcTanh[Sqrt[1 -
Sec[c + d*x]]]*Tan[c + d*x]))/(24*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x
]])
```

Maple [B] time = 0.227, size = 311, normalized size = 2.2

$$-\frac{a}{192\,d\sin(dx+c)(\cos(dx+c))^2}\left(33\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{5/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/192/d*a*(33*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)
*cos(d*x+c)^2+66*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+
c)*cos(d*x+c)+33*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+
c)+64*cos(d*x+c)^6+112*cos(d*x+c)^5+88*cos(d*x+c)^4-264*cos(d*x+c)^3)*(a*(c
os(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.09422, size = 803, normalized size = 5.58

$$\left[\frac{33\,(a\cos(dx+c)+a)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\,a\cos(dx+c)^3+22\,a\right.}{48\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(33*(a*cos(d*x + c) + a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos
(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a*cos(d*x + c)^3 + 22*a*cos(d*x +
 c)^2 + 33*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c) + d), -1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - (8*a*cos(d*x + c)^3 + 22*a*cos(d*x + c)^2 + 33*a*cos(d*x + c))*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.36423, size = 702, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/48*(33*sqrt(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 33*sqrt
(-a)*a*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*sqrt(2)*(33*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(-a)*
a^2*sgn(cos(d*x + c)) - 303*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 2394*(sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a^4*sg
n(cos(d*x + c)) - 1806*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 309*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^6*sgn(cos(
d*x + c)) - 19*sqrt(-a)*a^7*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2
*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.107 $\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}+\frac{46a^3\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}-56$$

```
[Out] (284*a^3*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) + (710*a^3*Sec[c + d
*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (46*a^3*Sec[c + d*x]
^4*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) - (568*a^2*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(693*d) + (2*a^2*Sec[c + d*x]^4*Sqrt[a + a*Sec[c +
d*x]]*Tan[c + d*x])/(11*d) + (284*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(231*d)
```

Rubi [A] time = 0.373957, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3814, 4016, 3803, 3800, 4001, 3792}

$$\frac{2a^2\tan(c+dx)\sec^4(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}+\frac{46a^3\tan(c+dx)\sec^4(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}-56$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (284*a^3*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) + (710*a^3*Sec[c + d
*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (46*a^3*Sec[c + d*x]
^4*Tan[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*x]]) - (568*a^2*Sqrt[a + a*Sec[
c + d*x]]*Tan[c + d*x])/(693*d) + (2*a^2*Sec[c + d*x]^4*Sqrt[a + a*Sec[c +
d*x]]*Tan[c + d*x])/(11*d) + (284*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(231*d)
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{11d} + \frac{1}{11}(2a)\int \sec^4(c+dx)\sqrt{a-}$$

$$= \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\tan(c+}{11d}$$

$$= \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c}{}$$

$$= \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sec^4(c}{}$$

$$= \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a^3\sec^4(c+dx)\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} - \frac{568a^2\sqrt{a-}}{}$$

$$= \frac{284a^3\tan(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} + \frac{710a^3\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}} + \frac{46a^3\sec^4(c+dx)}{99d\sqrt{a+a\,\mathrm{se}}}$$

Mathematica [A] time = 0.200337, size = 80, normalized size = 0.39

$$\frac{2a^3\tan(c+dx)\left(63\sec^5(c+dx)+224\sec^4(c+dx)+355\sec^3(c+dx)+426\sec^2(c+dx)+568\sec(c+dx)+1136\right)}{693d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(1136 + 568*Sec[c + d*x] + 426*Sec[c + d*x]^2 + 355*Sec[c + d*x]^3 +
 224*Sec[c + d*x]^4 + 63*Sec[c + d*x]^5)*Tan[c + d*x])/(693*d*Sqrt[a*(1 + S
ec[c + d*x])])
```

Maple [A] time = 0.169, size = 105, normalized size = 0.5

$$-\frac{2a^2\left(1136\,(\cos(dx+c))^6-568\,(\cos(dx+c))^5-142\,(\cos(dx+c))^4-71\,(\cos(dx+c))^3-131\,(\cos(dx+c))^2-161\right.}{693\,d\,(\cos(dx+c))^5\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/693/d*a^2*(1136*cos(d*x+c)^6-568*cos(d*x+c)^5-142*cos(d*x+c)^4-71*cos(d*
x+c)^3-131*cos(d*x+c)^2-161*cos(d*x+c)-63)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1
/2)/cos(d*x+c)^5/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.02099, size = 312, normalized size = 1.54

$$\frac{2\left(1136\,a^2\cos\left(dx+c\right)^5+568\,a^2\cos\left(dx+c\right)^4+426\,a^2\cos\left(dx+c\right)^3+355\,a^2\cos\left(dx+c\right)^2+224\,a^2\cos\left(dx+c\right)+63\,a^2\right)}{693\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(1136*a^2*cos(d*x + c)^5 + 568*a^2*cos(d*x + c)^4 + 426*a^2*cos(d*x +
 c)^3 + 355*a^2*cos(d*x + c)^2 + 224*a^2*cos(d*x + c) + 63*a^2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c
)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

[Out] Timed out

Giac [A] time = 5.50839, size = 282, normalized size = 1.39

$$-\frac{8\left(693\sqrt{2}a^8\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(1617\sqrt{2}a^8\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(3003\sqrt{2}a^8\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-25\left(99\sqrt{2}a^8\mathrm{sgn}\left(\cos\right.\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -8/693*(693*sqrt(2)*a^8*sgn(cos(d*x + c)) - (1617*sqrt(2)*a^8*sgn(cos(d*x +
 c)) - (3003*sqrt(2)*a^8*sgn(cos(d*x + c)) - 25*(99*sqrt(2)*a^8*sgn(cos(d*x
 + c)) + 4*(2*sqrt(2)*a^8*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 11*sqr
t(2)*a^8*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)
*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*t
an(1/2*d*x + 1/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.108 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=146

$$\frac{208a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{832a^3\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}-\frac{4\tan(c+dx)(a\ldots}{6\ldots}$$

```
[Out] (832*a^3*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (208*a^2*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (26*a*(a + a*Sec[c + d*x])^(3/2)*T
an[c + d*x])/(105*d) - (4*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) +
 (2*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.230472, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3800, 4001, 3793, 3792}

$$\frac{208a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{832a^3\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}-\frac{4\tan(c+dx)(a\ldots}{6\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (832*a^3*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (208*a^2*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (26*a*(a + a*Sec[c + d*x])^(3/2)*T
an[c + d*x])/(105*d) - (4*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) +
 (2*(a + a*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*a*d)
```


Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
```

```
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx &= \frac{2(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad}+\frac{2\int \sec(c+dx)\left(\frac{7a}{2}-a\sec(c+dx)\right)(a\ \cdots}{9a}\\ &= -\frac{4(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d}+\frac{2(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad}+\cdots\\ &= \frac{26a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d}-\frac{4(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d}+\cdots\\ &= \frac{208a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d}+\frac{26a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{105d}\cdots\\ &= \frac{832a^3\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{208a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{315d}+\frac{26a(a+a\ \cdots}{}\end{aligned}$$

Mathematica [A] time = 0.501923, size = 70, normalized size = 0.48

$$\frac{2a^3\tan(c+dx)\left(35\sec^4(c+dx)+130\sec^3(c+dx)+219\sec^2(c+dx)+292\sec(c+dx)+584\right)}{315d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(584 + 292*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 130*Sec[c + d*x]^3 +
35*Sec[c + d*x]^4)*Tan[c + d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.157, size = 95, normalized size = 0.7

$$-\frac{2\,a^2\left(584\,\left(\cos\left(dx+c\right)\right)^5-292\,\left(\cos\left(dx+c\right)\right)^4-73\,\left(\cos\left(dx+c\right)\right)^3-89\,\left(\cos\left(dx+c\right)\right)^2-95\,\cos\left(dx+c\right)-35\right)}{315\,d\left(\cos\left(dx+c\right)\right)^4\sin\left(dx+c\right)}\sqrt{\frac{a\,(\cos}{\mathrm{co}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/315/d*a^2*(584*cos(d*x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+
c)^2-95*cos(d*x+c)-35)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin
(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.97259, size = 277, normalized size = 1.9

$$\frac{2\left(584\,a^2\cos\left(dx+c\right)^4+292\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+130\,a^2\cos\left(dx+c\right)+35\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+\right.}{315\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.22365, size = 243, normalized size = 1.66

$$\frac{8\left(315\,\sqrt{2}a^{7}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)-\left(630\,\sqrt{2}a^{7}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)-13\left(63\,\sqrt{2}a^{7}\operatorname{sgn}\left(\cos\left(dx+c\right)\right)+4\left(2\,\sqrt{2}a^{7}\operatorname{sgn}\left(\cos\left(dx\right.\right.\right.\right.\right.}{315\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 8/315*(315*sqrt(2)*a^7*sgn(cos(d*x + c)) - (630*sqrt(2)*a^7*sgn(cos(d*x + c
)) - 13*(63*sqrt(2)*a^7*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^7*sgn(cos(d*x +
c))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*a^7*sgn(cos(d*x + c)))*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/
2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*
d)
```

3.109 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{64a^3\tan(c+dx)}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d}+\frac{2\tan(c+dx)(a\,\mathrm{se}}{7}$$

```
[Out] (64*a^3*Tan[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(7*d) + (2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.155143, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3798, 3793, 3792}

$$\frac{64a^3\tan(c+dx)}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d}+\frac{2\tan(c+dx)(a\,\mathrm{se}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Tan[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(7*d) + (2*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{5}{7}\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$$
$$= \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{7d}+\frac{2(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\ldots$$
$$= \frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{21d}+\frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{7d}+\ldots$$
$$= \frac{64a^3\tan(c+dx)}{21d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{21d}+\frac{2a(a+a\sec\ldots}{\ldots}$$

Mathematica [A] time = 0.165702, size = 60, normalized size = 0.52

$$\frac{2a^3\tan(c+dx)\left(3\sec^3(c+dx)+12\sec^2(c+dx)+23\sec(c+dx)+46\right)}{21d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(46 + 23*Sec[c + d*x] + 12*Sec[c + d*x]^2 + 3*Sec[c + d*x]^3)*Tan[c
+ d*x])/(21*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.135, size = 85, normalized size = 0.7

$$-\frac{2a^2\left(46(\cos(dx+c))^4-23(\cos(dx+c))^3-11(\cos(dx+c))^2-9\cos(dx+c)-3\right)}{21d(\cos(dx+c))^3\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*a^2*(46*cos(d*x+c)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3
)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.95026, size = 236, normalized size = 2.03

$$\frac{2\left(46\,a^2\cos\left(dx+c\right)^3+23\,a^2\cos\left(dx+c\right)^2+12\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c) +
 3*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c
)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 5.42586, size = 204, normalized size = 1.76

$$-\frac{8\Big(21\sqrt{2}a^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\Big(35\sqrt{2}a^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+4\Big(2\sqrt{2}a^6\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\sqrt{2}a^6\mathrm{sg}\cdots}{21\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -8/21*(21*sqrt(2)*a^6*sgn(cos(d*x + c)) - (35*sqrt(2)*a^6*sgn(cos(d*x + c))
 + 4*(2*sqrt(2)*a^6*sgn(cos(d*x + c))*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2)*a^
6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/
2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)*d)
```

3.110 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{64a^3\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (64*a^3*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(5*d)
```

Rubi [A] time = 0.0949433, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3793, 3792}

$$\frac{64a^3\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*Sqrt[a + a*
Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*(a + a*Sec[c + d*x])^(3/2)*Tan[c
+ d*x])/(5*d)
```

Rule 3793

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[(a*(2*m - 1))/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x]
 /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && IntegerQ[
2*m]
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{1}{5}(8a)\int \sec(c+dx)(a+a\sec(c+dx))$$
$$= \frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2a(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d} +$$
$$= \frac{64a^3\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2a(a+a\sec(}{}$$

Mathematica [A] time = 0.0907647, size = 50, normalized size = 0.56

$$\frac{2a^3\tan(c+dx)\left(3\sec^2(c+dx)+14\sec(c+dx)+43\right)}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*(43 + 14*Sec[c + d*x] + 3*Sec[c + d*x]^2)*Tan[c + d*x])/(15*d*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.128, size = 75, normalized size = 0.8

$$-\frac{2a^2\left(43(\cos(dx+c))^3-29(\cos(dx+c))^2-11\cos(dx+c)-3\right)}{15d(\cos(dx+c))^2\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d*a^2*(43*cos(d*x+c)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*(a*(cos(d*x+c
)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [A] time = 1.96229, size = 204, normalized size = 2.29

$$\frac{2\left(43\,a^{2}\cos\left(dx+c\right)^{2}+14\,a^{2}\cos\left(dx+c\right)+3\,a^{2}\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 4.91895, size = 165, normalized size = 1.85

$$\frac{8\left(15\,\sqrt{2}a^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+4\left(2\,\sqrt{2}a^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-5\,\sqrt{2}a^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 8/15*(15*sqrt(2)*a^5*sgn(cos(d*x + c)) + 4*(2*sqrt(2)*a^5*sgn(cos(d*x + c))
*tan(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/
2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*d)
```

3.111 $\int (a + a\sec(c + dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (14
*a^3*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.102234, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3775, 3915, 3774, 203, 3792}

$$\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{14a^3\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (14
*a^3*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Sec[c
 + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a + a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3}(2a)\int \sqrt{a+a\sec(c+dx)}\left(\frac{3a}{2}+\frac{7}{2}a\sec(c+dx)\right)$$

$$= \frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} + a^2\int \sqrt{a+a\sec(c+dx)}\,dx + \frac{1}{3}\left(7a^2\right)\int \sec(c+dx)$$

$$= \frac{14a^3\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d} - \frac{\left(2a^3\right)\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\right.}{d}$$

$$= \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{14a^3\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

Mathematica [C] time = 10.1788, size = 360, normalized size = 3.67

$$\sqrt{\frac{1}{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{1-2\sin^2\left(\frac{1}{2}(c+dx)\right)}\csc^3\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+1))^{5/2}\left(256\sin^6\left(\frac{1}{2}(c+dx)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]^5*(a*(1 + Sec[c + d*x]))^(5/2)*Sqrt[(1
 - 2*Sin[(c + d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[(c + d*x)/2]^2]*(256*Cos[(c +
 d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]
*Sin[(c + d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Sin[(c + d*x)/
2]^2]*Sin[(c + d*x)/2]^6*(2 - 3*Sin[(c + d*x)/2]^2 + Sin[(c + d*x)/2]^4) +
(21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[(c + d*x)/2]^2]]*(15 - 10*Sin[(c + d*x)
/2]^2 + 3*Sin[(c + d*x)/2]^4))/Sqrt[Sin[(c + d*x)/2]^2] - 14*Sqrt[1 - 2*Sin
[(c + d*x)/2]^2]*(45 + 30*Sin[(c + d*x)/2]^2 - 31*Sin[(c + d*x)/2]^4 + 12*S
in[(c + d*x)/2]^6)))/(672*d*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.164, size = 214, normalized size = 2.2

$$-\frac{a^2}{3\,d\,(\cos(dx+c)+1)\cos(dx+c)}\left(3\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d*a^2*(3*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*2^(1/2
)+3*cos(d*x+c)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-16*cos(d*x
+c)*sin(d*x+c)-2*sin(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(cos(d*x+c
)+1)/cos(d*x+c)
```

Maxima [B] time = 2.44421, size = 1883, normalized size = 19.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
```

```
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 2.1009, size = 792, normalized size = 8.08

$$\left[\frac{3\left(a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\,a^2\cos\right.}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x + c)
 + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)^2 + d*cos(d*x + c)), -2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - (8*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.112 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=94

$$-\frac{a^3\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{5a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (5*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
3*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*
x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.157041, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3814, 4015, 3774, 203}

$$-\frac{a^3\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{5a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (5*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d - (a^
3*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*
x]]*Sin[c + d*x])/d
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + (2a)\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\left(-\frac{a}{2}\right.$$

$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\left(5a^2\right)\int \sqrt{a+a$$

$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{}{a-}\right.}{}$$

$$= \frac{5a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{a^3\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(}{d}$$

Mathematica [C] time = 4.74327, size = 189, normalized size = 2.01

$$\frac{2\cos^{\frac{5}{2}}(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+1))^{5/2}\left(12\sin^2\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,\frac{5}{2}\right\},\left\{1,\frac{9}{2}\right\},2\sin\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2)*(12*HypergeometricPFQ[{3
/2, 2, 5/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[(c + d*x)/2]^2 + (Sec[(c +
```

```
d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F1
[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 24*(3 + Cos[c + d*x])*Hypergeometri
c2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2))/8)*Tan[(c + d*x)
/2])/(105*d)
```

Maple [A] time = 0.183, size = 128, normalized size = 1.4

$$-\frac{a^2}{2\,d\sin(dx+c)}\left(5\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)+2\,(\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/2/d*a^2*(5*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+
2*cos(d*x+c)^2+2*cos(d*x+c)-4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+
c)
```

Maxima [B] time = 2.44234, size = 1867, normalized size = 19.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x + 2*c)
^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x + 2
*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) + 4*a
^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c) +
(a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c) - a
^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
```

```
1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 +
 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(
a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

<u>Fricas</u> [A] time = 2.08373, size = 706, normalized size = 7.51

$$\left[\frac{5\left(a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(a^{2}\cos\left(dx+c\right)+2\,a^{2}\right)}{2\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*(5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*c
os(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(a^2*cos(d*x + c) + 2*a^2)*sqrt((a
```

```
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -(5*(a
^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.19396, size = 493, normalized size = 5.24

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a^3\mathrm{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+5\sqrt{-a}a^2\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*a^3*sgn(cos(d*x + c))*t
an(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 5*sqrt(-a)*a^2*log(abs
((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 5*sqrt(-a)*a^2*log(abs((sqrt(-a)*ta
n(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2)
- 3)))*sgn(cos(d*x + c)) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^3*sgn(cos(d*x + c)) - sqrt(
-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.113 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=106

$$\frac{9a^3\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{19a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

```
[Out] (19*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (9*a^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.164392, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3813, 4015, 3774, 203}

$$\frac{9a^3\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{19a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (9*a^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}+\frac{1}{2}a\int \cos(c+dx)\sqrt{a+a\sec(}$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}+\frac{1}{8}\left(1\right.$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}-\frac{(19a}{}$$

$$= \frac{19a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos(c+dx)\sqrt{a+a}}{}$$

Mathematica [C] time = 0.522436, size = 150, normalized size = 1.42

$$-\frac{a^2\cos(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-32\tan(c+dx)\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1-\sec(c+dx)\right)+\right.}{4d(\cos(c+dx)+1)\sqrt{1-\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -(a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[1 - Sec[c + d*x]]*(Sin[
c + d*x] + 3*Sin[2*(c + d*x)]) - 7*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c +
```

```
d*x] - 32*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]]*Tan[c + d*x]))/(4*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.189, size = 224, normalized size = 2.1

$$\frac{a^2}{16\,d\cos(dx+c)\sin(dx+c)}\left(19\sqrt{2}\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*a^2*(19*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)
*cos(d*x+c)+19*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)
-8*cos(d*x+c)^4-36*cos(d*x+c)^3+44*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+
c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.04913, size = 763, normalized size = 7.2

$$\left[\frac{19\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2\,a^2\cos(dx+c)^2+11\right.}{8\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/8*(19*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*
cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*a^2*cos(d*x + c)^2 + 11*a^2*co
s(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c) + d), -1/4*(19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*a^2*co
s(d*x + c)^2 + 11*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.114 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=144

$$\frac{25a^3\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{25a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{13a^3\sin(c+dx)}{12d\sqrt{a\sec(c+}}$$

```
[Out] (25*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
+ (25*a^3*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (13*a^3*Cos[c + d*
x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^2*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.234547, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3813, 4015, 3805, 3774, 203}

$$\frac{25a^3\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{25a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{13a^3\sin(c+dx)}{12d\sqrt{a\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (25*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
+ (25*a^3*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (13*a^3*Cos[c + d*
x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^2*Sqrt
[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}a\int \cos^2(c+dx)\sqrt{a+a\,\mathrm{se}}$$

$$= \frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d}$$

$$= \frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx)\sqrt{a+}}{}$$

$$= \frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\cos^2(c+dx)\sqrt{a+}}{}$$

$$= \frac{25a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{25a^3\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{13a^3\cos(c+dx)\sin(}{12d\sqrt{a+a\sec(c+}}$$

Mathematica [C] time = 0.771256, size = 151, normalized size = 1.05

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(192\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},1-\sec(c+dx)\right)+(159\cos(c+dx)\right.}{72d(\cos(c+dx)+1)\sqrt{1-\sec(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(165*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (31 + 159*Cos[c + d*x] + 31*Cos
[2*(c + d*x)] - 2*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]] + 192*Hypergeome
tric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 +
Sec[c + d*x])]*Sin[c + d*x])/(72*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]
])
```

Maple [B] time = 0.214, size = 313, normalized size = 2.2

$$-\frac{a^2}{192\,d\,(\cos(dx+c))^2\sin(dx+c)}\left(75\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{5/2}\sqrt{2}\text{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/192/d*a^2*(75*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/2*2
^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+
c)*cos(d*x+c)^2+150*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*2^(1/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d
*x+c)*cos(d*x+c)+75*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d
*x+c)+64*cos(d*x+c)^6+208*cos(d*x+c)^5+328*cos(d*x+c)^4-600*cos(d*x+c)^3)*(
a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

Fricas [A] time = 2.01304, size = 830, normalized size = 5.76

$$\left[\frac{75\left(a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8\,a^{2}\cos\left(dx+c\right)^{3}+3\right.}{48\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/48*(75*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a
*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*a^2*cos(d*x + c)^3 + 34*a^2*c
os(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(75*(a^2*cos(d*x + c) + a^2)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a^2*cos(d*x + c)^2 + 75*a^2*cos
(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2),x)

[Out] Timed out

Giac [B] time = 8.19531, size = 707, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/48*(75*sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 75*sq
rt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*sqrt(2)*(75*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(
-a)*a^3*sgn(cos(d*x + c)) - 1125*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 6174*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(-a)*a
^5*sgn(cos(d*x + c)) - 4314*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 807*(sqrt(-a)*tan
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a)*a^7*sgn
(cos(d*x + c)) - 49*sqrt(-a)*a^8*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.115 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=182

$$\frac{163a^3\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{163a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{17a^3\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

```
[Out] (163*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d
) + (163*a^3*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Cos[c
 + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (17*a^3*Cos[c + d*x
]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^3*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.294107, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3813, 4015, 3805, 3774, 203}

$$\frac{163a^3\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{163a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{17a^3\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d
) + (163*a^3*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Cos[c
 + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (17*a^3*Cos[c + d*x
]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Cos[c + d*x]^3*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^3(c+dx)\sqrt{a+a\,\mathrm{se}$$

$$= \frac{17a^3\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\cos^3(c+}{}$$

$$= \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c+dx)\,\mathrm{s}}{24d\sqrt{a+a\sec(}}$$

$$= \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\cos^2(c+dx)\,\mathrm{s}}{24d\sqrt{a+a\sec(}}$$

$$= \frac{163a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\cos(c+dx)\,\mathrm{s}}{96d\sqrt{a+a\sec(}}$$

Mathematica [C] time = 0.768638, size = 161, normalized size = 0.88

$$\frac{a^2\sin(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(512\sqrt{1-\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},5,\frac{3}{2},1-\sec(c+dx)\right)+(849\cos(c+\right.}{320d(\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(675*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + (231 + 849*Cos[c + d*x] + 233*C
os[2*(c + d*x)] + 58*Cos[3*(c + d*x)] + 2*Cos[4*(c + d*x)])*Sqrt[1 - Sec[c
+ d*x]] + 512*Hypergeometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(320*d*(1 + Cos[c + d*
x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.261, size = 402, normalized size = 2.2

$$\frac{a^2}{3072\,d\,(\cos(dx+c))^3\sin(dx+c)}\left(489\sqrt{2}\sin(dx+c)(\cos(dx+c))^3\,\mathrm{Artanh}\left(1/2\,\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3072/d*a^2*(489*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)+1467*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(7/2)+1467*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(7/2)+489*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2
)*sin(d*x+c)-768*cos(d*x+c)^8-2176*cos(d*x+c)^7-2272*cos(d*x+c)^6-2608*cos(
d*x+c)^5+7824*cos(d*x+c)^4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^
3/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.18428, size = 911, normalized size = 5.01

$$\left[\frac{489\left(a^2\cos(dx+c)+a^2\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48\,a^2\cos(dx+c)^4+\cdots\right.}{384\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(489*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) +
```

```
 a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*a^2*cos(d*x + c)^4 + 184*a
^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*
(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c)^4 + 1
84*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.98232, size = 856, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(489*sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))*sgn(cos(d*x + c)) - 489
*sqrt(-a)*a^2*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))*sgn(cos(d*x + c)) + 4*sqrt(2)*(489
*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*s
qrt(-a)*a^3*sgn(cos(d*x + c)) - 10269*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 69885*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sq
rt(-a)*a^5*sgn(cos(d*x + c)) - 259233*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 209979*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqr
t(-a)*a^7*sgn(cos(d*x + c)) - 55511*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^4*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 6687*(sqr
```

```
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(-a
)*a^9*sgn(cos(d*x + c)) - 299*sqrt(-a)*a^10*sgn(cos(d*x + c)))/((sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.116 $\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

Optimal. Leaf size=27

$$-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

```
[Out] (-2*a*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.0298956, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3792}

$$-\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (-2*a*Tan[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 3792

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a-a\sec(c+dx)}\,dx = -\frac{2a\tan(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

Mathematica [A] time = 0.110675, size = 30, normalized size = 1.11

$$\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*Cot[(c + d*x)/2]*Sqrt[a - a*Sec[c + d*x]])/d
```

Maple [A] time = 0.15, size = 42, normalized size = 1.6

$$-2\,\frac{\sin\left(dx+c\right)}{d\left(-1+\cos\left(dx+c\right)\right)}\sqrt{\frac{a\left(-1+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)/(-1+cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [A] time = 1.935, size = 107, normalized size = 3.96

$$\frac{2\sqrt{\frac{a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}\left(\cos\left(dx+c\right)+1\right)}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*(cos(d*x + c) + 1)/(d*sin(d*x + c
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\left(\sec\left(c+dx\right)-1\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sec(c + d*x) - 1))*sec(c + d*x), x)
```

Giac [B] time = 1.5077, size = 77, normalized size = 2.85

$$-\frac{2\sqrt{2}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\operatorname{sgn}\left(\cos\left(dx+c\right)\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*a*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x
 + c))/(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*d)
```

3.117 $\int \sqrt{a - a\sec(c + dx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d
```

Rubi [A] time = 0.0219381, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3774, 203}

$$\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a-a\sec(c+dx)}\,dx = \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

Mathematica [C] time = 3.43279, size = 188, normalized size = 4.95

$$-\frac{\sqrt{\cos(c)-i\sin(c)}\cos(c+dx)\left(\cot\left(\frac{1}{2}(c+dx)\right)+i\right)\sqrt{a-a\sec(c+dx)}\left(\tanh^{-1}\left(\frac{e^{idx}}{\sqrt{\cos(c)-i\sin(c)}\sqrt{e^{2idx}(\cos(c)+i\sin(c))-i\sin(c)+}}\right.\right.}{d\sqrt{i\sin(c)\left(-1+e^{2idx}\right)+\cos(c)\left(1+e^{2idx}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -(((ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*
(Cos[c] + I*Sin[c]) - I*Sin[c]])] + ArcTanh[Sqrt[Cos[c] + E^((2*I)*d*x)*(Co
s[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]])*Cos[c + d*x]*(I + Co
t[(c + d*x)/2])*Sqrt[a - a*Sec[c + d*x]]*Sqrt[Cos[c] - I*Sin[c]])/(d*Sqrt[(
1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c]]))
```

Maple [B] time = 0.138, size = 91, normalized size = 2.4

$$-\frac{\sqrt{2}\sin(dx+c)}{d\,(-1+\cos(dx+c))}\sqrt{\frac{a\,(-1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))/(-1+cos(d*x+c))
```

Maxima [B] time = 1.95359, size = 197, normalized size = 5.18

$$\frac{\sqrt{a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

Fricas [B] time = 2.37688, size = 464, normalized size = 12.21

$$\left[\frac{\sqrt{-a}\log\left(-\frac{4\left(2\,\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}+\left(8\,a\cos\left(dx+c\right)^2+8\,a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)}{\sin\left(dx+c\right)}\right)}{2\,d},-\frac{\sqrt{a}\arctan\left(\frac{2\left(\cos(dx+c\ldots\right.}{(2\,a\ldots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log(-(4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (8*a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))/d, -sqrt(a)*arctan(2*(cos(d
*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))/(
(2*a*cos(d*x + c) + a)*sin(d*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\sec\left(c+dx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sec(c + d*x) + a), x)
```

Giac [B] time = 1.51833, size = 88, normalized size = 2.32

$$-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}}{2\sqrt{a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))*s
gn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d*x + c))/d
```

3.118 $\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d) + (a
*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rubi [A] time = 0.0638623, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3805, 3774, 203}

$$\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/d) + (a
*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a-a\sec(c+dx)}\,dx &= \frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{1}{2}\int \sqrt{a-a\sec(c+dx)}\,dx\\ &= \frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,\frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.873431, size = 260, normalized size = 4.

$$\frac{\cos(c+dx)\sqrt{a-a\sec(c+dx)}\left(-2\sqrt{2}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+i\sin(dx))}+\sqrt{\cos(c)-i\sin(c)}\left(\cot\left(\frac{1}{2}(c\right.\right.\right.}{2d\sqrt{i\sin}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*Sqrt[a - a*Sec[c + d*x]]*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*
Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])]*(I +
Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + ArcTanh[Sqrt[Cos[c] + E^((2*I)*
d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]]*(I + Cot[(c +
 d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] - 2*Sqrt[2]*Cot[(c + d*x)/2]*Sqrt[Cos[c +
 d*x]*(Cos[d*x] + I*Sin[d*x])]))/(2*d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c] + I*(
-1 + E^((2*I)*d*x))*Sin[c]])
```

Maple [A] time = 0.204, size = 103, normalized size = 1.6

$$\frac{\sqrt{2}\sin(dx+c)}{2d(-1+\cos(dx+c))}\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{2}\cos(dx+c)+\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{\cos(dx+c}{\cos(dx+c)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c)*(2^(1/2)*cos(
d*x+c)+(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)))/(-1+cos(d*x+c))
```

Maxima [B] time = 2.1984, size = 1068, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c
) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1))) - 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

Fricas [B] time = 2.30585, size = 753, normalized size = 11.58

$$\left[\frac{\sqrt{-a}\log\left(\frac{4\left(2\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-4\left(\cos(dx\right.}{4d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a)*log((4*(2*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*
sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*(cos(d*x +
 c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(d*sin(d*x +
 c)), 1/2*(sqrt(a)*arctan(2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*sqrt((a
*cos(d*x + c) - a)/cos(d*x + c))/((2*a*cos(d*x + c) + a)*sin(d*x + c)))*sin
(d*x + c) - 2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos
(d*x + c)))/(d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\sec(c+dx)-1\right)}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sec(c + d*x) - 1))*cos(c + d*x), x)
```

Giac [A] time = 1.40868, size = 151, normalized size = 2.32

$$\frac{\sqrt{2}a\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\mathrm{sgn}\left(\cos(dx+c)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*a*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a
)/sqrt(a))/sqrt(a) - 2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/(a*tan(1/2*d*x +
1/2*c)^2 + a))*sgn(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*sgn(cos(d
*x + c))/d
```

3.119 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}+\frac{28\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(Sqrt[a]*d)) + (28*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Se
c[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Sqrt[a + a*S
ec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rubi [A] time = 0.275899, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3822, 4010, 4001, 3795, 203}

$$\frac{2\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}+\frac{28\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(Sqrt[a]*d)) + (28*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Se
c[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) - (2*Sqrt[a + a*S
ec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```


Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^2(c+dx)(4a-a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15ad} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{a^2}{2}+7a^2\sec(c+d\right.}{\sqrt{a+a\sec(c+dx)}}}{15a^2}$$

$$= \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15ad} -$$

$$= \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{15ad} +$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{28\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - 2\sqrt{a}$$

Mathematica [A] time = 0.202305, size = 106, normalized size = 0.76

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(3\sec^2(c+dx)-\sec(c+dx)+13\right)-15\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-15*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - Sec[c +
d*x]]*(13 - Sec[c + d*x] + 3*Sec[c + d*x]^2))*Tan[c + d*x])/(15*d*Sqrt[1 -
Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.19, size = 314, normalized size = 2.2

$$-\frac{1}{60\,ad\,(\cos(dx+c))^2\sin(dx+c)}\left(15\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(dx+c)+1\right)\right)\left(-2\frac{\text{co}}{\text{cos}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/60/d/a*(15*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c
)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)
^2+30*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)+15*ln((
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+104*cos(d*x+c)^3-112*cos(d*x
+c)^2+32*cos(d*x+c)-24)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^2/si
n(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 2.39997, size = 914, normalized size = 6.53

$$\left[\frac{15\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left(13\,\mathrm{c}\right.}{30\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-1/a)*log((2*s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(
d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) + 4*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^
```

```
2), 1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c
)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A] time = 10.0761, size = 277, normalized size = 1.98

$$-\frac{\sqrt{2}\left(\frac{15\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\left(\left(\frac{17a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{20a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{15a^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(2)*(15*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*((17*a
^2*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) - 20*a^2/sgn(tan(
1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 + 15*a^2/sgn(tan(1/2*d*x +
1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

3.120 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}-\frac{4\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[a]*d) - (4*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a
+ a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.157705, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3800, 4001, 3795, 203}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{2\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}-\frac{4\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[a]*d) - (4*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[a
+ a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad} + \frac{2\int \frac{\sec(c+dx)\left(\frac{a}{2}-a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= -\frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad} + \int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= -\frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\,\text{ta}}{\sqrt{a+a}}\right.}{d}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{4\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3ad}$$

Mathematica [A] time = 0.146578, size = 86, normalized size = 0.83

$$-\frac{\tan(c+dx)\left(\frac{2}{3}(1-\sec(c+dx))^{3/2}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((((-(Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]) + (2*(1 - Sec[c + d*
x])^(3/2))/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d
```

```
*x])]))
```

Maple [B] time = 0.173, size = 221, normalized size = 2.1

$$-\frac{1}{6\,ad\sin\left(dx+c\right)\cos\left(dx+c\right)}\left(3\,\ln\left(\frac{1}{\sin\left(dx+c\right)}\left(\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\sin\left(dx+c\right)-\cos\left(dx+c\right)+1\right)\right)\left(-2\,\frac{\cos\left(dx\right.}{\cos\left(dx+c\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/6/d/a*(3*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+
1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)*sin(d*x+c)+3
*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-4*cos(d*x+c)^2+8*cos(d*
x+c)-4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{3}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 2.25245, size = 846, normalized size = 8.13

$$\left[\frac{3\,\sqrt{2}\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\,\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\,\cos(dx+c)^{2}-2\,\cos(dx+c)+1}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)-4\,\sqrt{\frac{a\cos}{\cos}}}{6\left(ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*log(-(2*sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x
 + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*s
in(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/3*(2*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2)*(a*c
os(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)
^2 + a*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A] time = 9.51128, size = 184, normalized size = 1.77

$$\frac{\sqrt{2}\left(\frac{4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{3\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 3*log(a
```

```
bs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(
sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.121 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(Sqrt[a]*d)) + (2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0876906, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3798, 3795, 203}

$$\frac{2\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(Sqrt[a]*d)) + (2*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx\\ &= \frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}+\frac{2\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0773774, size = 83, normalized size = 1.14

$$-\frac{\tan(c+dx)\left(\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-2\sqrt{1-\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((((Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - 2*Sqrt[1 - Sec[c + d*
x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Maple [A] time = 0.132, size = 121, normalized size = 1.7

$$-\frac{1}{ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(dx+c)+1\right)\right)\sqrt{-2\frac{\text{co}}{\text{cos}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(ln(((-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1
```

```
))^(1/2)*sin(d*x+c)+2*cos(d*x+c)-2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 2.28855, size = 701, normalized size = 9.6

$$\left[\frac{\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx}{2(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x
+ c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
, (sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [B] time = 9.72297, size = 178, normalized size = 2.44

$$-\frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right|\right)}{\sqrt{-a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)))/(sqrt(-a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 2*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*
sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.122 $\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[a]*d)
```

Rubi [A] time = 0.0359379, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3795, 203}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])
/(Sqrt[a]*d)
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0467547, size = 64, normalized size = 1.39

$$\frac{\sqrt{2}\tan(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x])/(d*Sqrt[1 -
Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.127, size = 95, normalized size = 2.1

$$\frac{1}{ad}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(dx+c)+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d
*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 2.22211, size = 428, normalized size = 9.3

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x +
c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, -sqrt(2)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
/(sqrt(a)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A] time = 9.77395, size = 88, normalized size = 1.91

$$\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}d\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(-a)*d*sgn(tan(1/2*d*x + 1/2*c)^2 - 1))
```

3.123 $\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
(Sqrt[a]*d)
```

Rubi [A] time = 0.0664476, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/
(Sqrt[a]*d)
```


Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\int \sqrt{a+a\sec(c+dx)}\,dx}{a}-\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [C] time = 23.5607, size = 5416, normalized size = 63.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.134, size = 141, normalized size = 1.7

$$-\frac{1}{ad}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(dx+c)+1\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin
(d*x+c))+2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)/cos(d*x+c)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.24573, size = 784, normalized size = 9.22

$$\left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^{2}+2\cos(dx+c)-1}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)-2\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^{2}+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,c}{\cos(dx+}\right.}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x
+ c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 2*sqrt(-a)*log((2*a*cos(
d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d), (sqrt(2)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sec(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*sec(d*x + c) + a), x)
```

3.124 $\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/(Sqrt[a]*d)) + (S
qrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[a]*d) + Sin[c + d*x]/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.178732, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3823, 3904, 3887, 481, 203}

$$\frac{\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/(Sqrt[a]*d)) + (S
qrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(
Sqrt[a]*d) + Sin[c + d*x]/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[(-(a*c))^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
```

```
ntegerQ[n] && GtQ[m - n, 0])
```

Rule 3887

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[(-2*a^(m/2 + n + 1/2))/d, Subst[Int[(x^m*(2 + a*x^2)
^(m/2 + n - 1/2))/(1 + a*x^2), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}\\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}a\int \frac{\tan^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx\\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{a\,\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2\,\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.120172, size = 105, normalized size = 0.97

$$-\frac{\tan(c+dx)\left(-\cos(c+dx)\sqrt{1-\sec(c+dx)}+\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((ArcTanh[Sqrt[1 - Sec[c + d*x]]] - Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]
]/Sqrt[2]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 -
 Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Maple [B] time = 0.195, size = 201, normalized size = 1.9

$$\frac{1}{2\,ad\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\,\cos(dx+c)}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)+2\,\ln\left(\frac{1}{\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d/a*((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+2*ln(
((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*cos(d*x+c)^2+2*cos(d*x+c)
)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(dx+c)}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 2.38925, size = 1123, normalized size = 10.4

$$\left[\frac{\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\sqrt{-a}(\cos(dx+c)+1)\,l}{2\,(ad\cos(dx+c)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x
 + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - sqrt
(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(
cos(d*x + c) + 1)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), (sqrt(a)*(cos(d*x + c) + 1)*arctan
(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)
)) + sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - sq
rt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/sqrt(a))/(a*d*cos(d*x + c) + a*
d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.125 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=147

$$-\frac{\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (7*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sqrt[a]*d) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(Sqrt[a]*d) - Sin[c + d*x]/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (Cos[c + d*x]
*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.248374, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3823, 4022, 3920, 3774, 203, 3795}

$$-\frac{\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] (7*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sqrt[a]*d) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]
)/(Sqrt[a]*d) - Sin[c + d*x]/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (Cos[c + d*x]
*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)(a-3a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{-\frac{7a^2}{2}+\frac{1}{2}a^2\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{7\int \sqrt{a+a\sec(c+dx)}\,dx}{8a} - \int \frac{\sec(c}{\sqrt{a+a\,s}}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d} +$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)}{2d\sqrt{a+a\,s}}$$

Mathematica [A] time = 0.239557, size = 118, normalized size = 0.8

$$\frac{\tan(c+dx)\left(\cos(c+dx)(2\cos(c+dx)-1)\sqrt{1-\sec(c+dx)} + 7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right) - 4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((7*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*
x]]/Sqrt[2]] + Cos[c + d*x]*(-1 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])*T
an[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.226, size = 380, normalized size = 2.6

$$\frac{1}{16\,ad\cos(dx+c)\sin(dx+c)}\left(7\sqrt{2}\left(-2\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{3/2}\text{Artanh}\left(1/2\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d/a*(7*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*co
s(d*x+c)+7*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+8*l
n(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)*sin(d*x+c)+8*ln(((-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-8*cos(d*x+c)^4+12*cos(d*x+c)^3-4*cos
(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A] time = 2.53869, size = 1196, normalized size = 8.14

$$\left[\frac{4\sqrt{2}(a\cos(dx+c)+a)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-7\sqrt{-a}(\cos(dx+c)+}{8(ad\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) + 3*cos(d*
x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 7*s
```

```
qrt(-a)*(cos(d*x + c) + 1)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a
)/(cos(d*x + c) + 1)) + 2*(2*cos(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/4*(7*sq
rt(a)*(cos(d*x + c) + 1)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*cos(d*x + c)^2 - cos(d*x + c))*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*(a*cos(d*x + c)
 + a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.126 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{15\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{13\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{10a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{9\tan(c+dx)\sec^2(c+d\cdots}{10ad\sqrt{a\sec(c+dx)+\cdots}}$$

```
[Out] (-15*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2)) + (31*Tan[c + d*x])/(5*a*d*Sqrt[a + a*Sec[c + d*x]]) + (9*Sec[c +
d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) - (13*Sqrt[a + a*Sec
[c + d*x]]*Tan[c + d*x])/(10*a^2*d)
```

Rubi [A] time = 0.424642, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3816, 4021, 4010, 4001, 3795, 203}

$$-\frac{15\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{13\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{10a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{9\tan(c+dx)\sec^2(c+d\cdots}{10ad\sqrt{a\sec(c+dx)+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-15*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2)) + (31*Tan[c + d*x])/(5*a*d*Sqrt[a + a*Sec[c + d*x]]) + (9*Sec[c +
d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) - (13*Sqrt[a + a*Sec
[c + d*x]]*Tan[c + d*x])/(10*a^2*d)
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^3(c+dx)\left(3a-\frac{9}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{2a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\sec^2(c+dx)\left(-9a^2+\frac{39}{4}a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}}{5a^3}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{a+a\sec(c+dx)}\tan(c+}{10a^2d}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - 13\sqrt{}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec^2(c+dx)\tan(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - 13\sqrt{}$$

$$= -\frac{15\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{31\tan(c+dx)}{5ad\sqrt{a+a\sec(c+dx)}} + \frac{9\sec}{10a}$$

Mathematica [A] time = 0.43402, size = 124, normalized size = 0.68

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(4\sec^3(c+dx)-4\sec^2(c+dx)+36\sec(c+dx)+49\right)-75\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\right.}{20d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-75*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) +
2*Sqrt[1 - Sec[c + d*x]]*(49 + 36*Sec[c + d*x] - 4*Sec[c + d*x]^2 + 4*Sec[c
 + d*x]^3))*Tan[c + d*x])/(20*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]
))^(3/2))
```

Maple [B] time = 0.188, size = 417, normalized size = 2.3

$$\frac{1}{80\,da^2(\sin(dx+c))^3(\cos(dx+c))^2}\left(75\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(dx+c)+1\right)\right)\left(-2\frac{}{\mathrm{co}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/80/d/a^2*(75*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c
)^4+150*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/s
in(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)^3-150
*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+
c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)*cos(d*x+c)-75*ln(((-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+392*cos(d*x+c)^5-496*cos(d*x+c)^4
-216*cos(d*x+c)^3+384*cos(d*x+c)^2-96*cos(d*x+c)+32)*(a*(cos(d*x+c)+1)/cos(
d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 2.37225, size = 1095, normalized size = 5.98

$$\left[-\frac{75\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{40\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/40*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqr
t(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^3 + 36*cos(d*x + c)^2 -
 4*cos(d*x + c) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), 1/2
0*(75*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*
arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a
)*sin(d*x + c))) + 2*(49*cos(d*x + c)^3 + 36*cos(d*x + c)^2 - 4*cos(d*x + c
) + 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 10.149, size = 331, normalized size = 1.81

$$\frac{\frac{\left(\left(\left(\frac{5\sqrt{2}a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{127\sqrt{2}a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{175\sqrt{2}a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{85\sqrt{2}a}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{75\cdots}{\cdots}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/20*((((5*sqrt(2)*a*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1)
 - 127*sqrt(2)*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 +
175*sqrt(2)*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 - 85*
sqrt(2)*a/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2
```

```
*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 75*sqrt(2)*lo
g(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
)/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.127 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{11\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{13\tan(c+dx)}{3ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (11*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2)) - (13*Tan[c + d*x])/(3*a*d*Sqrt[a + a*Sec[c + d*x]]) + (7*Sqrt[a +
a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2*d)
```

Rubi [A] time = 0.288322, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3816, 4010, 4001, 3795, 203}

$$\frac{11\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}-\frac{\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{13\tan(c+dx)}{3ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (11*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2)) - (13*Tan[c + d*x])/(3*a*d*Sqrt[a + a*Sec[c + d*x]]) + (7*Sqrt[a +
a*Sec[c + d*x]]*Tan[c + d*x])/(6*a^2*d)
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^2(c+dx)\left(2a-\frac{7}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{2a^2}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{6a^2d} - \frac{\int \frac{\sec(c+dx)\left(-\frac{7a^2}{4}+\frac{13}{2}a^2\sec(c$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{6a^2d} -$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{6a^2d} -$$

$$= \frac{11\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{a+}}{}$$

Mathematica [A] time = 0.284934, size = 114, normalized size = 0.79

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(4\sec^2(c+dx)-12\sec(c+dx)-19\right)+33\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{12d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((33*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) + 2
*Sqrt[1 - Sec[c + d*x]]*(-19 - 12*Sec[c + d*x] + 4*Sec[c + d*x]^2))*Tan[c +
 d*x])/(12*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.167, size = 322, normalized size = 2.2

$$\frac{-1+\cos(dx+c)}{24\,da^2(\sin(dx+c))^3\cos(dx+c)}\left(33\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(dx+c)+1\right)\right)\left(-2\frac{\cos}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/24/d/a^2*(-1+cos(d*x+c))*(33*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(
d*x+c)*cos(d*x+c)^2+66*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)*s
in(d*x+c)+33*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)
+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-76*cos(d*x+
c)^3+28*cos(d*x+c)^2+64*cos(d*x+c)-16)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/
sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 2.20058, size = 1033, normalized size = 7.12

$$\Bigg[-\frac{33\sqrt{2}\big(\cos(dx+c)^3 + 2\cos(dx+c)^2 + \cos(dx+c)\big)\sqrt{-a}\log\Bigg(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx}{\cos(dx+c)^2+2\cos(dx+c)+1}}{24\big(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(
-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)) + 4*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3
 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/12*(33*sqrt(2)*(cos(d*x
```

```
+ c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*(19*
cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos
(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 10.5126, size = 279, normalized size = 1.92

$$-\frac{\frac{\left(\left(\frac{3\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{46\sqrt{2}}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{27\sqrt{2}}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{33\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan(\cdots}\right.\right.}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\cdots\right.}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/sgn(tan(1/2*d*x + 1/2*c)^2 - 1) -
 46*sqrt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)^2 + 27*sq
rt(2)/sgn(tan(1/2*d*x + 1/2*c)^2 - 1))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x
 + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 33*sqrt(2)*log(abs(
-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqr
t(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.128 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[
c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.168173, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3799, 4001, 3795, 203}

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-7*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (2*Tan[
c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
```

```
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\sec(c+dx)\left(-\frac{3a}{2}+2a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}\\
&= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}-\frac{7\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\
&= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}+\frac{7\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\
&= -\frac{7\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{2\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.306255, size = 104, normalized size = 0.99

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}(4\sec(c+dx)+5)-7\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{4d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-7*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*(1 + Sec[c + d*x]) + 2
*Sqrt[1 - Sec[c + d*x]]*(5 + 4*Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - S
ec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.152, size = 225, normalized size = 2.1

$$\frac{1}{4\,da^2\,(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(7\,\sin(dx+c)\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sin(dx+c)-\cos(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(7*sin(d*x+c)*ln(((-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-7*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)+10*cos(d*x+c)^3-12*cos(d*x+c)^2-6*cos(d*x+c)+8)/sin(d*x+c)^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 2.3264, size = 895, normalized size = 8.52

$$\left[-\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3\,a\cos(dx+c)^{2}-2\,a\cos(dx+c)+a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)-4}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x + c) +
 4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/
4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(d*x + c) + 4)*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 10.3394, size = 215, normalized size = 2.05

$$\frac{\frac{\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{9\,\sqrt{2}}{a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}-\frac{7\,\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) -
9*sqrt(2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a) - 7*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/
2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))/d
```

3.129 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sq
rt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0973793, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3797, 3795, 203}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sq
rt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```


Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{3\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\ &= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{3\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\ &= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.212403, size = 94, normalized size = 1.22

$$\frac{\tan(c+dx)\left(3\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-2\sqrt{1-\sec(c+dx)}\right)}{4d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-2*Sqrt[1 - Sec[c + d*x]] + 3*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt
[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 +
Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.117, size = 222, normalized size = 2.9

$$\frac{1}{4\,da^2\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(3\,\sin\left(dx+c\right)\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\ln\left(\frac{1}{\sin\left(dx+c\right)}\left(\sqrt{-2\,\frac{c}{\cos}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*sin(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-c
os(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+3*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+2*cos(d*x+c)^2-2*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 2.64114, size = 873, normalized size = 11.34

$$\left[-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\cdots}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(3*sqr
t(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(
```

```
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 9.1508, size = 165, normalized size = 2.14

$$\frac{\frac{3\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + sqrt(2)*s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(1/2*d*
x + 1/2*c)^2 - 1)))/d
```

3.130 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2
]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0720289, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3796, 3795, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]/(2*Sqrt[2
]*a^(3/2)*d) + Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\ &= \frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.110619, size = 93, normalized size = 1.21

$$\frac{\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}(\sec(c+dx)+1)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{4d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]
]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec
[c + d*x]))^(3/2))
```

Maple [B] time = 0.114, size = 220, normalized size = 2.9

$$\frac{1}{4\,da^2(\cos(dx+c)+1)\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\sin(dx+c)\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos}{\cos(}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(sin(d*x+c)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos
(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)-2*cos(d*x+c)^2+2*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 2.59357, size = 868, normalized size = 11.27

$$\left[-\frac{\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\sqrt{}}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c
) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)
*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [A] time = 9.01857, size = 165, normalized size = 2.14

$$\frac{\frac{\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}a\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - sqrt(2)*sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)))/d
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
5*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqr
t[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.115711, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3777, 3920, 3774, 203, 3795}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) - (
5*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqr
t[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
```

```
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{\int \frac{-2a+\frac{1}{2}a\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}\\
&= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \sqrt{a+a\sec(c+dx)}\,dx}{a^2}-\frac{5\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\
&= -\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}+\frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx,\right.}{2ad}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 23.8193, size = 5534, normalized size = 48.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.125, size = 370, normalized size = 3.3

$$-\frac{1}{4\,d a^{2} \left(\cos \left(dx+c\right)+1\right) \sin \left(dx+c\right)}\sqrt{\frac{a \left(\cos \left(dx+c\right)+1\right)}{\cos \left(dx+c\right)}}\left(4\,\sin \left(dx+c\right) \mathrm{Artanh}\left(1/2\,\frac{\sin \left(dx+c\right)\sqrt{2}}{\cos \left(dx+c\right)}\sqrt{-2\,\frac{\cos \left(dx+c\right)}{\cos \left(dx+c\right)+1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(4*sin(d*x+c)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*cos(d*x+c)+5*sin(d*x+c)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-c
os(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+4*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*sin(d*x+c)+5*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-2*cos(d*x+c)^2+2*cos(d*x+c))/(cos(d*x+c)+1)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sec \left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-3/2), x)
```

Fricas [B] time = 3.11438, size = 1319, normalized size = 11.57

$$\left[-\frac{5\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+8\left(\cdots\right.}{8\left(a^2d\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sqr
t(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) + 8*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 8*(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(
sqrt(a)*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(c+dx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(-3/2), x)
```

<u>Giac</u> [B] time = 11.4785, size = 375, normalized size = 3.29

$$\frac{\frac{5\sqrt{2}\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{8\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\cdots\right.}\right.\right.\right.}{\sqrt{-a}a\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\cdots\right.\right.}\cdots}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(5*sqrt(2)*sqrt(-a)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2)/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 2*sqrt(2)
*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/(a^2*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)) - 8*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a*sgn(tan(1/2
*d*x + 1/2*c)^2 - 1)) + 8*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a*sgn(tan(1/
2*d*x + 1/2*c)^2 - 1)))/d
```

3.132 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-3*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(9*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sq
rt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*Sin[c
 + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.257508, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3817, 4022, 3920, 3774, 203, 3795}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
(9*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sq
rt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*Sin[c
 + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(-3a+\frac{3}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{3a^2-\frac{3}{2}a^2\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^3}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{3\int\sqrt{a+a\sec(c+dx)}\,dx}{2a^2} + \frac{9\int \frac{\text{se}}{\sqrt{a+}}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{3\,\text{Subst}\left(\int\frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{ad}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\,\text{s}}{2ad\sqrt{a}}$$

Mathematica [A] time = 0.796091, size = 129, normalized size = 0.9

$$\frac{\tan(c+dx)\left(2(2\cos(c+dx)+3)\sqrt{1-\sec(c+dx)} - 12(\sec(c+dx)+1)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right) + 9\sqrt{2}(\sec(c+dx)+\right.}{4d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*(3 + 2*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 12*ArcTanh[Sqrt[1 - Sec[c
 + d*x]]]*(1 + Sec[c + d*x]) + 9*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqr
t[2]]*(1 + Sec[c + d*x]))*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 +
 Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.182, size = 384, normalized size = 2.7

$$-\frac{1}{4da^2(\sin(dx+c))^3}\left(6\sqrt{2}(\cos(dx+c))^2\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Artanh}\left(1/2\frac{\sqrt{2}\sin(dx+c)}{\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(6*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*sin(d*x+c)+9*sin(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
)^2-6*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)-4*cos(d*
x+c)^4-9*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/
sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^3+
8*cos(d*x+c)^2-6*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 2.43388, size = 1384, normalized size = 9.61

$$\left[-\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+12}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
```

```
 c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) + 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 4*(2*cos(d*x
+ c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*sqrt(2)*
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 12*(cos(d*
x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*cos(d*x + c)^2 + 3*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.133 $\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{7\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)\cos(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx}{2d(a\sec(c}$$

```
[Out] (19*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*a^(3/2)*d)
- (13*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2)) - (7*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) + (Cos[c + d*x]
*Sin[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.390697, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3817, 4022, 3920, 3774, 203, 3795}

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{7\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)\cos(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx}{2d(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*a^(3/2)*d)
- (13*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2)) - (7*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) + (Cos[c + d*x]
*Sin[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos^2(c+dx)\left(-4a+\frac{5}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{2a^2}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(7a^2-6a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{4a^3}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{-\frac{19a^3}{2}+ \ldots}{\sqrt{a+\ldots}}}{\ldots}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{19\int \sqrt{a \ldots}}{\ldots}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{19\,\mathrm{Subst}\ldots}{\ldots}$$

$$= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{13\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{7 \ldots}{4ad\sqrt{\ldots}}$$

Mathematica [C] time = 2.83766, size = 197, normalized size = 1.06

$$-\frac{\sin(2(c+dx)) - \frac{(\cos(c+dx)+1)\tan(c+dx)\sec(c+dx)\left(13\left(2\cos^2(c+dx)\sqrt{1-\sec(c+dx)}-\cos(c+dx)\sqrt{1-\sec(c+dx)}+7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}\tanh^{-1}\ldots\right.\right.}{4\sqrt{1-\sec(c+dx)}}}{4d(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -(Sin[2*(c + d*x)] - ((1 + Cos[c + d*x])*(13*(7*ArcTanh[Sqrt[1 - Sec[c + d*
x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - Cos[c + d*x]*Sqr
t[1 - Sec[c + d*x]] + 2*Cos[c + d*x]^2*Sqrt[1 - Sec[c + d*x]]) - 40*Hyperge
ometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sec[c + d
*x]*Tan[c + d*x])/(4*Sqrt[1 - Sec[c + d*x]]))/(4*d*(a*(1 + Sec[c + d*x]))^(
3/2))
```

Maple [B] time = 0.216, size = 560, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/16/d/a^2*(-1+cos(d*x+c))*(19*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*cos(d*x+c)^2*sin(d*x+c)+38*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos
(d*x+c))*sin(d*x+c)*cos(d*x+c)+26*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*s
in(d*x+c)*cos(d*x+c)^2+19*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
*sin(d*x+c)+52*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)*sin(d*x+c
)-8*cos(d*x+c)^5+26*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos
(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+20*c
os(d*x+c)^4+16*cos(d*x+c)^3-28*cos(d*x+c)^2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^
(1/2)/cos(d*x+c)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 2.82666, size = 1435, normalized size = 7.76

$$\left[-\frac{13\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+19\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log(-(2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*cos(d*
x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2
*d), 1/4*(13*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c))) - 19*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (2*c
os(d*x + c)^3 - 3*cos(d*x + c)^2 - 7*cos(d*x + c))*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
+ a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.134 $\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{163\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{95\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{197\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (163*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x
])^(5/2)) - (17*Sec[c + d*x]^2*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(
3/2)) - (197*Tan[c + d*x])/(24*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + (95*Sqrt[a
 + a*Sec[c + d*x]]*Tan[c + d*x])/(48*a^3*d)
```

Rubi [A] time = 0.423982, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3816, 4019, 4010, 4001, 3795, 203}

$$\frac{163\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{95\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}-\frac{197\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (163*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x
])^(5/2)) - (17*Sec[c + d*x]^2*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(
3/2)) - (197*Tan[c + d*x])/(24*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + (95*Sqrt[a
 + a*Sec[c + d*x]]*Tan[c + d*x])/(48*a^3*d)
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^3(c+dx)\left(3a-\frac{11}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^2(c+dx)\left(17a^2-\frac{95}{4}a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}} dx}{8a^4}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{95\sqrt{a+a\sec(c+dx)}\tan(c+ \ldots}{48a^3d}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{197\tan(c+dx)}{24a^2d\sqrt{a+a\sec(c+dx)}} + \frac{95 \ldots}{\ldots}$$

$$= -\frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{197\tan(c+dx)}{24a^2d\sqrt{a+a\sec(c+dx)}} + \frac{95 \ldots}{\ldots}$$

$$= \frac{163\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sec^2(c+dx)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\ldots}{24 \ldots}$$

Mathematica [A] time = 1.25889, size = 135, normalized size = 0.74

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(32\sec^3(c+dx)-160\sec^2(c+dx)-503\sec(c+dx)-299\right)+978\sqrt{2}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec \ldots\right.}{48d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((978*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Se
c[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-299 - 503*Sec[c + d*x] - 160*Sec[c
+ d*x]^2 + 32*Sec[c + d*x]^3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.179, size = 417, normalized size = 2.3

$$-\frac{(-1+\cos(dx+c))^2}{192\,da^3(\sin(dx+c))^5\cos(dx+c)}\left(489\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(dx+c)+1\right)\right)\left(-2\,\frac{\ldots}{\mathrm{co}\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/192/d/a^3*(-1+cos(d*x+c))^2*(489*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)
*cos(d*x+c)^3*sin(d*x+c)+1467*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d
*x+c)*cos(d*x+c)^2+1467*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*cos(d*x+c)*
sin(d*x+c)+489*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)-1196*cos(
d*x+c)^4-816*cos(d*x+c)^3+1372*cos(d*x+c)^2+768*cos(d*x+c)-128)*(a*(cos(d*x
+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.05429, size = 1220, normalized size = 6.67

$$\left[-\frac{489\sqrt{2}\left(\cos\left(dx+c\right)^4+3\cos\left(dx+c\right)^3+3\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)}{\cos(dx+c)^2+2\cos(}\right.}{192\left(a^3d\cos\left(dx+c\right)^4+3a^3d\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2
+ cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
```

```
cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x
+ c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(299*cos(d*x + c)^3 +
503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), -1/96*(489*sqrt(2)*(cos(d*x + c)
^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqr
t(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)
^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A] time = 10.284, size = 343, normalized size = 1.87

$$-\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{23\sqrt{2}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{668\sqrt{2}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{465\sqrt{2}}{a\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/(a*sgn(tan(1/2*d*x + 1/2*c)^2
- 1)) + 23*sqrt(2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c
)^2 - 668*sqrt(2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)
^2 + 465*sqrt(2)/(a*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 489*
```

```
sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{75\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{9\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-75*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x
])^(5/2)) + (13*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (9*Tan[
c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.308829, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3816, 4008, 4001, 3795, 203}

$$-\frac{75\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{9\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}-\frac{\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*x
])^(5/2)) + (13*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (9*Tan[
c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^2(c+dx)\left(2a-\frac{9}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)\left(-\frac{39a^2}{4}+9a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}dx}{8a^4}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{9\tan(c+dx)}{4a^2d\sqrt{a+a\sec(c+dx)}} - \frac{75\int}{}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{9\tan(c+dx)}{4a^2d\sqrt{a+a\sec(c+dx)}} + \frac{75\,\mathrm{Su}}{}$$

$$= -\frac{75\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{}{4a}$$

Mathematica [A] time = 0.707526, size = 125, normalized size = 0.86

$$\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}\left(32\sec^2(c+dx)+85\sec(c+dx)+49\right)-150\sqrt{2}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\tanh^{-1}\left(\sqrt{1}\right.\right.}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-150*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*S
ec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(49 + 85*Sec[c + d*x] + 32*Sec[c + d
*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(
5/2))
```

Maple [B] time = 0.164, size = 316, normalized size = 2.2

$$-\frac{(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(75\sin(dx+c)\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx+c)-\cos(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(75*sin(d
*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin
(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+150*sin(d*x+c)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+75*ln(((-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+98*cos(d*x+c)^3+72*cos(d*x+c)^2-106*cos(d*x+c)
-64)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.03906, size = 1079, normalized size = 7.44

$$\left[-\frac{75\,\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(-\frac{2\,\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3\,a\cos(dx+c)^{2}}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(49*cos(d*x + c)^2 + 85*cos(d*x + c)
 + 32)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
```

```
 + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(75*
sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) + 2*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A] time = 9.51116, size = 292, normalized size = 2.01

$$-\frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{17\sqrt{2}}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{83\sqrt{2}}{a^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+\frac{75\sqrt{2}\log\left(\left|-\sqrt{-}\cdots\right.\right.}{\sqrt{\cdots}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/(a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 17*sqrt(2)/(a^2*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)^2 - 83*sqrt(2)/(a^2*sgn(tan(1/2*d*x
+ 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + 75*
sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.136 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (19*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) + Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (13*Ta
n[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.1756, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3799, 4000, 3795, 203}

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) + Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (13*Ta
n[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
```

```
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{\int \frac{\sec(c+dx)\left(-\frac{5a}{2}+4a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}\\
&= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}+\frac{19\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}\\
&= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}-\frac{19\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(c+\cdots}{\sqrt{a+a\sec(c\cdots}}\right.}{16a^2d}\\
&= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{13\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.626845, size = 116, normalized size = 1.08

$$\frac{\tan(c+dx)\left(76\sqrt{2}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-2\sqrt{1-\sec(c+dx)}(13\sec(c+dx)+9)\right)}{32d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((76*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec
[c + d*x]^2 - 2*Sqrt[1 - Sec[c + d*x]]*(9 + 13*Sec[c + d*x]))*Tan[c + d*x])
/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.164, size = 323, normalized size = 3.

$$-\frac{-1+\cos(dx+c)}{32\,da^3(\cos(dx+c)+1)(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(19\,\sin(dx+c)\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(19*sin(d*x
+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d
*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+38*sin(d*x+c)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+19*ln(((-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+18*cos(d*x+c)^3+8*cos(d*x+c)^2-26*cos(d*x+c))/(co
s(d*x+c)+1)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.97066, size = 1062, normalized size = 9.93

$$\left[-\frac{19\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^{2}+2a\cdots}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(9*cos(d*x + c)^2 + 13*cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(19*sqrt(2
)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*
x + c))) + 2*(9*cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A] time = 10.562, size = 216, normalized size = 2.02

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{11\,\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{19\,\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\cdots\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,\cdots\right.\right.}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 11*sqrt(2)/(a^3*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + 19*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2
*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan
(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.137 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (5*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (5*Tan[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.136772, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3797, 3796, 3795, 203}

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (5*Tan[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

```
&& IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}\\
&= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}+\frac{5\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}\\
&= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx,x,-\frac{a\tan(\cdots}{\sqrt{a+a\,\mathrm{se}\cdots}}\right.}{16a^2d}\\
&= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.626717, size = 115, normalized size = 1.07

$$
\frac{\tan(c+dx)\left(\sqrt{1-\sec(c+dx)}(5\sec(c+dx)+1)+10\sqrt{2}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((10*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^4*Sec
[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(1 + 5*Sec[c + d*x]))*Tan[c + d*x])/(1
6*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.122, size = 315, normalized size = 2.9

$$\frac{1}{32\,da^3\left(\cos\left(dx+c\right)+1\right)^2\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(5\,\sin\left(dx+c\right)\ln\left(\frac{1}{\sin\left(dx+c\right)}\left(\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\sin\left(dx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(5*sin(d*x+c)*ln(((-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+10*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*cos(d*x+c)+5*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-2*cos(d*x+c)^3-8*cos(d*x+c)^2+10*cos(d*x+c))/(cos(d*x+c)+1)^2/sin(d*
x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^2}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 1.98814, size = 1052, normalized size = 9.83

$$\left[-\frac{5\,\sqrt{2}\left(\cos\left(dx+c\right)^3+3\,\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\,\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3\,a\cos(dx+c)^2+2\,a\right.}{\cos(dx+c)^2+2\,\cos(dx+c)+1}}{64\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*sqrt(2)*(co
s(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c
))) - 2*(cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A] time = 10.2834, size = 216, normalized size = 2.02

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{3\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{5\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\right.\right.}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) + 3*sqrt(2)/(a^3*sgn(tan(1/2*d*x +
1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - 5*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*
d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(
1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.138 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) + Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*Tan[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.113395, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3796, 3795, 203}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```



```
[Out] (3*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) + Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*Tan[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```


Rule 3796

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$
$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{3\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}$$
$$= \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{16a^2d}$$
$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{3\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

Mathematica [C] time = 0.0566871, size = 52, normalized size = 0.49

$$\frac{\tan(c+dx)\mathrm{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)}{4a^2d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Hypergeometric2F1[1/2, 3, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*x])/(4*a^2*
d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.113, size = 315, normalized size = 2.9

$$\frac{1}{32\,da^3\left(\cos(dx+c)+1\right)^2\sin(dx+c)}\sqrt{\frac{a\left(\cos(dx+c)+1\right)}{\cos(dx+c)}}\left(3\,\sin(dx+c)\ln\left(\frac{1}{\sin(dx+c)}\left(\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin(dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*sin(d*x+c)*ln(((-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+6*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c
)+1)/sin(d*x+c))*cos(d*x+c)+3*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)-14*cos(d*x+c)^3+8*cos(d*x+c)^2+6*cos(d*x+c))/(cos(d*x+c)+1)^2/sin(d*x
+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.02942, size = 1057, normalized size = 9.88

$$\left[-\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^{2}+2}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d
*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(7*cos(d*x + c)^2 + 3*cos(d*x + c))*sq
```

```
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(3*sqrt(2)*(
cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
 c))) - 2*(7*cos(d*x + c)^2 + 3*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3
*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (c + dx)}{(a (\sec (c + dx) + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [A] time = 10.1501, size = 216, normalized size = 2.02

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)} - \frac{5\,\sqrt{2}}{a^3\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{3\,\sqrt{2}\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{\cdot}\right.\right.}{\sqrt{-a}a^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 5*sqrt(2)/(a^3*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) + 3*sqrt(2)*log(abs(-sqrt(-a)*tan(1/2*d
*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(tan(1
/2*d*x + 1/2*c)^2 - 1)))/d
```

3.139 $\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{11\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
43*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (11*Tan
[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.176468, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3777, 3922, 3920, 3774, 203, 3795}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{11\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5/2),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) - (
43*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*S
qrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (11*Tan
[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
```

```
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-4a+\frac{3}{2}a\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$
$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{8a^2-\frac{11}{4}a^2\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{8a^4}$$
$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\sec(c+dx)}\,dx}{a^3} - \frac{43\int$$
$$= -\frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c}{\sqrt{a+a\sec(}}\right.}{a^2d}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{1}{16ad(a}$$

Mathematica [C] time = 23.7934, size = 5574, normalized size = 38.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.156, size = 550, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(32*2^(1/2)*
cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+43*sin(d*
x+c)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(
d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+64*sin(d*x+c)*arc
```

```
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*cos(d*x+c)+86*sin(d*x+c)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d*x+c)+32*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+43*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-30*cos(d*x+c)^3+8*cos(d*x+c)^2+22*cos(d*x+c))/(cos(d*x+c)+1
)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-5/2), x)
```

Fricas [B] time = 3.01786, size = 1565, normalized size = 10.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2
+ 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c)
 - a)/(cos(d*x + c) + 1)) + 4*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(cos(d
*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
```

```
 - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arct
an(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c))) - 2*(15*cos(d*x + c)^2 + 11*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(c+dx)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(-5/2), x)
```

Giac [B] time = 12.4774, size = 427, normalized size = 2.97

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\,\sqrt{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}-\frac{13\,\sqrt{2}}{a^{3}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{43\,\sqrt{2}\sqrt{-a}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{a^{3}\mathrm{sgn}\left(\tan\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c
)^2/(a^3*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)) - 13*sqrt(2)/(a^3*sgn(tan(1/2*d*x
 + 1/2*c)^2 - 1)))*tan(1/2*d*x + 1/2*c) - 43*sqrt(2)*sqrt(-a)*log((sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(a^3*sgn(ta
n(1/2*d*x + 1/2*c)^2 - 1)) + 64*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*a^2*sg
n(tan(1/2*d*x + 1/2*c)^2 - 1)) - 64*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*a^
2*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.140 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{35\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{15\sin(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\text{s}\ldots}{4d(a\,\text{se}\ldots}$$

```
[Out] (-5*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(115*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (15*S
in[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (35*Sin[c + d*x])/(16*a^
2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.371567, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3817, 4020, 4022, 3920, 3774, 203, 3795}

$$\frac{35\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{15\sin(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\text{s}\ldots}{4d(a\,\text{se}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
(115*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (15*S
in[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (35*Sin[c + d*x])/(16*a^
2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3920

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\cos(c+dx)\left(-5a+\frac{5}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(-\frac{35a^2}{2}+\frac{45}{4}a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{8a^4}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{20a^{\cdots}}{\cdots}}{\cdots}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} - \frac{5\int \cdots}{\cdots}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} + \frac{5\,\mathrm{Sub}\cdots}{\cdots}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\cdots}{16ad(\cdots}$$

Mathematica [A] time = 1.7662, size = 169, normalized size = 0.97

$$\frac{\sqrt{1-\sec(c+dx)}(16\sin(c+dx)+5\tan(c+dx)(7\sec(c+dx)+11))-80\tan(c+dx)(\sec(c+dx)+1)^2\tanh^{-1}\left(\sqrt{1-\mathrm{sec}\cdots}\right.}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (460*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Sec
[c + d*x]^3*Sin[(c + d*x)/2] - 80*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[
c + d*x])^2*Tan[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(16*Sin[c + d*x] + 5*(11
+ 7*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c
 + d*x]))^(5/2))
```

Maple [B] time = 0.202, size = 552, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*(-1+cos(d*x+c))^2*(80*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)/cos(d*x+c))*sin(d*x+c)+160*sin(d*x+c)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*2^(1/2)*cos(d*x+c)+115*sin(d*x+c)*ln(((-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*cos(d*x+c)^2+80*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*sin(d*x+c)+230*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ln(((-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*cos(d
*x+c)-32*cos(d*x+c)^4+115*ln(((-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-78*cos(d*x+c)^3+40*cos(d*x+c)^2+70*cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+
c))^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 3.02879, size = 1624, normalized size = 9.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
```

```
cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + 3*cos(d*x + c) + 1)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) - 4*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 35*
cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*
cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1
/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/
(sqrt(a)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)/(sqrt(a)*sin(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 3
5*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.141 $\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])
])/(Sqrt[a]*d))
```

Rubi [A] time = 0.0380545, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3795, 203}

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])
])/(Sqrt[a]*d))
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [C] time = 0.382189, size = 94, normalized size = 1.96

$$\frac{i\sqrt{2}\left(-1+e^{i(c+dx)}\right)\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*(-1 + E^(I*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))])])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*
Sec[c + d*x]])
```

Maple [B] time = 0.135, size = 83, normalized size = 1.7

$$-2\,\frac{-1+\cos(dx+c)}{d\sin(dx+c)}\arctan\left(\frac{1}{\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\frac{1}{\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}}\frac{1}{\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(-1+cos(d*x+c))*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(a*(-1+
cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{-a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(-a*sec(d*x + c) + a), x)
```

Fricas [A] time = 1.95033, size = 424, normalized size = 8.83

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2d}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqr
t((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*sin(
d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))/d, sqrt(2)*arctan(sqrt(2)*sqrt
((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/(s
qrt(a)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\sqrt{-a\left(\sec(c+dx)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(-a*(sec(c + d*x) - 1)), x)
```

Giac [C] time = 1.91456, size = 109, normalized size = 2.27

$$-\frac{\sqrt{2}\left(\frac{\arctan(-i)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}}{\sqrt{a}}\right)}{\sqrt{a}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(arctan(-I)*sgn(tan(1/2*d*x + 1/2*c))/sqrt(a) + arctan(sqrt(a*tan(
1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/(sqrt(a)*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*s
gn(tan(1/2*d*x + 1/2*c))))/d
```

3.142 $\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/
(Sqrt[a]*d)
```

Rubi [A] time = 0.0705244, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3776, 3774, 203, 3795}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/
(Sqrt[a]*d)
```

Rule 3776

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[
a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-a\sec(c+dx)}}\,dx &= \frac{\int \sqrt{a-a\sec(c+dx)}\,dx}{a}+\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx\\ &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [C] time = 0.45565, size = 127, normalized size = 1.46

$$-\frac{i\left(-1+e^{i(c+dx)}\right)\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] ((-I)*(-1 + E^(I*(c + d*x)))*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1
 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt
[1 + E^((2*I)*(c + d*x))]]))/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Se
c[c + d*x]])
```

Maple [A] time = 0.143, size = 119, normalized size = 1.4

$$-\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)}{d\sin\left(dx+c\right)}\left(\sqrt{2}\arctan\left(\frac{1}{\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+2\,\arctan\left(1/2\,\sqrt{2}\sqrt{-2\,\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\right)\right)\frac{1}{\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)*(-1+cos(d*x+c))*(2^(1/2)*arctan(1/(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))+2*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))/(a*(-
1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83487, size = 778, normalized size = 8.94

$$\left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\,\sqrt{2}\left(\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\,\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)-2\,\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cdots}{}}}{\sin\cdots}\right.}{2\,ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*s
in(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) - 2*sqrt(-a)*log((2*(cos(d*
x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) -
 (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)))/(a*d), (sqrt(2)*sqrt(a
)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt
(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\sec(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sec(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-a*sec(d*x + c) + a), x)
```

3.143 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=383

$$-\frac{19\, 3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}\text{EllipticF}\left(\cos^{-1}\right.}{80\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (-9*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(40*d) + (57*(a + a*Sec[c + d*
x])^(2/3)*Tan[c + d*x])/(80*d*(1 + Sec[c + d*x])) + (3*(a + a*Sec[c + d*x])
^(5/3)*Tan[c + d*x])/(8*a*d) - (19*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 -
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + S
ec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 +
 Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]
*Tan[c + d*x])/(80*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-((
(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (
1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.680069, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3800, 4001, 3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8ad}-\frac{9\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{40d}+\frac{57\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{80d(\sec(c+dx)+1)}-\frac{19\, 3^{3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```


```
[Out] (-9*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(40*d) + (57*(a + a*Sec[c + d*
x])^(2/3)*Tan[c + d*x])/(80*d*(1 + Sec[c + d*x])) + (3*(a + a*Sec[c + d*x])
^(5/3)*Tan[c + d*x])/(8*a*d) - (19*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 -
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + S
ec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 +
```

```
 Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]
*Tan[c + d*x])/(80*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-((
(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (
1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8ad} + \frac{3\int \sec(c+dx)\left(\frac{5a}{3}-a\sec(c+dx)\right)(a+}{8a}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8ad} + \frac{19}{40}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8ad} + \frac{(1}{}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8ad} - \frac{(1}{}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{80d(1+\sec(c+dx))} + \frac{3}{}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{80d(1+\sec(c+dx))} + \frac{3}{}$$

$$= -\frac{9(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40d} + \frac{57(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{80d(1+\sec(c+dx))} + \frac{3}{}$$

Mathematica [C] time = 0.255497, size = 105, normalized size = 0.27

$$\frac{\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\left(38\sqrt[6]{2}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{\sec(c+dx)+1}\left(5\sec\right.\right.}{40d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(38*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2,
 (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(1/6)*(2 + 7*Sec[c + d*x] + 5
*Sec[c + d*x]^2))*Tan[c + d*x])/(40*d*(1 + Sec[c + d*x])^(7/6))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^3 (a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a(\sec(c+dx)+1))^{\frac{2}{3}}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

3.144 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=353

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}\text{EllipticF}\left(\cos^{-1}\left(\ldots\right.\right.}{5\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (3*(a + a*Sec[c + d*x])
^(2/3)*Tan[c + d*x])/(5*d*(1 + Sec[c + d*x])) - (3^(3/4)*EllipticF[ArcCos[(
2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^
(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x]
)^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d
*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))
/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.352169, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3798, 3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}+\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d(\sec(c+dx)+1)}-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\ldots}}{5\sqrt[3]{2}d(1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```



```
[Out] (3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (3*(a + a*Sec[c + d*x])
^(2/3)*Tan[c + d*x])/(5*d*(1 + Sec[c + d*x])) - (3^(3/4)*EllipticF[ArcCos[(
2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^
(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x]
)^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d
```

```
*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))
/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{2}{5}\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{\left(2(a+a\sec(c+dx))^{2/3}\right)\int \sec(c+dx)(\ldots}{5(1+\sec(c+dx))^{2/\ldots}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{\left(2(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\mathrm{Su}\ldots}{5d\sqrt{1-\sec(c+dx)}(1+\ldots}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d(1+\sec(c+dx))} - \ldots$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d(1+\sec(c+dx))} - \ldots$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d(1+\sec(c+dx))} - \ldots$$

Mathematica [C] time = 0.127562, size = 85, normalized size = 0.24

$$\frac{\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\left(4\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3(\sec(c+dx)+1)^{7/6}\right)}{5d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(4*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2,
(1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(7/6))*Tan[c + d*x])/(5*d*(1 +
```

```
Sec[c + d*x])^(7/6))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{2} \left(a+a\sec \left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(dx+c\right)+a\right)^{\frac{2}{3}}\sec \left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec \left(dx+c\right)+a\right)^{\frac{2}{3}}\sec \left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

<u>Sympy</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (a (\sec (c + dx) + 1))^{\frac{2}{3}} \sec^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x)**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^{\frac{2}{3}} \sec (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

3.145 $\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=326

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{2d(\sec(c+dx)+1)}-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(}{2\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-}}$$

```
[Out] (3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])) - (3^(
3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2
^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*
Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^
(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1 - Sec[c
 + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^
2)])
```

Rubi [A] time = 0.269234, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{2d(\sec(c+dx)+1)}-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(}{2\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec}}{(}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])) - (3^(
3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2
^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*
Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^
(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1 - Sec[c
 + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
```

```
+ Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^
2)])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \sec(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}}{2d\sqrt{1-\sec(c+dx)}(1+}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{\left(3(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subs}}{d\sqrt{1-\sec(c+dx)}(1+}$$

$$= \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{2d(1+\sec(c+dx))} - \frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}\left(2\right.\right.}{2\sqrt[3]{2}d}$$

Mathematica [C] time = 0.0492302, size = 66, normalized size = 0.2

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
Sec[c + d*x]))^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \sec(dx+c)(a+a\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^{\frac{2}{3}} \sec (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a \sec (dx + c) + a)^{\frac{2}{3}} \sec (dx + c)\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a (\sec (c + dx) + 1))^{\frac{2}{3}} \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

3.146 $\int (a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

```
[Out] (3*Sqrt[2]*AppellF1[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] time = 0.0472338, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3779, 3778, 136}

$$\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^{2/3}\,dx &= \frac{(a+a\sec(c+dx))^{2/3}\int (1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}\\ &= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}x}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}}\\ &= \frac{3\sqrt{2}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{1-\sec(c+dx)}}\end{aligned}$$

Mathematica [B] time = 4.79151, size = 691, normalized size = 8.97

$$\overline{ad\left(135F_1\left(\frac{1}{2};\frac{2}{3},1;\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2\left(3\cos(c+dx)+2\tan^2(c+dx)+3\right)+40\sin^2\left(\frac{1}{2}(c+dx)\right)\mathrm{t}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (45*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(a*
(1 + Sec[c + d*x]))^(5/3)*Sin[c + d*x]*(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(a*d*(40*(3*AppellF1
[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/
2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])^2*Sec[c + d*x]*Si
n[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 6*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[c + d*x]^2*Sin[(c + d*x)/2]^2*(15*Appe
llF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-7 + 16*Co
s[c + d*x] - 3*Cos[2*(c + d*x)]) + 10*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*(7 - 16*Cos[c + d*x] + 3*Cos[2*(c + d*x)]) -
```

```
 24*(9*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
- 6*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 5
*AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c
 + d*x]*Tan[(c + d*x)/2]^2) + 135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]^2*(3 + 3*Cos[c + d*x] + 2*Tan[c + d*x]^2)))
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(dx+c\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(c+dx)+a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3), x)
```

3.147 $\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=77

$$-\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

```
[Out] (-3*Sqrt[2]*AppellF1[7/6, 1/2, 2, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```

Rubi [A] time = 0.10808, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3828, 3827, 136}

$$-\frac{3\sqrt{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```


```
[Out] (-3*Sqrt[2]*AppellF1[7/6, 1/2, 2, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]])
```


Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \cos(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}x^2}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}}$$

$$= -\frac{3\sqrt{2}F_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\text{ta}}{7d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] time = 16.0453, size = 2700, normalized size = 35.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(Sin[
c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(2/3)) - (2^(2/3)*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*((Sec[(c +
d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/6 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1
 + Sec[c + d*x])^(2/3))/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/
3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*d*(1 + Sec[c +
 d*x])^(2/3)*(-(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*
(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c
```

+ d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*2^(1/3)) - (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2] + (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (2*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (81*Cos[(c + d*x)/2]^2*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (81*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*(3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) - 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/9 - (2*2^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (81*AppellF1[1/2, 2/3, 1,

```
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*Appel
lF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*Appel
lF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1
[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/
2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2
]^2*Sec[c + d*x]*Tan[c + d*x]))/(27*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)
)))
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \cos (dx + c) (a + a \sec (dx + c))^{\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^{\frac{2}{3}} \cos (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*cos(d*x + c), x)
```

3.148 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=413

$$-\frac{343\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}\text{EllipticF}\left(\cos^{-}\right.}{880\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (147*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(440*d) + (1029*a*(a + a*Se
c[c + d*x])^(2/3)*Tan[c + d*x])/(880*d*(1 + Sec[c + d*x])) - (9*(a + a*Sec[
c + d*x])^(5/3)*Tan[c + d*x])/(88*d) + (3*(a + a*Sec[c + d*x])^(8/3)*Tan[c
+ d*x])/(11*a*d) - (343*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])
*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3
))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c +
d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c +
d*x])/(880*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec
[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.507738, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3800, 4001, 3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{8/3}}{11ad}-\frac{9\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{88d}+\frac{147a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{440d}+\frac{1029}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```


```
[Out] (147*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(440*d) + (1029*a*(a + a*Se
c[c + d*x])^(2/3)*Tan[c + d*x])/(880*d*(1 + Sec[c + d*x])) - (9*(a + a*Sec[
c + d*x])^(5/3)*Tan[c + d*x])/(88*d) + (3*(a + a*Sec[c + d*x])^(8/3)*Tan[c
+ d*x])/(11*a*d) - (343*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])
*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3
```

```
))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c +
d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c +
d*x])/(880*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec
[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+dx)}{11ad} + \frac{3\int \sec(c+dx)\left(\frac{8a}{3}-a\sec(c+dx)\right)(a\ldots}{11a}$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+dx)}{11ad} + \ldots$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+dx)}{11ad} + \ldots$$

$$= -\frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} + \frac{3(a+a\sec(c+dx))^{8/3}\tan(c+dx)}{11ad} - \ldots$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} - \frac{9(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{88d} \ldots$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx))^{2/3}\tan(c+\ldots}{880d(1+\sec(c+dx))}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx))^{2/3}\tan(c+\ldots}{880d(1+\sec(c+dx))}$$

$$= \frac{147a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{440d} + \frac{1029a(a+a\sec(c+dx))^{2/3}\tan(c+\ldots}{880d(1+\sec(c+dx))}$$

Mathematica [C] time = 0.302341, size = 96, normalized size = 0.23

$$\frac{a\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\left(196\sqrt[6]{2}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3(8\sec(c+dx)+5)\ldots\right.}{88d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (a*(a*(1 + Sec[c + d*x]))^(2/3)*(196*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3
/2, (1 - Sec[c + d*x])/2] + 3*(1 + Sec[c + d*x])^(13/6)*(5 + 8*Sec[c + d*x]
))*Tan[c + d*x])/(88*d*(1 + Sec[c + d*x])^(7/6))
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{3} (a+a\sec (dx+c))^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec (dx+c)^{4}+a\sec (dx+c)^{3}\right)(a\sec (dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(a*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

3.149 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=383

$$-\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}\text{EllipticF}\left(\cos^{-1}\left(\right.\right.}{16\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*d) + (21*a*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(16*d*(1 + Sec[c + d*x])) + (3*(a + a*Sec[c + d*x
])^(5/3)*Tan[c + d*x])/(8*d) - (7*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3) - (1
- Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 +
Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1
+ Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2
]*Tan[c + d*x])/(16*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(
((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) -
(1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.384297, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3798, 3828, 3827, 50, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{5/3}}{8d}+\frac{3a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{8d}+\frac{21a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{16d(\sec(c+dx)+1)}-\frac{7\,3^{3/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```


```
[Out] (3*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*d) + (21*a*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(16*d*(1 + Sec[c + d*x])) + (3*(a + a*Sec[c + d*x
])^(5/3)*Tan[c + d*x])/(8*d) - (7*3^(3/4)*a*EllipticF[ArcCos[(2^(1/3) - (1
- Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 +
Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1
```

```
+ Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2
]*Tan[c + d*x])/(16*2^(1/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(
((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) -
(1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{5}{8}\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$$

$$= \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} + \frac{\left(5a(a+a\sec(c+dx))^{2/3}\right)\int \sec(c+dx)(1}{8(1+\sec(c+dx))^{2/3}}$$

$$= \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} - \frac{\left(5a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\mathrm{Su}}{8d\sqrt{1-\sec(c+dx)}(1+}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{3(a+a\sec(c+dx))^{5/3}\tan(c+dx)}{8d} - \frac{(7}{}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{16d(1+\sec(c+dx))} +$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{16d(1+\sec(c+dx))} +$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{8d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{16d(1+\sec(c+dx))} +$$

Mathematica [C] time = 0.418734, size = 106, normalized size = 0.28

$$\frac{a\tan(c+dx)(a(\sec(c+dx)+1))^{2/3}\left(5\sqrt[6]{2}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c}{2d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (a*(a*(1 + Sec[c + d*x]))^(2/3)*(5*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2
, (1 - Sec[c + d*x])/2] + 3*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(1 + Sec[c +
d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(7/6))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^2 (a+a\sec (dx+c))^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)^{\frac{5}{3}} \sec (dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec (dx+c)^3 + a\sec (dx+c)^2\right)(a\sec (dx+c)+a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(a*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

3.150 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=356

$$-\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}\text{EllipticF}\left(\cos^{-1}\right.}{10\sqrt[3]{2}d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (21*a*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(10*d*(1 + Sec[c + d*x])) - (7*3^(3/4)*a*Elliptic
F[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])
^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Se
c[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)*d*(1 - Sec[c + d*x])*(1
+ Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.287137, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3828, 3827, 50, 63, 225}

$$\frac{3a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5d}+\frac{21a\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{10d(\sec(c+dx)+1)}-\frac{7\,3^{3/4}a\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)}\right.}{10\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```


```
[Out] (3*a*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (21*a*(a + a*Sec[c +
d*x])^(2/3)*Tan[c + d*x])/(10*d*(1 + Sec[c + d*x])) - (7*3^(3/4)*a*Elliptic
F[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])
^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Se
c[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 +
 Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)*d*(1 - Sec[c + d*x])*(1
```

```
+ Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{\left(a(a+a\sec(c+dx))^{2/3}\right)\int \sec(c+dx)(1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} - \frac{\left(7a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\mathrm{S}}{5d\sqrt{1-\sec(c+dx)}(1 -}$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{10d(1+\sec(c+dx))} -$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{10d(1+\sec(c+dx))} -$$

$$= \frac{3a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5d} + \frac{21a(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{10d(1+\sec(c+dx))} -$$

Mathematica [C] time = 0.0779026, size = 66, normalized size = 0.19

$$\frac{4\sqrt[6]{2}\tan(c+dx)(a(\sec(c+dx)+1))^{5/3}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*(a*(1 +
Sec[c + d*x]))^(5/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(13/6))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \sec(dx+c)(a+a\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)^{\frac{5}{3}}\sec (dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec (dx+c)^{2}+a\sec (dx+c)\right)(a\sec (dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^2 + a*sec(d*x + c))*(a*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

3.151 $\int (a+a\sec (c+dx))^{5/3}\,dx$

Optimal. Leaf size=86

$$\frac{3\sqrt{2}a\tan (c+dx)(\sec (c+dx)+1)(a\sec (c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(\sec (c+dx)+1),\sec (c+dx)+1\right)}{13d\sqrt{1-\sec (c+dx)}}$$

```
[Out] (3*Sqrt[2]*a*AppellF1[13/6, 1/2, 1, 19/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
 d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*Sq
rt[1 - Sec[c + d*x]])
```

Rubi [A] time = 0.0471119, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3779, 3778, 136}

$$\frac{3\sqrt{2}a\tan (c+dx)(\sec (c+dx)+1)(a\sec (c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(\sec (c+dx)+1),\sec (c+dx)+1\right)}{13d\sqrt{1-\sec (c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Sqrt[2]*a*AppellF1[13/6, 1/2, 1, 19/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
 d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*Sq
rt[1 - Sec[c + d*x]])
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/3}\,dx = \frac{\left(a(a+a\sec(c+dx))^{2/3}\right)\int (1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}x}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}}$$

$$= \frac{3\sqrt{2}aF_1\left(\frac{13}{6};\frac{1}{2},1;\frac{19}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(c+dx))(a+a\sec(c+d}{13d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] time = 15.7663, size = 2694, normalized size = 31.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*Tan
[(c + d*x)/2])/(2*d*(1 + Sec[c + d*x])^(5/3)) + ((Cos[(c + d*x)/2]^2*Sec[c
+ d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((3*Sec[(c + d*x)/2]^2*(1 + Sec[
c + d*x])^(2/3))/4 + (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2
/3))/2)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]
^2 + (135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*2^(1/3)*d*(1 + Sec[c + d*x])^(5/
3)*((Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(AppellF1[3
/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (135*AppellF1[1/2, 2/3, 1, 3/2,
```

```
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1
/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[
3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2
, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2
)))/(6*2^(1/3)) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]
*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2] + (C
os[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2,
 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (2*AppellF1[3/2,
 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(
-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (135*AppellF1[1/
2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*S
in[(c + d*x)/2])/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (135*Cos[(c + d*x)/2]^2*(-(AppellF1[3/2,
2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2, 2
/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2,
2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3
, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (
135*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos
[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-(AppellF1[3/2, 2/3,
 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*
(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 2, 7/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8/3, 1, 7/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*
Tan[(c + d*x)/2]^2)^2))/(3*2^(1/3)) + (2^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3
/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (135*AppellF1[1/2, 2/3, 1, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1
```

```
/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[
3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2
, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2
))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*
Sec[c + d*x]*Tan[c + d*x]))/(9*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3))))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int (a + a \sec(dx + c))^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec(dx + c) + a)^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3), x)
```

3.152 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=86

$$-\frac{3\sqrt{2}a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{13d\sqrt{1-\sec(c+dx)}}$$

```
[Out] (-3*Sqrt[2]*a*AppellF1[13/6, 1/2, 2, 19/6, (1 + Sec[c + d*x])/2, 1 + Sec[c
+ d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*S
qrt[1 - Sec[c + d*x]])
```

Rubi [A] time = 0.106585, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3828, 3827, 136}

$$-\frac{3\sqrt{2}a\tan(c+dx)(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{13d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```


```
[Out] (-3*Sqrt[2]*a*AppellF1[13/6, 1/2, 2, 19/6, (1 + Sec[c + d*x])/2, 1 + Sec[c
+ d*x]]*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(13*d*S
qrt[1 - Sec[c + d*x]])
```


Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/3}\,dx = \frac{\left(a(a+a\sec(c+dx))^{2/3}\right)\int \cos(c+dx)(1+\sec(c+dx))^{5/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$
$$= -\frac{\left(a(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1+x)^{7/6}}{\sqrt{1-x}x^2}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}}$$
$$= -\frac{3\sqrt{2}aF_1\left(\frac{13}{6};\frac{1}{2},2;\frac{19}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec(c+dx))(a+}{13d\sqrt{1-\sec(c+dx)}}$$

Mathematica [B] time = 15.9931, size = 2700, normalized size = 31.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*(Sin[
c + d*x] - Tan[(c + d*x)/2]))/(d*(1 + Sec[c + d*x])^(5/3)) - (2^(2/3)*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(5/3)*((2*Sec[(c
+ d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/3 + (5*Cos[c + d*x]*Sec[(c + d*x)/2]^
2*(1 + Sec[c + d*x])^(2/3))/6)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(-9*AppellF1[1/2, 2/3, 1, 3/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(9*d*(1 + Se
```

```
c[c + d*x])^(5/3)*(-(Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(
2/3)*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1[1
/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^
2)/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
- 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Ta
n[(c + d*x)/2]^2)))/(9*2^(1/3)) - (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(2/3)*Tan[(c + d*x)/2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/
3)*Tan[(c + d*x)/2] + (Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)
/2]^2*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
])/5) + (2*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(
1/3)) - (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2])/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (243*Cos[(c + d*x
)/2]^2*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
)/9))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])
*Tan[(c + d*x)/2]^2) - (243*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(3*AppellF1[3/2, 2/3, 2, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] -
9*(-(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9)
+ 2*Tan[(c + d*x)/2]^2*(3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF
1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2])/5) - 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + Appell
F1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))))/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/9 - (2*2^(2/3)*Tan[(c + d*x)/
```

```
2]*(AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (243*AppellF1[1/2
, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)
/(-9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
2*(3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[
(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(
c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(27*(Cos[(c + d*x)/2]^2*Sec[c + d
*x])^(1/3))))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \cos(dx+c)\left(a+a\sec(dx+c)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)

[Out] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec(dx+c)+a\right)^{\frac{5}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/3)*cos(d*x + c), x)
```

3.153 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=371

$$\frac{37\ 3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}\right)\right.}{80\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (99*Tan[c + d*x])/(80*d*(a + a*Sec[c + d*x])^(1/3)) + (3*Sec[c + d*x]^2*Tan
[c + d*x])/(8*d*(a + a*Sec[c + d*x])^(1/3)) - (3*(a + a*Sec[c + d*x])^(2/3)
*Tan[c + d*x])/(40*a*d) + (37*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3
) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) -
 (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(80*2^(1/3)*d*(1
- Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)
*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))^2)])
```

Rubi [A] time = 0.547911, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3824, 4010, 4001, 3828, 3827, 63, 225}

$$\frac{3\tan(c+dx)\sec^2(c+dx)}{8d\sqrt[3]{a\sec(c+dx)+a}}-\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{40ad}+\frac{99\tan(c+dx)}{80d\sqrt[3]{a\sec(c+dx)+a}}+\frac{37\ 3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\right.}{80}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```


```
[Out] (99*Tan[c + d*x])/(80*d*(a + a*Sec[c + d*x])^(1/3)) + (3*Sec[c + d*x]^2*Tan
[c + d*x])/(8*d*(a + a*Sec[c + d*x])^(1/3)) - (3*(a + a*Sec[c + d*x])^(2/3)
*Tan[c + d*x])/(40*a*d) + (37*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt
[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3
```

```
) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) -
 (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(80*2^(1/3)*d*(1
- Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)
*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c
+ d*x])^(1/3))^2)])
```

Rule 3824

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(m + n - 1)), x] + Dist[d^2/(b*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f*x
]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
```

```
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\int \frac{\sec^2(c+dx)\left(2a-\frac{1}{3}a\sec(c+dx)\right)}{\sqrt[3]{a+a\sec(c+dx)}}dx}{8a}$$

$$= \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad} + \frac{9\int \frac{\sec(c+dx)\left(-\frac{2a^2}{9}+\frac{11}{3}a^2\sec\right.}{\sqrt[3]{a+a\sec(c+dx)}}}{40a^2}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad}$$

$$= \frac{99\tan(c+dx)}{80d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{8d\sqrt[3]{a+a\sec(c+dx)}} - \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{40ad}$$

Mathematica [C] time = 0.349512, size = 155, normalized size = 0.42

$$\frac{\tan(c+dx)\left(-4\sqrt[6]{2}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+16\sqrt[6]{2}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\right.\right.}{8d\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a(\sec}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((-4*2^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, (1 - Sec[c + d*x])/2] + 16*2
^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Sec[c + d*x])/2] - 7*2^(1/6)*
Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*Sec[c + d*x]^2*(
1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(8*d*(1 + Sec[c + d*x])^(1/6)*(a*(1
+ Sec[c + d*x]))^(1/3))
```

Maple [F] time = 0.131, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^4 \frac{1}{\sqrt[3]{a+a\sec (dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^4}{(a\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (dx+c)^4}{(a\sec (dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(1/3), x)
```

3.154 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=336

$$-\frac{7\,3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}}\right)\right.}{10\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (-9*Tan[c + d*x])/(10*d*(a + a*Sec[c + d*x])^(1/3)) + (3*(a + a*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(5*a*d) - (7*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1
- Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(
2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)
*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.394185, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3800, 4001, 3828, 3827, 63, 225}

$$\frac{3\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{5ad}-\frac{9\tan(c+dx)}{10d\sqrt[3]{a\sec(c+dx)+a}}-\frac{7\,3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)\cdots}{\left(\sqrt[3]{2}-(1+\cdots\right.}}}{10\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(\cdots}}{\left(\sqrt[3]{\cdots}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```


```
[Out] (-9*Tan[c + d*x])/(10*d*(a + a*Sec[c + d*x])^(1/3)) + (3*(a + a*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(5*a*d) - (7*3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1
- Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(
2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(10*2^(1/3)
```

```
*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2)])
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{3\int \frac{\sec(c+dx)\left(\frac{2a}{3}-a\sec(c+dx)\right)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{7}{10}\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,d\ldots$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} + \frac{\left(7\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{\sec\ldots}{\sqrt[3]{1+\ldots}}}{10\sqrt[3]{a+a\sec(c+d\ldots}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{(7\tan(c+dx))\,\mathrm{Subst}\left(\int \ldots\right.}{10d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\ldots}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{(21\tan(c+dx))\,\mathrm{Subst}\left(\int \ldots\right.}{5d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\ldots}}$$

$$= -\frac{9\tan(c+dx)}{10d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{5ad} - \frac{7\,3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{1\ldots}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1\ldots}}\right.\right.}{10\ldots}$$

Mathematica [C] time = 0.161032, size = 95, normalized size = 0.28

$$\frac{\tan(c+dx)\left(7\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)+3\sqrt[6]{\sec(c+dx)+1}\,(2\sec(c+dx)-1)\right)}{10d\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((7*2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2] + 3*(1 +
 Sec[c + d*x])^(1/6)*(-1 + 2*Sec[c + d*x]))*Tan[c + d*x])/(10*d*(1 + Sec[c
+ d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^3 \frac{1}{\sqrt[3]{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(1/3), x)
```

3.155 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=306

$$\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right),\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*EllipticF[ArcC
os[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[
3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c +
d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c +
d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1
 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
+ Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.30231, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3798, 3828, 3827, 63, 225}

$$\frac{3\tan(c+dx)}{2d\sqrt[3]{a\sec(c+dx)+a}}+\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{}}{\sqrt[3]{2}-(1+\sqrt{}}\right.\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```


```
[Out] (3*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(1/3)) + (3^(3/4)*EllipticF[ArcC
os[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[
3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*
x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c +
d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c +
d*x])/(2*2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1
 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
```

```
+ Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} - \frac{1}{2}\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$

$$= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} - \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\sec(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{2\sqrt[3]{a+a\sec(c+dx)}}$$

$$= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

$$= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{(3\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^6}}\,dx, x, \sqrt[6]{1+\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

$$= \frac{3\tan(c+dx)}{2d\sqrt[3]{a+a\sec(c+dx)}} + \frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-(1-\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-(1+\sqrt{3})\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(}\right.}{2\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}}$$

Mathematica [C] time = 0.108103, size = 85, normalized size = 0.28

$$\frac{\tan(c+dx)\left(3\sqrt[6]{\sec(c+dx)+1}-\sqrt[6]{2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)\right)}{2d\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((-(2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]) + 3*(1
+ Sec[c + d*x])^(1/6))*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/6)*(a*(1 +
Sec[c + d*x]))^(1/3))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 \frac{1}{\sqrt[3]{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(1/3), x)
```

3.156 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=276

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right),\cdots}{\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] -((3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
 d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))
```

Rubi [A] time = 0.22795, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3828, 3827, 63, 225}

$$-\frac{3^{3/4}\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{\cdots}\right.\right.}{\sqrt[3]{2}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] -((3^(3/4)*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
 d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\sec(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}$$

$$= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

$$= -\frac{(6\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^6}}\,dx, x, \sqrt[6]{1+\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}$$

$$= -\frac{3^{3/4}F\left(\cos^{-1}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+dx)}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\left(\sqrt[3]{2}-\sqrt[3]{1+\sec(c+dx)}\right)\sqrt{\frac{2^{2/3}+\sqrt[3]{2}\sqrt[3]{1+\sec\cdots}}{\left(\sqrt[3]{2}-\left(1+\sqrt{\cdots}\right.\right.}}}{\sqrt[3]{2}d(1-\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}\sqrt{-\frac{\sqrt[3]{1+\sec(c+dx)}\left(\sqrt[3]{2}-\sqrt[3]{1+\sec\cdots}\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+\cdots}\right.}}}$$

Mathematica [C] time = 0.066838, size = 65, normalized size = 0.24

$$\frac{\sqrt[6]{2}\tan(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)}{d\sqrt[6]{\sec(c+dx)+1}\sqrt[3]{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2^(1/6)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sec[c + d*x])/2]*Tan[c + d*x
])/(d*(1 + Sec[c + d*x])^(1/6)*(a*(1 + Sec[c + d*x]))^(1/3))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \sec(dx+c)\frac{1}{\sqrt[3]{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.157 $\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (3*Sqrt[2]*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x
]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.042687, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3779, 3778, 136}

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x
]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}\\ &= -\frac{\tan(c+dx)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{5/6}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}\\ &= \frac{3\sqrt{2}F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [B] time = 4.51008, size = 718, normalized size = 9.57

$$\overline{d\sqrt[3]{a(\sec(c+dx)+1)}\left(40\sin^2\left(\frac{1}{2}(c+dx)\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3F_1\left(\frac{3}{2};-\frac{1}{3},2;\frac{5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-1/3),x]
```

```
[Out] (45*AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Co
s[c + d*x]*(1 + Sec[c + d*x])^2*Tan[(c + d*x)/2]*(9*AppellF1[1/2, -1/3, 1,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(3*AppellF1[3/2, -1/3, 2,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(d*(a*(1 +
Sec[c + d*x]))^(1/3)*(40*(3*AppellF1[3/2, -1/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])^2*Sec[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + 6*
AppellF1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[c
+ d*x]^2*Sin[(c + d*x)/2]^2*(-15*AppellF1[3/2, -1/3, 2, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*(1 - 10*Cos[c + d*x] + 3*Cos[2*(c + d*x)]) - 5*A
ppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 - 10*
```

```
Cos[c + d*x] + 3*Cos[2*(c + d*x)]) - 24*(9*AppellF1[5/2, -1/3, 3, 7/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2) + 135*Appell
F1[1/2, -1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*(3 + 3*Sec
[c + d*x] - 3*Sin[c + d*x]*Tan[c + d*x] - Tan[c + d*x]^2)))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a + a\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(1/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx + c) + a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a\sec(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-1/3), x)
```

3.158 $\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

```
[Out] (-3*Sqrt[2]*AppellF1[1/6, 1/2, 2, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*
x]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.0943261, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3828, 3827, 136}

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*Sqrt[2]*AppellF1[1/6, 1/2, 2, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*
x]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt[3]{1+\sec(c+dx)}\int \frac{\cos(c+dx)}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}}\\
&= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^2(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}\\
&= -\frac{3\sqrt{2}F_1\left(\frac{1}{6};\frac{1}{2},2;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [B] time = 1.97296, size = 240, normalized size = 3.2

$$
\frac{(a(\sec(c+dx)+1))^{2/3}\left(\frac{20\sin^3\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{3}{2};\frac{2}{3},1;\frac{5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{6(\cos(c+dx)-1)\left(3F_1\left(\frac{5}{2};\frac{2}{3},2;\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2F_1\left(\frac{5}{2};\frac{5}{3},1;\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)+45(\cos(c+dx \cdots}\right.}{ad}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*((20*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]^3)/(6*(3*Appe
llF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF
1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c +
 d*x]) + 45*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(1 + Cos[c + d*x])) + Sin[c + d*x] - Tan[(c + d*x)/2]))/(a*d)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,\frac{1}{\sqrt[3]{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(cos(c + d*x)/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(1/3), x)
```

3.159 $\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=766

$$-\frac{125\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,\text{Elliptic}}{28\,2^{2/3}a^2d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)}\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right.}}}$$

```
[Out] (-33*Tan[c + d*x])/(28*d*(a + a*Sec[c + d*x])^(5/3)) + (3*Sec[c + d*x]^2*Ta
n[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/3)) + (135*Tan[c + d*x])/(14*a*d*(
a + a*Sec[c + d*x])^(2/3)) + (375*(1 + Sqrt[3])*(a + a*Sec[c + d*x])^(1/3)*
Tan[c + d*x])/(28*a^2*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))) - (375*3^(1/4)*EllipticE[ArcCos[(2^(1/3) - (1 - S
qrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec
[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + S
ec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*T
an[c + d*x])/(14*2^(2/3)*a^2*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*
Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (125*3^(3/4)*(1 - Sqr
t[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a
*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2
^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(28*2^(2/3)*a^2*d*(1 -
 Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2
^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2)])
```

Rubi [A] time = 1.05862, antiderivative size = 766, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3824, 4008, 4000, 3828, 3827, 63, 308, 225, 1881}

$$\frac{375\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{28a^2d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}-\frac{125\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{28\,2^{2/3}a^2d(1-\text{sec}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-33*Tan[c + d*x])/(28*d*(a + a*Sec[c + d*x])^(5/3)) + (3*Sec[c + d*x]^2*Ta
n[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/3)) + (135*Tan[c + d*x])/(14*a*d*(
a + a*Sec[c + d*x])^(2/3)) + (375*(1 + Sqrt[3])*(a + a*Sec[c + d*x])^(1/3)*
Tan[c + d*x])/(28*a^2*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))) - (375*3^(1/4)*EllipticE[ArcCos[(2^(1/3) - (1 - S
qrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x
])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec
[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + S
ec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*T
an[c + d*x])/(14*2^(2/3)*a^2*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*
Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(
1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (125*3^(3/4)*(1 - Sqr
t[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a
*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2
^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 +
 Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(28*2^(2/3)*a^2*d*(1 -
 Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2
^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2)])
```

Rule 3824

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(m + n - 1)), x] + Dist[d^2/(b*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f*x
]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 4008

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(b^2*(2*m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*B*
(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A
*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
```

]

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{3\int \frac{\sec^2(c+dx)\left(2a-\frac{5}{3}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{5/3}}\,dx}{4a}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} - \frac{9\int \frac{\sec(c+dx)\left(-\frac{55a^2}{9}+\frac{35}{9}a^2\sec(c+dx)\right)}{(a+a\sec(c+dx))^{2/3}}}{28a^3}$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}} - 12$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}} - (12$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}} + (1$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}} + (3$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}} - (3$$

$$= -\frac{33\tan(c+dx)}{28d(a+a\sec(c+dx))^{5/3}} + \frac{3\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/3}} + \frac{135\tan(c+dx)}{14ad(a+a\sec(c+dx))^{2/3}} + \frac{}{28}$$

Mathematica [C] time = 0.55807, size = 111, normalized size = 0.14

$$\frac{\tan(c+dx)\left(3\left(7\sec^2(c+dx)+90\sec(c+dx)+79\right)-250\ 2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[6]{\sec(c+dx)+1}\text{Hyperge}}{28d(a(\sec(c+dx)+1))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-250*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Sec
[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6) + 3*(79 + 90*Sec[c + d*
x] + 7*Sec[c + d*x]^2))*Tan[c + d*x])/(28*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^4 (a+a\sec(dx+c))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^4}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(a*sec(d*x + c) + a)^(5/3), x)
```

3.160 $\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=731

$$\frac{19\ 3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,\text{EllipticF}\Big(}{7\ 2^{2/3}a^2d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right.}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}$$

```
[Out] (3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (36*Tan[c + d*x])/(7*a*
d*(a + a*Sec[c + d*x])^(2/3)) - (57*(1 + Sqrt[3])*(a + a*Sec[c + d*x])^(1/3
)*Tan[c + d*x])/(7*a^2*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))) + (57*2^(1/3)*3^(1/4)*EllipticE[ArcCos[(2^(1/3)
- (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) -
(1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3)
+ (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))^2]*Tan[c + d*x])/(7*a^2*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*S
qrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (19*3^(3/4)*(1 - Sqrt[
3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(
1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a^2*d*(1 - Se
c[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1
/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x]
)^(1/3))^2)])
```

Rubi [A] time = 0.770587, antiderivative size = 731, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3799, 4000, 3828, 3827, 63, 308, 225, 1881}

$$-\frac{57\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{7a^2d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}+\frac{19\ 3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\ldots}}{7\ 2^{2/3}a^2d(1-\sec\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) - (36*Tan[c + d*x])/(7*a*
d*(a + a*Sec[c + d*x])^(2/3)) - (57*(1 + Sqrt[3])*(a + a*Sec[c + d*x])^(1/3
)*Tan[c + d*x])/(7*a^2*d*(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))) + (57*2^(1/3)*3^(1/4)*EllipticE[ArcCos[(2^(1/3)
- (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) -
(1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3)
+ (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))^2]*Tan[c + d*x])/(7*a^2*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*S
qrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) + (19*3^(3/4)*(1 - Sqrt[
3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^
(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*S
ec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(
1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a^2*d*(1 - Se
c[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1
/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x]
)^(1/3))^2)])
```

Rule 3799

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4000

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[((A*b - a*B)*Cot[e +
 f*x]*(a + b*Csc[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
```

```
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
```

```
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{3\int \frac{\sec(c+dx)\left(-\frac{5a}{3}+\frac{7}{3}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{2/3}}\,dx}{7a^2}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{19\int \sec(c+dx)\sqrt[3]{a+a\sec(c+dx)}}{7a^2}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(19\sqrt[3]{a+a\sec(c+dx)}\right)\int \sec(c+dx}{7a^2\sqrt[3]{1+\sec(c+}}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(19\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)}{7a^2d\sqrt{1-\sec(c+d}}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(114\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)}{7a^2d\sqrt{1-\sec(c+}}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(57\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)}{7a^2d\sqrt{1-\sec}}$$

$$= \frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{36\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{57\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sec(c+}}{7a^2d(1+\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-\left(1+\right.\right.}$$

Mathematica [C] time = 0.269748, size = 98, normalized size = 0.13

$$\frac{\tan(c+dx)\left(38\,2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[6]{\sec(c+dx)+1}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)-3}{7d(a(\sec(c+dx)+1))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-33 - 36*Sec[c + d*x] + 38*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1
/6, 1/2, 3/2, (1 - Sec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6))*
Tan[c + d*x])/(7*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^3 (a+a\sec (dx+c))^{-\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a\sec (dx+c)+a)^{\frac{1}{3}}\sec (dx+c)^3}{a^2\sec (dx+c)^2+2\,a^2\sec (dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(a*sec(d*x + c) + a)^(5/3), x)
```

3.161 $\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=731

$$-\frac{5\,3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\text{EllipticF}\left(c\right.}{7\,2^{2/3}ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/\cdots}}$$

```
[Out] (-3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) + (15*Tan[c + d*x])/(7*a
*d*(a + a*Sec[c + d*x])^(2/3)) + (15*(1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3)
*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))) - (15*2^(1/3)*3^(1/4)*EllipticE[ArcCos[(2^(1/3) -
 (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[
c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (
1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
^2]*Tan[c + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt
[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (5*3^(3/4)*(1 - Sqrt[3])*
EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c +
 d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(
1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a*d*(1 - Sec[c + d*
x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) -
(1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3
))^2)])
```

Rubi [A] time = 0.673742, antiderivative size = 731, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3797, 3828, 3827, 51, 63, 308, 225, 1881}

$$\frac{15\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{15\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}}{7ad\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a\sec(c+dx)+a)^{2/3}}-\frac{3\tan(c+dx)}{7d(a\sec(c+dx)+a)^{5/3}}-\frac{5\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (-3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^(5/3)) + (15*Tan[c + d*x])/(7*a
*d*(a + a*Sec[c + d*x])^(2/3)) + (15*(1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3)
*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(
1 + Sec[c + d*x])^(1/3))) - (15*2^(1/3)*3^(1/4)*EllipticE[ArcCos[(2^(1/3) -
 (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[
c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1
+ Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (
1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
^2]*Tan[c + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt
[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (5*3^(3/4)*(1 - Sqrt[3])*
EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3
) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c +
 d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(
1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*2^(2/3)*a*d*(1 - Sec[c + d*
x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) -
(1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3
))^2)])
```

Rule 3797

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(2*m + 1)), x]
+ Dist[m/(b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{5\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx}{7a}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{\left(5(1+\sec(c+dx))^{2/3}\right)\int \frac{\sec(c+dx)}{(1+\sec(c+dx))^{2/3}}\,dx}{7a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} - \frac{\left(5\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)^{7/6}}\,dx, x, \text{se}\right.}{7ad\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(5\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\text{S}}{7ad\sqrt{1-\sec(c+dx}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{\left(30\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)}{7ad\sqrt{1-\sec(c+}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} - \frac{\left(15\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)}{7ad\sqrt{1-\sec(}}$$

$$= -\frac{3\tan(c+dx)}{7d(a+a\sec(c+dx))^{5/3}} + \frac{15\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{15\left(1+\sqrt{3}\right)\sqrt[3]{1+\sec(c+}}{7ad(a+a\sec(c+dx))^{2/3}\left(\sqrt[3]{2}-(1\right.}$$

Mathematica [C] time = 0.336507, size = 90, normalized size = 0.12

$$\frac{\tan(c+dx)\left(5\,2^{5/6}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt[6]{\sec(c+dx)+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)-3\right)}{7d(a(\sec(c+dx)+1))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] ((-3 + 5*2^(5/6)*Cos[(c + d*x)/2]^2*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - S
ec[c + d*x])/2]*Sec[c + d*x]*(1 + Sec[c + d*x])^(1/6))*Tan[c + d*x])/(7*d*(
a*(1 + Sec[c + d*x]))^(5/3))
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{2} \left(a+a\sec \left(dx+c\right)\right)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(dx+c\right)^{2}}{\left(a\sec \left(dx+c\right)+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a\sec \left(dx+c\right)+a\right)^{\frac{1}{3}}\sec \left(dx+c\right)^{2}}{a^{2}\sec \left(dx+c\right)^{2}+2\,a^{2}\sec \left(dx+c\right)+a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(a^2*sec(d*x + c)^2 + 2*
a^2*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(c+dx\right)}{\left(a\left(\sec \left(c+dx\right)+1\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/3), x)
```

3.162 $\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=744

$$-\frac{\sqrt[3]{2}3^{3/4}\left(1-\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\mathrm{EllipticF}\left(\right.}{7ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}}$$

```
[Out] (6*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)) + (3*Tan[c + d*x])/(7*a
*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)) + (6*(1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3)*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (6*2^(1/3)*3^(1/4)*EllipticE[
ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/
3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
+ d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (2^(1/3)
*3^(3/4)*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + S
qrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sq
rt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*
(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2)])
```

Rubi [A] time = 0.579813, antiderivative size = 744, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3828, 3827, 51, 63, 308, 225, 1881}

$$\frac{6\tan(c+dx)}{7ad(a\sec(c+dx)+a)^{2/3}}+\frac{3\tan(c+dx)}{7ad(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}+\frac{6\left(1+\sqrt{3}\right)\tan(c+dx)\sqrt[3]{\sec(c+dx)}}{7ad\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a\sec(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (6*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)) + (3*Tan[c + d*x])/(7*a
*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)) + (6*(1 + Sqrt[3])*(1 + S
ec[c + d*x])^(1/3)*Tan[c + d*x])/(7*a*d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3)
 - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (6*2^(1/3)*3^(1/4)*EllipticE[
ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + S
qrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1/
3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
+ d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
 d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (2^(1/3)
*3^(3/4)*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c
 + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + S
qrt[3])/4]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sq
rt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/
(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*a*d*
(1 - Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec
[c + d*x])^(1/3))^2)])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{\sec(c+dx)}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)^{13/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} - \frac{\left(2\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{7ad\sqrt{1-\sec(c+dx)}(a+a\sec}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{\left(2\sqrt[6]{1+\sec(c+d\right.}{7a}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{\left(12\sqrt[6]{1+\sec(c+\right.}{7}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} - \frac{\left(6\sqrt[6]{1+\sec(c+d\right.}{}$$

$$= \frac{6\tan(c+dx)}{7ad(a+a\sec(c+dx))^{2/3}} + \frac{3\tan(c+dx)}{7ad(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}} + \frac{6\left(1+\right.}{7ad(a+a\sec(c+}$$

Mathematica [C] time = 0.0621804, size = 68, normalized size = 0.09

$$\frac{\tan(c+dx)(\sec(c+dx)+1)^{7/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)}{2\sqrt[6]{2}d(a(\sec(c+dx)+1))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (Hypergeometric2F1[1/2, 13/6, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])
^(7/6)*Tan[c + d*x])/(2*2^(1/6)*d*(a*(1 + Sec[c + d*x]))^(5/3))
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(a+a\sec(dx+c))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a^
2*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

3.163 $\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=90

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

```
[Out] (-3*Sqrt[2]*AppellF1[-7/6, 1/2, 1, -1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(7*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a
*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.0476895, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3779, 3778, 136}

$$-\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(-5/3),x]
```

```
[Out] (-3*Sqrt[2]*AppellF1[-7/6, 1/2, 1, -1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(7*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a
*Sec[c + d*x])^(2/3))
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{1}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}x(1+x)^{13/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$

$$= -\frac{3\sqrt{2}F_1\left(-\frac{7}{6};\frac{1}{2},1;-\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

Mathematica [B] time = 16.3438, size = 3007, normalized size = 33.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(-5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((27*Sin[
c + d*x])/7 - (30*Tan[(c + d*x)/2])/7 + (3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) + (2^(1/3)*(1 + Sec[c + d*x])^(5
/3)*((16*(1 + Sec[c + d*x])^(1/3))/7 - (27*Cos[c + d*x]*(1 + Sec[c + d*x])^
(1/3))/7)*Tan[(c + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2
, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3
/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9
)))))/(7*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(
5/3)*((Sec[(c + d*x)/2]^2*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^
```

```
2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/
2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/
3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)
))))/(7*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) + (2^(1/3)*Tan[(c
+ d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) - (3*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (A
ppellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) +
(2*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[
(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3) - Cos[
(c + d*x)/2]*Sin[(c + d*x)/2]*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2,
 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/
2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/
3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)
)) + Cos[(c + d*x)/2]^2*((5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-1 + Tan[(c + d
*x)/2]^2)^2*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])*Tan[(c + d*x)/2]^2)/9)) - (5*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (Appe
llF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/9))/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[1/2, 1/3
, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, 1
/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/3, 1
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)) +
(5*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-(A
ppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9 + (2*(-3*
AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Appel
lF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/9 + (2*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 4/3,
 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/5 + (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - 3*((-6*AppellF1[5/2, 1
/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c
```

```
+ d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)))/9))/((-1 + Tan[(c +
d*x)/2]^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Tan[(c + d*x)/2]^2)/9)^2))))/(7*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3))
 - (2*2^(1/3)*Tan[(c + d*x)/2]*((-3*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x
)/2]^2)^(2/3) + Cos[(c + d*x)/2]^2*(-27 - (5*AppellF1[1/2, 1/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1
[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*Appell
F1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/
2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^
2)/9))))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)
/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(21*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/
3))))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^{-\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(1/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx + c) + a)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(c+dx\right)+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a*sec(c + d*x) + a)**(-5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(-5/3), x)
```

3.164 $\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=90

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

```
[Out] (3*Sqrt[2]*AppellF1[-7/6, 1/2, 2, -1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*Tan[c + d*x])/(7*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a*
Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.110847, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3828, 3827, 136}

$$\frac{3\sqrt{2}\tan(c+dx)F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7ad\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)(a\sec(c+dx)+a)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*Sqrt[2]*AppellF1[-7/6, 1/2, 2, -1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*Tan[c + d*x])/(7*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a*
Sec[c + d*x])^(2/3))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\sec(c+dx))^{5/3}}\,dx = \frac{(1+\sec(c+dx))^{2/3}\int \frac{\cos(c+dx)}{(1+\sec(c+dx))^{5/3}}\,dx}{a(a+a\sec(c+dx))^{2/3}}$$
$$= -\frac{\left(\sqrt[6]{1+\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}x^2(1+x)^{13/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}(a+a\sec(c+dx))^{2/3}}$$
$$= \frac{3\sqrt{2}F_1\left(-\frac{7}{6};\frac{1}{2},2;-\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{7ad\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))(a+a\sec(c+dx))^{2/3}}$$

Mathematica [B] time = 16.028, size = 3011, normalized size = 33.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + a*Sec[c + d*x])^(5/3),x]
```

```
[Out] (((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(5/3)*((-48*Sin
[c + d*x])/7 + (51*Tan[(c + d*x)/2])/7 - (3*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])/14))/(d*(a*(1 + Sec[c + d*x]))^(5/3)) - (5*2^(1/3)*(1 + Sec[c + d*x])
^(5/3)*((-30*(1 + Sec[c + d*x])^(1/3))/7 + (55*Cos[c + d*x]*(1 + Sec[c + d*
x])^(1/3))/7)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1, 3/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*(AppellF1[
1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF
1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2
, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2
```

)/9)))))/(63*d*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]
))^(5/3)*((-5*Sec[(c + d*x)/2]^2*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c +
d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*(AppellF
1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*Appel
lF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3
/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]
^2)/9)))))/(63*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) - (5*2^(1/3
)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c
 + d*x)/2]^2)^(2/3) - (11*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/5 + (AppellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3) + (22*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*
x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(5/3)) - 9*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*(-11 - AppellF1[1/2, 1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*
(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(
-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2)/9))) + 9*Cos[(c + d*x)/2]^2*((AppellF1[1/2, 1/3, 1, 3/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-
1 + Tan[(c + d*x)/2]^2)^2*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)) - (-(AppellF1[3/2, 1/3, 2, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
)/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9)/((-1 + Tan[(c + d*x)/2]^2)*(AppellF
1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*Appel
lF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3
/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]
^2)/9)) + (AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*(-(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9 +
 (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9 + (2*Tan[(c + d*x)/2]^2*((-3*AppellF1[5
/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/5 + (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - 3*((-6*AppellF

```
1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)))/9))/((-1 + T
an[(c + d*x)/2]^2)*(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] + (2*(-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2])*Tan[(c + d*x)/2]^2)/9)^2))))/(63*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(2/3)) + (10*2^(1/3)*Tan[(c + d*x)/2]*((-11*AppellF1[3/2, 1/3, 1, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*S
ec[(c + d*x)/2]^2)^(2/3) + 9*Cos[(c + d*x)/2]^2*(-11 - AppellF1[1/2, 1/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/((-1 + Tan[(c + d*x)/2]^2)*
(AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(
-3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Ap
pellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2)/9))))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[
(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(189*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^(5/3))))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(a+a\sec(dx+c))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

```
[Out] int(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(a\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(a*sec(d*x + c) + a)^(5/3), x)
```

3.165 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+6a$$

```
[Out] (-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0905414, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3768, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec^{\frac{5}{2}}(c+dx)\,dx + a\int \sec^{\frac{7}{2}}(c+dx)\,dx\\ &= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\\ &= \frac{6a\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(}{5d}\\ &= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6a\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2}{}\\ &= -\frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}\end{aligned}$$

Mathematica [A] time = 0.228212, size = 115, normalized size = 0.76

$$\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)\left(5\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+9\sin(c+dx)+5\tan(c+dx)-9\sqrt{\cos(c+}\right.}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(-9*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*Sin[c +
 d*x] + 5*Tan[c + d*x] + 3*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[Sec[c + d
*x]])
```

Maple [B] time = 2.175, size = 384, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/10*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d
*x+1/2*c)^2-1/2)^3-12/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1
/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+28/15*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6/5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))-1/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec(dx+c)^{3}+a\sec(dx+c)^{2}\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx+c)+a) \sec (dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.166 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2a}{}$$

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rubi [A] time = 0.0763306, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3768, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= a\int \sec^{\frac{3}{2}}(c+dx)\,dx + a\int \sec^{\frac{5}{2}}(c+dx)\,dx\\ &= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\\ &= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{}\right.\\ &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}\end{aligned}$$

Mathematica [A] time = 0.192793, size = 83, normalized size = 0.67

$$\frac{a\sec^{\frac{3}{2}}(c+dx)\left(2\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx)+3\sin(2(c+dx))-6\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*(-6*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*Sin[c + d*x] + 3*Sin[2*(c
+ d*x)]))/(3*d)
```

Maple [B] time = 2.24, size = 369, normalized size = 3.

$$\frac{2a}{3d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\,\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] 2/3*a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-
3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec(dx+c)^{2}+a\sec(dx+c)\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec(dx + c) + a) \sec(dx + c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.167 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{d}$$

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0644187, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3771, 2641, 3768, 2639}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx = a\int \sqrt{\sec(c+dx)}\,dx + a\int \sec^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$
$$= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(\right.$$
$$= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|\right.}{d}$$

Mathematica [A] time = 0.137726, size = 68, normalized size = 0.7

$$\frac{2a\sqrt{\sec(c+dx)}\left(\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]))/d
```

Maple [A] time = 1.266, size = 146, normalized size = 1.5

$$-2\,\frac{a\left(\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}+\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c\right.\right.}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2*a*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sec(dx+c)+a)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sqrt{\sec\left(c+dx\right)}\,dx+\int\sec^{\frac{3}{2}}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.168 $\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.0574911, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3787, 3771, 2639, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a\int \sqrt{\sec(c+dx)}\,dx$$
$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+$$
$$= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.0770941, size = 49, normalized size = 0.65

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 2] + EllipticF[(c + d*x)/2,
 2])*Sqrt[Sec[c + d*x]])/d
```

Maple [A] time = 1.331, size = 150, normalized size = 2.

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\,a\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(\text{EllipticF}\left(\text{co}\right.\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\,\sin(1/2\,dx+c/2)\,\sqrt{2\,(\cos(1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sec{(dx+c)} + a}{\sqrt{\sec{(dx+c)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a \sec{(dx+c)} + a}{\sqrt{\sec{(dx+c)}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{\sqrt{\sec{(c+dx)}}}\, dx + \int \sqrt{\sec{(c+dx)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.169 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0692706, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx \\
&= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx \\
&= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+
\\
&= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}
\end{aligned}$$

Mathematica [A] time = 0.132079, size = 73, normalized size = 0.72

$$\frac{a\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))+6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*S
qrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)]))/(3*d)
```

Maple [A] time = 1.156, size = 225, normalized size = 2.2

$$-\frac{2a}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a\sec\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)**(-3/2), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.170 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{5d}$$

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.082885, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}\\ &= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] time = 0.189308, size = 93, normalized size = 0.73

$$\frac{a\sqrt{\sec(c+dx)}\left(20\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\sin(c+dx)+10\sin(2(c+dx))+3\sin(3(c+dx))+36\sqrt{\cos(}\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(36*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*Sin[c + d*x] + 10*Sin[2*(
c + d*x)] + 3*Sin[3*(c + d*x)]))/(30*d)
```

Maple [A] time = 1.198, size = 219, normalized size = 1.7

$$-\frac{2a}{15d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,(\cos(1/2\,dx+c/2))^7-28\,(\cos(1/2\,dx+c/2))^5+5\sqrt{(\sin(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sec(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.171 $\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{6a\sqrt{}}{}$$

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0953708, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}(5a)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5a)\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}\left(3a\sqrt{\right. \\
&= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin}{21d\sqrt{\sec}} \\
&= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{21d}
\end{aligned}
$$

Mathematica [A] time = 0.263539, size = 103, normalized size = 0.68

$$
\frac{a\sqrt{\sec(c+dx)}\left(200\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+42\sin(c+dx)+130\sin(2(c+dx))+42\sin(3(c+dx))+15\,\mathrm{s}\right.}{420d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(504*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2
00*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 130*Sin
[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(420*d)
```

Maple [A] time = 1.284, size = 270, normalized size = 1.8

$$-\frac{2a}{105d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^8-528\,(\sin(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+25*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.172 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=187

$$\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+$$

```
[Out] (-12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(7*d) + (12*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(7*d) + (4*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.131782, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{7d}+\frac{12a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(7*d) + (12*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(7*d) + (4*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^{\frac{7}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}\left(6a^2\right)\int \sec^{\frac{3}{2}}(c$$

$$= \frac{12a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{8a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{4a^2\sec^{\frac{5}{2}}(c+dx}{5d}$$

$$= \frac{12a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{8a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{4a^2\sec^{\frac{5}{2}}(c+dx}{5d}$$

$$= -\frac{12a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{7d}$$

Mathematica [C] time = 2.26644, size = 287, normalized size = 1.53

$$\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(\frac{42\csc(c)\cos(dx)+(14\cos(c+dx)+10\cos(2(c+dx))+15)\tan(c+dx)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(21*(1 + E^((2*I)*(c + d*
x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((2*I)
*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*
I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (42*Cos[d*x]*Csc[c]
 + (15 + 14*Cos[c + d*x] + 10*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d*x]
)/Sec[c + d*x]^(3/2)))/(70*d)
```

Maple [B] time = 2.429, size = 439, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/5*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*
d*x+1/2*c)^2-1/2)^3-24/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+124/35*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12/5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/28*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-4/7*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d
*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\sec(dx+c)^4+2\,a^2\sec(dx+c)^3+a^2\sec(dx+c)^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^4 + 2*a^2*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.173 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=161

$$\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\cdots$$

```
[Out] (-16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (16*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d)
```

Rubi [A] time = 0.114471, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{16a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\cdots}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (16*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \int \sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2a^2\right)\int \sqrt{\mathrm{sec}}$$

$$= \frac{16a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\sec^{\frac{5}{2}}(c+}{5}$$

$$= \frac{4a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{16a^2\sqrt{\sec(c+dx)}\sin(c+dx}{5d}$$

$$= -\frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}$$

Mathematica [C] time = 1.88023, size = 269, normalized size = 1.67

$$\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(\frac{24\csc(c)\cos(dx)+\tan(c+dx)(3\sec(c+dx)+10)}{\sec^{\frac{3}{2}}(c+dx)}-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(12\left(-1+e^{2ic}\right)\sqrt{1+e^{2i}}\cdots\right.}{\cdots}\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(12*(1 + E^((2*I)*(c + d*
x))) + 12*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*
c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I
)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (24*Cos[d*x]*Csc[c]
+ (10 + 3*Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(3/2)))/(30*d)
```

Maple [B] time = 2.332, size = 386, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/10*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-32/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+68/15*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-16/5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)^2 \sec (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2 \sec (dx+c)^3 + 2\, a^2 \sec (dx+c)^2 + a^2 \sec (dx+c)\right)\sqrt{\sec (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^3 + 2*a^2*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)^2 \sec (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.174 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=131

$$\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-$$

```
[Out] (-4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (4*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.101867, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2\sqrt{\text{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (4*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx = \left(2a^2\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\sec(c+dx)}\left(a^2+a^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\right)\int \sqrt{\sec$$

$$= \frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\sqrt{\cos(c+$$

$$= -\frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{3d}$$

Mathematica [C] time = 1.58764, size = 264, normalized size = 2.02

$$\frac{1}{3}a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(\frac{\tan(c+dx)+6\csc(c)\cos(dx)}{2d\sec^{\frac{3}{2}}(c+dx)}-\frac{i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^2(c+dx)\left(3\left(-1+\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(((-I)*Sqrt[2]*Sqrt[E^(I*(c +
d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^2*(3*(1 + E^((2*I)*(c + d*x))
) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1
/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c
 + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (6*Cos[d*x]*Csc[c] + T
an[c + d*x])/(2*d*Sec[c + d*x]^(3/2))))/3
```

Maple [B] time = 2.417, size = 371, normalized size = 2.8

$$\frac{4a^2}{3d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 4/3*a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(sec(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.175 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=64

$$\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0736912, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3788, 3771, 2641, 4043}

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = \left(2a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}$$

Mathematica [A] time = 0.156184, size = 48, normalized size = 0.75

$$\frac{2a^2\sqrt{\sec(c+dx)}\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Sin[c + d*x]))/d
```

Maple [A] time = 1.347, size = 104, normalized size = 1.6

$$-4\,\frac{a^2\left(\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2-(\sin(1/2\,dx+c/2))^2\cos(\ldots}\right.}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -4*a^2*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sqrt(sec(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\int 2\sqrt{\sec(c+dx)}\,dx+\int \sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(sec(c + d*x)), x) + Integral(2*sqrt(sec(c + d*x)), x)
 + Integral(sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.176 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0917851, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3788, 3771, 2639, 4045, 2641}

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(2a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(4a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(}$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(4a^2\sqrt{\cos(c+dx)}\sqrt{}\right.$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{3d}$$

Mathematica [C] time = 1.28264, size = 156, normalized size = 1.46

$$\frac{a^2\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)-i\cos\left(\frac{c}{2}\right)\right)\left(-\frac{24\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+8\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hyperge}\right.}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(Cos[c/2] - I*Sin[c/2])*((-I)*Cos[c/2] + Sin[c/2])*(12 - (24*Hypergeom
etric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x)
)] + 8*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]*Sec[c + d*x] + (2*I)*Sin[c + d*x]))/(3*d*Sqrt[Sec[c + d*x]
])
```

Maple [A] time = 1.35, size = 228, normalized size = 2.1

$$-\frac{4a^2}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+2\,\mathrm{EllipticF}\left(\cos\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sec(d*x + c)^(3/2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{1}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{2}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\sqrt{\sec\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**(-3/2), x) + Integral(2/sqrt(sec(c + d*x)), x)
 + Integral(sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.177 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticE}\left(\frac{1}{2}(c+dx),2\right)}{5d}$$

```
[Out] (16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.105421, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```


```
[Out] (16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \left(2a^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(2a^2\right)\int \sqrt{\sec(c+dx)}\,dx+\frac{1}{5}\left(8a^2\right)\int \frac{1}{\sqrt{\sec(c+d}}\\
&= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,d\\
&= \frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3d}
\end{aligned}$$

Mathematica [C] time = 1.57618, size = 136, normalized size = 1.01

$$\frac{a^2\left(\frac{192i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-40i\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx)}\right)+40\sin(\right.}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*Sin
[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.414, size = 250, normalized size = 1.9

$$-\frac{4a^2}{15d}\sqrt{\left(2\left(\cos(1/2dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-12\left(\sin(1/2dx+c/2)\right)^6\cos(1/2dx+c/2)+32\left(\sin(1/2dx+c/2)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-13*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec(dx+c)+a\right)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sec(d*x + c)^(5/2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{1}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx+\int\frac{2}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{1}{\sqrt{\sec\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] a**2*(Integral(sec(c + d*x)**(-5/2), x) + Integral(2/sec(c + d*x)**(3/2), x
) + Integral(1/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.178 $\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{12a^2}{}$$

```
[Out] (12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]
])
```

Rubi [A] time = 0.114728, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```


```
[Out] (12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]
])
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a + a\sec(c + dx))^2}{\sec^{\frac{7}{2}}(c + dx)}\,dx = (2a^2)\int \frac{1}{\sec^{\frac{5}{2}}(c + dx)}\,dx + \int \frac{a^2 + a^2\sec^2(c + dx)}{\sec^{\frac{7}{2}}(c + dx)}\,dx$$

$$= \frac{2a^2\sin(c + dx)}{7d\sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2\sin(c + dx)}{5d\sec^{\frac{3}{2}}(c + dx)} + \frac{1}{5}(6a^2)\int \frac{1}{\sqrt{\sec(c + dx)}}\,dx + \frac{1}{7}(12a^2)\int \frac{1}{\sec^{\frac{3}{2}}(c +$$

$$= \frac{2a^2\sin(c + dx)}{7d\sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2\sin(c + dx)}{5d\sec^{\frac{3}{2}}(c + dx)} + \frac{8a^2\sin(c + dx)}{7d\sqrt{\sec(c + dx)}} + \frac{1}{7}(4a^2)\int \sqrt{\sec(c + dx)}\,dx + \frac{1}{5}($$

$$= \frac{12a^2\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c + dx)}}{5d} + \frac{2a^2\sin(c + dx)}{7d\sec^{\frac{5}{2}}(c + dx)} + \frac{4a^2\sin(c + dx)}{5d\sec^{\frac{3}{2}}(c + dx)} + \frac{8}{7}$$

$$= \frac{12a^2\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(c + dx)}}{5d} + \frac{8a^2\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{\sec(}}{7d}$$

Mathematica [C] time = 1.78498, size = 149, normalized size = 0.93

$$\frac{a^2\left(\frac{672i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2\left(-80i\sqrt{1 + e^{2i(c+dx)}}\sec(c + dx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx)}\right) + 8\right.\right.}{140d\sqrt{\sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 85
*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)])))/(140*d*Sqrt[Sec
[c + d*x]])
```

Maple [A] time = 1.2, size = 272, normalized size = 1.7

$$-\frac{4a^2}{35d}\sqrt{\left(2\left(\cos(1/2\,dx + c/2)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}\left(40\cos(1/2\,dx + c/2)\left(\sin(1/2\,dx + c/2)\right)^8 - 116\left(\sin(1/2\,dx + c/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sec(d*x + c)^(7/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

3.179 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=187

$$\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+$$

```
[Out] (-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.189881, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3768, 3771, 2639, 2641}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)\sqrt{\sec(c+}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+a^3\sec^{\frac{9}{2}}(c+dx)\right)$$

$$= a^3\int \sec^{\frac{3}{2}}(c+dx)\,dx + a^3\int \sec^{\frac{9}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \left(3a\right.$$

$$= \frac{2a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d} + \frac{6a^3\sec^{\frac{5}{2}}(c+d}{5a}$$

$$= \frac{28a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{52a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{6a^3\sec^{\frac{5}{2}}(c+}{}$$

$$= -\frac{2a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$

$$= -\frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{21d}$$

Mathematica [C] time = 2.47255, size = 287, normalized size = 1.53

$$a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(\frac{294\csc(c)\cos(dx)+(63\cos(c+dx)+65\cos(2(c+dx))+80)\tan(c+dx)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+}}}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(147*(1 + E^((2*I)*(c + d
*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (294*Cos[d*x]*Csc
[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c + d
*x])/Sec[c + d*x]^(5/2)))/(420*d)
```

Maple [B] time = 2.418, size = 439, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-3/10*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-56/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos
(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+848/105*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-28/5*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/28*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-26/21*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3\sec(dx+c)^4+3\,a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+a^3\sec(dx+c)\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] integral((a^3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + a^3*sec(d*x + c))*sqrt(sec(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)

3.180 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=157

$$\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+$$

```
[Out] (-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.160961, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3771, 2641, 3768, 2639}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d}+\frac{36a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx = \int \left(a^3\sqrt{\sec(c+dx)} + 3a^3\sec^{\frac{3}{2}}(c+dx) + 3a^3\sec^{\frac{5}{2}}(c+dx) + a^3\sec^{\frac{7}{2}}(c+dx)\right)$$

$$= a^3\int \sqrt{\sec(c+dx)}\,dx + a^3\int \sec^{\frac{7}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \left(3\right.$$

$$= \frac{6a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{5}{2}}(c+d}{5}$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{36a^3\sqrt{\sec(c+dx)}\sin(c+d}{5d}$$

$$= -\frac{6a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$

$$= -\frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$

Mathematica [C] time = 2.16752, size = 267, normalized size = 1.7

$$\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(\frac{18\csc(c)\cos(dx)+\tan(c+dx)(\sec(c+dx)+5)}{\sec^{\frac{5}{2}}(c+dx)} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^3(c+dx)\left(9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c}}\right.}{}\right.}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^3*(9*(1 + E^((2*I)*(c + d*x
))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[
-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*
(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (18*Cos[d*x]*Csc[c] +
(5 + Sec[c + d*x])*Tan[c + d*x])/Sec[c + d*x]^(5/2)))/(20*d)
```

Maple [B] time = 2.288, size = 386, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(56/5*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/
10*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^3-72/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(
-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-36/5*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))-cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3 \sec (dx + c)^3 + 3\,a^3 \sec (dx + c)^2 + 3\,a^3 \sec (dx + c) + a^3\right)\sqrt{\sec (dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^3 \sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.181 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-$$

```
[Out] (-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.141646, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3771, 2639, 2641, 3768}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3\sqrt{\text{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx &= \int \left(\frac{a^3}{\sqrt{\sec(c+dx)}} + 3a^3\sqrt{\sec(c+dx)} + 3a^3\sec^{\frac{3}{2}}(c+dx) + a^3\sec^{\frac{5}{2}}(c+dx)\right)dx \\
&= a^3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^3\int \sec^{\frac{5}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sqrt{\sec(c+dx)}\,dx + \left(3a^3\right)\int \sec^{\frac{3}{2}}(\\
&= \frac{6a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}a^3\int \sqrt{\sec(c+dx)}\,dx - (\\
&= \frac{2a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d} \\
&= -\frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\se}}{3d}
\end{aligned}$$

Mathematica [C] time = 1.77406, size = 187, normalized size = 1.43

$$\frac{a^3e^{-2i(c+dx)}\sec^{\frac{3}{2}}(c+dx)(\sin(2(c+dx))-i\cos(2(c+dx)))\left(6e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*I)*(c + d*
x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/E^((2*I
)*(c + d*x)) + 20*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hypergeometric
2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x] + (9*I)*Sin[2
*(c + d*x)])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*d*E^((2*I)*(c +
 d*x)))
```

Maple [B] time = 2.267, size = 371, normalized size = 2.8

$$\frac{4a^3}{3d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(10\,\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 4/3*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(10*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2-18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3 \sec(dx+c)^3 + 3\,a^3 \sec(dx+c)^2 + 3\,a^3 \sec(dx+c) + a^3}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.182 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+}}{}$$

```
[Out] (4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.137819, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```


```
[Out] (4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/d
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int\left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+3a^3\sqrt{\sec(c+dx)}+a^3\sec^{\frac{3}{2}}(c+dx)\right)dx$$

$$= a^3\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a^3\int\sec^{\frac{3}{2}}(c+dx)\,dx + \left(3a^3\right)\int\frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(3a^3\right)\int\sqrt{\sec\ldots}$$

$$= \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{3}a^3\int\sqrt{\sec(c+dx)}\,dx - a^3\int\frac{1}{\sqrt{\sec(c\ldots}}$$

$$= \frac{6a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c\ldots}}{d}$$

$$= \frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(\ldots}}{3d}$$

Mathematica [C] time = 1.48802, size = 169, normalized size = 1.29

$$\frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{24i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-10i\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hype}\right.\right.}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((24*I)*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] +
 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2,
 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c + d*x] + 3*Tan[c + d*x])))
/(3*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.462, size = 172, normalized size = 1.3

$$-\frac{4a^3}{3d}\left(2\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+5\,\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{3}{\sqrt{\sec(c+dx)}}\,dx+\int 3\sqrt{\sec(c+dx)}\,dx+\int\sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**(-3/2), x) + Integral(3/sqrt(sec(c + d*x)), x)
 + Integral(3*sqrt(sec(c + d*x)), x) + Integral(sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.183 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}...}{5d...}$$

```
[Out] (36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])
/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.139551, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2639, 2641}

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E...}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```



```
[Out] (36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])
/(d*Sqrt[Sec[c + d*x]])
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+a^3\sqrt{\sec(c+dx)}\right)dx$$

$$= a^3\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a^3\int \sqrt{\sec(c+dx)}\,dx + \left(3a^3\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right)\int \frac{1}{\sqrt{\sec(}}$$

$$= \frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{5}\left(3a^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^3\int \sqrt{\sec(c+dx)}\,dx +$$

$$= \frac{6a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{d}$$

$$= \frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

Mathematica [C] time = 1.44622, size = 171, normalized size = 1.31

$$\frac{a^3\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{144i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-20i\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hype}\right.\right.}{10d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^3*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(((144*I)*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))]
+ 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/
2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 10*Sin[c + d*x] + Sin[2*(c + d
*x)])))/(10*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.403, size = 250, normalized size = 1.9

$$-\frac{4\,a^3}{5\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+14\,\left(\sin\left(1/2\,dx+c/2\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int\frac{1}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx+\int\frac{3}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{3}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\sqrt{\sec\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] a**3*(Integral(sec(c + d*x)**(-5/2), x) + Integral(3/sec(c + d*x)**(3/2), x
) + Integral(3/sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

3.184 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{28\ldots}{}$$

```
[Out] (28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
d*x]])
```

Rubi [A] time = 0.167663, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2641, 2639}

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3\sqrt{\cos(\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```


```
[Out] (28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
d*x]])
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)} + \frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$= a^3\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a^3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(3a^3\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(3a^3\right)\int \frac{}{\sec^{\frac{3}{2}}(}$$

$$= \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^3\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a^3\int$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{52a}{21d}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{21d}$$

Mathematica [C] time = 1.90018, size = 146, normalized size = 0.91

$$\frac{a^3\left(\frac{4704i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-1040i\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx)}\right)+10\right.}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1
070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt
[Sec[c + d*x]])
```

Maple [A] time = 1.321, size = 272, normalized size = 1.7

$$-\frac{4a^3}{105d}\sqrt{\left(2\left(\cos(1/2dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\cos(1/2dx+c/2)\left(\sin(1/2dx+c/2)\right)^8-432\left(\sin(1/2dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+65*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3 \sec \left(dx+c\right)^3+3\,a^3 \sec \left(dx+c\right)^2+3\,a^3 \sec \left(dx+c\right)+a^3}{\sec \left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sec \left(dx+c\right)+a\right)^3}{\sec \left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

3.185 $\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3}{21d}$$

```
[Out] (68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
5*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]
^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.194308, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2639, 2641}

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```


```
[Out] (68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
5*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]
^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}\right)dx$$

$$= a^3\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(3a^3\right)\int \frac{}{\sec^{\frac{5}{2}}}$$

$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a^3\int \sqrt{\sec(c}$$

$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{15}\left(7a^3\right)\int$$

$$= \frac{18a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3d}$$

$$= \frac{68a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{44a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{21d}$$

Mathematica [C] time = 2.27767, size = 156, normalized size = 0.83

$$\frac{a^3\left(\frac{22848i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-5280i\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx)}\right)+58\right.}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*Sin[
4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.362, size = 260, normalized size = 1.4

$$-\frac{4a^3}{315d}\sqrt{\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(560(\cos(1/2dx+c/2))^{11}-600(\cos(1/2dx+c/2))^9+212(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3 \sec(dx+c)^3 + 3\,a^3 \sec(dx+c)^2 + 3\,a^3 \sec(dx+c) + a^3}{\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

3.186 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=213

$$\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{2a^4\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

```
[Out] (-152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(7*d) + (152*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (32*a^4*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(7*d) + (122*a^4*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(45*d) + (8*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + (2*a^4*Sec[
c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.251494, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3768, 3771, 2639, 2641}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{122a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{32a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```


```
[Out] (-152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(7*d) + (152*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (32*a^4*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(7*d) + (122*a^4*Sec[c + d*x]^(5/2)*Sin[c +
 d*x])/(45*d) + (8*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d) + (2*a^4*Sec[
c + d*x]^(9/2)*Sin[c + d*x])/(9*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^4\,dx = \int \Big(a^4\sec^{\frac{3}{2}}(c+dx)+4a^4\sec^{\frac{5}{2}}(c+dx)+6a^4\sec^{\frac{7}{2}}(c+dx)+4a^4\sec^{\frac{9}{2}}(c+dx)+$$

$$= a^4\int \sec^{\frac{3}{2}}(c+dx)\,dx + a^4\int \sec^{\frac{11}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \left(4a^4\right.$$

$$= \frac{2a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{12a^4\sec^{\frac{5}{2}}(c+dx}{5d}$$

$$= \frac{46a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{32a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{122a^4\sec^{\frac{5}{2}}(c}{4}$$

$$= -\frac{2a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{3d}$$

$$= -\frac{46a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{7d}$$

$$= -\frac{152a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{7d}$$

Mathematica [C] time = 3.86102, size = 289, normalized size = 1.36

$$a^4 \sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4 \left(\frac{1596\csc(c)\cos(dx)+\tan(c+dx)\left(35\sec^3(c+dx)+180\sec^2(c+dx)+427\sec(c+dx)+720\right)}{\sec^{\frac{7}{2}}(c+dx)} - \frac{12i\sqrt{2}e^{-i(c+dx)}\sqrt{\cdots}}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-12*I)*Sqrt[2]*Sqrt[E^(I*(c
 + d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(133*(1 + E^((2*I)*(c +
d*x))) + 133*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 60*E^(I*(c + d*x))*(-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^(
(2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (1596*Cos[d*x]*C
sc[c] + (720 + 427*Sec[c + d*x] + 180*Sec[c + d*x]^2 + 35*Sec[c + d*x]^3)*T
an[c + d*x])/Sec[c + d*x]^(7/2)))/(2520*d)
```

Maple [B] time = 2.649, size = 492, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x)
```

```
[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/72*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^5-61/90*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-304/15*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2
)^(1/2)+1544/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-152/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/7
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^4-16/7*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*
```

```
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4 \sec(dx+c)^5 + 4\,a^4 \sec(dx+c)^4 + 6\,a^4 \sec(dx+c)^3 + 4\,a^4 \sec(dx+c)^2 + a^4 \sec(dx+c)\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^5 + 4*a^4*sec(d*x + c)^4 + 6*a^4*sec(d*x + c)^3
+ 4*a^4*sec(d*x + c)^2 + a^4*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec(dx+c) + a)^4 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

3.187 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx$

Optimal. Leaf size=187

$$\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

```
[Out] (-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.209401, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3771, 2641, 3768, 2639}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{\sec(c+}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^4\,dx = \int \left(a^4\sqrt{\sec(c+dx)} + 4a^4\sec^{\frac{3}{2}}(c+dx) + 6a^4\sec^{\frac{5}{2}}(c+dx) + 4a^4\sec^{\frac{7}{2}}(c+dx)\right.$$

$$= a^4\int \sqrt{\sec(c+dx)}\,dx + a^4\int \sec^{\frac{9}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx + (4$$

$$= \frac{8a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{4a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{5}{2}}(c+d}{5}$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{64a^4\sqrt{\sec(c+dx)}\sin(c+d}{5d}$$

$$= -\frac{8a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$

$$= -\frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{21}$$

Mathematica [C] time = 2.7929, size = 279, normalized size = 1.49

$$a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\frac{672\csc(c)\cos(dx)+\tan(c+dx)\left(15\sec^2(c+dx)+84\sec(c+dx)+235\right)}{\sec^{\frac{7}{2}}(c+dx)} - \frac{4i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos^4(c+d}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(168*(1 + E^((2*I)*(c + d
*x))) + 168*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 85*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (672*Cos[d*x]*Csc
[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x])/Sec[c + d*x
]^(7/2)))/(840*d)
```

Maple [B] time = 2.765, size = 439, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x)
```

```
[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2024/105*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-2/5*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-128/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-64/5*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/28*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-47/21*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4\sec\left(dx+c\right)^4+4\,a^4\sec\left(dx+c\right)^3+6\,a^4\sec\left(dx+c\right)^2+4\,a^4\sec\left(dx+c\right)+a^4\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(dx+c\right)+a\right)^4\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.188 $\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+$$

```
[Out] (-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d)
```

Rubi [A] time = 0.181752, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3771, 2639, 2641, 3768}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \left(\frac{a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)} + 6a^4\sec^{\frac{3}{2}}(c+dx) + 4a^4\sec^{\frac{5}{2}}(c+dx) + a^4\sec^{\frac{7}{2}}(c+\right.$$

$$= a^4\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^4\int \sec^{\frac{7}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sqrt{\sec(c+dx)}\,dx + \left(4a^4\right)\int \sec^{\frac{5}{2}}($$

$$= \frac{12a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^4\sec^{\frac{5}{2}}(c+dx)\sin(c+d}{5d}$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{d}$$

$$= -\frac{10a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{se}{3d}$$

$$= -\frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{se}{3d}$$

Mathematica [C] time = 2.98429, size = 286, normalized size = 1.78

$$a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(\frac{30\cos(c)\sin(dx)-3(5\cos(2c)-61)\csc(c)\cos(dx)+2\tan(c+dx)(3\sec(c+dx)+20)}{\sec^{\frac{7}{2}}(c+dx)}-\frac{8i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*Cos[c + d*x]^4*(21*(1 + E^((2*I)*(c + d*
x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 20*E^(I*(c + d*x))*(-1 + E^((2*I)
*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*
I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + (-3*(-61 + 5*Cos[2*
c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x])*Tan[c +
d*x])/Sec[c + d*x]^(7/2)))/(240*d)
```

Maple [B] time = 2.393, size = 386, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x)
```

```
[Out] -a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-56/5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+328/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/10*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1
/2)^3-132/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c
)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-4/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4\sec(dx+c)^4+4\,a^4\sec(dx+c)^3+6\,a^4\sec(dx+c)^2+4\,a^4\sec(dx+c)+a^4}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.189 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{40a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.169367, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{40a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```


```
[Out] (40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\sec(c+dx)}} + 6a^4\sqrt{\sec(c+dx)} + 4a^4\sec^{\frac{3}{2}}(c+dx) + a^4\sec^{\frac{5}{2}}(c+dx)\right)$$

$$= a^4\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a^4\int \sec^{\frac{5}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(4a^4\right)\int \sec^{\frac{3}{2}}(c$$

$$= \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{8a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + 2\left(\frac{1}{3}a^4\int\right.$$

$$= \frac{8a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{12a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

$$= \frac{40a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{8a^4\sqrt{\sec(c+dx)}\sin(c}{d}$$

Mathematica [A] time = 0.301091, size = 70, normalized size = 0.59

$$\frac{a^4 \sec^{\frac{3}{2}}(c+dx) \left(80 \cos^{\frac{3}{2}}(c+dx) \text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5 \sin(c+dx)+24 \sin(2(c+dx))+\sin(3(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d)
```

Maple [B] time = 2.078, size = 292, normalized size = 2.5

$$\frac{8\,a^4}{3\,d}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+10\,\text{EllipticF}\left(\cos\left(1/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x)
```

```
[Out] 8/3*a^4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+7
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4 \sec(dx+c)^4 + 4\,a^4 \sec(dx+c)^3 + 6\,a^4 \sec(dx+c)^2 + 4\,a^4 \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a \sec(dx+c) + a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

3.190 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)} dx$

Optimal. Leaf size=159

$$\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] (56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.171876, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3791, 3769, 3771, 2639, 2641, 3768}

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```


```
[Out] (56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{6a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)} + a^4\sec^{\frac{3}{2}}(c+dx)\right) dx$$

$$= a^4\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a^4\int \sec^{\frac{3}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(4a^4\right)\int \sqrt{\sec(c$$

$$= \frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{5}\left(3a^4\right)\int \frac{1}{\sqrt{\sec(c+d}}$$

$$= \frac{12a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

$$= \frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3d}$$

Mathematica [C] time = 1.63219, size = 184, normalized size = 1.16

$$\frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{672i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-320i\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hyper}\right.}{30d\sqrt{\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-336*I + ((672*I)*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c
+ d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2
, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 80*Sin[c + d*x] + 3*Sec[c + d*x
]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.669, size = 194, normalized size = 1.2

$$\frac{8\,a^4}{15\,d}\left(6\,\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)-26\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+21\,\text{EllipticE}\left(\cos\left(1/2\,dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x)
```

```
[Out] 8/15*a^4*(6*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-26*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-20*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
+19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4\sec\left(dx+c\right)^4+4\,a^4\sec\left(dx+c\right)^3+6\,a^4\sec\left(dx+c\right)^2+4\,a^4\sec\left(dx+c\right)+a^4}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int\frac{1}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx+\int\frac{4}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{6}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int4\sqrt{\sec\left(c+dx\right)}\,dx+\int\sec^{\frac{3}{2}}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

```
[Out] a**4*(Integral(sec(c + d*x)**(-5/2), x) + Integral(4/sec(c + d*x)**(3/2), x
) + Integral(6/sqrt(sec(c + d*x)), x) + Integral(4*sqrt(sec(c + d*x)), x) +
 Integral(sec(c + d*x)**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^4}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

3.191 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{64a}{}$$

```
[Out] (64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]])
```

Rubi [A] time = 0.187004, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2641, 2639}

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```


```
[Out] (64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]])
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \int\left(\frac{a^4}{\sec^{\frac{7}{2}}(c+dx)}+\frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)}+\frac{6a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}+a^4\sqrt{\sec(c+dx)}\right)dx$$

$$= a^4\int\frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a^4\int\sqrt{\sec(c+dx)}\,dx + \left(4a^4\right)\int\frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(4a^4\right)\int\frac{}{\sqrt{\sec}}$$

$$= \frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^4\right)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(2a^4\right.$$

$$= \frac{8a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

$$= \frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{6a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{d}$$

$$= \frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{21d}$$

Mathematica [C] time = 1.69522, size = 180, normalized size = 1.12

$$\frac{a^4\left(\cos\left(\frac{c}{2}\right)-i\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\left(\frac{10752i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-2720i\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hyp}\right.}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^4*(Cos[c/2] - I*Sin[c/2])*(Cos[c/2] + I*Sin[c/2])*(-5376*I + ((10752*I)*
Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*
(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4,
 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1910*Sin[c + d*x] + 336*Sin
[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.352, size = 272, normalized size = 1.7

$$-\frac{8a^4}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(60\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8-258\left(\sin\left(1/2\,dx+c/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x)
```

```
[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/
2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)-168*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4 \sec(dx+c)^4 + 4a^4 \sec(dx+c)^3 + 6a^4 \sec(dx+c)^2 + 4a^4 \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

3.192 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32}{7}$$

```
[Out] (152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x
]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.221984, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2639, 2641}

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```


```
[Out] (152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x
]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{9}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{a^4}{\sqrt{\sec(c+dx)}}\right) dx$$

$$= a^4 \int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^4 \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(4a^4\right) \int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(4a^4\right) \int \frac{}{\sec^{\frac{3}{2}}(}$$

$$= \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{12a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{9}\left(7a^4\right) \int \frac{}{\sec^{\frac{5}{2}}}$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{122a}{45d}$$

$$= \frac{46a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{3d}$$

$$= \frac{152a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{7d}$$

Mathematica [C] time = 2.25967, size = 156, normalized size = 0.83

$$\frac{a^4\left(\frac{51072i\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-11520i\sqrt{1+e^{2i(c+dx)}}\sec(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-e^{2i(c+dx)}\right)+\right.}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
+ 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 70*Si
n[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.535, size = 260, normalized size = 1.4

$$-\frac{8a^4}{315d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(280\left(\cos\left(1/2\,dx+c/2\right)\right)^{11}-120\left(\cos\left(1/2\,dx+c/2\right)\right)^9+34\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x)
```

```
[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos
(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(
1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4 \sec(dx+c)^4 + 4\,a^4 \sec(dx+c)^3 + 6\,a^4 \sec(dx+c)^2 + 4\,a^4 \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

3.193 $\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{904a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+$$

```
[Out] (128*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (904*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(231*d) + (2*a^4*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (8*a^4*Si
n[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (150*a^4*Sin[c + d*x])/(77*d*Sec[c +
 d*x]^(5/2)) + (128*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (904*a^4*
Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.258238, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3791, 3769, 3771, 2641, 2639}

$$\frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)}+\frac{904a^4\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{904a^4\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```


```
[Out] (128*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (904*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(231*d) + (2*a^4*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2)) + (8*a^4*Si
n[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (150*a^4*Sin[c + d*x])/(77*d*Sec[c +
 d*x]^(5/2)) + (128*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (904*a^4*
Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]])
```


Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\sec^{\frac{11}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{9}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}\right) dx$$

$$= a^4 \int \frac{1}{\sec^{\frac{11}{2}}(c+dx)}\,dx + a^4 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(4a^4\right) \int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \left(4a^4\right) \int \frac{}{\sec^{\frac{5}{2}}(}$$

$$= \frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{12a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^4\sin(c+dx}{3d\sqrt{\sec(c+dx}}$$

$$= \frac{2a^4\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{150a^4\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{128a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{74a^4\sin(c}{21d\sqrt{\sec(c}}$$

$$= \frac{24a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{3d}$$

$$= \frac{128a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{74a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{21d}$$

$$= \frac{128a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{904a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{se}}{231d}$$

Mathematica [C] time = 3.2189, size = 306, normalized size = 1.44

$$-\frac{ia^4e^{-6i(c+dx)}\sec^8\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^4\left(-946176e^{5i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] ((-I/1774080)*a^4*(-315 - 3080*E^(I*(c + d*x)) - 14760*E^((2*I)*(c + d*x))
- 48664*E^((3*I)*(c + d*x)) - 137055*E^((4*I)*(c + d*x)) + 427504*E^((5*I)*
(c + d*x)) + 518672*E^((7*I)*(c + d*x)) + 137055*E^((8*I)*(c + d*x)) + 4866
4*E^((9*I)*(c + d*x)) + 14760*E^((10*I)*(c + d*x)) + 3080*E^((11*I)*(c + d*
x)) + 315*E^((12*I)*(c + d*x)) - 946176*E^((5*I)*(c + d*x))*Sqrt[1 + E^((2*
I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 43
3920*E^((6*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/
4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4
)/(d*E^((6*I)*(c + d*x))*Sec[c + d*x]^(7/2))
```

Maple [A] time = 1.388, size = 273, normalized size = 1.3

$$-\frac{8a^4}{3465d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5040\left(\cos\left(1/2\,dx+c/2\right)\right)^{13}-5320\left(\cos\left(1/2\,dx+c/2\right)\right)^{11}+174\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x)
```

```
[Out] -8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*c
os(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9+3
26*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)^3+
1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*cos(1/2*
d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4\sec\left(dx+c\right)^4+4\,a^4\sec\left(dx+c\right)^3+6\,a^4\sec\left(dx+c\right)^2+4\,a^4\sec\left(dx+c\right)+a^4}{\sec\left(dx+c\right)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(d*x + c)^4 + 4*a^4*sec(d*x + c)^3 + 6*a^4*sec(d*x + c)^2
+ 4*a^4*sec(d*x + c) + a^4)/sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

3.194 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=164

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+$$

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d
) - (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (5*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*a*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
))
```

Rubi [A] time = 0.126484, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3818, 3787, 3768, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d
) - (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (5*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*a*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
))
```


Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{5}{2}a\sec(c+dx)\right)dx}{a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{5\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a}$$

$$= -\frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{5\int \sqrt{}}{}$$

$$= -\frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(5\sqrt{c\right.}}{}$$

$$= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad}$$

Mathematica [C] time = 3.32114, size = 291, normalized size = 1.77

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\sqrt{\sec(c+dx)}\left(18\csc(c)\cos(dx)+\sec(c+dx)\left(\tan\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+dx)\right)\sec\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
 + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x
))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec[c + d
*x]*(-5*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2]))))/(3*a*d
*(1 + Sec[c + d*x]))
```

Maple [B] time = 2.526, size = 413, normalized size = 2.5

$$\frac{1}{3\,ad}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(10\sqrt{2}\left(\sin(1/2\,d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*x
+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)+9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d
*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{7}{2}}}{a \sec (dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (dx+c)^{\frac{7}{2}}}{a \sec (dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

3.195 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{3\sqrt{\cos(c}{}$$

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.112525, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3818, 3787, 3771, 2641, 3768, 2639}

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3\sqrt{\cos(c+dx)}\sqrt{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x]))
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{3}{2}a\sec(c+dx)\right)dx}{a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a}$$

$$= \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s\right.}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin}{d(a+a\sec(c}$$

$$= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}$$

Mathematica [C] time = 2.00561, size = 262, normalized size = 1.93

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(\frac{\sqrt{\sec(c+dx)}\left(6\csc(c)\cos(dx)-2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2}\right.}{}\right.}{a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(
c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*
x))*(-1 + E^((2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*Tan[(c
 + d*x)/2]))/d))/(a*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.535, size = 253, normalized size = 1.9

$$-\frac{1}{ad}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\left(\text{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/cos
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

3.196 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{ad}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.102041, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3818, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx &= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \frac{-\frac{a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}\\ &= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a}+\frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}\\ &= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.}}{}\\ &= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{}}{}\end{aligned}$$

Mathematica [C] time = 24.8612, size = 201, normalized size = 1.83

$$-\frac{2ie^{-i(c+dx)}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(-\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+}\right.}{ad\left(1+e^{i(c+dx)}\right)(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((-2*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x)))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*
```

```
(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]
^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.449, size = 200, normalized size = 1.8

$$\frac{1}{ad}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

3.197 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{ad}$$

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d))
+ (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.100047, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3820, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```


```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d))
+ (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{2a^2}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}\\ &= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \cdots\end{aligned}$$

Mathematica [C] time = 6.267, size = 202, normalized size = 1.84

$$-\frac{2ie^{-i(c+dx)}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{i(c+dx)}\cdots\right.}{ad\left(1+e^{i(c+dx)}\right)(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((-2*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x))
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)
```

```
*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x
]^(3/2))/(a*d*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.377, size = 198, normalized size = 1.8

$$-\frac{1}{ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\mathrm{Elli}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.198 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+$$

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.101886, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3819, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx &= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{(3\sqrt{}
\\
&= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{ad}
\end{aligned}$$

Mathematica [C] time = 1.79868, size = 317, normalized size = 2.83

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+2\cos\left(\frac{1}{2}(c+3dx)\right)+\cos\left(\frac{1}{2}(5c+3dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{2d} + \frac{2i}{}\right.}{a(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
```

```
*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E
^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*d*x)/2
] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d
*x]])/(2*d))*Sec[c + d*x])/(a*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.322, size = 199, normalized size = 1.8

$$\frac{1}{ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.\left(\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/
2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*s
in(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{2}+a\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*sec(d*x + c)^2 + a*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.199 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=140

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}-\frac{3\sqrt{\cos(c}$$

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
+ (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*
d) + (5*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Sec
[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.116845, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3819, 3787, 3769, 3771, 2641, 2639}

$$\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
+ (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*
d) + (5*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Sec
[c + d*x]]*(a + a*Sec[c + d*x]))
```


Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{5\int \sqrt{\sec(c+dx)}\,dx}{6a} - \cdots$$

$$= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\cdots}{d\sqrt{\sec(c+\cdots}}$$

$$= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\cdots}{3ad}$$

Mathematica [C] time = 4.3001, size = 318, normalized size = 2.27

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2\sqrt{\sec(c+dx)}\left(\sin(2c)\cos(2dx)-6\cos(c)\sin(dx)+\cos(2c)\sin(2dx)+3(\cos(2c)+2)\mathrm{cs}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(
c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*
x))*(-1 + E^((2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[d*x]*C
sc[c] + Cos[2*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 6*
Cos[c]*Sin[d*x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2])))/(3*a*d*(1 + Sec[c + d
*x]))
```

Maple [A] time = 1.475, size = 215, normalized size = 1.5

$$-\frac{1}{3ad}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5\,\mathrm{E}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/3/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{3}+a\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*sec(d*x + c)^3 + a*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^{\frac{5}{2}}\left(c+dx\right)+\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.200 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=168

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] (21*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a*d) + (7*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d
*x]))
```

Rubi [A] time = 0.129467, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3819, 3787, 3769, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (21*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a*d) + (7*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d
*x]))
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}dx = -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx}{2a} + \frac{7\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx}{2a}$$

$$= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{5\int$$

$$= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{(5\sqrt{}}{}$$

$$= \frac{21\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}$$

Mathematica [C] time = 2.77552, size = 347, normalized size = 2.07

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-\sqrt{\sec(c+dx)}\left(18(11\cos(2c)+17)\csc(c)\cos(dx)+4\left(10\sin(2c)\cos(2dx)-3\sin(3c)\cos(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 +
E^((2*I)*(c + d*x)))]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*
(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c +
d*x))*(-1 + E^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])*Cos[d
*x]*Csc[c] + 4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*Sec[c/2
]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*Sin[2*d*
x] - 3*Cos[3*c]*Sin[3*d*x] - 30*Tan[c/2]))))/(60*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.399, size = 229, normalized size = 1.4

$$-\frac{1}{15\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/15/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d
*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)
^4+23*sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a\sec\left(dx+c\right)^{4}+a\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*sec(d*x + c)^4 + a*sec(d*x + c)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+\sec^{\frac{5}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.201 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=202

$$\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin\ldots}{}$$

```
[Out] (7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) - (7*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (10*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a^2*d) - (7*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(
1 + Sec[c + d*x])) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```

Rubi [A] time = 0.229149, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\ldots}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) - (7*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (10*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a^2*d) - (7*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(
1 + Sec[c + d*x])) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}dx}{3a^2}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{21a^2}{2}-15a^2\sec(\right.}{3a^4}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{7\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a^2} + \frac{5\int \sec^{\frac{5}{2}}(}{a}$$

$$= -\frac{7\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} -$$

$$= -\frac{7\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} -$$

$$= \frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{3a^2d}$$

Mathematica [C] time = 3.69732, size = 287, normalized size = 1.42

$$-\frac{\left(-1+e^{ic}\right)\csc\left(\frac{c}{2}\right)e^{-\frac{1}{2}i(4c+3dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(7e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(1+e^{i(c+dx)}\right)^3\text{Hypergeometric2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x)) - 65*E
^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x)) - 53*E
^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c + d*x)))^
3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)))^(3/2)*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^(5/2))/
(12*a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + Sec[c + d*
x])^2)
```

Maple [A] time = 2.365, size = 413, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(1/3*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)-22/3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1
/2)^2+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2
+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(9/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^2, x)
```

3.202 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{4\sqrt{\cos}}{}$$

```
[Out] (-4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - (5*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.213548, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - (5*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}dx}{3a^2}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{5a^2}{2}-6a^2\sec(c+\right.}{3a^4}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int\sqrt{\sec(c+dx)}\,dx}{6a^2} + \frac{2\int \sec^{\frac{3}{2}}(c}{a^2}$$

$$= \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{2\int \frac{}{\sqrt{s}}}{}$$

$$= -\frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx}{3a^2d(1+\mathrm{se}}$$

$$= -\frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{3a^2d}$$

Mathematica [C] time = 1.35322, size = 252, normalized size = 1.43

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-4ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\mathrm{Hype}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-4*I)*(1 + E^(I*(c + d*x)))^3*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 + 50*Cos[c
 + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*(c + d*x)
]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Se
c[c + d*x])^2)
```

Maple [A] time = 1.584, size = 405, normalized size = 2.3

$$-\frac{1}{6a^2d}\left(2\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin(1/2\,dx+c/2)\right)^2}\left(5\,\mathrm{EllipticF}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(7/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

3.203 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+a\right.}{a^2d}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) +
 (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2
*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.198649, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3816, 4019, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) +
 (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2
*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{5}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}}{2a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{}}}{3a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}$$

Mathematica [C] time = 1.24231, size = 242, normalized size = 1.62

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3 \text{Hyper}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c + d
*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin
[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 1.394, size = 257, normalized size = 1.7

$$\frac{1}{6\,a^2d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^6-4\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.204 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.0609247, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3815, 21, 3771, 2641}

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}\\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \sqrt{\sec(c+dx)}\,dx}{6a^2}\\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{6a^2}\\
&= \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 0.348232, size = 98, normalized size = 1.27

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(4\sqrt{\cos(c+dx)}\cos^3\left(\frac{1}{2}(c+dx)\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx\right.\right.}{3a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3
*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [B] time = 1.546, size = 188, normalized size = 2.4

$$-\frac{1}{6a^2d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.205 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{a^2d}$$

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.201147, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3817, 4019, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{5a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}dx}{3a^2}$$

$$= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{3a^4}$$

$$= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2a^2}$$

$$= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(\ldots}}}{3a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}$$

Mathematica [C] time = 1.42233, size = 239, normalized size = 1.6

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\text{Hyperg}\ldots\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(
-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c
+ 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 1.651, size = 257, normalized size = 1.7

$$-\frac{1}{6a^2d}\sqrt{\left(2(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12(\cos(1/2\,dx+c/2))^6+4\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2(\cos(1/2\,\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/
2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x
+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.206 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{a^2d}$$

```
[Out] (4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a
^2*d) - (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.202154, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a
^2*d) - (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx = -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{7a}{2}+\frac{3}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}dx}{3a^2}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-6a^2+\frac{5}{2}a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}}{3a^4}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int \sqrt{\sec(c+dx)}\,d}{6a^2}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{s}\right.}{}$$

$$= \frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3a^2d}$$

Mathematica [C] time = 0.975865, size = 260, normalized size = 1.71

$$-\frac{i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(4e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-5ie^{i(c+dx)}\left(1+\right.\right.}{3a^2d\left(1+e^{i\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((-I/3)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-3 - 16*E^
(I*(c + d*x)) - 23*E^((2*I)*(c + d*x)) - 25*E^((3*I)*(c + d*x)) - 20*E^((4*
I)*(c + d*x)) - 9*E^((5*I)*(c + d*x)) - (5*I)*E^(I*(c + d*x))*(1 + E^(I*(c
+ d*x)))^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 4*E^((2*I)*(c + d
*x))*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F
1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(a^2*d*E^(I*(c + d*x))*(1 + E^(I*(
c + d*x)))^3)
```

Maple [A] time = 1.584, size = 257, normalized size = 1.7

$$\frac{1}{6a^2d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\left(\cos(1/2\,dx+c/2)\right)^6+10\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2
*d*x+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d
*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^2\sec(dx+c)^3+2\,a^2\sec(dx+c)^2+a^2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^3 + 2*a^2*sec(d*x + c)^2 + a^
2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)+2\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x)
)), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.207 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=178

$$\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}-\frac{7\sqrt{\text{co}}}{}$$

```
[Out] (-7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (7*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.224789, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (-7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (7*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{9a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \cdots$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \cdots$$

$$= \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+\cdots}{3d\sqrt{\sec(c+dx)}(a+\cdots}$$

$$= -\frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{\cdots}{3a^2d\sqrt{\sec\cdots}}$$

$$= -\frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{3a^2d}$$

Mathematica [C] time = 1.7424, size = 257, normalized size = 1.44

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(7ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\text{Hypergeometric2F1}\left(\frac{1}{2},\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-84*I)*Cos[(
c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] + 8
0*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*I)*
(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*x)/2]
 + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]
))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 1.421, size = 270, normalized size = 1.5

$$-\frac{1}{6\,a^{2}d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(16\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{8}+12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{6}+20\,\sqrt{\left(\sin\left(1/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/
2*d*x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2
*d*x+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*
d*x+1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{4}+2\,a^{2}\sec\left(dx+c\right)^{3}+a^{2}\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^4 + 2*a^2*sec(d*x + c)^3 + a^
2*sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\frac{1}{\sec^{\frac{7}{2}}(c+dx)+2\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}}{a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(5/2) + sec(c + d*x)**(3/
2)), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.208 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\,\text{s}}{a^2d}$$

```
[Out] (56*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^2
*d) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (56*Sin[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])
/(a^2*d*Sqrt[Sec[c + d*x]]) - (3*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^2)
```

Rubi [A] time = 0.239756, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (56*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^2
*d) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (56*Sin[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])
/(a^2*d*Sqrt[Sec[c + d*x]]) - (3*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^2)
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
```

```
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{11a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \cdots$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \cdots$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \cdots$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \cdots$$

$$= \frac{56\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\cdots\right.}{a^2d}$$

Mathematica [C] time = 1.83598, size = 271, normalized size = 1.36

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(-112ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\text{Hypergeometric2F}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((1344*I)*Cos[
(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/2]
 - 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (
(112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) - 34*Sin[(c
+ d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*Sin[(7
*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + Sec[c +
d*x])^2)
```

Maple [A] time = 1.63, size = 283, normalized size = 1.4

$$-\frac{1}{30\,a^2d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(96\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{10}-352\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+120\,\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1
/2*d*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\sec\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^5+2\,a^2\sec\left(dx+c\right)^4+a^2\sec\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*sec(d*x + c)^5 + 2*a^2*sec(d*x + c)^4 + a^
2*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec(dx + c) + a)^2 \sec(dx + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.209 $\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=247

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}-\frac{119\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2a^3d}-\frac{119}{}$$

```
[Out] (119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(2*a^3*d) - (119*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + (11*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(2*a^3*d) - (Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5
*d*(a + a*Sec[c + d*x])^3) - (2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a
+ a*Sec[c + d*x])^2) - (119*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a
^3*Sec[c + d*x]))
```

Rubi [A] time = 0.35645, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{119\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(2*a^3*d) - (119*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) + (11*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(2*a^3*d) - (Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5
*d*(a + a*Sec[c + d*x])^3) - (2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a
+ a*Sec[c + d*x])^2) - (119*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a
^3*Sec[c + d*x]))
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
```

```
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(\frac{7a}{2}-\frac{13}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(25a^2-\frac{69}{2}a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{30d\left(a^3+a^3\sec(c+dx)\right)} - \frac{\int \text{s}}{}$$

$$= -\frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{30d\left(a^3+a^3\sec(c+dx)\right)} - \frac{11}{}$$

$$= -\frac{119\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d} - \frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2}{}$$

$$= -\frac{119\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a^3d} - \frac{\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2}{}$$

$$= \frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{2a^3d}$$

Mathematica [C] time = 4.70638, size = 378, normalized size = 1.53

$$\csc\left(\frac{c}{2}\right)e^{-idx}\left(\frac{\left(-1+e^{ic}\right)e^{-\frac{3}{2}i(2c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(-165i\left(1+e^{2i(c+dx)}\right)\left(1+e^{i(c+dx)}\right)^5\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+944e^{i(c+dx)}+2476e^{2i(c+dx)}+4}{16\left(1+e^{2i(c+dx)}\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(11/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c/2]*(-119*Sqrt[2]*E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x
))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2
]^6*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]*Sec[c/2]*Sec[c +
 d*x]^3 + ((-1 + E^(I*c))*Cos[(c + d*x)/2]*(165 + 944*E^(I*(c + d*x)) + 247
6*E^((2*I)*(c + d*x)) + 4148*E^((3*I)*(c + d*x)) + 5134*E^((4*I)*(c + d*x))
```

```
 + 4664*E^((5*I)*(c + d*x)) + 3340*E^((6*I)*(c + d*x)) + 1620*E^((7*I)*(c +
 d*x)) + 357*E^((8*I)*(c + d*x)) - (165*I)*(1 + E^(I*(c + d*x)))^5*(1 + E^(
(2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sec[c + d*x
]^(7/2))/(16*E^(((3*I)/2)*(2*c + d*x))*(1 + E^((2*I)*(c + d*x))))))/(15*a^3
*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 3.016, size = 453, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(32/15*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+11
8/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4/3*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)^5+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2
*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{11}{2}}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(11/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3
*a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{11}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(11/2)/(a*sec(d*x + c) + a)^3, x)
```

3.210 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-$$

```
[Out] (-49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(6*a^3*d) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) - (Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.33954, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(6*a^3*d) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) - (Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
```

```
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{11}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(12a^2-\frac{41}{2}a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} - \frac{\int}{}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} - 13$$

$$= \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13}{6}$$

$$= -\frac{13\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+}{5d(a+a}$$

$$= -\frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{6a^3d}$$

Mathematica [C] time = 2.32517, size = 371, normalized size = 1.68

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}(c+3dx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 1243
*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2] +
```

```
 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[
(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c
 + d*x])^3)
```

Maple [B] time = 1.579, size = 555, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(9/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{9}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^3, x)
```

3.211 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{10a^3d}$$

```
[Out] (9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3
*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*
a^3*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (9*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.324677, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3816, 4019, 3787, 3771, 2639, 2641}

$$-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3
*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*
a^3*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (9*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(3a^2-\frac{21}{2}a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)} - \frac{\int -\frac{2}{}}{}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)} + \frac{\int \sqrt{}}{}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)} + \frac{\left(\sqrt{c}\right.}{}$$

$$= \frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{2a^3d}$$

Mathematica [C] time = 4.69128, size = 274, normalized size = 1.41

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-3ie^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5\text{Hyper}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(((-3*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*(34 +
69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin[c +
d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2
] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.5, size = 268, normalized size = 1.4

$$\frac{1}{20\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\Bigg(36\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8-10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos(1/\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(7/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3, x)
```

3.212 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{10a^3d}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d
) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^
3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (4*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.320014, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3816, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d
) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^
3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (4*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-2a^2-\frac{9}{2}a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx}{15a^4}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} - \frac{\int \frac{-}{-}}{}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} + \frac{\int \frac{-}{\sqrt{}}}{}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} + \frac{(\sqrt{c}}{}$$

$$= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}$$

Mathematica [C] time = 2.18751, size = 371, normalized size = 1.9

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+7\cos\left(\frac{1}{2}(c+3dx)\right)+26\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] -
 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^
((2*I)*c))) - ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 7*Cos[(c + 3*d
*x)/2] + 26*Cos[(5*c + 3*d*x)/2] + 10*Cos[(3*c + 5*d*x)/2] + 5*Cos[(7*c + 5
*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqr
t[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.453, size = 270, normalized size = 1.4

$$\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8-10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

3.213 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{10a^3d}$$

```
[Out] -(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*
d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a
^3*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.319463, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3815, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\mathrm{si}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*
d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a
^3*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{a^2}{2}+3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx}{15a^4}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} + \frac{\int \frac{-\frac{3a^3}{4}+\ldots}{\sqrt{s\ldots}}}{\ldots}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} - \frac{\int \frac{1}{\sqrt{\sec(c\ldots}}}{20a\ldots}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} - \frac{\left(\sqrt{\cos(c\ldots}\right.}{\ldots}$$

$$= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}$$

Mathematica [C] time = 2.06881, size = 371, normalized size = 1.9

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+17\cos\left(\frac{1}{2}(c+3dx)\right)+16\cos\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
+ 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E
^((2*I)*c))) + ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 17*Cos[(c + 3
*d*x)/2] + 16*Cos[(5*c + 3*d*x)/2] + 20*Cos[(3*c + 5*d*x)/2] - 5*Cos[(7*c +
 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*S
qrt[Sec[c + d*x]])/32)*Sec[c + d*x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.621, size = 270, normalized size = 1.4

$$-\frac{1}{60\,a^{3}d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{8}+10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

3.214 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}-\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{10a^3d}$$

```
[Out] (-9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2
*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.329034, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2
*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{9a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{3a^2-\frac{9}{2}a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}dx}{15a^4}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d\left(a^3+a^3\sec(c+dx)\right)} - \ldots$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d\left(a^3+a^3\sec(c+dx)\right)} + \ldots$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d\left(a^3+a^3\sec(c+dx)\right)} + \ldots$$

$$= -\frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d\ldots}}{2a^3d}$$

Mathematica [C] time = 5.25159, size = 272, normalized size = 1.39

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(3e^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5\text{Hyp}\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*
(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3*(1
 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d*x] +
 (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*
Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.547, size = 270, normalized size = 1.4

$$-\frac{1}{20\,a^{3}d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(36\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{8}+10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1
/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2
*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

3.215 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{10a^3d}$$

```
[Out] (49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(6*a^3*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3)
 - (8*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.321851, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] (49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(6*a^3*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3)
 - (8*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx = -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{11a}{2}+\frac{5}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{41a^2}{2}+12a^2\sec(c+d}{\sqrt{\sec(c+dx)}(a+a\sec(c+}}{15a^4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)}\sin(}{6d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)}\sin(}{6d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)}\sin(}{6d\left(a^3+a^3\sec(c+\right.}$$

$$= \frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{6a^3d}$$

Mathematica [C] time = 2.28516, size = 386, normalized size = 1.98

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1134\cos\left(\frac{1}{2}(c-dx)\right)+1071\cos\left(\frac{1}{2}(3c+dx)\right)+923\cos\left(\frac{1}{2}(c+3dx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2] + 923*
Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x)/2] +
265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c + 7*d*x)/
2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32)*Sec[c + d*
```

```
x]^3)/(15*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.612, size = 270, normalized size = 1.4

$$\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(348\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+130\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1
/2*d*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*
c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos
(1/2*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^4+3\,a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+a^3\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^4 + 3*a^3*sec(d*x + c)^3 + 3*
a^3*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+3\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(1/(sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**(
3/2) + sqrt(sec(c + d*x))), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.216 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\cdots$$

```
[Out] (-119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*
a^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x
]/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d
*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Sqrt
[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.35179, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119\sqrt{\cos\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] (-119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*
a^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x
]/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d
*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Sqrt
[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{13a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c}{3ad\sqrt{\sec(c+dx)}(}$$

$$= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{}{5d\sqrt{s}}$$

$$= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{2a^3d}$$

Mathematica [C] time = 2.41702, size = 285, normalized size = 1.29

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(119ie^{-\frac{3}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5 \text{Hypergeometric2F1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Cos
[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x))/
2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x))/2]
```

```
+ 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c + d*x)
)/2]))/(120*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.557, size = 283, normalized size = 1.3

$$-\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{10}+468\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+330\,\sqrt{(\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(
1/2*d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*co
s(1/2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*
cos(1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^5+3\,a^3\sec\left(dx+c\right)^4+3\,a^3\sec\left(dx+c\right)^3+a^3\sec\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^5 + 3*a^3*sec(d*x + c)^4 + 3*
a^3*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.217 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=247

$$-\frac{21\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}-\frac{63\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)}-$$

```
[Out] (231*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (21*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(2*a^3*d) + (77*Sin[c + d*x])/(10*a^3*d*Sec[c + d*x]^(3/2)) - (21*Sin[c +
 d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(5*d*Sec[c + d*x]^(3/2)*
(a + a*Sec[c + d*x])^3) - (4*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)*(a + a
*Sec[c + d*x])^2) - (63*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3*S
ec[c + d*x]))
```

Rubi [A] time = 0.373765, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{63\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{21\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] (231*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (21*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(2*a^3*d) + (77*Sin[c + d*x])/(10*a^3*d*Sec[c + d*x]^(3/2)) - (21*Sin[c +
 d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(5*d*Sec[c + d*x]^(3/2)*
(a + a*Sec[c + d*x])^3) - (4*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)*(a + a
*Sec[c + d*x])^2) - (63*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3 + a^3*S
ec[c + d*x]))
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
```

```
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{15a}{2}+\frac{9}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= -\frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= \frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}$$

$$= \frac{77\sin(c+dx)}{10a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{21\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}$$

$$= \frac{231\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{21\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|\right.}{2a^3d}$$

Mathematica [C] time = 2.66608, size = 297, normalized size = 1.2

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(77ie^{-\frac{3}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5 \text{Hypergeometric2F1}\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])*((-3465*I)*Co
s[(c + d*x)/2] - (2541*I)*Cos[(3*(c + d*x))/2] - (1155*I)*Cos[(5*(c + d*x))
/2] - (231*I)*Cos[(7*(c + d*x))/2] + 3360*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d
```

```
*x]]*EllipticF[(c + d*x)/2, 2] + ((77*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
/E^(((3*I)/2)*(c + d*x)) + 125*Sin[(c + d*x)/2] + 359*Sin[(3*(c + d*x))/2]
+ 350*Sin[(5*(c + d*x))/2] + 138*Sin[(7*(c + d*x))/2] + 5*Sin[(9*(c + d*x))
/2] - Sin[(11*(c + d*x))/2]))/(40*a^3*d*E^(I*d*x)*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.65, size = 296, normalized size = 1.2

$$-\frac{1}{20\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(64\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{12}-288\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{10}-76\,(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1
/2*d*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos
(1/2*d*x+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^3\sec\left(dx+c\right)^6+3\,a^3\sec\left(dx+c\right)^5+3\,a^3\sec\left(dx+c\right)^4+a^3\sec\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*sec(d*x + c)^6 + 3*a^3*sec(d*x + c)^5 + 3*
a^3*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^3\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.218 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{3a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

```
[Out] (3*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (3*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.167241, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3803, 3801, 215}

$$\frac{a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{3a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (3*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{3}{4}\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$
$$= \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{3}{8}\int \sqrt{\sec(c+dx)}\sqrt{a}$$
$$= \frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx\right.}{4d}$$
$$= \frac{3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{3a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a\sec^{\frac{5}{2}}(c+dx)\sin(c}{2d\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 0.495665, size = 100, normalized size = 0.86

$$\frac{2a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\left(\frac{1}{8}\cos(c+dx)(3\cos(c+dx)+2)+\frac{3\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{8\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\right)}{d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*((Cos[c + d*x]*(2 + 3*Cos[c + d*x]))/8 + (3*ArcSin[Sqrt[1 - Sec[c + d*
x]]])/(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)))*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.26, size = 221, normalized size = 1.9

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{16\,d\,(\sin(dx+c))^2}\left(3\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(3*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))*cos(d*x+c)^2-3*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+6*(-2/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*(
1/cos(d*x+c))^(5/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.80039, size = 1706, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(
2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
```

```
- 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.79729, size = 953, normalized size = 8.22

$$\left[\frac{3\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\sqrt{\frac{a\cos(dx+c)}{\cos(dx+c)}}}{}}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*c
os(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos
```

```
(d*x + c)), 1/8*(3*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec (dx + c) + a} \sec (dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.219 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.112704, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3803, 3801, 215}

$$\frac{a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.220046, size = 75, normalized size = 1.04

$$\frac{a\tan(c+dx)\left(\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}+\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (a*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x
])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.215, size = 186, normalized size = 2.6

$$-\frac{(-1+\cos(dx+c))\cos(dx+c)}{2d(\sin(dx+c))^2}\left(\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1+\sin(dx+c))}{4}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{2}\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)-arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)+2*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(cos(d*x+c)+1)/cos(
d*x+c))^(1/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.83542, size = 894, normalized size = 12.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)
/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [B] time = 1.81148, size = 814, normalized size = 11.31

$$\left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\,(d\cos(dx+c)+d)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(sqrt(-a)*(cos(d*x + c) + 1)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.220 $\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

Rubi [A] time = 0.0583492, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3801, 215}

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

Mathematica [A] time = 0.101556, size = 54, normalized size = 1.46

$$-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])
/(d*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.214, size = 147, normalized size = 4.

$$\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))}{d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{(\cos(dx+c))^{-1}}\left(\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1-\sin(dx+c))}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*cos(d*
x+c)*(-1+cos(d*x+c))*(arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))-arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
+1+sin(d*x+c))))/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.98589, size = 325, normalized size = 8.78

$$\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))/d
```

Fricas [B] time = 1.77684, size = 495, normalized size = 13.38

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{2\,d},\ \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.221 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0541663, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {3804}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.0856468, size = 39, normalized size = 1.08

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.188, size = 52, normalized size = 1.4

$$-2\,{\frac {-1+\cos \left(dx+c \right) }{d\sin \left(dx+c \right) \sqrt { \left(\cos \left(dx+c \right) \right) ^{-1}}}\sqrt {{\frac {a \left(\cos \left(dx+c \right) +1 \right) }{\cos \left(dx+c \right) }}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/d*(-1+cos(d*x+c))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/(1/cos(
d*x+c))^(1/2)
```

Maxima [A] time = 2.59423, size = 27, normalized size = 0.75

$$\frac{2 \, \sqrt{2} \sqrt{a} \sin\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [A] time = 1.62283, size = 130, normalized size = 3.61

$$\frac{2 \, \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right)}{d \cos\left(d x + c\right) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.222 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.108971, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3805, 3804}

$$\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2}{3}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{4a\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.154257, size = 49, normalized size = 0.64

$$\frac{2(\cos(c+dx)+2)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*Sqr
t[Sec[c + d*x]])
```

Maple [A] time = 0.21, size = 68, normalized size = 0.9

$$-\frac{\left(2\,(\cos(dx+c))^2+2\,\cos(dx+c)-4\right)(\cos(dx+c))^2}{3\,d\sin(dx+c)}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+cos(d*x+c)-2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(
d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [A] time = 2.54467, size = 153, normalized size = 1.99

$$\frac{\sqrt{2}\left(3\,\cos\left(\frac{2}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3\,\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\sin\left(\frac{2}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx\right.\right.\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.65563, size = 181, normalized size = 2.35

$$\frac{2\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(d*x + c)^2 + 2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.223 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a
*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.163738, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3805, 3804}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a
*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{4}{5}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8}{15}\int \frac{\sqrt{a+a\,\mathrm{s}}}{\sqrt{\mathrm{sec}}}$$
$$= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{8a\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a\sqrt{\sec(c+d}}{15d\sqrt{a+a\,}}$$

Mathematica [A] time = 0.183567, size = 61, normalized size = 0.53

$$\frac{(8\cos(c+dx)+3\cos(2(c+dx))+19)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[
(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.217, size = 80, normalized size = 0.7

$$-\frac{\left(6\,(\cos(dx+c))^3+2\,(\cos(dx+c))^2+8\,\cos(dx+c)-16\right)(\cos(dx+c))^3}{15\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^3+cos(d*x+c)^2+4*cos(d*x+c)-8)*(a*(cos(d*x+c)+1)/cos(
d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 3.07753, size = 274, normalized size = 2.38

$$\frac{\sqrt{2}\left(30 \cos\left(\frac{4}{5} \arctan\left(\sin\left(\frac{5}{2} dx + \frac{5}{2} c\right), \cos\left(\frac{5}{2} dx + \frac{5}{2} c\right)\right)\right) \sin\left(\frac{5}{2} dx + \frac{5}{2} c\right) + 5 \cos\left(\frac{2}{5} \arctan\left(\sin\left(\frac{5}{2} dx + \frac{5}{2} c\right), \cos\left(\frac{5}{2} \right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x +
 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.55611, size = 211, normalized size = 1.83

$$\frac{2\left(3 \cos\left(dx+c\right)^{3} + 4 \cos\left(dx+c\right)^{2} + 8 \cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + 8*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x +
c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.224 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(c+dx)\sqrt{\sec(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{16}{35d\sqrt{\sec(c+}}$$

```
[Out] (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (12*
a*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (16*a*
Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (32*a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.215677, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3805, 3804}

$$\frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(c+dx)\sqrt{\sec(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{16}{35d\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (12*
a*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (16*a*
Sin[c + d*x])/(35*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (32*a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
```

```
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{6}{7}\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{24}{35}\int \frac{\sqrt{a+a\,\mathrm{s}}}{\sec^{\frac{3}{2}}}$$
$$= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{16a}{35d\sqrt{\sec(c+}}$$
$$= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{12a\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{16a}{35d\sqrt{\sec(c+}}$$

Mathematica [A] time = 0.240278, size = 71, normalized size = 0.46

$$\frac{(47\cos(c+dx)+12\cos(2(c+dx))+5\cos(3(c+dx))+76)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{70d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((76 + 47*Cos[c + d*x] + 12*Cos[2*(c + d*x)] + 5*Cos[3*(c + d*x)])*Sqrt[a*(
1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(70*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.213, size = 90, normalized size = 0.6

$$-\frac{\left(10\,(\cos(dx+c))^4+2\,(\cos(dx+c))^3+4\,(\cos(dx+c))^2+16\,\cos(dx+c)-32\right)(\cos(dx+c))^4}{35\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/35/d*(5*cos(d*x+c)^4+cos(d*x+c)^3+2*cos(d*x+c)^2+8*cos(d*x+c)-16)*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [B] time = 2.69414, size = 396, normalized size = 2.59

$$\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2*c)
*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7/2*
d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.6068, size = 238, normalized size = 1.56

$$\frac{2\left(5\cos\left(dx+c\right)^{4}+6\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+16\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 16*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c)
+ d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.225 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=160

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a\sec(c+dx)+}}\right.}{8d}$$

```
[Out] (11*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
 + (11*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ (11*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.234375, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3814, 21, 3803, 3801, 215}

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a\sec(c+dx)+}}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
 + (11*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ (11*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{1}{3}a\int\frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{11a}{2}+\frac{11}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{1}{6}(11a)\int\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{1}{8}(11a)\int\sec^{\frac{3}{2}}($$

$$= \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{7}{2}}(c+d}{3d\sqrt{a+a\,s}}$$

$$= \frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{11a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{7}{2}}(c+d}{3d\sqrt{a+a\,s}}$$

$$= \frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{11a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{11a^2\sec^{\frac{5}{2}}(c+}{12d\sqrt{a+a}}$$

Mathematica [A] time = 0.556477, size = 112, normalized size = 0.7

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(54\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)+66\sqrt{2}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])]*(66*Sqrt[
2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] +
 11*(Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

Maple [A] time = 0.233, size = 246, normalized size = 1.5

$$\frac{a\left((\cos(dx+c))^2-1\right)}{96\,d\,(\sin(dx+c))^2}\left(-33\,\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)\sqrt{2}\,(\cos(dx+c))^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/96/d*a*(-33*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))*2^(1/2)*cos(d*x+c)^3+33*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^3+66*(-2/(cos(d*x+c)+1))
^(1/2)*cos(d*x+c)^2*sin(d*x+c)+44*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(
d*x+c)+16*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c
))^(1/2)*(1/cos(d*x+c))^(5/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(cos(d
*x+c)^2-1)
```

Maxima [B] time = 3.30047, size = 3187, normalized size = 19.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2
*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*
a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos
(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d
*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x
 + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c)
```

```
+ a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a
*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*c
os(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin
(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x +
2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*
c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) +
 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 +
 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 1
8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos
(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d
*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4
*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqr
t(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x +
4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*
cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) +
3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*
x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sq
rt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2
)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x
 + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sqrt(a)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*d)
```

Fricas [A] time = 1.84128, size = 1052, normalized size = 6.58

$$\left[\frac{33\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+\frac{4\left(33\right.}{}}{96\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(33*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x +
 c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(33*(a*cos(d*x + c)^3
+ a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x +
 c) - 2*a)) + 2*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

3.226 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=120

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

```
[Out] (7*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (7*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
(a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.17516, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3814, 21, 3803, 3801, 215}

$$\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}+\frac{7a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
+ (7*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
(a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}a\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{7a}{2}+\frac{7}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{4}(7a)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{8}(7a)\int \sqrt{\sec(c$$

$$= \frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}-\frac{(7a)\,\mathrm{Subst}\left(\int \frac{}{\sqrt{}}\right.}{}$$

$$= \frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{7a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{5}{2}}(c+dx)\,s}{2d\sqrt{a+a\sec}}$$

Mathematica [A] time = 0.407595, size = 99, normalized size = 0.82

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)+7\sqrt{2}\cos^2(c+dx)\tanh^{-1}\left(\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])]*(7*Sqrt[2
]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7
*Sin[(3*(c + d*x))/2]))/(8*d)
```

Maple [B] time = 0.218, size = 214, normalized size = 1.8

$$-\frac{a(-1+\cos(dx+c))}{8d(\sin(dx+c))^2}\left(7\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d*a*(-1+cos(d*x+c))*(7*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-7*2^(1/2)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+14*(-2/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(-2/(cos(d*
x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 3.55083, size = 3029, normalized size = 25.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] -1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(

```
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*
cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/((2*(2*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*d)
```

Fricas [A] time = 1.80685, size = 975, normalized size = 8.12

$$\left[\frac{7\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{16\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}+\frac{4\,(7\,a\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3
- 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos
(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2
 + d*cos(d*x + c)), 1/8*(7*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-a)*arc
tan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(7*a*cos(d*x + c
) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.227 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.117463, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3814, 21, 3801, 215}

$$\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx &= \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+a\int \frac{\sqrt{\sec(c+dx)}\left(\frac{3a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx\\
&= \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{1}{2}(3a)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx\\
&= \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.263817, size = 75, normalized size = 1.

$$\frac{a^2\tan(c+dx)\left(\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}-3\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^2*(-3*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*
x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.211, size = 184, normalized size = 2.5

$$\frac{a\left((\cos(dx+c))^2-1\right)}{4\,d\,(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*a*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(
1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*co
s(d*x+c)-3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d
*x+c)))*2^(1/2)*cos(d*x+c)+2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos
(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.97324, size = 1543, normalized size = 20.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x + 3/
2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
```

```
(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 4
*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*
x + 2*c))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*d)
```

Fricas [B] time = 1.79298, size = 830, normalized size = 11.07

$$\left[\frac{3\,(a\cos(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+}{\sqrt{\cos(dx+c)}}}{4\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)) + 4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(3*(a*cos(d*x + c) + a)*sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*a*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
 + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^{\frac{3}{2}} \sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.228 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.11753, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3813, 21, 3801, 215}

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (2
*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+(2a)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx\\
&= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+a\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx\\
&= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}-\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,-\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\
&= \frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.32885, size = 86, normalized size = 1.13

$$\frac{2a^2\left(\sin(c+dx)\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}+\tan(c+dx)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a^2*(Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + ArcSin[Sqr
t[1 - Sec[c + d*x]]]*Tan[c + d*x]))/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + S
ec[c + d*x])])
```

Maple [B] time = 0.187, size = 174, normalized size = 2.3

$$\frac{a}{2\,d\sin\left(dx+c\right)}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\cos\left(dx+c\right)+1-\sin\left(dx+c\right)\right)}{4}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\right)\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*a*(2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
-sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2^(1/2)*arctan(1/4*2^(1/
2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-4*cos(d*x+c)+4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d
*x+c)/(1/cos(d*x+c))^(1/2)
```

Maxima [B] time = 3.42125, size = 370, normalized size = 4.87

$$\frac{\sqrt{2}\left(\sqrt{2}a\log\left(2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)-\sqrt{2}a\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*sqrt(
a)/d
```

Fricas [B] time = 1.86648, size = 819, normalized size = 10.78

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\,c}{c}}}{\sqrt{\cos(dx+c)}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{2\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
 c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) +
d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.229 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] (8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.108923, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3809, 3804}

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(4a)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{8a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.193369, size = 50, normalized size = 0.63

$$\frac{2a(\cos(c+dx)+5)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(3*d*S
qrt[Sec[c + d*x]])
```

Maple [A] time = 0.19, size = 71, normalized size = 0.9

$$-\frac{2\,a\left((\cos(dx+c))^2+4\,\cos(dx+c)-5\right)(\cos(dx+c))^2}{3\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d*a*(cos(d*x+c)^2+4*cos(d*x+c)-5)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*
cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [A] time = 2.88006, size = 51, normalized size = 0.65

$$\frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

Fricas [A] time = 1.61669, size = 186, normalized size = 2.35

$$\frac{2\left(a\cos\left(dx+c\right)^{2}+5\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c)^2 + 5*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.230 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sqrt{\sec(c+dx)}}$$

```
[Out] (8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]) + (2*(a
+ a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.172158, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3812, 3809, 3804}

$$\frac{8a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{5d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]) + (2*(a
+ a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
```

```
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{3}{5}\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(4a)\int \frac{\sqrt{a+}}{\sqrt{\mathrm{se}}}$$
$$= \frac{8a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2(a+a\sec(c+dx))^{3/}}{5d\sec^{\frac{3}{2}}(c+}$$

Mathematica [A] time = 0.257362, size = 60, normalized size = 0.52

$$\frac{a(6\cos(c+dx)+\cos(2(c+dx))+13)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{5d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[
(c + d*x)/2])/(5*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.184, size = 81, normalized size = 0.7

$$-\frac{2\,a\left((\cos(dx+c))^3+2\,(\cos(dx+c))^2+3\,\cos(dx+c)-6\right)(\cos(dx+c))^3}{5\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left((\cos(dx+c))^{-1}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/5/d*a*(cos(d*x+c)^3+2*cos(d*x+c)^2+3*cos(d*x+c)-6)*(a*(cos(d*x+c)+1)/cos
(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 2.82425, size = 284, normalized size = 2.45

$$\frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\cos\left(\frac{2}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/
2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin
(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*sqrt(a)/d
```

Fricas [A] time = 1.56229, size = 215, normalized size = 1.85

$$\frac{2\left(a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)^{2}+6\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 + 6*a*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x
 + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

3.231 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{1}{105d\sqrt{\sec}}$$

```
[Out] (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
6*a^2*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (1
04*a^2*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(208*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.232306, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3813, 21, 3805, 3804}

$$\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{1}{105d\sqrt{\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
6*a^2*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (1
04*a^2*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(208*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{1}{7}(2a)\int \frac{\frac{13a}{2}+\frac{13}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx\\
&= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{1}{7}(13a)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{1}{35}(52a)\int \\
&= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{10}{105d\sqrt{\sec(c}}\\
&= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{26a^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{10}{105d\sqrt{\sec(c}}
\end{aligned}$$

Mathematica [A] time = 0.337498, size = 72, normalized size = 0.45

$$\frac{a(253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx))+494)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{210d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqr
t[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.205, size = 93, normalized size = 0.6

$$-\frac{2\,a\left(15\,(\cos(dx+c))^4+24\,(\cos(dx+c))^3+13\,(\cos(dx+c))^2+52\,\cos(dx+c)-104\right)(\cos(dx+c))^4}{105\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d*a*(15*cos(d*x+c)^4+24*cos(d*x+c)^3+13*cos(d*x+c)^2+52*cos(d*x+c)-1
04)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/s
in(d*x+c)
```

Maxima [B] time = 2.82408, size = 409, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*
x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 1
75*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.65348, size = 255, normalized size = 1.58

$$\frac{2\left(15\,a\cos\left(dx+c\right)^{4}+39\,a\cos\left(dx+c\right)^{3}+52\,a\cos\left(dx+c\right)^{2}+104\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^4 + 39*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 10
4*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*
cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

3.232 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{68a^2\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{544}{3}$$

```
[Out] (2*a^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (3
4*a^2*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (6
8*a^2*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (
272*a^2*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
 (544*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.291753, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3813, 21, 3805, 3804}

$$\frac{68a^2\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{544}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (3
4*a^2*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (6
8*a^2*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (
272*a^2*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
 (544*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(2a)\int \frac{\frac{17a}{2}+\frac{17}{2}a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx \\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}(17a)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{21}(34a)\int \\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{68}{105d\sec^{\frac{3}{2}}(c} \\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{68}{105d\sec^{\frac{3}{2}}(c} \\
&= \frac{2a^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{34a^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{68}{105d\sec^{\frac{3}{2}}(c}
\end{aligned}
$$

Mathematica [A] time = 0.520719, size = 80, normalized size = 0.4

$$\frac{2a^2\sin(c+dx)\left(272\sec^4(c+dx)+136\sec^3(c+dx)+102\sec^2(c+dx)+85\sec(c+dx)+35\right)}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(35 + 85*Sec[c + d*x] + 102*Sec[c + d*x]^2 + 136*Sec[c + d*x]^3 + 27
2*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c
 + d*x])])
```

Maple [A] time = 0.196, size = 103, normalized size = 0.5

$$-\frac{2\,a\left(35\,(\cos(dx+c))^5+50\,(\cos(dx+c))^4+17\,(\cos(dx+c))^3+34\,(\cos(dx+c))^2+136\,\cos(dx+c)-272\right)(\cos(dx}{315\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*a*(35*cos(d*x+c)^5+50*cos(d*x+c)^4+17*cos(d*x+c)^3+34*cos(d*x+c)^2
+136*cos(d*x+c)-272)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/co
s(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [B] time = 3.37442, size = 535, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5040*sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(9
/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2/
```

```
9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c)
 - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/
2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
+ 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arcta
n2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*c
)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(9
/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) +
 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*sqrt(
a)/d
```

Fricas [A] time = 1.65599, size = 288, normalized size = 1.43

$$\frac{2\left(35\,a\cos\left(dx+c\right)^{5}+85\,a\cos\left(dx+c\right)^{4}+102\,a\cos\left(dx+c\right)^{3}+136\,a\cos\left(dx+c\right)^{2}+272\,a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\mathrm{s}}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a*cos(d*x + c)^5 + 85*a*cos(d*x + c)^4 + 102*a*cos(d*x + c)^3 + 1
36*a*cos(d*x + c)^2 + 272*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2), x)
```

3.233 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=200

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{17a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+163$$

```
[Out] (163*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*
d) + (163*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (163*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (17*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*
x]]) + (a^2*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.337087, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3814, 4016, 3803, 3801, 215}

$$\frac{a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{17a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+163$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*
d) + (163*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (163*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (17*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*
x]]) + (a^2*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}a\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}$$

$$= \frac{17a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+d}{4d}$$

$$= \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{7}{2}}(c}{}$$

$$= \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\sec}{24d\sqrt{}}$$

$$= \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{17a^3\sec}{24d\sqrt{}}$$

$$= \frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d} + \frac{163a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sec}{96d\sqrt{}}$$

Mathematica [C] time = 8.25729, size = 582, normalized size = 2.91

$$-\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(7824i\tan^{-1}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(\sqrt{2}-1\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)+7824i\tan^{-1}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(1+\sqrt{2}\right)\mathrm{s}}{\left(\sqrt{2}-1\right)\cos\left(\frac{1}{4}(c+dx)\right)-\mathrm{s}}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -(a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*((7824*I)*ArcTan[(Cos[(c
+ d*x)/4] - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c + d*x)/4
] - Sin[(c + d*x)/4])] + (7824*I)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sqrt[2])*
Sin[(c + d*x)/4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4])] + S
ec[c + d*x]^4*(-2934*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] + 1467*Log[2 - Sqrt[
2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - 1956*Cos[2*(c + d*x)]*(2*
Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] - Log[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt
[2]*Sin[(c + d*x)/2]] - Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c +
 d*x)/2]]) - 489*Cos[4*(c + d*x)]*(2*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] - Lo
g[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - Log[2 + Sqrt[2
]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]]) + 1467*Log[2 + Sqrt[2]*Cos[
```

```
(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] + 2060*Sqrt[2]*Sin[(c + d*x)/2] -
6204*Sqrt[2]*Sin[(3*(c + d*x))/2] - 652*Sqrt[2]*Sin[(5*(c + d*x))/2] - 1956
*Sqrt[2]*Sin[(7*(c + d*x))/2])))/(6144*Sqrt[2]*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.265, size = 286, normalized size = 1.4

$$\frac{a^2\left((\cos(dx+c))^2-1\right)}{768\,d\,(\sin(dx+c))^2\cos(dx+c)}\left(489\,\sqrt{2}\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx+c))\right)(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/768/d*a^2*(489*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^4-489*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^4+978*(-2/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)^3*sin(d*x+c)+652*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
)^2*sin(d*x+c)+368*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+96*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos
(d*x+c))^(5/2)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c
)^2-1)
```

Maxima [B] time = 3.81894, size = 5211, normalized size = 26.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) +
 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x
+ 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*si
n(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x +
```

```
 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2
*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*
sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x
 + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(
2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*si
n(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x
 + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin
(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 4
8*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2
*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c
) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4
*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*
x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x +
 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos
(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a
^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^
2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x +
2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x
+ 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*
x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2
*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*
x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*
d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2
+ 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16
*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x
 + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x
+ 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*
x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*si
n(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x
```

+ 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c

```
))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c
) + 1)*d)
```

Fricas [A] time = 1.87487, size = 1160, normalized size = 5.8

$$\left[\frac{489\left(a^2\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\cdots}{768\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*
x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(489*a^2*cos(d*x + c)^3 + 326*a
^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(
d*x + c)^3), 1/384*(489*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(489*a^2*cos(
d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2), x)
```

3.234 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{13a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{25a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{25a^{5/}}{}$$

```
[Out] (25*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
 + (25*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ (13*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.274975, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3814, 4016, 3803, 3801, 215}

$$\frac{13a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{25a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}+\frac{25a^{5/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (25*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(8*d)
 + (25*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]])
+ (13*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]])
+ (a^2*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
```

```
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}a\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}$$

$$= \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+d}{3d}$$

$$= \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+}{}$$

$$= \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sec^{\frac{5}{2}}(c+}{}$$

$$= \frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d} + \frac{25a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{13a^3\sec^{\frac{5}{2}}(c}{12d\sqrt{a+}}$$

Mathematica [C] time = 7.83101, size = 458, normalized size = 2.86

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(-600i\tan^{-1}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(\sqrt{2}-1\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)-600i\tan^{-1}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(1+\sqrt{2}\right)\sin(}{\left(\sqrt{2}-1\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin(}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*((-600*I)*ArcTan[(Cos[(c +
 d*x)/4] - (-1 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c + d*x)/4]
 - Sin[(c + d*x)/4])] - (600*I)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sqrt[2])*Si
n[(c + d*x)/4])/((-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4])] + Sec
[c + d*x]^3*(225*Cos[c + d*x]*(2*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]] - Log[2
- Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]] - Log[2 + Sqrt[2]*Co
s[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]]) + 75*Cos[3*(c + d*x)]*(2*Log[Sq
rt[2] + 2*Sin[(c + d*x)/2]] - Log[2 - Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Si
n[(c + d*x)/2]] - Log[2 + Sqrt[2]*Cos[(c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/
2]]) + 4*Sqrt[2]*(114*Sin[(c + d*x)/2] - 7*Sin[(3*(c + d*x))/2] + 75*Sin[(5
*(c + d*x))/2]))))/(384*Sqrt[2]*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.238, size = 254, normalized size = 1.6

$$-\frac{a^{2}\left(-1+\cos\left(dx+c\right)\right)}{48\,d\cos\left(dx+c\right)\left(\sin\left(dx+c\right)\right)^{2}}\left(75\,\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\left(\cos\left(dx+c\right)+1+\sin\left(dx+c\right)\right)\right)\sqrt{2}\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/48/d*a^2*(-1+cos(d*x+c))*(75*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^3-75*arctan(1/4*2^(1/2)*(-2
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^3+150*
(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+68*(-2/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)*sin(d*x+c)+16*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(
d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/cos(d*x+c)/sin(d*x+c)^2/(-
2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 3.29901, size = 4683, normalized size = 29.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)
*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6
*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12
*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -
 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4
56*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan
```

```
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*
d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^
2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*
x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*
a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*
x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sq
rt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7
5*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^
2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2
+ 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*
a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) +
a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*
sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
```

+ 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*l
og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(
6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*
cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2) + 28*(sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*
sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ sqrt(2)*a^2)*sin(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/
2*c) - 114*sqrt(2)*a^2*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 114*sqrt(2)*a^2*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x + 6*
c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + sqrt(2)*a^2)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
c))) + 12(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x
+ 3/2*c) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/((cos(6*d*x + 6*c)^2 + 6*(cos(
6*d*x + 6*c) + 3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*co
s(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(6*d*x
+ 6*c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
9*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(6*d

```
*x + 6*c)^2 + 6*(sin(6*d*x + 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(6*d*x + 6*c) + 1)*d)
```

Fricas [A] time = 1.8581, size = 1079, normalized size = 6.74

$$\left[\frac{75\left(a^2\cos(dx+c)^3+a^2\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4(}{}}{96\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(75*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a^2*cos(d*x + c)^2 + 34*a^2*c
os(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(75*(a^2*cos
(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 - a*cos(d*x + c) - 2*a)) + 2*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c)
+ 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

3.235 $\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=120

$$\frac{9a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

```
[Out] (19*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
 + (9*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a^2*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.215084, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3814, 4016, 3801, 215}

$$\frac{9a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (19*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*d)
 + (9*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a^2*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
```

```
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}+\frac{1}{2}a\int \sqrt{\sec(c+dx)}\sqrt{a+a\,\mathrm{se}$$

$$=\frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}$$

$$=\frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d}$$

$$=\frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{9a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2\sec^{\frac{3}{2}}(c+dx}{}$$

Mathematica [A] time = 0.450677, size = 106, normalized size = 0.88

$$\frac{a^3\tan(c+dx)\left(2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+11\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}-19\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^3*(-19*ArcSin[Sqrt[Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x
]^(3/2) + 11*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(4*d*
Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.225, size = 226, normalized size = 1.9

$$\frac{a^2\left((\cos(dx+c))^2-1\right)}{16\,d\,(\sin(dx+c))^2\cos(dx+c)}\left(19\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx+c))\right)(\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d*a^2*(19*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-19*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+22*(-2/(cos(d*x+c)+1)
)^(1/2)*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(c
os(d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/
2)/sin(d*x+c)^2/cos(d*x+c)*(cos(d*x+c)^2-1)
```

Maxima [B] time = 24.0529, size = 3815, normalized size = 31.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a^
2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 - 7
6*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
```

cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*
c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)
^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)
- 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
 - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2

```
*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22
*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*
(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) +
 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) -
 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*d*x +
 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d*x +
3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sqrt(a)/((2
*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d
*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*
sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.81515, size = 1002, normalized size = 8.35

$$\left[\frac{19\left(a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-\frac{4\left(\cos(dx+c)^{2}-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+\frac{4(1\cdots}{\cdots}}{16\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqr
```

```
t((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c)^2 + d*cos(d*x + c)), 1/8*(19*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c
))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(
11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

3.236 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=112

$$\frac{a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}+\frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (5*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqr
t[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.216975, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3814, 4015, 3801, 215}

$$\frac{a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}+\frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (5*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (a
^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqr
t[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{a^2\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + a\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{a}{2}+\frac{5}{2}a\sec(c+d\right.}{\sqrt{\sec(c+dx)}}$$

$$= \frac{a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\left(5a^2\right)\int$$

$$= \frac{a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d} - \frac{\left(5a^2\right)\text{Subs}}{}$$

$$= \frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c-}}{d}$$

Mathematica [A] time = 0.656345, size = 91, normalized size = 0.81

$$\frac{a^3\tan(c+dx)\left((2\cos(c+dx)+1)\sqrt{(\cos(c+dx)-1)\sec^2(c+dx)}+5\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]] + (1 + 2*Cos[c + d*x])*Sqrt[(-1 + Co
s[c + d*x])*Sec[c + d*x]^2])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.226, size = 199, normalized size = 1.8

$$-\frac{a^2}{4\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(5\,\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)\sqrt{\cdot}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(5*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*2^
(1/2)*cos(d*x+c)*sin(d*x+c)-5*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)*sin
(d*x+c)+8*cos(d*x+c)^2-4*cos(d*x+c)-4)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.85792, size = 909, normalized size = 8.12

$$\left[\frac{5\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(2\,a^2\cos(dx+c)+a^2\right)\sqrt{\cdot}}{\sqrt{\cos}}}{4\,(d\cos(dx+c)+d)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(
5*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*co
s(d*x + c) - 2*a)) + 2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.237 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{14a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
4*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.219059, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3813, 4015, 3801, 215}

$$\frac{14a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}+\frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (1
4*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
```

```
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(2a)\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{7a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + a^2\int \sqrt{\sec(c+dx)}\sqrt{}$$

$$= \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+}}\right.}{}$$

$$= \frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(}{3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.397374, size = 103, normalized size = 0.87

$$\frac{2a^3\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}(8\sec(c+dx)+1)+3\sec^{\frac{3}{2}}(c+dx)\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{3d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^3*(3*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2) + Sqrt[1 - Sec[
c + d*x]]*(1 + 8*Sec[c + d*x]))*Sin[c + d*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x
])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.223, size = 195, normalized size = 1.7

$$\frac{a^2(\cos(dx+c))^2}{6d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/6/d*a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*2^(1/2)*arctan(1/4*2^(1/2)
*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-3*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*cos(d*x+c)^2-
28*cos(d*x+c)+32)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 2.23743, size = 801, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
```

```
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x
 + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*sqrt(a)/d
```

Fricas [A] time = 1.7961, size = 949, normalized size = 8.04

$$\left[\frac{3\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(a^2\cos(dx+c)^2+8a^2\text{co}\ldots}{\sqrt{\ldots}}}{6\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x + c))*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c) + d), 1/3*(3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(a^2*cos(d*x + c)^2 + 8*a^2*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.238 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (64*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) +
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.169419, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3809, 3804}

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (64*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]) +
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
```

```
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(8a)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{15}\left(32a^2\right)\int$$
$$= \frac{64a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+}{5d\sec^{\frac{3}{2}}}$$

Mathematica [A] time = 0.28466, size = 64, normalized size = 0.54

$$\frac{a^2(28\cos(c+dx)+3\cos(2(c+dx))+89)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{15d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]
*Tan[(c + d*x)/2])/(15*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.183, size = 85, normalized size = 0.7

$$-\frac{2a^2\left(3(\cos(dx+c))^3+11(\cos(dx+c))^2+29\cos(dx+c)-43\right)(\cos(dx+c))^3}{15d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left((\cos(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d*a^2*(3*cos(d*x+c)^3+11*cos(d*x+c)^2+29*cos(d*x+c)-43)*(a*(cos(d*x+c
)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [A] time = 2.25436, size = 81, normalized size = 0.68

$$\frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.65321, size = 230, normalized size = 1.93

$$\frac{2\left(3\,a^2\cos\left(dx+c\right)^3+14\,a^2\cos\left(dx+c\right)^2+43\,a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^3 + 14*a^2*cos(d*x + c)^2 + 43*a^2*cos(d*x + c))*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*s
qrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

3.239 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx}{7d}$$

```
[Out] (64*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) +
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(3/2)) + (2
*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.236141, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3812, 3809, 3804}

$$\frac{64a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{21d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sqrt{\sec(c+dx)}}+\frac{2a\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{7d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (64*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) +
(2*a*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(3/2)) + (2
*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
```

```
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{5}{7}\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}(8a)\int \frac{(a+ \\
&= \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(a+a\sec(}{7d \\
&= \frac{64a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a(a+a\sec(c+d}{7d\sec^{\frac{3}{2}}(
\end{aligned}$$

Mathematica [A] time = 0.336332, size = 74, normalized size = 0.47

$$\frac{a^2(101\cos(c+dx)+24\cos(2(c+dx))+3\cos(3(c+dx))+208)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{42d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(208 + 101*Cos[c + d*x] + 24*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)])*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(42*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.187, size = 95, normalized size = 0.6

$$-\frac{2a^{2}\left(3\left(\cos\left(dx+c\right)\right)^{4}+9\left(\cos\left(dx+c\right)\right)^{3}+11\left(\cos\left(dx+c\right)\right)^{2}+23\cos\left(dx+c\right)-46\right)\left(\cos\left(dx+c\right)\right)^{4}}{21\,d\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+\right.}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/21/d*a^2*(3*cos(d*x+c)^4+9*cos(d*x+c)^3+11*cos(d*x+c)^2+23*cos(d*x+c)-46
)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin
(d*x+c)
```

Maxima [B] time = 2.16188, size = 436, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*cos
(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*s
in(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.70664, size = 262, normalized size = 1.68

$$\frac{2\left(3a^{2}\cos\left(dx+c\right)^{4}+12a^{2}\cos\left(dx+c\right)^{3}+23a^{2}\cos\left(dx+c\right)^{2}+46a^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin\left(dx+c\right)}{21\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^4 + 12*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2
+ 46*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

3.240 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1168\cdots}{3\cdots}$$

```
[Out] (38*a^3*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) +
(146*a^3*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
+ (584*a^3*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
) + (1168*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
))
```

Rubi [A] time = 0.332743, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3813, 4015, 3805, 3804}

$$\frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1168\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (38*a^3*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) +
(146*a^3*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
+ (584*a^3*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
) + (1168*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2
))
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
```

```
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{1}{9}(2a)\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{19a}{2}+\frac{15}{2}a\sec(c+dx)\right)}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{1}{21}\left(73a^2\right)\int$$

$$= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+}}{9}$$

$$= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{315d\sqrt{\sec}}$$

$$= \frac{38a^3\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{315d\sqrt{\sec}}$$

Mathematica [A] time = 0.564219, size = 80, normalized size = 0.4

$$\frac{2a^3\sin(c+dx)\left(584\sec^4(c+dx)+292\sec^3(c+dx)+219\sec^2(c+dx)+130\sec(c+dx)+35\right)}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(35 + 130*Sec[c + d*x] + 219*Sec[c + d*x]^2 + 292*Sec[c + d*x]^3 + 5
84*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[
c + d*x])])
```

Maple [A] time = 0.194, size = 105, normalized size = 0.5

$$-\frac{2\,a^2\left(35\,(\cos(dx+c))^5+95\,(\cos(dx+c))^4+89\,(\cos(dx+c))^3+73\,(\cos(dx+c))^2+292\,\cos(dx+c)-584\right)(\cos(d\ldots}{315\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d*a^2*(35*cos(d*x+c)^5+95*cos(d*x+c)^4+89*cos(d*x+c)^3+73*cos(d*x+c)
^2+292*cos(d*x+c)-584)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/
cos(d*x+c))^(9/2)/sin(d*x+c)
```

Maxima [B] time = 2.18681, size = 570, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/
2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^
```

```
2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
+ 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)
*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos
(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos
(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.89736, size = 302, normalized size = 1.5

$$\frac{2\left(35\,a^{2}\cos\left(dx+c\right)^{5}+130\,a^{2}\cos\left(dx+c\right)^{4}+219\,a^{2}\cos\left(dx+c\right)^{3}+292\,a^{2}\cos\left(dx+c\right)^{2}+584\,a^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos(dx}{\cos(d}}}{315\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^5 + 130*a^2*cos(d*x + c)^4 + 219*a^2*cos(d*x + c
)^3 + 292*a^2*cos(d*x + c)^2 + 584*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

3.241 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=241

$$\frac{284a^3\sin(c+dx)}{231d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

```
[Out] (46*a^3*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) +
(710*a^3*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]])
+ (284*a^3*Sin[c + d*x])/(231*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
) + (1136*a^3*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*
x]]) + (2272*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11*d*Sec[c + d*x]^
(9/2))
```

Rubi [A] time = 0.403243, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3813, 4015, 3805, 3804}

$$\frac{284a^3\sin(c+dx)}{231d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```


```
[Out] (46*a^3*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) +
(710*a^3*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]])
+ (284*a^3*Sin[c + d*x])/(231*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
) + (1136*a^3*Sin[c + d*x])/(693*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*
x]]) + (2272*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11*d*Sec[c + d*x]^
(9/2))
```


Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
```

```
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{1}{11}(2a)\int \frac{\sqrt{a+a\sec(c+dx)}\left(\frac{23a}{2}+\frac{19}{2}a\sec(c+dx)\right.}{\sec^{\frac{9}{2}}(c+dx)}$$

$$= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{1}{99}\left(355a^2\right)$$

$$= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{a+}}{1}$$

$$= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{231d\sec^{\frac{3}{2}}}$$

$$= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{231d\sec^{\frac{3}{2}}}$$

$$= \frac{46a^3\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{710a^3\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{}{231d\sec^{\frac{3}{2}}}$$

Mathematica [A] time = 0.358853, size = 90, normalized size = 0.37

$$\frac{2a^3\sin(c+dx)\left(1136\sec^5(c+dx)+568\sec^4(c+dx)+426\sec^3(c+dx)+355\sec^2(c+dx)+224\sec(c+dx)+63\right)}{693d\sec^{\frac{9}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*a^3*(63 + 224*Sec[c + d*x] + 355*Sec[c + d*x]^2 + 426*Sec[c + d*x]^3 + 5
68*Sec[c + d*x]^4 + 1136*Sec[c + d*x]^5)*Sin[c + d*x])/(693*d*Sec[c + d*x]^
(9/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.207, size = 115, normalized size = 0.5

$$-\frac{2a^2\left(63(\cos(dx+c))^6+161(\cos(dx+c))^5+131(\cos(dx+c))^4+71(\cos(dx+c))^3+142(\cos(dx+c))^2+568\,\mathrm{co}\right.}{693\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/693/d*a^2*(63*cos(d*x+c)^6+161*cos(d*x+c)^5+131*cos(d*x+c)^4+71*cos(d*x+
c)^3+142*cos(d*x+c)^2+568*cos(d*x+c)-1136)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1
/2)*cos(d*x+c)^6*(1/cos(d*x+c))^(11/2)/sin(d*x+c)
```

Maxima [B] time = 2.49706, size = 703, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/22176*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 3465*a
^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11
/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11
/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 31878*a
^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/
2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*c
)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1287*
a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/
2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2*c)
+ 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))
 + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)
)) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
11/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.93911, size = 338, normalized size = 1.4

$$\frac{2\left(63\,a^2\cos\left(dx+c\right)^6+224\,a^2\cos\left(dx+c\right)^5+355\,a^2\cos\left(dx+c\right)^4+426\,a^2\cos\left(dx+c\right)^3+568\,a^2\cos\left(dx+c\right)^2+1136\,a^2}{693\,\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^2*cos(d*x + c)^6 + 224*a^2*cos(d*x + c)^5 + 355*a^2*cos(d*x + c
)^4 + 426*a^2*cos(d*x + c)^3 + 568*a^2*cos(d*x + c)^2 + 1136*a^2*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) +
 d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/2), x)
```

3.242 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=38

$$\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (4*a^2*Sec[c + d*x]^(3/4)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.0564871, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3814, 8}

$$\frac{4a^2\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

```
[Out] (4*a^2*Sec[c + d*x]^(3/4)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt[4]{\sec(c+dx)}}\,dx = \frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + (4a)\int 0\,dx$$
$$= \frac{4a^2\sec^{\frac{3}{4}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.0806643, size = 45, normalized size = 1.18

$$\frac{4\sin(c+dx)\sec^{\frac{3}{4}}(c+dx)(a(\sec(c+dx)+1))^{3/2}}{d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/4),x]
```

```
[Out] (4*Sec[c + d*x]^(3/4)*(a*(1 + Sec[c + d*x]))^(3/2)*Sin[c + d*x])/(d*(1 + Se
c[c + d*x])^2)
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^{\frac{3}{2}}\frac{1}{\sqrt[4]{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

```
[Out] int((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x)
```

Maxima [B] time = 1.91563, size = 163, normalized size = 4.29

$$\frac{4\left(\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] 4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^(1/4))
```

Fricas [A] time = 2.08256, size = 132, normalized size = 3.47

$$\frac{4 a \sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}} \cos(dx+c)^{\frac{1}{4}} \sin(dx+c)}{d \cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)^(1/4)*sin(d*x + c)
/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a \sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/4), x)
```

3.243 $\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f
```

Rubi [A] time = 0.0576645, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3801, 215}

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a + a*Sec[e + f*x]]])/f
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.13311, size = 54, normalized size = 1.46

$$-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sin^{-1}\left(\sqrt{\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-2*ArcSin[Sqrt[Sec[e + f*x]]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])
/(f*Sqrt[1 - Sec[e + f*x]])
```

Maple [B] time = 0.222, size = 147, normalized size = 4.

$$\frac{\sqrt{2}\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)}{f\left(\sin\left(fx+e\right)\right)^2}\sqrt{\frac{a\left(1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}}\sqrt{\left(\cos\left(fx+e\right)\right)^{-1}}\left(\arctan\left(\frac{\sqrt{2}\left(\cos\left(fx+e\right)+1-\sin\left(fx\right.\right.}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*2^(1/2)*(1/cos(f*x+e)*a*(1+cos(f*x+e)))^(1/2)*(1/cos(f*x+e))^(1/2)*cos(
f*x+e)*(-1+cos(f*x+e))*(arctan(1/4*2^(1/2)*(-2/(1+cos(f*x+e)))^(1/2)*(cos(f
*x+e)+1-sin(f*x+e)))-arctan(1/4*2^(1/2)*(-2/(1+cos(f*x+e)))^(1/2)*(cos(f*x+
e)+1+sin(f*x+e))))/sin(f*x+e)^2/(-2/(1+cos(f*x+e)))^(1/2)
```

Maxima [B] time = 2.24869, size = 325, normalized size = 8.78

$$\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\sqrt{2}\sin\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*sq
rt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) - log(2*co
s(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2*e)^2 + 2*sqrt(2)*cos(1/2*f*x + 1
/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2) + log(2*cos(1/2*f*x + 1/2*e)^2
+ 2*sin(1/2*f*x + 1/2*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) + 2*sqrt(2)*sin
(1/2*f*x + 1/2*e) + 2) - log(2*cos(1/2*f*x + 1/2*e)^2 + 2*sin(1/2*f*x + 1/2
*e)^2 - 2*sqrt(2)*cos(1/2*f*x + 1/2*e) - 2*sqrt(2)*sin(1/2*f*x + 1/2*e) + 2
))/f
```

Fricas [B] time = 2.00403, size = 495, normalized size = 13.38

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^3-7\,a\cos\left(fx+e\right)^2-\frac{4\left(\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sin\left(fx+e\right)}{\sqrt{\cos\left(fx+e\right)}}+8\,a}{\cos\left(fx+e\right)^3+\cos\left(fx+e\right)^2}\right)}{2\,f},\ \frac{\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\cos\left(fx+e\right)}\sin}{a\cos\left(fx+e\right)^2-a\cos\left(fx+e\right)-2}\right.}{f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2
 - 2*cos(f*x + e))*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x
+ e)/sqrt(cos(f*x + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))/f, sqrt(-
a)*arctan(2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x +
```

```
e))*sin(f*x + e)/(a*cos(f*x + e)^2 - a*cos(f*x + e) - 2*a))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(1/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(sec(e + f*x)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(1/2)*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.244 $\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a - a*Sec[e + f*x]]])/f
```

Rubi [A] time = 0.0633156, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3801, 215}

$$\frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[e + f*x])/Sqrt[a - a*Sec[e + f*x]]])/f
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{-\sec(e+fx)}\sqrt{a-a\sec(e+fx)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, \frac{a\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a-a\sec(e+fx)}}\right)}{f}$$

Mathematica [C] time = 1.8488, size = 299, normalized size = 7.87

$$\csc\left(\frac{1}{2}(e+fx)\right)\sqrt{a-a\sec(e+fx)}\left(\log\left(-\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)-\sqrt{2}\cos\left(\frac{1}{2}(e+fx)\right)+2\right)-\log\left(-\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Sec[e + f*x]]*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]*((-2*I)*ArcTan[(Cos[(e + f*x)/4] - (-1 + Sqrt[2])*Sin[(e
+ f*x)/4])/((1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])] + (2*I)*Arc
Tan[(Cos[(e + f*x)/4] - (1 + Sqrt[2])*Sin[(e + f*x)/4])/((-1 + Sqrt[2])*Cos
[(e + f*x)/4] - Sin[(e + f*x)/4])] - 4*ArcTanh[Sqrt[2]*Cos[(2*e + f*x)/4]*S
ec[(f*x)/4] + Tan[(f*x)/4]] + Log[2 - Sqrt[2]*Cos[(e + f*x)/2] - Sqrt[2]*Si
n[(e + f*x)/2]] - Log[2 + Sqrt[2]*Cos[(e + f*x)/2] - Sqrt[2]*Sin[(e + f*x)/
2]])*Sqrt[a - a*Sec[e + f*x]])/(2*Sqrt[2]*f*Sqrt[-Sec[e + f*x]])
```

Maple [B] time = 0.252, size = 127, normalized size = 3.3

$$-\frac{\cos(fx+e)}{f\sin(fx+e)}\left(\text{Artanh}\left(\frac{-\cos(fx+e)-1+\sin(fx+e)}{2}\sqrt{\left(1+\cos(fx+e)\right)^{-1}}\right)-\text{Artanh}\left(\frac{\cos(fx+e)+1+\sin}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(arctanh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)-1+sin(f*x+e)))-arct
anh(1/2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)+1+sin(f*x+e))))*cos(f*x+e)*(-1
```

```
/cos(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(1/(1+co
s(f*x+e)))^(1/2)
```

Maxima [B] time = 2.43655, size = 477, normalized size = 12.55

$$-\frac{\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)\right)\right)^{2}+2\sin\left(\frac{1}{2}\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)\right)\right)^{2}+2\sqrt{2}\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*(log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) + log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2) - log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(
1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e))) + 2))/f
```

Fricas [B] time = 1.80491, size = 545, normalized size = 14.34

$$\left[\frac{\sqrt{a}\log\left(\frac{4\left(\cos\left(fx+e\right)^{3}+3\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)}}\sqrt{-\frac{1}{\cos\left(fx+e\right)}}+\left(a\cos\left(fx+e\right)^{2}+8a\cos\left(fx+e\right)+8a\right)\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}\right)}{2f},-\frac{\sqrt{-a}\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((4*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + 2*cos(f*x + e))*sq
rt(a)*sqrt((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e)) + (a*co
s(f*x + e)^2 + 8*a*cos(f*x + e) + 8*a)*sin(f*x + e))/(cos(f*x + e)^2*sin(f*
x + e)))/f, -sqrt(-a)*arctan(2*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-a)*sqr
t((a*cos(f*x + e) - a)/cos(f*x + e))*sqrt(-1/cos(f*x + e))/((a*cos(f*x + e)
 + 2*a)*sin(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sec\left(e+fx\right)}\sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**(1/2)*(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-sec(e + f*x))*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Giac [B] time = 2.41662, size = 124, normalized size = 3.26

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^(1/2)*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)/sqrt
(-a))/sqrt(-a) - sqrt(2)*arctan(sqrt(a)/sqrt(-a))/sqrt(-a))*abs(a)*sgn(tan(
1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

3.245 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=128

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/(Sqrt[a]*d)) + (
Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt
[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.266661, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3822, 4023, 3808, 206, 3801, 215}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]/(Sqrt[a]*d)) + (
Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(Sqrt[a]*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}(a-a\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{2a} + \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,d\right.}{}$$

$$= -\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.235714, size = 125, normalized size = 0.98

$$\frac{\tan(c+dx)\left(\sqrt{-(\sec(c+dx)-1)\sec(c+dx)} + \sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right) + 2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right) - \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+}}{\sqrt{1-\sec(c+}}\right.\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*A
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + S
ec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [B] time = 0.224, size = 222, normalized size = 1.7

$$\frac{(\cos(dx+c))^2(-1+\cos(dx+c))}{2\,ad\,(\sin(dx+c))^2}\left((\cos(dx+c))^{-1}\right)^{\frac{5}{2}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1+\sin(dx+c)}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d/a*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/
2)*(-1+cos(d*x+c))*(arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)-arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)-4*arctan(1/2*sin(d*x+c)
*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.3591, size = 1183, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
```

```
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

Fricas [A] time = 1.91325, size = 1310, normalized size = 10.23

$$\left[\frac{\sqrt{a}(\cos(dx+c)+1)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(d}{}\right.}{}}{4\,(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*(cos(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/s
qrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a)
 + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(a*d*cos(d*x + c) + a*d), -1/2*(2*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a
)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(co
s(d*x + c))/sin(d*x + c)) + sqrt(-a)*(cos(d*x + c) + 1)*arctan(2*sqrt(-a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*c
os(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)
```

3.246 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.163848, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3821, 3801, 215, 3808, 206}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rule 3821

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
 + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{a} - \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx \\
&= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad} \\
&= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.0930124, size = 89, normalized size = 0.94

$$\frac{\tan(c+dx)\left(\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*
(1 + Sec[c + d*x])])
```

Maple [B] time = 0.197, size = 181, normalized size = 1.9

$$\frac{(-1+\cos(dx+c))(\cos(dx+c))^2}{ad(\sin(dx+c))^2}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1-\sin(dx+c))}{4}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)-\sqrt{2}\,\mathrm{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))-2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c)))+2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^2*(1/cos(d*x+c))^(
3/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.26744, size = 643, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
```

```
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

Fricas [A] time = 2.16169, size = 936, normalized size = 9.85

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)}{\sqrt{\cos}}}{\cos(dx+c)^3+\cos(dx+c)^2}\right.}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c
) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arctan(sqrt
(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/s
in(d*x + c)) + sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
 2*a)))/(a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)
```

3.247 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0594848, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3808, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.064988, size = 75, normalized size = 1.34

$$-\frac{\sqrt{2}\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[
c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]))
```

Maple [B] time = 0.185, size = 99, normalized size = 1.8

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{ad(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(1/cos(d*x+c))^(1/2)*cos(d*x+c)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*a
rctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)/s
in(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [A] time = 2.22194, size = 122, normalized size = 2.18

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\right.}{2\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Fricas [A] time = 2.04311, size = 439, normalized size = 7.84

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d}, -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
 c))/sin(d*x + c))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(a*sec(d*x + c) + a), x)
```

3.248 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.111831, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3812, 3808, 206}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx\\ &= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.208703, size = 102, normalized size = 1.1

$$\frac{\tan(c+dx)\left(\frac{2\sqrt{1-\sec(c+dx)}}{\sqrt{\sec(c+dx)}} + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + (2*
Sqrt[1 - Sec[c + d*x]])/Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c
 + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.168, size = 100, normalized size = 1.1

$$\frac{1}{ad\sin(dx+c)}\left(\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\sin(dx+c) - 2\,\cos(dx+c) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-2*cos(d*x+c)+2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(1/co
s(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 2.26689, size = 140, normalized size = 1.51

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{2\,\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/(sq
rt(a)*d)
```

Fricas [A] time = 2.00059, size = 764, normalized size = 8.22

$$\left[\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)},\ \frac{\sqrt{2}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) -
```

```
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*c
os(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(
d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.249 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=131

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]])/(Sqrt[a]*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]
*Sqrt[a + a*Sec[c + d*x]]) - (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[
a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.21943, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3823, 4013, 3808, 206}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]])/(Sqrt[a]*d) + (2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]
*Sqrt[a + a*Sec[c + d*x]]) - (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[
a + a*Sec[c + d*x]])
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
```

```
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \int \frac{\sqrt{\sec(c}}{\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{2\,\mathrm{Subst}\left(\int \frac{}{2}\right.}{}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\sec}}{3d\sqrt{}}$$

Mathematica [A] time = 0.248948, size = 120, normalized size = 0.92

$$\frac{\tan(c+dx)\left(2(\cos(c+dx)-1)\sqrt{1-\sec(c+dx)}-3\sqrt{2}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*
Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x
])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])
])
```

Maple [A] time = 0.21, size = 120, normalized size = 0.9

$$-\frac{(\cos(dx+c))^2}{3ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\arctan\left(1/2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx+c)+1)^{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4
*cos(d*x+c)+2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 2.26055, size = 381, normalized size = 2.91

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Fricas [A] time = 2.13793, size = 863, normalized size = 6.59

$$\left[\frac{\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos\left(dx+c\right)+ad\right)}, -\frac{3\sqrt{2}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a)
- 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*(c
os(d*x + c)^2 - cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*(a*co
s(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*(cos(d*x + c)^2 - co
s(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.250 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{}\right.}{}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3
/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[
a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.34495, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3823, 4022, 4013, 3808, 206}

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3
/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[
a + a*Sec[c + d*x]])
```


Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\int}{}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{26}{1}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{26}{1}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{}{15d}$$

Mathematica [A] time = 1.08245, size = 117, normalized size = 0.69

$$\frac{\frac{15\sqrt{2}\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{\sqrt{1-\sec(c+dx)}} + \sin(c+dx)(-2\cos(c+dx)+3\cos(2(c+dx))+29)\sqrt{\sec(c+dx)}}{15d\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((29 - 2*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
 + (15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*
Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.212, size = 130, normalized size = 0.8

$$-\frac{(\cos(dx+c))^3}{15\,ad\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(6\,(\cos(dx+c))^3-15\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(6*cos(d*x+c)^3-15*arctan(1/2
*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
-8*cos(d*x+c)^2+28*cos(d*x+c)-26)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x
+c)
```

Maxima [B] time = 2.2817, size = 482, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

Fricas [A] time = 2.05869, size = 926, normalized size = 5.48

$$\left[\frac{\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+\frac{4\left(3\cos(dx+c)^3-\cos(dx+c)^2+13\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+}{\cos(dx+c)}}}{\sqrt{\cos(dx+c)}}}{30\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a
) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*
(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d
), 1/15*(15*sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))
 + 2*(3*cos(d*x + c)^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.251 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (-3*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
 (9*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.419276, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3816, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) +
 (9*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
2*d*(a + a*Sec[c + d*x])^(3/2)) + (3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-3a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{3a^2}{2}+3a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{2a^3}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{3\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c}}{2a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\,\text{ta}}{\sqrt{a+a}}\right.}{a^2 d}$$

$$= -\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.588843, size = 252, normalized size = 1.45

$$4\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx) + 6\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) - 9\sqrt{2}\tan(c+dx)\sec(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 4*Sqrt[1 - Sec[
c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[
Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 9*Sqrt[2]*ArcTan[(Sqr
t[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x]
+ 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x] + 18*Arc
Sin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.215, size = 281, normalized size = 1.6

$$\frac{(-1+\cos(dx+c))(\cos(dx+c))^3}{2\,da^2(\sin(dx+c))^3}\left(3\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)\sqrt{2}\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d/a^2*(-1+cos(d*x+c))*(3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)*sin(d*x+c)-3*arctan(1/4*2^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)*s
in(d*x+c)+3*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-9*arctan(1/2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-(-2/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)-2*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)
*cos(d*x+c)^3*(1/cos(d*x+c))^(7/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [B] time = 3.63063, size = 6661, normalized size = 38.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sin(4
*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqr
t(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sq
rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
```

```
))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*s
in(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqrt(
2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(
2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqr
t(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*
x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt
(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*
c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(
2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
 + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c)
+ 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
```

2) - 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c
)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2
*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*s
in(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 9*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4
*c) + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos
(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 9*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4
*c) + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x

```
+ 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos
(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(4*d*x + 4*c) + 2*c
os(2*d*x + 2*c) + 2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(cos(5/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 3*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x +
2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(
2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*
sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))/((sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt
(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(4*d*
x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*si
n(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqr
t(2)*a)*cos(4*d*x + 4*c) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(
2*d*x + 2*c) + 2*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + sqrt(2)*a)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*
(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*sin(4*d*x
+ 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c) + 2*sqrt(2)*a*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

Fricas [A] time = 2.2883, size = 1555, normalized size = 8.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 6*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*(3*cos(d*x + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(9*sqrt(2)*(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 6*(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*
cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x
 + c) + 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d
*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.252 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
(5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.282968, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3816, 4023, 3808, 206, 3801, 215}

$$-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
(5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-2a\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{5\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^2 d} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\right.}{}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.541061, size = 220, normalized size = 1.64

$$\frac{-2\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) + 5\sqrt{2}\tan(c+dx)\sec(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right) + 5\sqrt{2}\tan(c+dx)\tan}{4d\sqrt{1-\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 5*Sqrt[2]*ArcT
an[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 5*Sq
rt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d
*x]*Tan[c + d*x] - 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[
c + d*x] - 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(
4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] time = 0.195, size = 237, normalized size = 1.8

$$\frac{(-1+\cos(dx+c))(\cos(dx+c))^3}{2\,da^2(\sin(dx+c))^3}\left(2\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d/a^2*(2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))*sin(d*x+c)-2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)-(-2/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)+5*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+(-2/
(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*
cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [B] time = 3.25082, size = 2865, normalized size = 21.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
+ 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
```

```
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d
*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*s
in(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*s
in(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sq
rt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*
a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

Fricas [B] time = 2.28541, size = 1497, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^
2*d*cos(d*x + c) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) -
 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.253 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.122896, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3810, 3808, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
 + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2))
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
```

```
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\ &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [B] time = 0.565315, size = 220, normalized size = 2.27

$$\frac{2\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-\sqrt{2}\tan(c+dx)\sec(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-\sqrt{2}\tan(c+dx)\tan^{-1}\left(\ldots\right.}{4d\sqrt{1-\sec(c+d\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - Sqrt[2]
*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*T
an[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d
*x] + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sq
rt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.178, size = 146, normalized size = 1.5

$$\frac{(-1+\cos(dx+c))(\cos(dx+c))^2}{2\,da^2(\sin(dx+c))^3}\left(\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\cos(dx+c)-\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d/a^2*((-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*sin(d*x+c)-(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+
1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/(-2/
(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.00296, size = 903, normalized size = 9.31

$$\left[\frac{\sqrt{2}\left(\cos(dx+c)^2+2\,\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)+4\sqrt{\frac{a\cos}{\cos}}}{8\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/
4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*
x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.254 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.121989, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3811, 3808, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx &= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{3\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}\\ &= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{2ad}\\ &= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.16234, size = 120, normalized size = 1.24

$$\frac{-2\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-3\sqrt{2}\tan(c+dx)(\sec(c+dx)+1)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{4d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 3*Sqrt[2]*ArcT
an[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*
Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.186, size = 146, normalized size = 1.5

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{4\,da^2(\sin(dx+c))^3}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\cos(dx+c)+3\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d/a^2*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+
c)*((-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*arctan(1/2*sin(d*x+c)*(-2/(cos(d
*x+c)+1))^(1/2))*sin(d*x+c)-(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^
(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.1884, size = 1392, normalized size = 14.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*si
n(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
+ c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(
d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2*
sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c)
*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) +
 4*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(d
```

```
*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*sin
(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2*sq
rt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a)*d)
```

Fricas [A] time = 2.06276, size = 909, normalized size = 9.37

$$\left[\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\sqrt{}}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -
1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*si
n(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)
```

3.255 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
2*d*(a + a*Sec[c + d*x])^(3/2)) + (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.234774, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3817, 4013, 3808, 206}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
2*d*(a + a*Sec[c + d*x])^(3/2)) + (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*
d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{5a}{2}+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{7\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,d}{4a}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{7\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,d\right.}{}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx}}{2ad\sqrt{a+a\,s}}$$

Mathematica [A] time = 0.479293, size = 145, normalized size = 1.06

$$\frac{2\sin(c+dx)\left(5\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+4\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\right)+7\sqrt{2}\tan(c+dx)(\sec(c+dx)+1)\,\mathrm{ta}}{4d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*(5*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*Sqrt[-((-1 + Sec[c + d*
x])*Sec[c + d*x])])*Sin[c + d*x] + 7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d
*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])*Tan[c + d*x])/(4*d*Sqrt[1
- Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.196, size = 175, normalized size = 1.3

$$\frac{1}{4\,da^{2}\left(\sin\left(dx+c\right)\right)^{3}}\left(-7\,\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)\arctan\left(1/2\,\sin\left(dx+c\right)\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\right)\sqrt{-2\,\left(\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d/a^2*(-7*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)
+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+8*cos(d*x+c)^3+7*arctan(1/2*sin(d*x+c
)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*cos(d*x
+c)^2-12*cos(d*x+c)+10)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^3/(1
/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.52793, size = 9688, normalized size = 70.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)
^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) -
```

8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x
+ 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*
x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2
c)^3 + 4(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2
*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin(
1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/
2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*
cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(
1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2
+ 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*s
in(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/2
*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c
))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1

/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (
7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x +
 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)
 - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x +
1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + (8*(7*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 259*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(1/2*d*x + 1/2*c)^2 + 91*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^
2 - 104*sin(1/2*d*x + 1/2*c)^3 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*
c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 8*
(37*cos(1/2*d*x + 1/2*c)^2 + 21)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)
^2 + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*
c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^3 + 7*(log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*
d*x + 1/2*c)^3 + 13*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d
*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + (2*(7*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 7*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co

s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(3
/2*d*x + 3/2*c)^2 + 2*(84*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 7*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 - 16*(6*cos
(1/2*d*x + 1/2*c)^2 + 1)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*
(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d
*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x
+ 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2
*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*cos(3/2*d*x + 3/2*c) - 8*cos(1/2*d*x +
1/2*c))*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^3 + 2*cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(147*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*cos(1/2*d*x + 1/2*c)^3 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin
(1/2*d*x + 1/2*c)^2 - 40*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^3 - 56*(
3*cos(1/2*d*x + 1/2*c)^3 + cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(
3/2*d*x + 3/2*c) + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(
3/2*d*x + 3/2*c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*
x + 1/2*c) + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^3 - 8*sin(1/2*d*x + 1
/2*c)^4 + (7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c
)^2 - 4)*cos(3/2*d*x + 3/2*c)^2 + (35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c
) - 40*sin(1/2*d*x + 1/2*c)^2 - 36)*sin(3/2*d*x + 3/2*c)^2 - 4*(18*cos(1/2*
d*x + 1/2*c)^2 + 5)*sin(1/2*d*x + 1/2*c)^2 + 6*(7*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2
*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1

/2*c)^2 - 4*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 36*cos(1/2*d*x + 1
/2*c)^2 + 2*((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3
/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 14*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*sin(1/2*d*x + 1/2*c)^2 - 16*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(
3/2*d*x + 3/2*c) - 4*(18*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c))*
sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(133*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1
/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 21*(log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x +
 1/2*c)^3 - 24*sin(1/2*d*x + 1/2*c)^4 + 2*(21*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x
 + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*cos(3/2*d*x + 3/2*c)^2 - 8*(19*
cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c)^2 + 16*(7*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/
2*d*x + 1/2*c)^2 - 5*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 80*cos(1/
2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^4 + 11*c
os(1/2*d*x + 1/2*c)^2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/((4*sqrt(2)*a^2*cos(3/
2*d*x + 3/2*c)^4 + 28*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3*cos(1/2*d*x + 1/2*
c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^4 + 4*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c)^4 + 12*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3*sin(1/2*d*x + 1/2*c) + 10*sqr
t(2)*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/
2*d*x + 1/2*c)^4 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^
2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(5/2*d*x + 5
/2*c)^2 + (61*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^2*sin(1/2*d*
x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2
+ 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*
sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c
)^2)*sin(5/2*d*x + 5/2*c)^2 + (8*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 28*sq

```
rt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 37*sqrt(2)*a^2*cos(1/
2*d*x + 1/2*c)^2 + 13*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2
*c)^2 + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3 + 13*sqrt(2)*a^2*cos(3/2*d*
x + 3/2*c)^2*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 +
sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + (2*sqrt(2)*a^2*co
s(3/2*d*x + 3/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)
^2 + 2*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1
/2*c)^2)*cos(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin(1
/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*si
n(3/2*d*x + 3/2*c))*cos(5/2*d*x + 5/2*c) + 2*(21*sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^3 + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2)*cos(3
/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3 + sqrt(2)*a^2*cos
(3/2*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*
c)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2*sin(1/
2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 + 2*(sqrt(2)*a^2*cos(3/
2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c)
+ 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)
^2)*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(6*sqrt(2)*a^2*cos(3/2*d
*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 16*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 19*sqrt(2)*a^2*cos(1/2*d*x + 1/2*
c)^2*sin(1/2*d*x + 1/2*c) + 3*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3)*sin(3/2*d
*x + 3/2*c))*d)
```

Fricas [A] time = 2.18804, size = 1004, normalized size = 7.33

$$\left[\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(4\cos\right.}{}}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 + 5*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^
```

```
2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*cos(d*x + c)^2 + 5*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a(\sec(c+dx)+1))^{\frac{3}{2}}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.256 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{19\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sin}{2d\sqrt{\sec(c+dx)}(}$$

```
[Out] (11*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sqrt[Sec[c + d*x]
]*(a + a*Sec[c + d*x])^(3/2)) + (7*Sin[c + d*x])/(6*a*d*Sqrt[Sec[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) - (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqr
t[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.366936, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3817, 4022, 4013, 3808, 206}

$$\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{19\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sin}{2d\sqrt{\sec(c+dx)}(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (11*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sqrt[Sec[c + d*x]
]*(a + a*Sec[c + d*x])^(3/2)) + (7*Sin[c + d*x])/(6*a*d*Sqrt[Sec[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) - (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqr
t[a + a*Sec[c + d*x]])
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{7a}{2}+2a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{}{6a}$$

Mathematica [A] time = 0.907231, size = 150, normalized size = 0.85

$$\frac{\sqrt{1-\sec(c+dx)}(4\sin(c+dx)-\tan(c+dx)(19\sec(c+dx)+12))-33\sqrt{2}\sin(c+dx)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\tan}{6d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-33*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(
4*Sin[c + d*x] - (12 + 19*Sec[c + d*x])*Tan[c + d*x]))/(6*d*Sqrt[-((-1 + Se
c[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.202, size = 193, normalized size = 1.1

$$\frac{(\cos(dx+c))^2}{12\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(33\,(\cos(dx+c))^2\sin(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(33*cos(d*x+c)^2*sin(d*x+c)*
arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+
8*cos(d*x+c)^4-40*cos(d*x+c)^3-33*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))
^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+18*cos(d*x+c)^2+52*cos(d*x+c)-
38)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.30944, size = 1068, normalized size = 6.03

$$\left[\frac{33\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4(4}{24\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*sqrt(2)
*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) -
```

```
2*(4*cos(d*x + c)^3 - 12*cos(d*x + c)^2 - 19*cos(d*x + c))*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x +
c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.257 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{49}{1}$$

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sec[c + d*x]^(3/
2)*(a + a*Sec[c + d*x])^(3/2)) + (9*Sin[c + d*x])/(10*a*d*Sec[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]]) - (13*Sin[c + d*x])/(10*a*d*Sqrt[Sec[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.515508, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3817, 4022, 4013, 3808, 206}

$$-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{49}{1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (-15*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sec[c + d*x]^(3/
2)*(a + a*Sec[c + d*x])^(3/2)) + (9*Sin[c + d*x])/(10*a*d*Sec[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]]) - (13*Sin[c + d*x])/(10*a*d*Sqrt[Sec[c + d*x]]*
Sqrt[a + a*Sec[c + d*x]]) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a*d*Sq
rt[a + a*Sec[c + d*x]])
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
```

```
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{9a}{2}+3a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}$$

$$= -\frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} +$$

Mathematica [A] time = 1.25475, size = 163, normalized size = 0.75

$$\frac{(39\cos(c+dx)-2\cos(2(c+dx))+\cos(3(c+dx))+47)\tan(c+dx)\sqrt{1-\sec(c+dx)}\sec(c+dx)+75\sqrt{2}\sin(c+dx)\,c}{10d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos
[(c + d*x)/2]^2*Sec[c + d*x]^(5/2)*Sin[c + d*x] + (47 + 39*Cos[c + d*x] - 2
*Cos[2*(c + d*x)] + Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*T
an[c + d*x])/(10*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c
+ d*x]))^(3/2))
```

Maple [A] time = 0.211, size = 203, normalized size = 0.9

$$\frac{(\cos (dx+c))^3}{20\, da^2 (\sin (dx+c))^3}\sqrt{\frac{a(\cos (dx+c)+1)}{\cos (dx+c)}}\left(8\, (\cos (dx+c))^5 - 75\, (\cos (dx+c))^2 \sin (dx+c) \arctan \left(1/2\, \sin (dx+c) \sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/20/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(8*cos(d*x+c)^5-75*cos(d*x+c
)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*
x+c)+1))^(1/2)-24*cos(d*x+c)^4+96*cos(d*x+c)^3+75*arctan(1/2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-54*cos(d*x+c)^
2-124*cos(d*x+c)+98)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.13614, size = 1118, normalized size = 5.15

$$\left[\frac{75\sqrt{2}\left(\cos (dx+c)^2 + 2\, \cos (dx+c) + 1\right)\sqrt{a} \log \left(-\frac{a\cos(dx+c)^2 + 2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) - 2\, a\cos(dx+c) - 3\, a}{\cos(dx+c)^2 + 2\, \cos(dx+c) + 1}\right) + \frac{4\left(4\, c\right.}{}}{40\left(a^2 d \cos (dx+c)^2 + 2\, a^2 d \cos (dx+c) + a^2 d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos(d*x +
c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
, 1/20*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(
a*sin(d*x + c))) + 2*(4*cos(d*x + c)^4 - 4*cos(d*x + c)^3 + 36*cos(d*x + c)
^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.258 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$\frac{35\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-$$

```
[Out] (-5*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
 (115*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (15*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (35*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.562065, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3816, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{35\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-5*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) +
 (115*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a
*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (15*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
16*a*d*(a + a*Sec[c + d*x])^(3/2)) + (35*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-5a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{45a^2}{4}-\frac{35}{2}a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{8a^4}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} - $$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} - $$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{15\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{35\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16a^2d\sqrt{a+a\sec(c+dx)}} + $$

$$= -\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [A] time = 1.18125, size = 340, normalized size = 1.59

$$\frac{32\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{7}{2}}(c+dx)+110\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)+70\sin(c+dx)\sqrt{1-\sec(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (70*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 110*Sqrt[1 - S
ec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*Se
c[c + d*x]^(7/2)*Sin[c + d*x] - 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*
x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 230*Sqrt[2]*ArcTan[(Sqrt[2]*Sqr
t[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 115*Sq
rt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d
*x]^2*Tan[c + d*x] + 70*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2
*Tan[c + d*x] + 230*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c +
 d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.221, size = 454, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d/a^3*(1/cos(d*x+c))^(9/2)*cos(d*x+c)^4*(a*(cos(d*x+c)+1)/cos(d*x+c))
^(1/2)*(-1+cos(d*x+c))^2*(40*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)-40*2^(1/2)*arctan
(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c
)^2*sin(d*x+c)+40*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)*sin(d*x+c)-40*arctan(1/4*2^(1/2)*(-2/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)*sin(d*x+c)+
35*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-115*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)+20*(-2/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)^2-115*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*
sin(d*x+c)-39*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-16*(-2/(cos(d*x+c)+1))^(
1/2))/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^5
```

Maxima [B] time = 18.3205, size = 12215, normalized size = 57.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
6*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*c) + 7*si
n(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*(24*sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 96*(
sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 300*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*
d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 140*(sin(6*d*x + 6*c) + 7*
sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x +
4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c
)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x +
 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*s
qrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c)
 + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*
cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c)
+ 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c)
+ 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt
```

(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqr
t(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*s
qrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(
sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x
 + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt(2)*cos
(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*
c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*
d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4
*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*s
qrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2
)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*sin(6*d*
x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt
(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(
2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(

```
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4
*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*
x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(
2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7
*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x
 + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(s
qrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x
+ 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*co
s(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x
+ 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) +
 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c
) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c
) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4
*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*si
n(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt
(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d
*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)
*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x
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+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c
) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sq
rt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*s
in(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) +
 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(6*d*x + 6*c) +
7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) - 115*(2*(7*cos(4*d*x + 4*c) + 7*cos(2*d*x +
 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 14*(7*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 49*cos(4*d*x + 4*c)^2 + 49*cos(2*d*x + 2*c)^2 + 8*(c
os(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 16*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(cos(
6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 64*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 49*sin(4*d*x + 4*c)^2 + 98*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sin(2*d*x + 2*c)^2 + 8*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(sin(6*d*x + 6*c) + 7*s
in(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
64*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(6*d*x +
6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x +
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2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 115*(2*(7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 14*(7*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 49*cos(4*d*x + 4*c)^2 + 49*cos(2*d*x + 2*c)^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 49*sin(4*d*x + 4*c)^2 + 98*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sin(2*d*x + 2*c)^2 + 8*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 140*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(75*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 35*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 96*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 32*(24*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 560*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 140*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 560*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((sqrt(2)*a^2*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*a^2*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + 8*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c) + 8*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*d)

Fricas [A] time = 2.43082, size = 1796, normalized size = 8.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 80*(cos(d*x + c)^3 + 3*cos(d*x + c
)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d
*x + c)^2)) + 4*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(115*sq
rt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arc
tan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))/(a*sin(d*x + c))) + 80*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
+ c) + 1)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
) - 2*(35*cos(d*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.259 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
(43*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(4*d*(a + a*Sec[c + d*x])^(5/2)) - (11*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16
*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.429807, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3816, 4019, 4023, 3808, 206, 3801, 215}

$$-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
(43*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(4*d*(a + a*Sec[c + d*x])^(5/2)) - (11*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16
*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-4a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{11a^2}{4}-8a^2\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{8a^4}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+}}{a^3}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{11\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan}{\sqrt{a+a\,s}}\right.}{a^3d}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{1}{}$$

Mathematica [A] time = 0.673583, size = 308, normalized size = 1.77

$$\frac{-30\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx) - 22\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) + 43\sqrt{2}\tan(c+dx)\sec^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-22*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 30*Sqrt[1 - S
ec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*S
qrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 86*Sqrt[2]*ArcTan
[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c +
d*x] + 43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]
]]*Sec[c + d*x]^2*Tan[c + d*x] - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec
[c + d*x])^2*Tan[c + d*x] - 86*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x]
)^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)
)
```

Maple [B] time = 0.208, size = 406, normalized size = 2.3

$$-\frac{(-1+\cos(dx+c))^2(\cos(dx+c))^4}{16\,da^3(\sin(dx+c))^5}\left(16\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d/a^3*(-1+cos(d*x+c))^2*(16*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)*sin(d*x+c)-16*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*
x+c)*sin(d*x+c)+16*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)-16*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)-11*(-2/(cos(d*x+c)+1)
)^(1/2)*cos(d*x+c)^2+43*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)*sin(d*x+c)-4*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+43*arctan(1/2*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+15*(-2/(cos(d*x+c)+1))^(1/2)
)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin
(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 3.90356, size = 6734, normalized size = 38.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(19*
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2
```

)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*s
qrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4
*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x +
4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 +
 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqr
t(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4
*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*s
in(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2
)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2
*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)
*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqr
t(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16
*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6
*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos

(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 43*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 43*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c

) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*co
s(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x
 + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 1
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x
+ 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*
sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*
c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x +
2*c) + 4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*(19*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 19*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
1*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 76*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*
c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 76*(cos(4*d*x + 4*c) + 6*cos(2
*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*s
in(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 176*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 176*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 +
16*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*
sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*a^2*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 12*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*
a^2 + 2*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8
*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)
*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d
*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*s
qrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/

```
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*d)
```

Fricas [B] time = 2.78523, size = 1778, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*
x + c)^2)) - 4*(11*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(c
os(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt
(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*
sin(d*x + c))) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*(1
1*cos(d*x + c)^2 + 15*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
 c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.260 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.186849, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3810, 3808, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\int\frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}\\
&= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}+\frac{3\int\frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}\\
&= \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}-\frac{3\,\mathrm{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,-\frac{a\sqrt{\sec(c+dx}}{\sqrt{a+a\,\mathrm{se}}}\right.}{16a^2d}\\
&= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [B] time = 0.69831, size = 308, normalized size = 2.25

$$\frac{14\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)+6\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-3\sqrt{2}\tan(c+dx)\sec^2(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (6*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 14*Sqrt[1 - Sec
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt
[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 6*Sqrt[2]*ArcTan[(Sq
rt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x]
 - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Se
c[c + d*x]^2*Tan[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d
```

```
*x])^2*Tan[c + d*x] + 6*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan
[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.191, size = 210, normalized size = 1.5

$$-\frac{(-1+\cos(dx+c))^2(\cos(dx+c))^3}{16\,da^3(\sin(dx+c))^5}\left(3\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c))^2-3\arctan\left(1/2\sin(dx+c)\sqrt{-2\,(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d/a^3*(-1+cos(d*x+c))^2*(3*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-3*a
rctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+4*(-2
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2))*sin(d*x+c)-7*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x
+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(
d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.08945, size = 1127, normalized size = 8.23

$$\left[\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,a\cos\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x + c)^2 + 7*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/
(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3
*d), -1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) +
 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(3*cos(d*x + c)^2 + 7*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{(a\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.261 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (5*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.188127, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3811, 3810, 3808, 206}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + (5*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx &= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}\\
&= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}+\frac{5\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}\\
&= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}-\frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sqrt{\sec(c+}{\sqrt{a+a}}\right.}{16a^2d}\\
&= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 1.74039, size = 266, normalized size = 1.94

$$-\frac{10\sin(c+dx)\sqrt{1-\sec(c+dx)}(\sec(c+dx)+1)\sec^{\frac{5}{2}}(c+dx)+8\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)-10\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 10*Sqrt[1 - Se
c[c + d*x]]*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])*Sin[c + d*x] - 10*Sqrt[1
- Sec[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x] - 10*A
rcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] - 10*ArcSin
[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x] + 5*Sqrt[2]*ArcTan[(
Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Ta
n[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.196, size = 210, normalized size = 1.5

$$-\frac{(-1+\cos(dx+c))^2(\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\left(5\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c))^2-5\arctan\left(1/2\sin(dx+c)\sqrt{-2\,(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d/a^3*(-1+cos(d*x+c))^2*(5*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-5*a
rctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-4*(-2
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-5*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2))*sin(d*x+c)-(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c
))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*
x+c)^5
```

Maxima [B] time = 4.64965, size = 3881, normalized size = 28.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*sin
```

(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x +
 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*c
) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x +
 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*c
os(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*d
*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 1
2*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*
(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*
sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/

2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c) +
48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*c
os(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x + 3*c) + 1)*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*sin(3/2*d*x + 3/2*c))/(
(16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*cos(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*sin(3*d*x +
3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c) +
sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a^2*cos(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2*co
s(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2)*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*(2*sqrt(2)*a^2*s
in(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/

```
2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*d)
```

Fricas [A] time = 2.01401, size = 1122, normalized size = 8.19

$$\left[\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,a\cos(}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(5*cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
), -1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(5*cos(d*x + c)^2 + cos(d*x + c))*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.262 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(4*d*(a + a*Sec[c + d*x])^(5/2)) - (9*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*
a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.242129, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3817, 4012, 3808, 206}

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/
(4*d*(a + a*Sec[c + d*x])^(5/2)) - (9*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*
a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx &= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{7a}{2}+a\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}\\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{19\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx}{32a^2}\\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{19\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec \ldots}}{\sqrt{a \ldots}}\right.}{16a^2d}\\
&= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{9\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.860733, size = 146, normalized size = 1.07

$$\frac{-\sin(c+dx)(13\cos(c+dx)+9)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx) - 76\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\,\mathrm{ta}\ldots}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-76*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] - (9 + 13*Cos[c + d*x])*Sq
rt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c
+ d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.194, size = 208, normalized size = 1.5

$$\frac{\cos(dx+c)(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(13\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c))^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d/a^3*(1/cos(d*x+c))^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x
+c)*(-1+cos(d*x+c))^2*(13*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+19*arctan(
1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-4*(-2/(cos(
d*x+c)+1))^(1/2)*cos(d*x+c)+19*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1
/2))*sin(d*x+c)-9*(-2/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)^5/(-2/(cos(d*x+c)+1
))^(1/2)
```

Maxima [B] time = 5.66685, size = 4116, normalized size = 30.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3
*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
```

2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 +
 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x +
 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d
*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
+ 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4
c) + 104(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7/
2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10*
sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*si
n(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*s
in(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(1
/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2

```
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) +
 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
- 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*co
s(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 5*cos(5/2*
d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(3*d*x
+ 3*c) - 20*(6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)
+ 24*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1
/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 20*(4*cos(d*x + c) + 1)*sin(3/2*d*x + 3
/2*c) - 80*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 208*cos(1/2*d*x + 1/2*c)*sin
(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 52*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a^2*
cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(4*d*x + 4
*c)^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)
^2 + 48*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 16*sqrt(2)*a^2*sin(d*x
+ c)^2 + 8*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*
d*x + 3*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c) +
sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(
2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 12*(4*sqrt(2)*a^2*cos
(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*
c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x
 + 4*c) + 16*(3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*
sin(3*d*x + 3*c))*sqrt(a)*d)
```

Fricas [A] time = 2.10525, size = 1133, normalized size = 8.27

$$\left[\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(13*cos(d*x + c)^2 + 9*cos(d*x +
c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a
^3*d), -1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c
) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(13*cos(d*x + c)^2 + 9*cos(d*
x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
 + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)
```

3.263 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=177

$$\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (13*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
6*a*d*(a + a*Sec[c + d*x])^(3/2)) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
6*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.37984, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3817, 4020, 4013, 3808, 206}

$$\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (13*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
6*a*d*(a + a*Sec[c + d*x])^(3/2)) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(1
6*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{9a}{2}+2a\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-\frac{49a^2}{4}+\frac{13}{2}a^2\text{se}}{\sqrt{\sec(c+dx)}\sqrt{a+a}}}{8a^4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a+a\,\text{s}}}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a+a\,\text{s}}}$$

$$= -\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c-}}{16ad(a+}$$

Mathematica [A] time = 1.23222, size = 186, normalized size = 1.05

$$\frac{\sin(c+dx)\left(49\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)+85\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+32\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\right)}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (300*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + (85*Sqrt[1 - Sec[c + d*x
]]*Sec[c + d*x]^(3/2) + 49*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2) + 32*S
qrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.199, size = 236, normalized size = 1.3

$$-\frac{(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\left(-75\,(\cos(dx+c))^2\sin(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/32/d/a^3*(-1+cos(d*x+c))^2*(-75*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-150*cos(d*x+c)*s
in(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+
1))^(1/2)+64*cos(d*x+c)^3-75*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2
))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+106*cos(d*x+c)^2-72*cos(d*x+c)-98)*
(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/(1/cos(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.16138, size = 1188, normalized size = 6.71

$$\left[\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos\ldots}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 + 85*cos(d*x +
 c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x +
```

```
c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*cos(d
*x + c)^3 + 85*cos(d*x + c)^2 + 49*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.264 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{95\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{}{16ad\sqrt{\sec}}$$

```
[Out] (163*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Sqrt[Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(5/2)) - (17*Sin[c + d*x])/(16*a*d*Sqrt[Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(3/2)) + (95*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (299*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48
*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.50717, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3817, 4020, 4022, 4013, 3808, 206}

$$-\frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{95\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{}{16ad\sqrt{\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (163*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Sqrt[Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(5/2)) - (17*Sin[c + d*x])/(16*a*d*Sqrt[Sec[c + d*
x]]*(a + a*Sec[c + d*x])^(3/2)) + (95*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (299*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48
*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-\frac{11a}{2}+3a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}$$

$$= \frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{}{1}$$

Mathematica [A] time = 2.2171, size = 165, normalized size = 0.76

$$-\frac{\sec(c+dx)\left(\tan(c+dx)\sqrt{1-\sec(c+dx)}(379\sec(c+dx)+16\cos(2(c+dx))(5\sec(c+dx)-1)+487)+978\sqrt{2}\sin(c\right.}{48d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -(Sec[c + d*x]*(978*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 -
Sec[c + d*x]]*(487 + 379*Sec[c + d*x] + 16*Cos[2*(c + d*x)]*(-1 + 5*Sec[c +
 d*x]))*Tan[c + d*x]))/(48*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(
1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.208, size = 254, normalized size = 1.2

$$-\frac{(-1+\cos(dx+c))^2(\cos(dx+c))^2}{96\,da^3(\sin(dx+c))^5}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(489\,(\cos(dx+c))^2\sin(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/96/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*cos(
d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(c
os(d*x+c)+1))^(1/2)+978*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(co
s(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+64*cos(d*x+c)^4+489*arctan(1/
2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-384*cos(d*x+c)^3-686*cos(d*x+c)^2+408*cos(d*x+c)+598)*cos(d*x+c)^2*(1/cos
(d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.21058, size = 1256, normalized size = 5.79

$$\left[\frac{489 \sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right.}{192\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^4 - 160*cos(d*
```

```
x + c)^3 - 503*cos(d*x + c)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt(2)*(co
s(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*s
in(d*x + c))) - 2*(32*cos(d*x + c)^4 - 160*cos(d*x + c)^3 - 503*cos(d*x + c
)^2 - 299*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^
3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a \sec(dx+c)+a)^{\frac{5}{2}} \sec(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.265 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}} dx$

Optimal. Leaf size=126

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{4d}$$

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (7*ArcSinh[Tan[c
+ d*x]/Sqrt[1 + Sec[c + d*x]]])/(4*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
4*d*Sqrt[1 + Sec[c + d*x]]) + (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[1
 + Sec[c + d*x]])
```

Rubi [A] time = 0.285309, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3822, 4021, 4023, 3807, 215, 3801}

$$\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{\sec(c+dx)+1}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}+\frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (7*ArcSinh[Tan[c
+ d*x]/Sqrt[1 + Sec[c + d*x]]])/(4*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
4*d*Sqrt[1 + Sec[c + d*x]]) + (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[1
 + Sec[c + d*x]])
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx = \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{1}{4}\int \frac{(3-\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{1}{4}\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{2}+\frac{7}{2}\sec(c+\right.}{\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} + \frac{7}{8}\int \sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\sec(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx}}\right.}{4d}$$

$$= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{7\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{4d} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{1+\sec(c+dx)}} + \frac{\sec^{\frac{5}{2}}(c+dx}{2d\sqrt{1+s}}$$

Mathematica [A] time = 0.400099, size = 140, normalized size = 1.11

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(-2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}+\sin^{-1}\left(\sqrt{1-\sec(c\right.\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (Cot[c + d*x]*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 8*ArcSin[Sqrt[Sec[c + d*x]]
] - 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] -
 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Sqrt[-((-1 + Sec[c + d*x])*S
ec[c + d*x])])*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [B] time = 0.239, size = 255, normalized size = 2.

$$\frac{(\cos(dx+c))^2\left((\cos(dx+c))^2-1\right)}{16\,d\,(\sin(dx+c))^2}\left(7\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx+c))\right)(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(7*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))*cos(d*x+c)^2-7*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2-2*(-2/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)*sin(d*x+c)-16*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*
cos(d*x+c)^2+4*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*((cos(d*x+c)+1)/cos(d*
x+c))^(1/2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^2*(-2/(cos(d*x+c)+1))^(1/2)/sin
(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.34053, size = 2218, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2
)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*co
s(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
```

```
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
 + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(
sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d
*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) +
 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) +
2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [B] time = 2.08105, size = 936, normalized size = 7.43

$$\frac{8\left(\sqrt{2}\cos\left(dx+c\right)^2+\sqrt{2}\cos\left(dx+c\right)\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-7\left(\cos\left(dx+c\right)^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*log(-(2*sqrt(2)*sqr
t((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*
x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 7*(
cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos
(d*x + c) + 1)) + 7*(cos(d*x + c)^2 + cos(d*x + c))*log(-(cos(d*x + c)^2 -
2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - c
```

```
os(d*x + c) - 2)/(cos(d*x + c) + 1)) - 4*sqrt((cos(d*x + c) + 1)/cos(d*x +
c))*(cos(d*x + c) - 2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{\sec(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(sec(d*x + c) + 1), x)
```

3.266 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d - ArcSinh[Tan[c + d*x]
/Sqrt[1 + Sec[c + d*x]]]/d + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 +
Sec[c + d*x]])
```

Rubi [A] time = 0.185332, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3822, 4023, 3807, 215, 3801}

$$\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}-\frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d - ArcSinh[Tan[c + d*x]
/Sqrt[1 + Sec[c + d*x]]]/d + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 +
Sec[c + d*x]])
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
```

```
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx &= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}+\frac{1}{2}\int \frac{(1-\sec(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx\\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}-\frac{1}{2}\int \sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}\,dx+\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx\\
&= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}-\frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,-\right.}{d}\\
&= \frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}-\frac{\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}+\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.285565, size = 111, normalized size = 1.31

$$\frac{\tan(c+dx)\left(\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}+\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+}}{\sqrt{1-\sec(c+}}\right.\right.}{d\sqrt{-\tan^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] ((ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*A
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + S
ec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x])/(d*Sqrt[-Tan[c + d*x]^2])
```

Maple [B] time = 0.201, size = 218, normalized size = 2.6

$$\frac{(\cos(dx+c))^2(-1+\cos(dx+c))}{2d(\sin(dx+c))^2}\left((\cos(dx+c))^{-1}\right)^{\frac{5}{2}}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\left(\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1+\sin(dx+c))}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*((cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(
-1+cos(d*x+c))*(arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))*2^(1/2)*cos(d*x+c)-arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)-4*arctan(1/2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))/
(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.217, size = 1179, normalized size = 13.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
```

```
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [B] time = 2.12773, size = 826, normalized size = 9.72

$$\frac{2\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\left(\cos\left(dx+c\right)+1\right)\log\left(-\frac{\cos(dx+c)^2}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c) +
 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(
d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + (cos(d*x + c) + 1)*l
og(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) - (cos(d*x + c)
+ 1)*log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1)) + 4*sqrt((
cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
```

```
+ c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(sec(d*x + c) + 1), x)
```

3.267 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*ArcSinh[Tan[c
+ d*x]/Sqrt[1 + Sec[c + d*x]]])/d
```

Rubi [A] time = 0.10941, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3821, 3801, 215, 3807}

$$\frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{\sec(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*ArcSinh[Tan[c
+ d*x]/Sqrt[1 + Sec[c + d*x]]])/d
```

Rule 3821

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
 + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\sec(c+dx)}}\,dx &= -\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx+\int \sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}\,dx\\ &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,-\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}+\frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,-\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\\ &= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sinh^{-1}\left(\frac{\tan(c+dx)}{\sqrt{1+\sec(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0823011, size = 76, normalized size = 1.41

$$\frac{\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)-\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] ((2*ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]]
)/Sqrt[1 - Sec[c + d*x]]])*Cot[c + d*x]*Sqrt[-Tan[c + d*x]^2])/d
```

Maple [B] time = 0.193, size = 180, normalized size = 3.3

$$\frac{(\cos(dx+c))^2\left((\cos(dx+c))^2-1\right)}{2\,d\,(\sin(dx+c))^2}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(\cos(dx+c)+1+\sin(dx+c))}{4}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)-\sqrt{2}\,\text{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+s
in(d*x+c)))-2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1-sin(d*x+c)))-2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*((cos(
d*x+c)+1)/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2*(-2/(cos(d*x+
c)+1))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.18536, size = 639, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/d
```

Fricas [B] time = 2.10775, size = 614, normalized size = 11.37

$$\frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\log\left(-\frac{\cos(dx+c)^2+2\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)}{\cos(dx+c)+1}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos(d*x + c) - 3)/(cos(d*x + c
)^2 + 2*cos(d*x + c) + 1)) - log(-(cos(d*x + c)^2 + 2*sqrt((cos(d*x + c) +
1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d
*x + c) + 1)) + log(-(cos(d*x + c)^2 - 2*sqrt((cos(d*x + c) + 1)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c) - 2)/(cos(d*x + c) + 1))
)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{\sec(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(sec(c + d*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(sec(d*x + c) + 1), x)
```

3.268 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d
```

Rubi [A] time = 0.0391462, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3807, 215}

$$\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]

[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx = -\frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, -\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}$$
$$= \frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}$$

Mathematica [A] time = 0.0265921, size = 40, normalized size = 1.48

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\cos(c+dx)+1}}\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Sec[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]*Sqrt[(1 + Cos[c + d*x])^(-1)]
)/d
```

Maple [B] time = 0.182, size = 95, normalized size = 3.5

$$\frac{\cos(dx+c)\left((\cos(dx+c))^2-1\right)}{d(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*cos(d*x+c)*((cos(d*x+c)+1)/cos(d*x+c))^(1/2)*arcta
n(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)/sin(d
*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 1.94075, size = 117, normalized size = 4.33

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/d
```

Fricas [B] time = 2.01411, size = 240, normalized size = 8.89

$$\frac{\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((2*sqrt(2)*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(d*x + c) + 3)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\sec(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(sec(c + d*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{\sec(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(sec(d*x + c) + 1), x)
```

3.269 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]])
```

Rubi [A] time = 0.078271, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3812, 3807, 215}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]])
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx &= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}-\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx\\ &= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}+\frac{\sqrt{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,-\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}\\ &= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d}+\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.207902, size = 90, normalized size = 1.45

$$\frac{2\sin(c+dx)\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}+\sqrt{2}\tan(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{-\tan^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/(d*Sqrt[
-Tan[c + d*x]^2])
```

Maple [A] time = 0.155, size = 96, normalized size = 1.6

$$\frac{1}{d\sin(dx+c)}\left(\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\sin(dx+c)-2\,\cos(dx+c)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-2*cos(d*x+c)+2)*((cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(1/cos(d*
x+c))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 2.20209, size = 136, normalized size = 2.19

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 1.93437, size = 404, normalized size = 6.52

$$\frac{\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(-\frac{2\,\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)+\cos(dx+c)^2-2\,\cos(dx+c)-3}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)+4\,\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(}{2\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x + c) +
1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*cos(d
*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((cos(d*x + c)
+ 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sec(c + d*x) + 1)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(dx+c)+1}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sqrt(sec(d*x + c))), x)
```

3.270 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\sec(c+dx)+1}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d + (2*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) - (2*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*d*Sqrt[1 + Sec[c + d*x]])
```

Rubi [A] time = 0.151497, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3823, 4013, 3807, 215}

$$-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\sec(c+dx)+1}}+\frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}+\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d + (2*Sin[c + d*x])/(3*
d*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) - (2*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*d*Sqrt[1 + Sec[c + d*x]])
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
```

```
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{1}{3}\int \frac{1-2\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}} + \int \frac{\sqrt{\sec(c+}}{\sqrt{1+\sec(c}}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\sec(c+dx)}} - \frac{\sqrt{2}\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2\sqrt{\sec(c+dx)}\sin}{3d\sqrt{1+\sec(c}}$$

Mathematica [A] time = 0.225538, size = 118, normalized size = 1.2

$$\frac{\tan(c+dx)\left(2(\cos(c+dx)-1)\sqrt{1-\sec(c+dx)}-3\sqrt{2}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{\sec(c+dx)+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] ((2*(-1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*ArcTan[(Sqrt[2]*
Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x
```

```
])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[1 + Sec[c + d*x]])
```

Maple [A] time = 0.194, size = 116, normalized size = 1.2

$$-\frac{(\cos(dx+c))^2}{3d\sin(dx+c)}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\left(3\arctan\left(1/2\sin(dx+c)\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx+c)+1)^{-1}}\sin($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d*((cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(cos(
d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4*cos
(d*x+c)+2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [B] time = 1.89255, size = 377, normalized size = 3.85

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\sqrt{2}\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/d
```

Fricas [A] time = 1.97408, size = 448, normalized size = 4.57

$$\frac{3\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\log\left(\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\cos(dx+c)^2+2\cos(dx+c)+3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(\cos(dx+c)^2-\cos(dx+c)\right)\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\,s}{\sqrt{\cos(dx+c)}}}{6\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(sqrt(2)*cos(d*x + c) + sqrt(2))*log((2*sqrt(2)*sqrt((cos(d*x + c) +
 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - cos(d*x + c)^2 + 2*cos(
d*x + c) + 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^2 -
cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d
*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec\left(c+dx\right)+1}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(sec(c + d*x) + 1)*sec(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec\left(dx+c\right)+1}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

3.271 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$

Optimal. Leaf size=134

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{\sec(c+dx)+1}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c}{\sec(c+}\right.}{d}$$

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Sq
rt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(15*d*Sqrt[1 + Sec[c + d*x]])
```

Rubi [A] time = 0.234701, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3823, 4022, 4013, 3807, 215}

$$\frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{\sec(c+dx)+1}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{\sec(c+dx)+1}}-\frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}}-\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c}{\sec(c+}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSinh[Tan[c + d*x]/(1 + Sec[c + d*x])])/d) + (2*Sin[c + d*x])/
(5*d*Sec[c + d*x]^(3/2)*Sqrt[1 + Sec[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Sq
rt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(15*d*Sqrt[1 + Sec[c + d*x]])
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3807

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> -Dist[(Sqrt[2]*Sqrt[a])/(b*f), Subst[Int[1/Sqrt[1 + x
^2], x], x, (b*Cot[e + f*x])/(a + b*Csc[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d - a/b, 0] && GtQ[a, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{1}{5}\int \frac{1-4\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}}\,dx$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} - \frac{2}{15}\int \frac{}{\sqrt{\mathrm{s}}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} + \frac{26\sqrt{\sec(}}{15d\sqrt{}}$$

$$= \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{1+\sec(c+dx)}} + \frac{26\sqrt{\sec(}}{15d\sqrt{}}$$

$$= -\frac{\sqrt{2}\sinh^{-1}\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{1+\sec(c+dx)}} - \frac{2\sin(c}{15d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.289526, size = 122, normalized size = 0.91

$$\frac{\sin(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(13\sec^2(c+dx)-\sec(c+dx)+3\right)+15\sqrt{2}\sec^{\frac{5}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{15d\sqrt{-\tan^2(c+dx)}\sec^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[1 + Sec[c + d*x]]),x]
```

```
[Out] ((15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Se
c[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c + d*x] + 13*Sec[c +
d*x]^2))*Sin[c + d*x])/(15*d*Sec[c + d*x]^(3/2)*Sqrt[-Tan[c + d*x]^2])
```

Maple [A] time = 0.21, size = 126, normalized size = 0.9

$$-\frac{(\cos(dx+c))^3}{15\,d\sin(dx+c)}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\left(6\,(\cos(dx+c))^3-15\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\mathrm{co}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x)
```

```
[Out] -1/15/d*((cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(6*cos(d*x+c)^3-15*arctan(1/2*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*c
os(d*x+c)^2+28*cos(d*x+c)-26)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [B] time = 1.98978, size = 478, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/d
```

Fricas [A] time = 1.97565, size = 482, normalized size = 3.6

$$\frac{15\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\log\left(-\frac{2\sqrt{2}\sqrt{\frac{\cos(dx+c)+1}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\cos(dx+c)^2-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left(3\cos(dx+c)^3-\cos(dx+c)^2+13\cos(\cdots\right.}{\sqrt{\cos(dx\cdots}}}{30\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(15*(sqrt(2)*cos(d*x + c) + sqrt(2))*log(-(2*sqrt(2)*sqrt((cos(d*x + c
) + 1)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + cos(d*x + c)^2 - 2*c
os(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(3*cos(d*x + c)
```

```
^3 - cos(d*x + c)^2 + 13*cos(d*x + c))*sqrt((cos(d*x + c) + 1)/cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(1+sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sec(dx+c)+1}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(1+sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(sec(d*x + c) + 1)*sec(d*x + c)^(5/2)), x)
```

3.272 $\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2e\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(}\right.\right.}{5d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] (6*a*e*(e*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
+ (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*e*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3
) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3)
)], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1
/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3)
 - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*d*(a - a*Sec[c + d*x])*Sqrt[
a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 +
 Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rubi [A] time = 0.383354, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3806, 50, 63, 218}

$$\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2e\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right|-}{5d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (6*a*e*(e*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
+ (4*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*e*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3
) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3)
)], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1
/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3)
 - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*d*(a - a*Sec[c + d*x])*Sqrt[
a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 +
 Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int (e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{ex}}{\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2a^2e^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{2/3}\sqrt{a-ax}}\,d\right.}{5d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(6a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ax^3}{e}}}\,dx,x,\right.}{5d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(}}$$

$$= \frac{6ae\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2eF\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\,\text{se}}}{(1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\,\text{se}}}\right.\right.}{5d(a-a}$$

Mathematica [C] time = 0.230085, size = 71, normalized size = 0.22

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(e\sec(c+dx))^{4/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)}{d\sec^{\frac{4}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(4/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(4/3))
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{4}{3}}\sqrt{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec(dx+c) + a}\,(e \sec(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a \sec(dx+c) + a}\,(e \sec(dx+c))^{\frac{1}{3}} e \sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)*e*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(4/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(4/3), x)
```

3.273 $\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=280

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))],
-7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*
(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (
e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a + a*
Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[
3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rubi [A] time = 0.198076, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3806, 63, 218}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))],
-7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*
(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (
e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a + a*
Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[
3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2 e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{2/3}\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ax^3}{e}}}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a^2F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c\right.}{d(a-a\sec(c+dx))\sqrt{a+a\sec(c+dx)}\sqrt{\frac{}{((1-}}}$$

Mathematica [C] time = 0.1366, size = 71, normalized size = 0.25

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\sqrt[3]{e\sec(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},1-\sec(c+dx)\right)}{d\sqrt[3]{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/3
)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(1/3))
```

Maple [F] time = 0.337, size = 0, normalized size = 0.

$$\int \sqrt[3]{e\sec(dx+c)}\sqrt{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\sqrt[3]{e\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3), x)
```

3.274 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=326

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right.\right.}{2de(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] (3*a*Tan[c + d*x])/(2*d*(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]) +
(3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e
*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7
 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e
*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*
Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*d*e*(a - a*Sec[c + d*x])*Sqrt[a +
a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rubi [A] time = 0.243619, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3806, 51, 63, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right|-7-}{2de(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(2*d*(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]) +
(3^(3/4)*Sqrt[2 + Sqrt[3]]*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e
*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7
 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e
*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*
Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*d*e*(a - a*Sec[c + d*x])*Sqrt[a +
a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqr
```

```
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{2/3}}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{5/3}\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} - \frac{\left(a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{2/3}\sqrt{a-ax}}\,dx,x,\sec(c+\right.}{4d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} - \frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\frac{ax^3}{e}}}\,dx,x,\sqrt[3]{e\sec(c-\right.}{4de\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{2d(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}a^2F\left(\sin^{-1}\left(\frac{(1-\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{(1+\sqrt{3})\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7\right.}{2de(a-a\sec(c+dx)}$$

Mathematica [C] time = 0.148028, size = 71, normalized size = 0.22

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{2}{3}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{3}{2},1-\sec(c+dx)\right)}{d(e\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 5/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(2/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.24, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sec(dx+c)}\,(e\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{e\sec\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)/(e*sec(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(2/3), x)
```

3.275 $\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=716

$$-\frac{80\sqrt{2}3^{3/4}a^2e^{7/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+}}\right.\right.}{91d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] (60*a*e^2*(e*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(91*d*Sqrt[a + a*Sec[c + d*x
]]) + (6*a*e*(e*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(13*d*Sqrt[a + a*Sec[c +
d*x]]) - (240*a*e^3*Tan[c + d*x])/(91*d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt
[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))) + (120*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^
2*e^(7/3)*EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))
/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3
) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3)
+ (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^
2]*Tan[c + d*x])/(91*d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(
e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))^2]) - (80*Sqrt[2]*3^(3/4)*a^2*e^(7/3)*EllipticF[ArcSin[((
1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*
Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*S
qrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1
 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(91*d*(a - a
*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c +
 d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rubi [A] time = 0.560383, antiderivative size = 716, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3806, 50, 63, 303, 218, 1877}

$$-\frac{80\sqrt{2}3^{3/4}a^2e^{7/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right|-7}{91d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (60*a*e^2*(e*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(91*d*Sqrt[a + a*Sec[c + d*x
]]) + (6*a*e*(e*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(13*d*Sqrt[a + a*Sec[c +
d*x]]) - (240*a*e^3*Tan[c + d*x])/(91*d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt
[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))) + (120*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^
2*e^(7/3)*EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))
/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3
) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3)
+ (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^
2]*Tan[c + d*x])/(91*d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(
e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))^2]) - (80*Sqrt[2]*3^(3/4)*a^2*e^(7/3)*EllipticF[ArcSin[((
1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*
Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*S
qrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1
 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(91*d*(a - a
*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c +
 d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
```

```
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{8/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{(ex)^{5/3}}{\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}} - \frac{\left(10a^2e^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{(ex)^{2/3}}{\sqrt{a-ax}}\,dx, x\right.}{13d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}} - \frac{(40}{}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}} - \frac{(12}{}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}} + \frac{(12}{}$$

$$= \frac{60ae^2(e\sec(c+dx))^{2/3}\tan(c+dx)}{91d\sqrt{a+a\sec(c+dx)}} + \frac{6ae(e\sec(c+dx))^{5/3}\tan(c+dx)}{13d\sqrt{a+a\sec(c+dx)}} - \frac{}{91d}$$

Mathematica [C] time = 0.22952, size = 71, normalized size = 0.1

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(e\sec(c+dx))^{8/3}\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)}{d\sec^{\frac{8}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(8/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-5/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(8/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(8/3))
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{8}{3}}\sqrt{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{2}{3}}e^2\sec(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)*e^2*sec(d*x + c)^2
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(8/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec(dx + c) + a}\,(e \sec(dx + c))^{\frac{8}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(8/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(8/3), x)
```

3.276 $\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=673

$$-\frac{8\sqrt{2}3^{3/4}a^2e^{4/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+d}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+d}}\right.\right.}{7d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] (6*a*e*(e*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
- (24*a*e^2*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1
/3) - (e*Sec[c + d*x])^(1/3))) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*e^(4/3)*
EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec
[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c
 + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c +
 d*x])/(7*d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^
(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]) - (8*Sqrt[2]*3^(3/4)*a^2*e^(4/3)*EllipticF[ArcSin[((1 - Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x]
)^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3)
 + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*
e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*d*(a - a*Sec[c + d*x]
)*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rubi [A] time = 0.4783, antiderivative size = 673, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3806, 50, 63, 303, 218, 1877}

$$-\frac{8\sqrt{2}3^{3/4}a^2e^{4/3}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7\right.}{7d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (6*a*e*(e*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
- (24*a*e^2*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1
/3) - (e*Sec[c + d*x])^(1/3))) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*e^(4/3)*
EllipticE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec
[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c
 + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c +
 d*x])/(7*d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^
(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]) - (8*Sqrt[2]*3^(3/4)*a^2*e^(4/3)*EllipticF[ArcSin[((1 - Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x]
)^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3)
 + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*
e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(7*d*(a - a*Sec[c + d*x]
)*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
```

```
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{(ex)^{2/3}}{\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{\left(4a^2e^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{ex}\sqrt{a-ax}}\,dx\right.}{7d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{\left(12a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{ax^3}{e}}}\,dx,\right.}{7d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{\left(12a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1-\sqrt{3})\sqrt[3]{e}-x}{\sqrt{a-\frac{ax^3}{e}}}\right.}{7d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{6ae(e\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} - \frac{24ae^2\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e}\right.}$$

Mathematica [C] time = 0.220646, size = 71, normalized size = 0.11

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(e\sec(c+dx))^{5/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)}{d\sec^{\frac{5}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[-2/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(5/
3)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(5/3))
```

Maple [F] time = 0.192, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{5}{3}}\sqrt{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{2}{3}}\,e\sec(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)*e*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(5/3), x)
```

3.277 $\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=624

$$-\frac{2\sqrt{2}3^{3/4}a^2\sqrt[3]{e}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] (-6*a*e*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*e^(1/3)*Ellipti
cE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e
^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*
x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x]
)^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/
(d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (
e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]
) - (2*Sqrt[2]*3^(3/4)*a^2*e^(1/3)*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3)
- (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))]
, -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3
)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) -
 (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a +
a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rubi [A] time = 0.435255, antiderivative size = 624, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {3806, 63, 303, 218, 1877}

$$-\frac{2\sqrt{2}3^{3/4}a^2\sqrt[3]{e}\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7\ldots\right.}{d(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-6*a*e*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/3) -
(e*Sec[c + d*x])^(1/3))) + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*e^(1/3)*Ellipti
```

```
cE[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e
^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*
x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x]
)^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/
(d*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (
e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]
) - (2*Sqrt[2]*3^(3/4)*a^2*e^(1/3)*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3)
- (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))]
, -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3
)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) -
 (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(a - a*Sec[c + d*x])*Sqrt[a +
a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqr
t[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}\,dx = -\frac{\left(a^2 e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{ex}\sqrt{a-ax}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{ax^3}{e}}}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-x}{\sqrt{a-\frac{ax^3}{e}}}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(3\sqrt{2\left(2-\right.}\right.}{}$$

$$= -\frac{6ae\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)} + \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^2\sqrt[3]{}}{}$$

Mathematica [C] time = 0.138841, size = 71, normalized size = 0.11

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}(e\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},1-\sec(c+dx)\right)}{d\sec^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/3, 1/2, 3/2, 1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/3
)*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sec[c + d*x]^(2/3))
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{2}{3}}\sqrt{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2/3)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3), x)
```

3.278 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=662

$$\frac{\sqrt{2}3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7}{de^{2/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] (3*a*Tan[c + d*x])/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]) + (3
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*EllipticE[ArcSin[((1
 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*S
ec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sq
rt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1
+ Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*d*e^(2/3)*
(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*S
ec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]) +
 (Sqrt[2]*3^(3/4)*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c +
d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt
[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c +
d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(d*e^(2/3)*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec
[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rubi [A] time = 0.483359, antiderivative size = 662, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3806, 51, 63, 303, 218, 1877}

$$\frac{\sqrt{2}3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{de^{2/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]) + (3
*a*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*EllipticE[ArcSin[((1
 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*S
ec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sq
rt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1
+ Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*d*e^(2/3)*
(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*S
ec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]) +
 (Sqrt[2]*3^(3/4)*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c +
d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt
[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c +
d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(d*e^(2/3)*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec
[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])
*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
```

```
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt[3]{e\sec(c+dx)}}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{4/3}\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{ex}\sqrt{a-ax}}\,dx,x,\sec(c+dx\right)}{2d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{x}{\sqrt{a-\frac{ax^3}{e}}}\,dx,x,\sqrt[3]{e\sec(c+\right.}{2de\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(3a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-x}{\sqrt{a-\frac{ax^3}{e}}}\,dx,x,\sqrt[3]{e\sec(c\right.}{2de\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{d\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{3a\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}$$

Mathematica [C] time = 0.14724, size = 71, normalized size = 0.11

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt[3]{\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{3}{2},1-\sec(c+dx)\right)}{d\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(1/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(1/3))
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sec(dx+c)}\frac{1}{\sqrt[3]{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}{e\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)/(e*sec(d*x + c)),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt[3]{e\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(1/3), x)
```

3.279 $\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=715

$$\frac{5\,3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right),-7-\right.}{4\sqrt{2}de^{5/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}$$

```
[Out] (3*a*Tan[c + d*x])/(4*d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]]) +
(15*a*Tan[c + d*x])/(8*d*e*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]])
 + (15*a*Tan[c + d*x])/(8*d*e*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/
3) - (e*Sec[c + d*x])^(1/3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*EllipticE
[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(
1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x]
)^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^
(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(1
6*d*e^(5/3)*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^
(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]) + (5*3^(3/4)*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 -
4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Se
c[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(4*Sqrt[2]*d*e^(5/3)*(a - a*Sec[c + d*x]
)*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rubi [A] time = 0.53989, antiderivative size = 715, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3806, 51, 63, 303, 218, 1877}

$$\frac{5\,3^{3/4}a^2\tan(c+dx)\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)\sqrt{\frac{\sqrt[3]{e}\sqrt[3]{e\sec(c+dx)}+(e\sec(c+dx))^{2/3}+e^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}{\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}de^{5/3}(a-a\sec(c+dx))\sqrt{a\sec(c+dx)+a}\sqrt{\frac{\sqrt[3]{e}\left(\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{e}-\sqrt[3]{e\sec(c+dx)}\right)^2}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*a*Tan[c + d*x])/(4*d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + a*Sec[c + d*x]]) +
(15*a*Tan[c + d*x])/(8*d*e*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]])
 + (15*a*Tan[c + d*x])/(8*d*e*Sqrt[a + a*Sec[c + d*x]]*((1 + Sqrt[3])*e^(1/
3) - (e*Sec[c + d*x])^(1/3))) - (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^2*EllipticE
[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(
1/3) - (e*Sec[c + d*x])^(1/3))], -7 - 4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x]
)^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Sec[c + d*x])^(1/3) + (e*Sec[c + d*x])^
(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(1
6*d*e^(5/3)*(a - a*Sec[c + d*x])*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^
(1/3) - (e*Sec[c + d*x])^(1/3)))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^
(1/3))^2]) + (5*3^(3/4)*a^2*EllipticF[ArcSin[((1 - Sqrt[3])*e^(1/3) - (e*Se
c[c + d*x])^(1/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))], -7 -
4*Sqrt[3]]*(e^(1/3) - (e*Sec[c + d*x])^(1/3))*Sqrt[(e^(2/3) + e^(1/3)*(e*Se
c[c + d*x])^(1/3) + (e*Sec[c + d*x])^(2/3))/((1 + Sqrt[3])*e^(1/3) - (e*Sec
[c + d*x])^(1/3))^2]*Tan[c + d*x])/(4*Sqrt[2]*d*e^(5/3)*(a - a*Sec[c + d*x]
)*Sqrt[a + a*Sec[c + d*x]]*Sqrt[(e^(1/3)*(e^(1/3) - (e*Sec[c + d*x])^(1/3))
)/((1 + Sqrt[3])*e^(1/3) - (e*Sec[c + d*x])^(1/3))^2])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
```

```
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{(e\sec(c+dx))^{4/3}}\,dx = -\frac{\left(a^2e\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{7/3}\sqrt{a-ax}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} - \frac{\left(5a^2\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{(ex)^{4/3}\sqrt{a-ax}}\,dx,x,\sec(c\right.}{8d\sqrt{a-a\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(5a^2\tan(\right.}{16}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(15a^2\tan\right.}{16}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(15a^2\tan\right.}{}$$

$$= \frac{3a\tan(c+dx)}{4d(e\sec(c+dx))^{4/3}\sqrt{a+a\sec(c+dx)}} + \frac{15a\tan(c+dx)}{8de\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{}{8de\sqrt{a+}}$$

Mathematica [C] time = 0.174693, size = 71, normalized size = 0.1

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{4}{3}}(c+dx)\sqrt{a(\sec(c+dx)+1)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{3},\frac{3}{2},1-\sec(c+dx)\right)}{d(e\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/(e*Sec[c + d*x])^(4/3),x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 7/3, 3/2, 1 - Sec[c + d*x]]*Sec[c + d*x]^(4/3)*Sq
rt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*(e*Sec[c + d*x])^(4/3))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sec(dx+c)}\,(e\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a \sec (dx+c)+a}}{(e \sec (dx+c))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a \sec (dx+c)+a}\,(e \sec (dx+c))^{\frac{2}{3}}}{e^2 \sec (dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)/(e^2*sec(d*x + c)^
2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a (\sec (c+dx)+1)}}{(e \sec (c+dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/(e*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/(e*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec(dx+c)+a}}{(e\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/(e*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/(e*sec(d*x + c))^(4/3), x)
```

3.280 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{3\tan(c+dx)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (-3*AppellF1[2/3, 1/2, 1, 5/3, Sec[c + d*x], -Sec[c + d*x]]*(e*Sec[c + d*x]
)^(2/3)*Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.164334, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3828, 3827, 130, 510}

$$-\frac{3\tan(c+dx)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-3*AppellF1[2/3, 1/2, 1, 5/3, Sec[c + d*x], -Sec[c + d*x]]*(e*Sec[c + d*x]
)^(2/3)*Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt{1+\sec(c+dx)}\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(e\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt[3]{ex}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(3\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{3F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\sec(c+dx),-\sec(c+dx)\right)(e\sec(c+dx))^{2/3}\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 6.74803, size = 760, normalized size = 9.74

$$\frac{}{ad\left(270(2\cos(c+dx)+1)\cos^4\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{1}{2};\frac{1}{6},\frac{1}{3};\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2-3\tan^2\left(\frac{1}{2}(c+dx)\right)F\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (90*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*C
os[(c + d*x)/2]*Cos[c + d*x]^2*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d
*x])]*Sin[(c + d*x)/2]*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))/(a*d*(270*AppellF1[1/2, 1/6, 1/3,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]^4*(1 + 2*C
os[c + d*x]) + 10*(-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - 3*App
ellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(10*Appe
llF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c +
d*x)/2]^2*(2 - 9*Cos[c + d*x] + Cos[2*(c + d*x)]) - 5*AppellF1[3/2, 7/6, 1/
3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2 - 9*
Cos[c + d*x] + Cos[2*(c + d*x)]) + 6*(16*AppellF1[5/2, 1/6, 7/3, 7/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*AppellF1[5/2, 7/6, 4/3, 7/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 7*AppellF1[5/2, 13/6, 1/3, 7/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Sin[(c + d*x)/2]^2)*Tan[(c
 + d*x)/2]^2))
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int (e \sec (dx+c))^{\frac{2}{3}} \frac{1}{\sqrt{a+a \sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec (dx+c))^{\frac{2}{3}}}{\sqrt{a \sec (dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(c+dx))^{\frac{2}{3}}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(2/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(2/3)/sqrt(a*sec(d*x + c) + a), x)
```

3.281 $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{3\tan(c+dx)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (-3*AppellF1[1/3, 1/2, 1, 4/3, Sec[c + d*x], -Sec[c + d*x]]*(e*Sec[c + d*x]
)^(1/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.142169, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3828, 3827, 130, 429}

$$-\frac{3\tan(c+dx)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-3*AppellF1[1/3, 1/2, 1, 4/3, Sec[c + d*x], -Sec[c + d*x]]*(e*Sec[c + d*x]
)^(1/3)*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt{1+\sec(c+dx)}\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(e\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}(ex)^{2/3}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(3\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};\sec(c+dx),-\sec(c+dx)\right)\sqrt[3]{e\sec(c+dx)}\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [B] time = 7.51445, size = 749, normalized size = 9.86

$$
\overline{d\sqrt{a(\sec(c+dx)+1)}(e\sec(c+dx))^{2/3}\left(4320(4\cos(c+dx)-1)\cos^6\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{1}{2};-\frac{1}{6},\frac{2}{3};\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan\right.\right.}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (720*e*AppellF1[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x])^2*Sin[(c + d*x)/2]*(9*AppellF1[1/2,
-1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - (4*AppellF1[3/2,
 -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 5
/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)
)/(d*(e*Sec[c + d*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])]*(4320*AppellF1[1/2,
-1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]^2*Cos[(c + d*x)/2]
^6*(-1 + 4*Cos[c + d*x]) + 160*(4*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])^2*Cos[c + d*x]*Sin[(c + d*x)/2]^4 + 12*AppellF1
[1/2, -1/6, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[(c + d*x
)/2]^2*(20*AppellF1[3/2, -1/6, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*(7 + 14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] - 2*Cos[3*(c + d*x)]) + 5*
AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(7 +
14*Cos[c + d*x] + 5*Cos[2*(c + d*x)] - 2*Cos[3*(c + d*x)]) - 24*(40*AppellF
1[5/2, -1/6, 8/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 8*AppellF
1[5/2, 5/6, 5/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF1
[5/2, 11/6, 2/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x
]*Sin[(c + d*x)/2]^2)))
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int \sqrt[3]{e \sec (dx + c)} \frac{1}{\sqrt{a + a \sec (dx + c)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec (dx + c))^{\frac{1}{3}}}{\sqrt{a \sec (dx + c) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{e \sec(c + dx)}}{\sqrt{a (\sec(c + dx) + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(1/3)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec(dx + c))^{\frac{1}{3}}}{\sqrt{a \sec(dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(1/3)/sqrt(a*sec(d*x + c) + a), x)
```

3.282 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{3\tan(c+dx)F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}}$$

```
[Out] (3*AppellF1[-1/3, 1/2, 1, 2/3, Sec[c + d*x], -Sec[c + d*x]]*Tan[c + d*x])/(
d*Sqrt[1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.15897, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3828, 3827, 130, 510}

$$\frac{3\tan(c+dx)F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (3*AppellF1[-1/3, 1/2, 1, 2/3, Sec[c + d*x], -Sec[c + d*x]]*Tan[c + d*x])/(
d*Sqrt[1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt{1+\sec(c+dx)}\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(e\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}(ex)^{4/3}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= -\frac{(3\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\\
&= \frac{3F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt[3]{e\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 20.0723, size = 3346, normalized size = 44.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -(((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]*(-1 + Tan[(c +
d*x)/2]^2)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
```

*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*
(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/
3)))/(d*(e*Sec[c + d*x])^(1/3)*Sqrt[a*(1 + Sec[c + d*x])]*(-(Sec[(c + d*x)/
2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]^2*((2*AppellF
1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x
)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*AppellF1[1/2,
1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*
x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3))) - (Sec[(c + d*x)/
2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/6)*(-1 + Tan[(c + d*x)/2]^2)*((2*
AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(
c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*AppellF
1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[
(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3)))/2 - (Cos[(
c + d*x)/2]^2*Sec[c + d*x])^(1/6)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2
)*((2*AppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/
6) + (2*Tan[(c + d*x)/2]^2*(-(AppellF1[5/2, 1/6, 4/3, 7/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1
[5/2, 7/6, 1/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/10))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) - (5*A
ppellF1[3/2, 1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c
 + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(11/6)) - (
Tan[(c + d*x)/2]*(1 + (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3,
3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(Sec[(c
 + d*x)/2]^2)^(1/3) + (3*((-3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((-1 + Tan[(c
 + d*x)/2]^2)^2*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])*Tan[(c + d*x)/2]^2)) + (3*(-(AppellF1[3/2, 1/6, 4/3, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9

```
 + (AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/18))/((-1 + Tan[(c + d*x)/2]^2)*(9*Appe
llF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*App
ellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Appell
F1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d
*x)/2]^2)) - (3*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*((-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-(AppellF1[3/2, 1/6, 4/3,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/9 + (AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/18) + Tan[(c + d*x)/2]^2*(-(A
ppellF1[5/2, 7/6, 4/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (7*AppellF1[5/2, 13/6, 1/3, 7/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/10 -
 2*((-4*AppellF1[5/2, 1/6, 7/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 7/6, 4/3, 7/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
)/10))))/((-1 + Tan[(c + d*x)/2]^2)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2)))/(Sec[(c + d*x)/2]
^2)^(1/3)) - (Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)*((2*AppellF1[3/2,
1/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)
/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6) + (3*(1 + (3*AppellF1[1/2, 1/6, 1/
3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/((-1 + Tan[(c + d*x)/2]^2
)*(9*AppellF1[1/2, 1/6, 1/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ (-2*AppellF1[3/2, 1/6, 4/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
 + AppellF1[3/2, 7/6, 1/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*T
an[(c + d*x)/2]^2))))/(Sec[(c + d*x)/2]^2)^(1/3))*(-(Cos[(c + d*x)/2]*Sec[c
 + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/
(6*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/6)))))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{e \sec (dx+c)}}\frac{1}{\sqrt{a+a \sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a(\sec(c+dx)+1)}\,\sqrt[3]{e\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(1/3)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\,(e\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(1/3)), x)
```

3.283 $\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{3\tan(c+dx)F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{2/3}}$$

```
[Out] (3*AppellF1[-2/3, 1/2, 1, 1/3, Sec[c + d*x], -Sec[c + d*x]]*Tan[c + d*x])/(
2*d*Sqrt[1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.168797, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {3828, 3827, 130, 510}

$$\frac{3\tan(c+dx)F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)}{2d\sqrt{1-\sec(c+dx)}\sqrt{a\sec(c+dx)+a}(e\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (3*AppellF1[-2/3, 1/2, 1, 1/3, Sec[c + d*x], -Sec[c + d*x]]*Tan[c + d*x])/(
2*d*Sqrt[1 - Sec[c + d*x]]*(e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}\,dx &= \frac{\sqrt{1+\sec(c+dx)}\int \frac{1}{(e\sec(c+dx))^{2/3}\sqrt{1+\sec(c+dx)}}\,dx}{\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(e\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}(ex)^{5/3}(1+x)}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{(3\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-\frac{x^3}{e}}\left(1+\frac{x^3}{e}\right)}\,dx, x, \sqrt[3]{e\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= \frac{3F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};\sec(c+dx),-\sec(c+dx)\right)\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}(e\sec(c+dx))^{2/3}\sqrt{a+a\sec(c+dx)}}
\end{aligned}
$$

Mathematica [B] time = 6.81286, size = 585, normalized size = 7.5

$$
\sec^{\frac{7}{6}}(c+dx)\left(\frac{5\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\cos(c+dx)+1}}(3\cos(c+dx)-1)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{5/6}\left(2\tan^2\left(\frac{1}{2}(c+\cdots\right.\right.}{5\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)\left(3-4\sqrt{2}\left(\frac{1}{\cos(c+dx)+1}\right)^{2/3}\left(\frac{\cos(c+dx)}{\cos(c+dx)+1}\right)^{5/6}\tan^4\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{5}{2};\frac{11}{6},\frac{2}{3};\frac{7}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)+32\sin\left(\frac{1}{2}(c+\cdots\right.}\cdots\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((e*Sec[c + d*x])^(2/3)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sec[c + d*x]^(7/6)*((-3*Cos[(c + d*x)/2]*Sec[c + d*x]^(5/6)*(Sin[(c + d*x)
/2] - Sin[(3*(c + d*x))/2]))/2 + (5*Sqrt[(1 + Cos[c + d*x])^(-1)]*(-1 + 3*C
os[c + d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(5/6)*Sin[(c + d*x)/2]*(-3*C
os[c + d*x]^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, 2*Sin[(c + d*x)/2]^2]*(S
ec[(c + d*x)/2]^2)^(1/3) + 2*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/6)*Tan[(c + d*
x)/2]^2))/(-120*AppellF1[3/2, 5/6, 2/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(Cos[c + d*x]/(1 + Cos[c + d*x])
)^(5/6)*Sin[(c + d*x)/2]*Tan[(c + d*x)/2] + 32*AppellF1[5/2, 5/6, 5/3, 7/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(
Cos[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Sin[(c + d*x)/2]*Tan[(c + d*x)/2]^3
+ 5*Sqrt[2]*Cos[(c + d*x)/2]*(3 - 4*Sqrt[2]*AppellF1[5/2, 11/6, 2/3, 7/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*((1 + Cos[c + d*x])^(-1))^(2/3)*(Co
s[c + d*x]/(1 + Cos[c + d*x]))^(5/6)*Tan[(c + d*x)/2]^4))))/(d*(e*Sec[c + d
*x])^(2/3)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int (e \sec (dx + c))^{-\frac{2}{3}} \frac{1}{\sqrt{a + a \sec (dx + c)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a \sec (dx + c) + a} (e \sec (dx + c))^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(e\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(2/3)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*(e*sec(c + d*x))**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\left(e\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(2/3)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*(e*sec(d*x + c))^(2/3)), x)
```

3.284 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{2^{5/6}\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}F_1\left(\frac{1}{2};-\frac{1}{3},\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

```
[Out] (2^(5/6)*AppellF1[1/2, -1/3, 1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*x])
/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6))
```

Rubi [A] time = 0.109453, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3825, 133}

$$\frac{2^{5/6}\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}F_1\left(\frac{1}{2};-\frac{1}{3},\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2^(5/6)*AppellF1[1/2, -1/3, 1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*x])
/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\sec(c+dx)}\,dx = \frac{\sqrt[3]{a+a\sec(c+dx)}\int \sec^{\frac{4}{3}}(c+dx)\sqrt[3]{1+\sec(c+dx)}\,dx}{\sqrt[3]{1+\sec(c+dx)}}$$

$$= \frac{\left(\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x}}{\sqrt[6]{2-x}\sqrt{x}}\,dx, x, 1-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{5/6}}$$

$$= \frac{2^{5/6}F_1\left(\frac{1}{2}; -\frac{1}{3}, \frac{1}{6}; \frac{3}{2}; 1-\sec(c+dx), \frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+dx)}\tan(}{d(1+\sec(c+dx))^{5/6}}$$

Mathematica [B] time = 14.654, size = 1982, normalized size = 25.41

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Sec[c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[c
 + d*x]))^(1/3)*Sin[c + d*x])/(2*d*(1 + Sec[c + d*x])^(1/3)) + (3*(a*(1 + S
ec[c + d*x]))^(1/3)*(-((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(2/3)) + (Sec[
c + d*x]^(1/3)*(1 + Sec[c + d*x])^(1/3))/2)*Tan[(c + d*x)/2]*(-1 + (3*Appel
lF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*Appel
lF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*Ap
pellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Hype
rgeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2)))
/(2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1
/3)*(1 + Sec[c + d*x])^(1/3)*((3*(Sec[(c + d*x)/2]^2)^(1/3)*(-1 + (3*Appell
F1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*Appell
F1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*App
```

```
ellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + Hyper
geometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2)))/
(2*2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)) - (2^(1/3)*Tan[(c + d*x
)/2]^2*(-1 + (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*
Tan[(c + d*x)/2]^2)))/((Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c
 + d*x])^(1/3)) + (3*Tan[(c + d*x)/2]*((3*((-2*AppellF1[3/2, -1/3, 5/3, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/9 - (HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4}, T
an[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2) - (3*AppellF1[1/2, -1/3, 2/3, 3/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-2*(2*AppellF1[3/2, -1/3, 5/3, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4},
 {7/4}, Tan[(c + d*x)/2]^4])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*((-2*A
ppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])/9 - (HypergeometricPFQ[{2/3, 3/4}, {7/4}, T
an[(c + d*x)/2]^4]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) - 2*Tan[(c + d*x
)/2]^2*(2*(-(AppellF1[5/2, -1/3, 8/3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) - (AppellF1[5/2, 2/3, 5/3, 7/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2])/5) + (3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-HypergeometricPFQ[{2/3, 3
/4}, {7/4}, Tan[(c + d*x)/2]^4] + (1 - Tan[(c + d*x)/2]^4)^(-2/3)))/2)))/(9
*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2
*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
+ HypergeometricPFQ[{2/3, 3/4}, {7/4}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2
]^2)^2))/(2^(2/3)*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*
x])^(1/3)) - (Tan[(c + d*x)/2]*(-1 + (3*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(9*AppellF1[1/2, -1/3, 2/3, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*(2*AppellF1[3/2, -1/3, 5/3, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + HypergeometricPFQ[{2/3, 3/4}, {7/4
}, Tan[(c + d*x)/2]^4])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*
x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(2^(2
/3)*(Sec[(c + d*x)/2]^2)^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(4/3))))
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{\frac{4}{3}} \sqrt[3]{a+a\sec \left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^(4/3), x)
```

3.285 $\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

```
[Out] (2*2^(1/6)*AppellF1[1/2, -1/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*
x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6)
)
```

Rubi [A] time = 0.123868, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3825, 133}

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -1/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*
x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6)
)
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
```

```
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{4}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx &= \frac{(a+a\sec(c+dx))^{2/3}\int \sec^{\frac{4}{3}}(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}} \\ &= \frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{\sqrt[3]{1-x}\,\sqrt[6]{2-x}}{\sqrt{x}}\,dx,x,1-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}} \\ &= \frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};-\frac{1}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+a\sec(c+dx))^{2/3}}{d(1+\sec(c+dx))^{7/6}}\end{aligned}$$

Mathematica [C] time = 19.2487, size = 2618, normalized size = 33.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sec[c + d*x]^(1/3)*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c +
 d*x]))^(2/3)*Tan[(c + d*x)/2])/(d*(1 + Sec[c + d*x])^(2/3)) + (15*AppellF1
[1/2, 2/3, 1/3, 3/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*(a*(1 + Sec[c
 + d*x]))^(2/3)*(9*AppellF1[1/2, 2/3, 1/3, 3/2, Tan[(c + d*x)/4]^2, -Tan[(c
 + d*x)/4]^2] - 2*(AppellF1[3/2, 2/3, 4/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c
 + d*x)/4]^2] - 2*AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c
+ d*x)/4]^2])*Tan[(c + d*x)/4]^2)*Tan[(c + d*x)/2])/(d*(Sec[c + d*x]/(1 + S
ec[c + d*x]))^(2/3)*(1 + Sec[c + d*x])^(2/3)*((135*AppellF1[1/2, 2/3, 1/3,
3/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]^2*(5 + Cos[c + d*x]))/2 + 20*
(AppellF1[3/2, 2/3, 4/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*
AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])^2*Co
```

```
s[(c + d*x)/2]*Tan[(c + d*x)/4]^4 - 3*AppellF1[1/2, 2/3, 1/3, 3/2, Tan[(c +
 d*x)/4]^2, -Tan[(c + d*x)/4]^2]*Tan[(c + d*x)/4]^2*(5*AppellF1[3/2, 2/3, 4
/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*(5 - 12*Cos[(c + d*x)/2]
+ Cos[c + d*x]) - 10*AppellF1[3/2, 5/3, 1/3, 5/2, Tan[(c + d*x)/4]^2, -Tan[
(c + d*x)/4]^2]*(5 - 12*Cos[(c + d*x)/2] + Cos[c + d*x]) + 24*(2*AppellF1[5
/2, 2/3, 7/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*AppellF1[5/
2, 5/3, 4/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 5*AppellF1[5/2
, 8/3, 1/3, 7/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Cos[(c + d*x)/2]
*Tan[(c + d*x)/4]^2))) - ((Sec[(c + d*x)/2]^2)^(4/3)*Sec[c + d*x]^(1/3)*(a*
(1 + Sec[c + d*x]))^(2/3)*(AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1 + T
an[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2
])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*
x)/2]))^(1/3)) - AppellF1[-2/3, -1/3, -1/3, 1/3, (1 - I)/(1 + Tan[(c + d*x)
/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(1 + Tan[(c
 + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3))))
/(2^(1/3)*d*(-(((Sec[(c + d*x)/2]^2)^(1/3)*Tan[(c + d*x)/2]*(AppellF1[-2/3,
 -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c +
 d*x)/2])]/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)*((I + T
an[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)) - AppellF1[-2/3, -1/3, -1/
3, 1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]/(((
-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2]
)/(1 + Tan[(c + d*x)/2]))^(1/3))))/2^(1/3)) - (3*(Sec[(c + d*x)/2]^2)^(1/3)
*((((1/3 - I/3)*AppellF1[1/3, -1/3, 2/3, 4/3, (-1 - I)/(-1 + Tan[(c + d*x)/
2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1 + Tan[(c + d
*x)/2])^2 + ((1/3 + I/3)*AppellF1[1/3, 2/3, -1/3, 4/3, (-1 - I)/(-1 + Tan[(
c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1 + T
an[(c + d*x)/2])^2)/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3
)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(1/3)) - (AppellF1[-2/3,
 -1/3, -1/3, 1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c +
 d*x)/2])]*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2])) - (Sec[(c + d*x)
/2]^2*(-I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2])^2)))/(3*((-I + Ta
n[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(4/3)*((I + Tan[(c + d*x)/2])/(-1
+ Tan[(c + d*x)/2]))^(1/3)) - (AppellF1[-2/3, -1/3, -1/3, 1/3, (-1 - I)/(-1
 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*(Sec[(c + d*x)/2]^2
/(2*(-1 + Tan[(c + d*x)/2])) - (Sec[(c + d*x)/2]^2*(I + Tan[(c + d*x)/2]))/
(2*(-1 + Tan[(c + d*x)/2])^2)))/(3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c +
d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(4/3)) - (
((-1/3 - I/3)*AppellF1[1/3, -1/3, 2/3, 4/3, (1 - I)/(1 + Tan[(c + d*x)/2]),
 (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2])
^2 - ((1/3 - I/3)*AppellF1[1/3, 2/3, -1/3, 4/3, (1 - I)/(1 + Tan[(c + d*x)/
2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)
/2])^2)/(((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)) + (AppellF1[-2/3, -1/3, -1/3, 1
/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*(-(Sec[
(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(2*(1 + Tan[(c + d*x)/2])^2) + Sec[
```

```
(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((-I + Tan[(c + d*x)/2])/(1
+ Tan[(c + d*x)/2]))^(4/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^
(1/3)) + (AppellF1[-2/3, -1/3, -1/3, 1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (
1 + I)/(1 + Tan[(c + d*x)/2])]*(-(Sec[(c + d*x)/2]^2*(I + Tan[(c + d*x)/2])
)/(2*(1 + Tan[(c + d*x)/2])^2) + Sec[(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2
]))))/(3*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(4/3))))/2^(1/3)))
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{4}{3}} (a+a\sec (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)

[Out] int(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(4/3), x)
```

3.286 $\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=327

$$-\frac{5\tan^3(c+dx)(a(\sec(c+dx)+1))^{2/3}\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{4},\frac{7}{4},\tan^4\left(\frac{1}{2}(c+dx)\right)\right)}{8d\sqrt[3]{\frac{1}{\cos(c+dx)+1}}(\sec(c+dx)+1)^{10/3}}+$$

```
[Out] (-3*a*Sec[c + d*x]^(5/3)*Sin[c + d*x])/(2*d*(a*(1 + Sec[c + d*x]))^(1/3)) +
 (9*Sec[c + d*x]^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*Sin[c + d*x])/(4*d) - (
9*(a*(1 + Sec[c + d*x]))^(2/3)*Tan[c + d*x])/(4*d*((1 + Cos[c + d*x])^(-1))
^(1/3)*(1 + Sec[c + d*x])^(7/3)) + (Hypergeometric2F1[1/4, 1/3, 5/4, Tan[(c
 + d*x)/2]^4]*(Cos[c + d*x]*Sec[(c + d*x)/2]^4)^(1/3)*(a*(1 + Sec[c + d*x])
)^(2/3)*Tan[c + d*x])/(8*d*((1 + Cos[c + d*x])^(-1))^(1/3)*(1 + Sec[c + d*x
])^(4/3)) - (5*Hypergeometric2F1[1/3, 3/4, 7/4, Tan[(c + d*x)/2]^4]*(Cos[c
+ d*x]*Sec[(c + d*x)/2]^4)^(1/3)*(a*(1 + Sec[c + d*x]))^(2/3)*Tan[c + d*x]^
3)/(8*d*((1 + Cos[c + d*x])^(-1))^(1/3)*(1 + Sec[c + d*x])^(10/3))
```

Rubi [C] time = 0.122051, antiderivative size = 79, normalized size of antiderivative = 0.24, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3825, 133}

$$\frac{2\sqrt[6]{2}\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{2}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{7/6}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -2/3, -1/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d*
x])/2]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(7/6)
)
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+a\sec(c+dx))^{2/3}\,dx = \frac{(a+a\sec(c+dx))^{2/3}\int \sec^{\frac{5}{3}}(c+dx)(1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}}$$

$$= \frac{\left((a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{2/3}\sqrt[6]{2-x}}{\sqrt{x}}\,dx, x, 1-\sec(c+d\right.}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{7/6}}$$

$$= \frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};-\frac{2}{3},-\frac{1}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)(a+a\sec(c+dx))^{2}}{d(1+\sec(c+dx))^{7/6}}$$

Mathematica [A] time = 7.39605, size = 274, normalized size = 0.84

$$\frac{(a(\sec(c+dx)+1))^{2/3}\left(-5\sqrt[3]{2}\tan^3\left(\frac{1}{2}(c+dx)\right)\sqrt[3]{\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)}\sqrt[3]{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\text{Hypergeo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^(2/3)*(-3*Sec[(c + d*x)/2]^3*Sec[c + d*x]*(1 + Sec[
c + d*x])^(1/3)*(Sin[(c + d*x)/2] - 2*Sin[(3*(c + d*x))/2]) + 2^(1/3)*Hyper
geometric2F1[1/4, 1/3, 5/4, Tan[(c + d*x)/2]^4]*(Cos[c + d*x]*Sec[(c + d*x)
/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[(c + d*x)/2] - 5*2
^(1/3)*Hypergeometric2F1[1/3, 3/4, 7/4, Tan[(c + d*x)/2]^4]*(Cos[c + d*x]*S
ec[(c + d*x)/2]^4)^(1/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(1/3)*Tan[(c + d
*x)/2]^3))/(8*d*((1 + Cos[c + d*x])^(-1))^(1/3)*(1 + Sec[c + d*x])^(2/3))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{5}{3}} (a+a\sec (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^{\frac{5}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+a*sec(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^(5/3), x)
```

3.287 $\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2\,2^{5/6} a \tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}F_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

```
[Out] (2*2^(5/6)*a*AppellF1[1/2, 4/3, -5/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d
*x])/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6
))
```

Rubi [A] time = 0.13262, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3825, 133}

$$\frac{2\,2^{5/6} a \tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}F_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)}{d(\sec(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

```
[Out] (2*2^(5/6)*a*AppellF1[1/2, 4/3, -5/6, 3/2, 1 - Sec[c + d*x], (1 - Sec[c + d
*x])/2]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[c + d*x])^(5/6
))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
```

```
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx = \frac{\left(a\sqrt[3]{a+a\sec(c+dx)}\right)\int \frac{(1+\sec(c+dx))^{4/3}}{\sqrt[3]{\sec(c+dx)}}\,dx}{\sqrt[3]{1+\sec(c+dx)}}$$

$$= \frac{\left(a\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{(2-x)^{5/6}}{(1-x)^{4/3}\sqrt{x}}\,dx, x, 1-\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}(1+\sec(c+dx))^{5/6}}$$

$$= \frac{2\,2^{5/6}aF_1\left(\frac{1}{2};\frac{4}{3},-\frac{5}{6};\frac{3}{2};1-\sec(c+dx),\frac{1}{2}(1-\sec(c+dx))\right)\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{d(1+\sec(c+dx))^{5/6}}$$

Mathematica [C] time = 14.7493, size = 2325, normalized size = 29.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(4/3)/Sec[c + d*x]^(1/3),x]
```

```
[Out] (-3*(a*(1 + Sec[c + d*x]))^(4/3)*((1 + Sec[c + d*x])^(1/3)/Sec[c + d*x]^(1/
3) + Sec[c + d*x]^(2/3)*(1 + Sec[c + d*x])^(1/3))*(-8*Tan[(c + d*x)/2] + (A
ppellF1[-4/3, -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/
(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/(-1
+ Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))
^(2/3)) - AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)/2]),
(1 + I)/(1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)
/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[
(c + d*x)/2])^2))/(4*2^(2/3)*d*(Sec[(c + d*x)/2]^2)^(1/3)*(1 + Sec[c + d*x]
)^(4/3)*((Tan[(c + d*x)/2]*(-8*Tan[(c + d*x)/2] + (AppellF1[-4/3, -2/3, -2/
3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])
```

]*Sec[(c + d*x)/2]^2)/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2
/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)) - AppellF1[-4/3
, -2/3, -2/3, -1/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d
x)/2])]((-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(
c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2]^2))/(4*2^
(2/3)*(Sec[(c + d*x)/2]^2)^(1/3)) - (3*(-4*Sec[(c + d*x)/2]^2 + (Sec[(c + d
x)/2]^2(((-4/3 + (4*I)/3)*AppellF1[-1/3, -2/3, 1/3, 2/3, (-1 - I)/(-1 + T
an[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2)/(-1
 + Tan[(c + d*x)/2])^2 - ((4/3 + (4*I)/3)*AppellF1[-1/3, 1/3, -2/3, 2/3, (-
1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c +
d*x)/2]^2)/(-1 + Tan[(c + d*x)/2])^2))/(((-I + Tan[(c + d*x)/2])/(-1 + Tan[
(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)
) + (AppellF1[-4/3, -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1
 + I)/(-1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(((-I +
 Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*x)/2])/(
-1 + Tan[(c + d*x)/2]))^(2/3)) - AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)/(
1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x)/2]^2*(
(-I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2
])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2]) - (2*AppellF1[-4/3,
 -2/3, -2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c
+ d*x)/2])]*Sec[(c + d*x)/2]^2*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2
])) - (Sec[(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2
])^2)))/(3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(5/3)*((I + Ta
n[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)) - (2*AppellF1[-4/3, -2/3, -
2/3, -1/3, (-1 - I)/(-1 + Tan[(c + d*x)/2]), (-1 + I)/(-1 + Tan[(c + d*x)/2
])]*Sec[(c + d*x)/2]^2*(Sec[(c + d*x)/2]^2/(2*(-1 + Tan[(c + d*x)/2])) - (S
ec[(c + d*x)/2]^2*(I + Tan[(c + d*x)/2]))/(2*(-1 + Tan[(c + d*x)/2])^2)))/(
3*((-I + Tan[(c + d*x)/2])/(-1 + Tan[(c + d*x)/2]))^(2/3)*((I + Tan[(c + d*
x)/2])/(-1 + Tan[(c + d*x)/2]))^(5/3)) - ((-I + Tan[(c + d*x)/2])/(1 + Tan[
(c + d*x)/2]))^(1/3)*((I + Tan[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*
(1 + Tan[(c + d*x)/2])^2*(((4/3 + (4*I)/3)*AppellF1[-1/3, -2/3, 1/3, 2/3, (
1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec[(c + d*x
)/2]^2)/(1 + Tan[(c + d*x)/2])^2 + ((4/3 - (4*I)/3)*AppellF1[-1/3, 1/3, -2/
3, 2/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*Sec
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2])^2) - (AppellF1[-4/3, -2/3, -2/3, -1
/3, (1 - I)/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*((I + T
an[(c + d*x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])^2*(-(
Sec[(c + d*x)/2]^2*(-I + Tan[(c + d*x)/2]))/(2*(1 + Tan[(c + d*x)/2])^2) +
Sec[(c + d*x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((-I + Tan[(c + d*x)/2])
/(1 + Tan[(c + d*x)/2]))^(2/3)) - (AppellF1[-4/3, -2/3, -2/3, -1/3, (1 - I)
/(1 + Tan[(c + d*x)/2]), (1 + I)/(1 + Tan[(c + d*x)/2])]*((-I + Tan[(c + d*
x)/2])/(1 + Tan[(c + d*x)/2]))^(1/3)*(1 + Tan[(c + d*x)/2])^2*(-(Sec[(c + d
x)/2]^2(I + Tan[(c + d*x)/2]))/(2*(1 + Tan[(c + d*x)/2])^2) + Sec[(c + d*
x)/2]^2/(2*(1 + Tan[(c + d*x)/2]))))/(3*((I + Tan[(c + d*x)/2])/(1 + Tan[(c
 + d*x)/2]))^(2/3))))/(4*2^(2/3)*(Sec[(c + d*x)/2]^2)^(1/3))))

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^{\frac{4}{3}} \frac{1}{\sqrt[3]{\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

```
[Out] int((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx + c) + a)^{\frac{4}{3}}}{\sec(dx + c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(4/3)/sec(d*x+c)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{4}{3}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(4/3)/sec(d*x + c)^(1/3), x)
```

3.288 $\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$

Optimal. Leaf size=304

$$-\frac{a^4\left(8n^2+24n+3\right)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)(n+3)\sqrt{\sin^2(e+fx)}}+\frac{4a^4(2n+3)\sin(e+$$

```
[Out] (a^4*(30 + 21*n + 4*n^2)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)*(2 +
 n)*(3 + n)) + (Sec[e + f*x]^(1 + n)*(a^2 + a^2*Sec[e + f*x])^2*Sin[e + f*x
])/(f*(3 + n)) + (2*(4 + n)*Sec[e + f*x]^(1 + n)*(a^4 + a^4*Sec[e + f*x])*S
in[e + f*x])/(f*(2 + n)*(3 + n)) - (a^4*(3 + 24*n + 8*n^2)*Hypergeometric2F
1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e +
f*x])/(f*(1 - n)*(1 + n)*(3 + n)*Sqrt[Sin[e + f*x]^2]) + (4*a^4*(3 + 2*n)*H
ypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e
 + f*x])/(f*n*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.484646, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3814, 4018, 3997, 3787, 3772, 2643}

$$-\frac{a^4\left(8n^2+24n+3\right)\sin(e+fx)\sec^{n-1}(e+fx)\,_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)(n+3)\sqrt{\sin^2(e+fx)}}+\frac{4a^4(2n+3)\sin(e+fx)\sec^n(e+fx)\,_2}{fn(n+2)\sqrt{\sin^2(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
```


```
[Out] (a^4*(30 + 21*n + 4*n^2)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)*(2 +
 n)*(3 + n)) + (Sec[e + f*x]^(1 + n)*(a^2 + a^2*Sec[e + f*x])^2*Sin[e + f*x
])/(f*(3 + n)) + (2*(4 + n)*Sec[e + f*x]^(1 + n)*(a^4 + a^4*Sec[e + f*x])*S
in[e + f*x])/(f*(2 + n)*(3 + n)) - (a^4*(3 + 24*n + 8*n^2)*Hypergeometric2F
1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e +
f*x])/(f*(1 - n)*(1 + n)*(3 + n)*Sqrt[Sin[e + f*x]^2]) + (4*a^4*(3 + 2*n)*H
ypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e
 + f*x])/(f*n*(2 + n)*Sqrt[Sin[e + f*x]^2])
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4018

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*C
ot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n)/(f*(m + n)), x
] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^n
*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*Csc
[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx = \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)} + \frac{a\int \sec^n(e+fx)(a+a\sec$$

$$= \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(e+fx)\right)^2\sin(e+fx)}{f(3+n)} + \frac{2(4+n)\sec^{1+n}(e+fx)\left(a\right.}{f(2+}$$

$$= \frac{a^4\left(30+21n+4n^2\right)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\left(6+5n+n^2\right)} + \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(\right.}{f(3+n}$$

$$= \frac{a^4\left(30+21n+4n^2\right)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\left(6+5n+n^2\right)} + \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(\right.}{f(3+n}$$

$$= \frac{a^4\left(30+21n+4n^2\right)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\left(6+5n+n^2\right)} + \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(\right.}{f(3+n}$$

$$= \frac{a^4\left(30+21n+4n^2\right)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)\left(6+5n+n^2\right)} + \frac{\sec^{1+n}(e+fx)\left(a^2+a^2\sec(\right.}{f(3+n}$$

Mathematica [F] time = 0.590919, size = 0, normalized size = 0.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4,x]
```

```
[Out] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^4, x]
```

Maple [F] time = 0.962, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^n \left(a+a\sec\left(fx+e\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(fx+e)+a)^4 \sec(fx+e)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4\sec(fx+e)^4 + 4\,a^4\sec(fx+e)^3 + 6\,a^4\sec(fx+e)^2 + 4\,a^4\sec(fx+e) + a^4\right)\sec(fx+e)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*sec(f*x + e)^4 + 4*a^4*sec(f*x + e)^3 + 6*a^4*sec(f*x + e)^2
+ 4*a^4*sec(f*x + e) + a^4)*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int 4\sec(e+fx)\sec^n(e+fx)\,dx + \int 6\sec^2(e+fx)\sec^n(e+fx)\,dx + \int 4\sec^3(e+fx)\sec^n(e+fx)\,dx + \int \sec^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**4,x)
```

```
[Out] a**4*(Integral(4*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(6*sec(e + f*x)
**2*sec(e + f*x)**n, x) + Integral(4*sec(e + f*x)**3*sec(e + f*x)**n, x) +
Integral(sec(e + f*x)**4*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^4\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^4*sec(f*x + e)^n, x)
```

3.289 $\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$

Optimal. Leaf size=230

$$-\frac{a^3(4n+1)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{a^3(4n+7)\sin(e+fx)\sec^n(e}{}$$

```
[Out] (a^3*(5 + 2*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)*(2 + n)) + (Se
c[e + f*x]^(1 + n)*(a^3 + a^3*Sec[e + f*x])*Sin[e + f*x])/(f*(2 + n)) - (a^
3*(1 + 4*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Se
c[e + f*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[e + f*x]^2]) + (a^3
*(7 + 4*n)*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^n*Sin[e + f*x])/(f*n*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.276106, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3814, 3997, 3787, 3772, 2643}

$$-\frac{a^3(4n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{a^3(4n+7)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2}{}\right.}{fn(n+2)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```


```
[Out] (a^3*(5 + 2*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)*(2 + n)) + (Se
c[e + f*x]^(1 + n)*(a^3 + a^3*Sec[e + f*x])*Sin[e + f*x])/(f*(2 + n)) - (a^
3*(1 + 4*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Se
c[e + f*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[e + f*x]^2]) + (a^3
*(7 + 4*n)*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^n*Sin[e + f*x])/(f*n*(2 + n)*Sqrt[Sin[e + f*x]^2])
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
```

```
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx = \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin(e+fx)}{f(2+n)} + \frac{a\int \sec^n(e+fx)(a+a\sec(e$$

$$= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin}{f(2+n)}$$

$$= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin}{f(2+n)}$$

$$= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin}{f(2+n)}$$

$$= \frac{a^3(5+2n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)(2+n)} + \frac{\sec^{1+n}(e+fx)\left(a^3+a^3\sec(e+fx)\right)\sin}{f(2+n)}$$

Mathematica [F] time = 2.0934, size = 0, normalized size = 0.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^3, x]
```

Maple [F] time = 2.337, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^n \left(a+a\sec\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{3}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{3}\sec\left(fx+e\right)^{3}+3\,a^{3}\sec\left(fx+e\right)^{2}+3\,a^{3}\sec\left(fx+e\right)+a^{3}\right)\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int 3\sec\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int 3\sec^{2}\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int \sec^{3}\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int \sec^{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(3*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(3*sec(e + f*x)
**2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**3*sec(e + f*x)**n, x) + In
tegral(sec(e + f*x)**n, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{3}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)^n, x)
```

3.290 $\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$

Optimal. Leaf size=172

$$-\frac{a^2(2n+1)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)}{}$$

```
[Out] (a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)) - (a^2*(1 + 2*n)*Hyperg
eometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n
)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[e + f*x]^2]) + (2*a^2*Hypergeometric2
F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])/(f*n*
Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.139675, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3788, 3772, 2643, 4046}

$$-\frac{a^2(2n+1)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```


```
[Out] (a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + n)) - (a^2*(1 + 2*n)*Hyperg
eometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n
)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[e + f*x]^2]) + (2*a^2*Hypergeometric2
F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])/(f*n*
Sqrt[Sin[e + f*x]^2])
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx &= \left(2a^2\right)\int \sec^{1+n}(e+fx)\,dx+\int \sec^n(e+fx)\left(a^2+a^2\sec^2(e+fx)\right)\,dx\\
&= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)}+\frac{\left(a^2(1+2n)\right)\int \sec^n(e+fx)\,dx}{1+n}+\left(2a^2\cos^n(e+\right.\\
&= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)}+\frac{2a^2\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)\sec^n(e+fx)}{fn\sqrt{\sin^2(e+fx)}}\\
&= \frac{a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+n)}-\frac{a^2(1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1-}}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [F] time = 0.899763, size = 0, normalized size = 0.

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^2, x]
```

Maple [F] time = 1.121, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(fx+e\right)+a\right)^{2} \sec \left(fx+e\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2}\sec \left(fx+e\right)^{2}+2\,a^{2}\sec \left(fx+e\right)+a^{2}\right)\sec \left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int 2\sec\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int\sec^{2}\left(e+fx\right)\sec^{n}\left(e+fx\right)dx+\int\sec^{n}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**
2*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(fx+e\right)+a\right)^{2}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)^n, x)
```

3.291 $\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$\frac{a\sin(e+fx)\sec^n(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeom}\ldots}{f(1-n)\sqrt{\sin\ldots}}$$

```
[Out] -((a*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f
*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeom
etric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])
/(f*n*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0865364, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3787, 3772, 2643}

$$\frac{a\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] -((a*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f
*x]^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (a*Hypergeom
etric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e + f*x])
/(f*n*Sqrt[Sin[e + f*x]^2])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))\,dx = a\int \sec^n(e+fx)\,dx + a\int \sec^{1+n}(e+fx)\,dx$$
$$= \left(a\cos^n(e+fx)\sec^n(e+fx)\right)\int \cos^{-1-n}(e+fx)\,dx + \left(a\cos^n(e+fx)\sec^n(e+f$$
$$= -\frac{a\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}} + \frac{a\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2}\right.}{}$$

Mathematica [A] time = 0.146162, size = 106, normalized size = 0.8

$$\frac{a\sqrt{-\tan^2(e+fx)}\csc(e+fx)\sec^{n-1}(e+fx)\left((n+1)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\sec^2(e+fx)\right)+n\sec(e+fx)\text{Hy}\right.}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] (a*Csc[e + f*x]*Sec[e + f*x]^(-1 + n)*((1 + n)*Hypergeometric2F1[1/2, n/2,
(2 + n)/2, Sec[e + f*x]^2] + n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
 Sec[e + f*x]^2]*Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

Maple [F] time = 0.508, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(fx+e)+a)\sec(fx+e)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(fx+e)+a)\sec(fx+e)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sec(e+fx)\sec^n(e+fx)\,dx+\int \sec^n(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e)),x)
```

```
[Out] a*(Integral(sec(e + f*x)*sec(e + f*x)**n, x) + Integral(sec(e + f*x)**n, x)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (fx+e)+a)\sec (fx+e)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.292 $\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=174

$$\frac{(1-n)\sin(e+fx)\sec^{n-2}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-n}{2},\frac{4-n}{2},\cos^2(e+fx)\right)}{af(2-n)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)\sec^{n-1}(e+fx)\text{Hyper}}{af\sqrt{}}$$

```
[Out] (Sec[e + f*x]^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) + ((1 - n)*Hypergeom
etric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-2 + n)*S
in[e + f*x])/(a*f*(2 - n)*Sqrt[Sin[e + f*x]^2]) - (Hypergeometric2F1[1/2, (
1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e + f*x])/(a
*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.157535, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3820, 3787, 3772, 2643}

$$\frac{(1-n)\sin(e+fx)\sec^{n-2}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(e+fx)\right)}{af(2-n)\sqrt{\sin^2(e+fx)}}-\frac{\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+\right.}{af\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Sec[e + f*x]^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) + ((1 - n)*Hypergeom
etric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-2 + n)*S
in[e + f*x])/(a*f*(2 - n)*Sqrt[Sin[e + f*x]^2]) - (Hypergeometric2F1[1/2, (
1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^(-1 + n)*Sin[e + f*x])/(a
*f*Sqrt[Sin[e + f*x]^2])
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))}-\frac{(1-n)\int \sec^{-1+n}(e+fx)(a-a\sec(e+fx))\,dx}{a^2}\\ &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))}-\frac{(1-n)\int \sec^{-1+n}(e+fx)\,dx}{a}+\frac{(1-n)\int \sec^n(e+fx)\,dx}{a}\\ &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))}-\frac{\left((1-n)\cos^n(e+fx)\sec^n(e+fx)\right)\int \cos^{1-n}(e+fx)\,dx}{a}+\frac{\left((1-n)\right.\cdots}{\cdots}\\ &= \frac{\sec^n(e+fx)\sin(e+fx)}{f(a+a\sec(e+fx))}+\frac{(1-n)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(e+fx)\right)\sec^{-2+n}(e+fx)\sin(e+fx)}{af(2-n)\sqrt{\sin^2(e+fx)}}\cdots\end{aligned}$$

Mathematica [F] time = 0.946683, size = 0, normalized size = 0.

$$\int \frac{\sec^n(e+fx)}{a+a\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x]),x]
```

[Out] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x]), x]

Maple [F] time = 0.746, size = 0, normalized size = 0.

$$\int \frac{\left(\sec \left(fx+e\right)\right)^{n}}{a+a\sec \left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)

[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e)),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(fx+e\right)^{n}}{a\sec \left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec \left(fx+e\right)^{n}}{a\sec \left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="fricas")

[Out] integral(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^{n}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)**n/(sec(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{n}}{a\sec\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a), x)
```

3.293 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=217

$$-\frac{(3-2n)\sin(e+fx)\sec^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{2(2-n)\sin(e+fx)\sec^n(e+fx)}{}$$

```
[Out] (-2*(2 - n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
 - (Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - ((3 -
 2*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^(-1 + n)*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) + (2*(2 - n)*Hyp
ergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e +
 f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.308924, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3817, 4020, 3787, 3772, 2643}

$$-\frac{(3-2n)\sin(e+fx)\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{2(2-n)\sin(e+fx)\sec^n(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\right.}{3a^2f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```


```
[Out] (-2*(2 - n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x]))
 - (Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - ((3 -
 2*n)*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e +
f*x]^(-1 + n)*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) + (2*(2 - n)*Hyp
ergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*Sec[e + f*x]^n*Sin[e +
 f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2])
```


Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
```

```
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx = -\frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\int \frac{\sec^n(e+fx)(a(-3+n)-a(-1+n)\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^2}$$

$$= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\int \sec^n(e+fx)\left(-a^2(\right.}{}$$

$$= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{((3-2n)(1-n))\int \mathrm{se}}{3a^2}$$

$$= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left((3-2n)(1-n)\cos^{n}\right.}{}$$

$$= -\frac{2(2-n)\sec^{1+n}(e+fx)\sin(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{\sec^{1+n}(e+fx)\sin(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(3-2n)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\right.}{}$$

Mathematica [F] time = 1.46734, size = 0, normalized size = 0.

$$\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \frac{\left(\sec\left(fx+e\right)\right)^n}{\left(a+a\sec\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec\left(fx+e\right)^n}{a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^n\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^n}{\left(a\sec\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^2, x)
```

3.294 $\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=162

$$\frac{2\left(16n^2+24n+3\right)\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^n}{f(2n+3)}$$

```
[Out] (2*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt
[1 + Sec[e + f*x]]) + (2*Sec[e + f*x]^(1 + n)*Sqrt[1 + Sec[e + f*x]]*Sin[e
+ f*x])/(f*(3 + 2*n)) + (2*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n
, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[1 + Sec[
e + f*x]])
```

Rubi [A] time = 0.241929, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3814, 4016, 3806, 65}

$$\frac{2\left(16n^2+24n+3\right)\tan(e+fx)\,_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sqrt{\sec(e+fx)+1}\sec^{n+1}(e+fx)}{f(2n+3)}+\frac{2(4n}{f(2n}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt
[1 + Sec[e + f*x]]) + (2*Sec[e + f*x]^(1 + n)*Sqrt[1 + Sec[e + f*x]]*Sin[e
+ f*x])/(f*(3 + 2*n)) + (2*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n
, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[1 + Sec[
e + f*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{5/2}\,dx = \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\sin(e+fx)}{f(3+2n)} + \frac{2\int \sec^n(e+fx)\sqrt{1+\sec(e+\cdots}{\cdots}$$

$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\sin(\cdots}{f(3+2n)}$$

$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\sin(\cdots}{f(3+2n)}$$

$$= \frac{2(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\sec^{1+n}(e+fx)\sqrt{1+\sec(e+fx)}\sin(\cdots}{f(3+2n)}$$

Mathematica [C] time = 58.2682, size = 398, normalized size = 2.46

$$-\frac{i2^{n-\frac{5}{2}}e^{-\frac{1}{2}i(2n+3)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{3}{2}}\sec^5\left(\frac{1}{2}(e+fx)\right)(\sec(e+fx)+1)^{5/2}\left(\frac{10e^{i(n+2)(e+fx)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-n-1),\frac{n+4}{2},-e^{2i(e\cdots}\right)}{n+2}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2 + n)*((
10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (4 + n)/2, -E^(
(2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n)*(e + f*x))*Hypergeometric2F1[1
, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/(4 + n) + (E^(I*n*(e + f*x))
*Hypergeometric2F1[1, -3/2 - n/2, 1 + n/2, -E^((2*I)*(e + f*x))])/n + (5*E^
(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3 + n)/2, -E^((2*I)*(
e + f*x))])/(1 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[1, 1 - n/2
, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) + (10*E^(I*(3 + n)*(e + f*x))*H
ypergeometric2F1[1, -n/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*Sec[(e
 + f*x)/2]^5*(1 + Sec[e + f*x])^(5/2))/(E^((I/2)*(3 + 2*n)*(e + f*x))*f*Sec
[e + f*x]^(5/2))
```

Maple [F] time = 0.153, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{n} \left(1+\sec\left(fx+e\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{n} \left(\sec\left(fx+e\right)+1\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\sec\left(fx+e\right)^2+2\,\sec\left(fx+e\right)+1\right)\sec\left(fx+e\right)^n\sqrt{\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((sec(f*x + e)^2 + 2*sec(f*x + e) + 1)*sec(f*x + e)^n*sqrt(sec(f*x
+ e) + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^n\left(\sec\left(fx+e\right)+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(5/2), x)
```

3.295 $\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{2(4n+1)\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

```
[Out] (2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
+ (2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e +
 f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] time = 0.116606, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3814, 21, 3806, 65}

$$\frac{2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{\sec(e+fx)+1}}+\frac{2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```


```
[Out] (2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
+ (2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e +
 f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]])
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sec^n(e+fx)(1+\sec(e+fx))^{3/2}\,dx = \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}+\frac{2\int \frac{\sec^n(e+fx)\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\sec(e+fx)\right)}{\sqrt{1+\sec(e+fx)}}\,dx}{1+2n}$$

$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}+\frac{(1+4n)\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx}{1+2n}$$

$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}-\frac{((1+4n)\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{1-x}}\,dx,x,\sec\right.}{f(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+}$$

$$= \frac{2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}}+\frac{2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\tan(}{f(1+2n)\sqrt{1+\sec(e+fx)}}$$

Mathematica [A] time = 0.410474, size = 83, normalized size = 0.85

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}\sec^n(e+fx)\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},n+\frac{3}{2},\frac{3}{2},2\sin^2\left(\frac{1}{2}(e\right.\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x
)/2])/(f*n)
```

Maple [F] time = 0.148, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{n} \left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)^{n} \left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sec\left(fx+e\right)^{n} \left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sec \left(fx+e\right)^{n} \left(\sec \left(fx+e\right)+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.296 $\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx$

Optimal. Leaf size=45

$$\frac{2\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[1 + Sec[e + f*x]])
```

Rubi [A] time = 0.0517975, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3806, 65}

$$\frac{2\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[1 + Sec[e + f*x]])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sec^n(e+fx)\sqrt{1+\sec(e+fx)}\,dx = -\frac{\tan(e+fx)\,\mathrm{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{1-x}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{2\,{}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; 1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [A] time = 0.0372123, size = 45, normalized size = 1.

$$\frac{2\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2}, 1-n, \frac{3}{2}, 1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[1 + Sec[e + f*x]])
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int (\sec(fx+e))^n \sqrt{1+\sec(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sec(fx+e)^n \sqrt{\sec(fx+e)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sec\left(fx+e\right)^{n}\sqrt{\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sec\left(e+fx\right)+1}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(e + f*x) + 1)*sec(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sec\left(fx+e\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1), x)
```

3.297 $\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=59

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] time = 0.0769604, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= \frac{(2\tan(e+fx))\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{2-x^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= \frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 15.6912, size = 2938, normalized size = 49.8

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f*(3*AppellF1[1/2, -1/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(
1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]
*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*Appe
llF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(S
```

ec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f
*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Appel
lF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e +
 f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + S
ec[e + f*x]]*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3 + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[
1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-
1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2
 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Se
c[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*
x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-1/
2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1
 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)
*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*AppellF
1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 -
n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2
+ (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e +
f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/2
 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Ap

```
pellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n,
1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e +
 f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e + f
*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 + n, 1
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[
3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 +
 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{n}\frac{1}{\sqrt{1+\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{n}\left(e+fx\right)}{\sqrt{\sec\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**n/sqrt(sec(e + f*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(fx+e\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/sqrt(sec(f*x + e) + 1), x)
```

3.298 $\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2f\sqrt{\sec(e+fx)+1}}$$

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*f*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] time = 0.0802565, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*f*Sqrt[1 + Sec[e + f*x]])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 16.7738, size = 2990, normalized size = 48.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f*x
)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2
)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/
2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2))
/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2
```

, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/

```
(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (6*(3/2 + n)*A
ppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2
 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(-(Cos[(e + f*x)/2]*Sec[
e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))
/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{n}\left(1+\sec\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^{n}\sqrt{\sec\left(fx+e\right)+1}}{\sec\left(fx+e\right)^{2}+2\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(f*x
+ e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{n}\left(e+fx\right)}{\left(\sec\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**n/(sec(e + f*x) + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(fx+e\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.299 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(-\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] (2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) - (
(1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*
Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] time = 0.129473, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3814, 21, 3806, 64}

$$\frac{2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(-\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) - (
(1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*
Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int(-\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\int\frac{(-\sec(e+fx))^n\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\sec(e+fx)\right)}{\sqrt{1+\sec(e+fx)}}\,dx}{1+2n}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{(1+4n)\int(-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{((1+4n)\tan(e+fx))\,\text{Subst}\left(\int\frac{(-x)^{-1+n}}{\sqrt{1-x}}\,dx,\right.}{f(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(}$$

$$= \frac{2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(1+4n)\,{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(-\sec}{fn(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+}$$

Mathematica [A] time = 0.385606, size = 85, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}(-\sec(e+fx))^n\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},n+\frac{3}{2},\frac{3}{2},2\sin^2\left(\frac{1}{2}(\right.\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e +
f*x)/2])/(f*n)
```

Maple [F] time = 0.16, size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n} \left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n} \left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\sec\left(fx+e\right)\right)^{n} \left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(\sec \left(fx+e\right)+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.300 $\int (-\sec(e+fx))^n \sqrt{1+\sec(e+fx)}\,dx$

Optimal. Leaf size=64

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*Tan[e +
 f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.0548234, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3806, 64}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n\, {}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(-Sec[e + f*x])^n*Tan[e +
 f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (-\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{1-x}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [A] time = 0.0461574, size = 67, normalized size = 1.05

$$\frac{2\sin(e+fx)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*S
ec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^n\sqrt{1+\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^n\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\sec\left(e+fx\right)\right)^{n}\sqrt{\sec\left(e+fx\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

3.301 $\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.0693781, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3826, 136}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}x}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 6.21233, size = 2951, normalized size = 40.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2 -
n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f*
(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -
1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*
(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + S
ec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2,
-1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n
)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(
```

e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]
*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[
e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[3/2
, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt
[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])
^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2
+ n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Appe
llF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2])/3 + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan
[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n
)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*Appel
lF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/
2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*Ap
pellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]
*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*Ap
pellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 +
n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x
]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*App

```
ellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \frac{1}{\sqrt{1+\sec \left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\sqrt{\sec \left(fx+e\right)+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\sqrt{\sec \left(fx+e\right)+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(e+fx\right)\right)^{n}}{\sqrt{\sec \left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\sqrt{\sec \left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

3.302 $\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.0736466, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3826, 136}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
```

```
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}x^2}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 6.24974, size = 3003, normalized size = 41.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3 +
 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
 Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^
(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1
+ Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*S
ec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
llF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
```

```
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e
 + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) +
 (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/
2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
+ n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)
/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-((1
 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[3/2,
-1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2
]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + T
an[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3
/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(
-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 +
n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*Appe
llF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n,
 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2
]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(-
```

```
(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e
 + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)))
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(1+\sec \left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\left(\sec \left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(-\sec \left(fx+e\right)\right)^{n}\sqrt{\sec \left(fx+e\right)+1}}{\sec \left(fx+e\right)^{2}+2\,\sec \left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(f
*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec\left(e+fx\right)\right)^{n}}{\left(\sec\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.303 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] (2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) -
((1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]
])
```

Rubi [A] time = 0.125207, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3814, 21, 3806, 64}

$$\frac{2\tan(e+fx)(d\sec(e+fx))^n}{f(2n+1)\sqrt{\sec(e+fx)+1}}-\frac{(4n+1)\tan(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn(2n+1)\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[1 + Sec[e + f*x]]) -
((1 + 4*n)*Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*(1 + 2*n)*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]
])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^{3/2}\,dx = \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{2\int\frac{(d\sec(e+fx))^n\left(\frac{1}{2}+2n+\left(\frac{1}{2}+2n\right)\sec(e+fx)\right)}{\sqrt{1+\sec(e+fx)}}\,dx}{1+2n}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} + \frac{(1+4n)\int(d\sec(e+fx))^n\sqrt{1+\sec(e+fx}}{1+2n}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(d(1+4n)\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{(dx)^{-1+n}}{\sqrt{1-x}}\,d\right.}{f(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+\mathrm{se}}}$$

$$= \frac{2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{1+\sec(e+fx)}} - \frac{(1+4n)\,{}_2F_1\left(\frac{1}{2},n;1+n;\sec(e+fx)\right)(d\,\mathrm{sec}}{fn(1+2n)\sqrt{1-\sec(e+fx)}\sqrt{1+}}$$

Mathematica [A] time = 0.365603, size = 85, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)+1}(d\sec(e+fx))^n\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\mathrm{Hypergeometric2F1}\left(\frac{1}{2},n+\frac{3}{2},\frac{3}{2},2\sin^2\left(\frac{1}{2}\right.\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3/2
, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e +
 f*x)/2])/(f*n)
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(1+\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\sec\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (d\sec(fx+e))^n (\sec(fx+e)+1)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^(3/2), x)
```

3.304 $\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx$

Optimal. Leaf size=64

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},n,n+1,\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^n*Tan[e
+ f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.0566329, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3806, 64}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},n;n+1;\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1/2, n, 1 + n, Sec[e + f*x]]*(d*Sec[e + f*x])^n*Tan[e
+ f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int (d\sec(e+fx))^n\sqrt{1+\sec(e+fx)}\,dx = -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{{}_2F_1\left(\frac{1}{2}, n; 1+n; \sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [A] time = 0.0398213, size = 67, normalized size = 1.05

$$\frac{2\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2}, 1-n, \frac{3}{2}, 1-\sec(e+fx)\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 - n
)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Maple [F] time = 0.173, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^n\sqrt{1+\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^n\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(e+fx\right)\right)^{n}\sqrt{\sec\left(e+fx\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*sqrt(sec(e + f*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{n}\sqrt{\sec\left(fx+e\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1), x)
```

3.305 $\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.0685992, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3827, 133}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx = -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}(1+x)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 6.19341, size = 2951, normalized size = 40.42

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[1 + Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 - n + (-1 + 2*n)/2)*
(d*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2])/(f
*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2,
-1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]
*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 +
Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2
 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2,
 -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 +
n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
```

```
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[3/
2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqr
t[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2
 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*App
ellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/3 + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Ta
n[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*
n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*Appe
llF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1
/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*A
ppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Se
c[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]
]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*A
ppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1
+ n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*
x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*Ap
pellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.177, size = 0, normalized size = 0.

$$\int \left(d \sec \left(f x+e\right)\right)^{n} \frac{1}{\sqrt{1+\sec \left(f x+e\right)}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(f x+e\right)\right)^{n}}{\sqrt{\sec \left(f x+e\right)+1}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d \sec \left(f x+e\right)\right)^{n}}{\sqrt{\sec \left(f x+e\right)+1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{n}}{\sqrt{\sec\left(e+fx\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/sqrt(sec(e + f*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{\sec\left(fx+e\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(sec(f*x + e) + 1), x)
```

3.306 $\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.07366, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3827, 133}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx = -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}(1+x)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 6.21731, size = 3003, normalized size = 41.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(1 + Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[e
+ f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1
+ Tan[(e + f*x)/2]^2)^2)/(f*(1 + Sec[e + f*x])^(3/2)*(3*AppellF1[1/2, -3/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)
^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1
 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/
2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*
Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*App
ellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(
e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Ta
```

```
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n,
 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
+ (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1
/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x
)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-((
1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[3/2,
 -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2,
-3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n
)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/
2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-
3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*
(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 +
 n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*App
ellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n
, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(
-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[
e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/
```

```
2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2)))
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int \left(d\sec \left(fx+e\right)\right)^{n} \left(1+\sec \left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec \left(fx+e\right)\right)^{n}}{\left(\sec \left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sec \left(fx+e\right)\right)^{n}\sqrt{\sec \left(fx+e\right)+1}}{\sec \left(fx+e\right)^{2}+2\,\sec \left(fx+e\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*sqrt(sec(f*x + e) + 1)/(sec(f*x + e)^2 + 2*sec(
f*x + e) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{n}}{\left(\sec\left(e+fx\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(1+sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/(sec(e + f*x) + 1)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(\sec\left(fx+e\right)+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(1+sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(sec(f*x + e) + 1)^(3/2), x)
```

3.307 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=177

$$\frac{2a^3\left(16n^2+24n+3\right)\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)}}{f(2n+3)}$$

```
[Out] (2*a^3*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*
Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*Sec[e + f*x]^(1 + n)*Sqrt[a + a*Sec[e +
f*x]]*Sin[e + f*x])/(f*(3 + 2*n)) + (2*a^3*(3 + 24*n + 16*n^2)*Hypergeometr
ic2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*
n)*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.295454, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3814, 4016, 3806, 65}

$$\frac{2a^3\left(16n^2+24n+3\right)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)(2n+3)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sqrt{a\sec(e+fx)+a}\sec^{n+1}(e+fx)}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a^3*(7 + 4*n)*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*
Sqrt[a + a*Sec[e + f*x]]) + (2*a^2*Sec[e + f*x]^(1 + n)*Sqrt[a + a*Sec[e +
f*x]]*Sin[e + f*x])/(f*(3 + 2*n)) + (2*a^3*(3 + 24*n + 16*n^2)*Hypergeometr
ic2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*
n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^{5/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx)}\sin(e+fx)}{f(3+2n)} + \frac{(2a)\int \sec^n(e+fx)\sqrt{a+a}}{}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx}}{f(3+2n)}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx}}{f(3+2n)}$$

$$= \frac{2a^3(7+4n)\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)(3+2n)\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sec^{1+n}(e+fx)\sqrt{a+a\sec(e+fx}}{f(3+2n)}$$

Mathematica [C] time = 8.0659, size = 400, normalized size = 2.26

$$-\frac{i2^{n-\frac{5}{2}}e^{-\frac{1}{2}i(2n+3)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{3}{2}}\sec^5\left(\frac{1}{2}(e+fx)\right)(a(\sec(e+fx)+1))^{5/2}\left(\frac{10e^{i(n+2)(e+fx)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-n-1),\frac{n+4}{2},-e^{2i}\right.}{n+2}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(3/2 + n)*((
10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F1[1, (-1 - n)/2, (4 + n)/2, -E^(
(2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n)*(e + f*x))*Hypergeometric2F1[1
, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x))])/(4 + n) + (E^(I*n*(e + f*x))
*Hypergeometric2F1[1, -3/2 - n/2, 1 + n/2, -E^((2*I)*(e + f*x))])/n + (5*E^
(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3 + n)/2, -E^((2*I)*(
e + f*x))])/(1 + n) + (E^(I*(5 + n)*(e + f*x))*Hypergeometric2F1[1, 1 - n/2
, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) + (10*E^(I*(3 + n)*(e + f*x))*H
ypergeometric2F1[1, -n/2, (5 + n)/2, -E^((2*I)*(e + f*x))])/(3 + n))*Sec[(e
 + f*x)/2]^5*(a*(1 + Sec[e + f*x]))^(5/2))/(E^((I/2)*(3 + 2*n)*(e + f*x))*f
*Sec[e + f*x]^(5/2))
```

Maple [F] time = 0.164, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(fx+e\right)+a\right)^{\frac{5}{2}} \sec \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(5/2)*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2\right)\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*sqrt(a*sec(f*x + e
) + a)*sec(f*x + e)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.308 $\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=108

$$\frac{2a^2(4n+1)\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f
*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x
]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.144604, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3814, 21, 3806, 65}

$$\frac{2a^2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+fx)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f
*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x
]]*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \sec^n(e+fx)(a+a\sec(e+fx))^{3/2}\,dx &= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{(2a)\int \frac{\sec^n(e+fx)\left(a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right)\sec(e+fx)\right)}{\sqrt{a+a\sec(e+fx)}}\,dx}{1+2n}\\ &= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{(a(1+4n))\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx}}{1+2n}\\ &= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}-\frac{\left(a^3(1+4n)\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a-ax}}\,dx,\right.}{f(1+2n)\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec}}\\ &= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}+\frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(1+2n)\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.362795, size = 86, normalized size = 0.8

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sec^n(e+fx)\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},n+\frac{3}{2},\frac{3}{2},2\sin^2\left(\frac{1}{2}\right.\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*Sec[e + f*x]^n*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e
 + f*x)/2])/(f*n)
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}} \sec \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}} \sec \left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.309 $\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=48

$$\frac{2a\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*
Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.0659588, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3806, 65}

$$\frac{2a\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Tan[e + f*x])/(f*
Sqrt[a + a*Sec[e + f*x]])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sec^n(e+fx)\sqrt{a+a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{2a\,{}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; 1-\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.0910787, size = 51, normalized size = 1.06

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\text{Hypergeometric2F1}\left(\frac{1}{2}, 1-n, \frac{3}{2}, 1-\sec(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sqrt[a*(1 + Sec[e +
 f*x])]*Tan[(e + f*x)/2])/f
```

Maple [F] time = 0.193, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^n \sqrt{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)**n, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.310 $\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=61

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.133754, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3828, 3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^n(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx &= \frac{\sqrt{1+\sec(e+fx)}\int \frac{\sec^n(e+fx)}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}}\\ &= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{2-x^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

Mathematica [B] time = 6.21738, size = 2964, normalized size = 48.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 + (-1 + 2*n)/2)*(Cos
```

```
[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2])/(f
*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2,
 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]*Tan[(e
+ f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqr
t[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, -1/2 + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF
1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1
 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2
)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x
)/2]*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-1/2 + n)*Appe
llF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2
 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*App
ellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*((2*
(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((1 - n
)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-1/2 + n)*AppellF1[3/2, 1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*Appell
F1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[
(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 2
```

```
- n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*AppellF1[5/2, 1/2 + n, 2 - n, 7/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*Appel
lF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^
2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2]*Tan[e + f*x])/(Sq
rt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2
, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]*(-(
Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e
+ f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2)))
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{n} \frac{1}{\sqrt{a+a\sec\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(fx+e\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(fx+e\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{n}\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.311 $\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.14846, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3828, 3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{2af\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 - Sec[e + f*x], (1 - Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^n(e+fx)}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1+\sec(e+fx)}\int \frac{\sec^n(e+fx)}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}\\
&= \frac{\tan(e+fx)\operatorname{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(2\tan(e+fx))\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1-\sec(e+fx)}\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [B] time = 6.23303, size = 2992, normalized size = 44.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 + (-3 + 2*n)/2)*(Cos[(e + f*x
```

)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2
)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF
1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]
^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
x)/2]^2] + (2(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*AppellF1
[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e
+ f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2
+ n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF
1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f
*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2)/(
3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*n*AppellF1[1/2
, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*
x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(
e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2,
 -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n
)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e +
 f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[3/2
, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/
2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*Ap
pellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)
^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e +

```
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(
-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[3
/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
 f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 -
n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 + n)*AppellF1[5/2, -
1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*AppellF1[5/2, -1/2 + n
, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2 + n, 1 - n, 7/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)
)))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (6*(3/2 +
n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^
(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(-(Cos[(e + f*x)/2]*
Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*
x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(fx+e\right)^{n}}{\left(a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)^n/(a^2*sec(f*x + e)^2 + 2*a^
2*sec(f*x + e) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{n}\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.312 $\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(4n+1)\sin(e+fx)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+}{f(2n+1)}$$

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec
[e + f*x])^n*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec
[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a +
a*Sec[e + f*x]])
```

Rubi [A] time = 0.161286, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3814, 21, 3806, 67, 65}

$$\frac{2a^2(4n+1)\sin(e+fx)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec
[e + f*x])^n*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Sec
[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a +
a*Sec[e + f*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (-\sec(e+fx))^n (a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(2a)\int \frac{(-\sec(e+fx))^n\left(a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right)\sec(e\ldots\right.}{\sqrt{a+a\sec(e+fx)}}}{1+2n}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(a(1+4n))\int(-\sec(e+fx))^n\sqrt{a+a\,\mathrm{se}\ldots}}{1+2n}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^3(1+4n)\tan(e+fx)\right)\mathrm{Subst}\left(\int\frac{(-x\ldots}{\sqrt{a\ldots}}\right.}{f(1+2n)\sqrt{a-a\sec(e+fx)}\sqrt{a\ldots}}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3(1+4n)(-\sec(e+fx))^n\sec^{1-n}(e+\ldots\right.}{f(1+2n)\sqrt{a-a\ldots}}$$

$$= \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+f\ldots}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.344008, size = 88, normalized size = 0.68

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}(-\sec(e+fx))^n\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},n+\frac{3}{2},\frac{3}{2},2\sin^2\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*(-Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*Tan
[(e + f*x)/2])/(f*n)
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.313 $\int (-\sec(e+fx))^n \sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n
*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.0764576, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3806, 67, 65}

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^n \sec^{1-n}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n
*Sec[e + f*x]^(1 - n)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (-\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx = \frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(-x)^{-1+n}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2(-\sec(e+fx))^n\sec^{1-n}(e+fx)\sin(e+fx)\right)\text{Subst}\left(\int\frac{x^{-1+n}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)(-\sec(e+fx))^n\sec^{1-n}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.120182, size = 71, normalized size = 1.01

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}(-\sec(e+fx))^n\sec^{-n}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(-Sec[e + f*x])^n*S
qrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

Maple [F] time = 0.189, size = 0, normalized size = 0.

$$\int\left(-\sec\left(fx+e\right)\right)^n\sqrt{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(fx+e\right)+a}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\sec\left(fx+e\right)+a}\left(-\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(e+fx\right)\right)^{n}\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)

[Out] Integral((-sec(e + f*x))**n*sqrt(a*(sec(e + f*x) + 1)), x)

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.314 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rubi [A] time = 0.130989, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3826, 136}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int \frac{(-\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx &= \frac{\sqrt{1+\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}} \\
&= \frac{\tan(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}x}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 6.22115, size = 2977, normalized size = 39.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(-1/2 -
n + (-1 + 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Tan[(e + f*x)/2])/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appell
F1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-
1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 +
 n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(3
```

*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*AppellF
1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n,
 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1
[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
+ f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*A
ppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f*
x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[
e + f*x]]*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
)/3 + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2
, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 +
 n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(
e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]
*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e +
 f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-1/2 +
 n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 +
n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*Ap
pellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*AppellF1[5
/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 - n,
7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 -

```
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x
)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*
x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e + f*x]
]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos
[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2
, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*
n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.18, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \frac{1}{\sqrt{a+a\sec \left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\sqrt{a\sec \left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\sec\left(e+fx\right)\right)^{n}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.315 $\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rubi [A] time = 0.144541, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3826, 136}

$$-\frac{\tan(e+fx)(-\sec(e+fx))^n F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(-Sec[e + f*x])^
n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int \frac{(-\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1+\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}\\
&= \frac{\tan(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^{-1+n}}{\sqrt{2-x}x^2}\,dx, x, 1+\sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(-\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 6.20618, size = 3005, normalized size = 38.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*(-Sec[e + f*x])^n*Sec[e + f*x]^(1/2 - n + (-3 +
 2*n)/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 +
 Tan[(e + f*x)/2]^2)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -3
/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*
AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]
^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*
```

(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*

```
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)*
AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2
+ n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -
1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x
)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^
2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*S
ec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
-1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)))
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\left(a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\left(-\sec\left(fx+e\right)\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 + 2
*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-\sec\left(e+fx\right)\right)^{n}}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.316 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+}{f(2n+1)}$$

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e
 + f*x]^(1 - n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Se
c[e + f*x]]) + (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a
+ a*Sec[e + f*x]])
```

Rubi [A] time = 0.161721, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3814, 21, 3806, 67, 65}

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f(2n+1)\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)(d\sec(e+fx))}{f(2n+1)\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```


```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e
 + f*x]^(1 - n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a + a*Se
c[e + f*x]]) + (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a
+ a*Sec[e + f*x]])
```


Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(2a)\int \frac{(d\sec(e+fx))^n\left(a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right)\sec(e+\cdots}{\sqrt{a+a\sec(e+fx)}}}{1+2n}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} + \frac{(a(1+4n))\int (d\sec(e+fx))^n\sqrt{a+a\,\mathrm{se}\cdots}}{1+2n}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3d(1+4n)\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(dx\cdots}{\sqrt{\cdots}}\right.}{f(1+2n)\sqrt{a-a\sec(e+fx)}\sqrt{a\cdots}}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^3(1+4n)\sec^{1-n}(e+fx)(d\sec(e+f\cdots\right.}{f(1+2n)\sqrt{a-a\,\mathrm{s}\cdots}}$$

$$= \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))\cdots}{f(1+2n)\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.315141, size = 88, normalized size = 0.68

$$\frac{a\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}(d\sec(e+fx))^n\left((4n+1)\cos^{n+\frac{1}{2}}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},n+\frac{3}{2},\frac{3}{2},2\sin^2\right.\right.}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(-1 + (1 + 4*n)*Cos[e + f*x]^(1/2 + n)*Hypergeometric2F1[1/2, 3/2 + n, 3
/2, 2*Sin[(e + f*x)/2]^2])*(d*Sec[e + f*x])^n*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(f*n)
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.317 $\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{2a\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 -
 n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rubi [A] time = 0.0762857, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3806, 67, 65}

$$\frac{2a\sin(e+fx)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*Sec[e + f*x]^(1 -
 n)*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sec(e+fx))^n\sqrt{a+a\sec(e+fx)}\,dx = -\frac{\left(a^2d\tan(e+fx)\right)\text{Subst}\left(\int\frac{(dx)^{-1+n}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{\left(a^2\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+fx)\right)\text{Subst}\left(\int\frac{x^{-1+n}}{\sqrt{a-ax}}\,dx,x,\sec(e\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1-\sec(e+fx)\right)\sec^{1-n}(e+fx)(d\sec(e+fx))^n\sin(e+f}{f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] time = 0.119744, size = 71, normalized size = 1.01

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sec(e+fx)+1)}\sec^{-n}(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},1-\sec(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Sec[e + f*x]]*(d*Sec[e + f*x])^n*
Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sec[e + f*x]^n)
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^n\sqrt{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(d\sec\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(d*sec(e + f*x))**n, x)
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.318 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rubi [A] time = 0.131517, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3827, 133}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},1;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{fn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((AppellF1[n, 1/2, 1, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{(d\sec(e+fx))^n}{\sqrt{1+\sec(e+fx)}}\,dx}{\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{(d\tan(e+fx))\operatorname{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}(1+x)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},1;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{fn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 6.19344, size = 2977, normalized size = 39.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[2]*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1/2 - n + (-1 + 2*n)/2)*
(d*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x
]]*Tan[(e + f*x)/2])/(f*Sqrt[a*(1 + Sec[e + f*x])]*(3*AppellF1[1/2, -1/2 +
n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appel
lF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1
+ n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]])/(Sqrt[2]*(
3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*Sqrt[2]*Appell
F1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec
```

```
[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x
]]*Sin[e + f*x]*Tan[(e + f*x)/2])/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF
1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]]*Tan[(e + f*x)/2]^2)/(3*
AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*Sqrt[2]*Cos[e + f
*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec
[e + f*x]]*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
])/3 + ((-1/2 + n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/
2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
+ n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sqrt[2]*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Sqrt[1 + Sec[e + f*x]
]*Tan[(e + f*x)/2]*((2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-1/2
+ n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(2*(-1 +
 n)*((-3*(2 - n)*AppellF1[5/2, -1/2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*A
ppellF1[5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-1 + 2*n)*((-3*(1 - n)*AppellF1[
5/2, 1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
 f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1/2 + n)*AppellF1[5/2, 3/2 + n, 1 - n,
 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/5))))/(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 +
(3*AppellF1[1/2, -1/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*
x)/2]*Tan[e + f*x])/(Sqrt[2]*Sqrt[1 + Sec[e + f*x]]*(3*AppellF1[1/2, -1/2 +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*Appe
```

```
llF1[3/2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(-1 + 2*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*Sqrt[2]*n*AppellF1[1/2, -1/2 + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f
*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Sqrt[1 + Sec[e + f*x
]]*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Co
s[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -1/2 + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/
2, -1/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-1 + 2
*n)*AppellF1[3/2, 1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\frac{1}{\sqrt{a+a\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{a\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sec\left(e+fx\right)\right)^{n}}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/sqrt(a*(sec(e + f*x) + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.319 $\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rubi [A] time = 0.145099, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3827, 133}

$$-\frac{\tan(e+fx)F_1\left(n;\frac{1}{2},2;n+1;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n}{afn\sqrt{1-\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((AppellF1[n, 1/2, 2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e + f*x])
^n*Tan[e + f*x])/(a*f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{1+\sec(e+fx)}\int \frac{(d\sec(e+fx))^n}{(1+\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{1-x}(1+x)^2}\,dx, x, \sec(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2},2;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx)}{afn\sqrt{1-\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] time = 6.19471, size = 3005, normalized size = 38.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (6*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(1/2 - n + (-3 + 2*n)/2)*(d*Sec[e
+ f*x])^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-1
+ Tan[(e + f*x)/2]^2)^2)/(f*(a*(1 + Sec[e + f*x]))^(3/2)*(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((12*AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2
]^2)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2
*(-1 + Tan[(e + f*x)/2]^2))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
 -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2) + (3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
```

```
[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2
]^2*Sec[e + f*x])^(3/2 + n)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -
3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)
*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-1 + T
an[(e + f*x)/2]^2)^2)/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2
+ n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]
^2) + (6*n*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e
+ f*x])^(3/2 + n)*Tan[(e + f*x)/2]^2*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*Appell
F1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2
*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (6*Cos[e + f*x]*(Sec[(e +
 f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*
(-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((-3/2 + n)*AppellF1[
3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])/3)*(-1 + Tan[(e + f*x)/2]^2)^2)/(3*AppellF1[1
/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1
 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (6*AppellF1[1/2, -3/2 + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f
*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2 + n)*Tan[(e + f*x)/2]*(-
1 + Tan[(e + f*x)/2]^2)^2*((2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2, -1/2 +
n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2] + 3*(-((1 - n)*AppellF1[3/2, -3/2 + n, 2 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 +
 ((-3/2 + n)*AppellF1[3/2, -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2
*(2*(-1 + n)*((-3*(2 - n)*AppellF1[5/2, -3/2 + n, 3 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-3
/2 + n)*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (-3 + 2*n)*((-3*(1 - n)
*AppellF1[5/2, -1/2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(-1/2 + n)*AppellF1[5/2, 1/2
 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
```

```
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 - n
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
-1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)^2 + (6*(3/2 + n)*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x]*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(1/2 + n)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)
^2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*
Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, -3/2 + n, 1 - n, 3/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-1 + n)*AppellF1[3/2, -3/2 + n, 2 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (-3 + 2*n)*AppellF1[3/2,
 -1/2 + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2)))
```

Maple [F] time = 0.162, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/(a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^n}{a^2\sec\left(fx+e\right)^2+2\,a^2\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(a^2*sec(f*x + e)^2 +
2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\sec\left(e+fx\right)\right)^n}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/(a*(sec(e + f*x) + 1))**(3/2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.320 $\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=178

$$\frac{2a^3(16n^2+24n+3)\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)(2n+3)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+f}{f(2n+3)}$$

```
[Out] (2*a^3*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f
*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^
3*(7 + 4*n)*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a -
 a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]]*Tan[e
 + f*x])/(f*(3 + 2*n))
```

Rubi [A] time = 0.334391, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3814, 4016, 3806, 65}

$$\frac{2a^3(16n^2+24n+3)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)(2n+3)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n}{f(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a^3*(3 + 24*n + 16*n^2)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f
*x]]*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^
3*(7 + 4*n)*(-Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(3 + 2*n)*Sqrt[a -
 a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]]*Tan[e
 + f*x])/(f*(3 + 2*n))
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (-\sec(e+fx))^n(a-a\sec(e+fx))^{5/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\tan(e+fx)}{f(3+2n)} - \frac{(2a)\int(-\sec(e+fx}{}$$

$$= \frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\sqrt{a-a\sec}}{f(3+2n)}$$

$$= \frac{2a^3(7+4n)(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)(3+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+fx))^n\sqrt{a-a\sec}}{f(3+2n)}$$

$$= \frac{2a^3\left(3+24n+16n^2\right)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f\left(3+8n+4n^2\right)\sqrt{a-a\sec(e+fx)}} + \frac{2a^3}{f}$$

Mathematica [C] time = 24.7991, size = 429, normalized size = 2.41

$$2^{n-\frac{5}{2}}e^{-i\left(n-\frac{1}{2}\right)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n-\frac{1}{2}}\csc^5\left(\frac{e}{2}+\frac{fx}{2}\right)(a-a\sec(e+fx))^{5/2}(-\sec(e+fx))^n\sec^{-n-\frac{5}{2}}(e+fx)\left(\frac{e^{in(e+fx)}\text{Hypergeome}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2^(-5/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(-1/2 + n)*Csc[e/
2 + (f*x)/2]^5*((E^(I*n*(e + f*x))*Hypergeometric2F1[1, (-3 - n)/2, (2 + n)
/2, -E^((2*I)*(e + f*x))])/n + (10*E^(I*(2 + n)*(e + f*x))*Hypergeometric2F
1[1, (-1 - n)/2, (4 + n)/2, -E^((2*I)*(e + f*x))])/(2 + n) + (5*E^(I*(4 + n
)*(e + f*x))*Hypergeometric2F1[1, (1 - n)/2, (6 + n)/2, -E^((2*I)*(e + f*x)
)])/(4 + n) - (5*E^(I*(1 + n)*(e + f*x))*Hypergeometric2F1[1, -1 - n/2, (3
+ n)/2, -E^((2*I)*(e + f*x))])/(1 + n) - (E^(I*(5 + n)*(e + f*x))*Hypergeom
etric2F1[1, 1 - n/2, (7 + n)/2, -E^((2*I)*(e + f*x))])/(5 + n) - (10*E^(I*(
3 + n)*(e + f*x))*Hypergeometric2F1[1, -n/2, (5 + n)/2, -E^((2*I)*(e + f*x)
)])/(3 + n))*(-Sec[e + f*x])^n*Sec[e + f*x]^(-5/2 - n)*(a - a*Sec[e + f*x])
^(5/2))/(E^(I*(-1/2 + n)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^2*f)
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n} \left(a-a\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}} \left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2 \sec\left(fx+e\right)^2 - 2\,a^2 \sec\left(fx+e\right) + a^2\right)\sqrt{-a \sec\left(fx+e\right) + a}\left(-\sec\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 - 2*a^2*sec(f*x + e) + a^2)*sqrt(-a*sec(f*x +
e) + a)*(-sec(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a \sec\left(fx+e\right) + a\right)^{\frac{5}{2}} \left(-\sec\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(5/2)*(-sec(f*x + e))^n, x)
```

3.321 $\int (-\sec (e+fx))^n (a-a\sec (e+fx))^{3/2} \, dx$

Optimal. Leaf size=108

$$\frac{2a^2(4n+1)\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e
 + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*
Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]])
```

Rubi [A] time = 0.154762, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3814, 21, 3806, 65}

$$\frac{2a^2(4n+1)\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(-\sec(e+fx))^n}{f(2n+1)\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e
 + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]]) + (2*a^2*(-Sec[e + f*x])^n*
Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{(-\sec(e+fx))^n\left(-a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right)\sec(}{\sqrt{a-a\sec(e+fx)}}}{1+2n}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int(-\sec(e+fx))^n\sqrt{a-a\,\mathrm{se}}}{1+2n}$$

$$= \frac{2a^2(-\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{\left(a^3(1+4n)\tan(e+fx)\right)\mathrm{Subst}\left(\int\frac{(-x}{\sqrt{a}}\right.}{f(1+2n)\sqrt{a-a\sec(e+fx)}\sqrt{a}}$$

$$= \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(-\sec(e+}{f(1+2n)\sqrt{}}$$

Mathematica [C] time = 13.498, size = 346, normalized size = 3.2

$$-\frac{2^{n-\frac{3}{2}}e^{-\frac{1}{2}i(2n+1)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{1}{2}}\csc^3\left(\frac{1}{2}(e+fx)\right)(a-a\sec(e+fx))^{3/2}(-\sec(e+fx))^n\sec^{-n-\frac{3}{2}}(e+fx)\left(\left(n^3+6n^2+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*Csc[(
e + f*x)/2]^3*(E^(I*n*(e + f*x))*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1
[1, (-1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(2 + n)*(e + f*x)
)*n*(3 + 4*n + n^2)*Hypergeometric2F1[1, (1 - n)/2, (4 + n)/2, -E^((2*I)*(e
 + f*x))] - n*(2 + n)*(E^(I*(3 + n)*(e + f*x))*(1 + n)*Hypergeometric2F1[1,
 1 - n/2, (5 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(1 + n)*(e + f*x))*(3 +
 n)*Hypergeometric2F1[1, -n/2, (3 + n)/2, -E^((2*I)*(e + f*x))]))*(-Sec[e +
 f*x])^n*Sec[e + f*x]^(-3/2 - n)*(a - a*Sec[e + f*x])^(3/2))/(E^((I/2)*(1 +
 2*n)*(e + f*x))*f*n*(1 + n)*(2 + n)*(3 + n)))
```

Maple [F] time = 0.172, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(a-a\sec \left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}} \left(-\sec \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a\sec \left(fx+e\right)-a\right)\sqrt{-a\sec \left(fx+e\right)+a}\left(-\sec \left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(-sec(f*x + e))^n, x)
```

3.322 $\int (-\sec(e+fx))^n \sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=47

$$\frac{2a\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e + f*x])/(f*
Sqrt[a - a*Sec[e + f*x]])
```

Rubi [A] time = 0.0718413, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3806, 65}

$$\frac{2a\tan(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Tan[e + f*x])/(f*
Sqrt[a - a*Sec[e + f*x]])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (-\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx = \frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{2a\,{}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; 1+\sec(e+fx)\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

Mathematica [C] time = 70.3183, size = 213, normalized size = 4.53

$$\frac{2^{n-\frac{1}{2}}e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n-\frac{1}{2}}\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{a-a\sec(e+fx)}(-\sec(e+fx))^n\sec^{-n-\frac{1}{2}}(e+fx)\left((n+1)e^{ifnx}\text{Hyperg}\right.}{fn(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2^(-1/2 + n)*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^(-1/2 + n)*Csc[e/2 + (f*x)/2]*(E^(I*f*n*x)*(1 + n)*Hypergeomet
ric2F1[1, (1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] - E^(I*(e + f*(1 + n)
*x))*n*Hypergeometric2F1[1, 1 - n/2, (3 + n)/2, -E^((2*I)*(e + f*x))])*(-Se
c[e + f*x])^n*Sec[e + f*x]^(-1/2 - n)*Sqrt[a - a*Sec[e + f*x]])/(f*n*(1 + n
))
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^n\sqrt{a-a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sec\left(fx+e\right)+a}\left(-\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a\sec\left(fx+e\right)+a}\left(-\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\sec\left(e+fx\right)\right)^{n}\sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\sec\left(fx+e\right)+a}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n, x)
```

3.323 $\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}} dx$

Optimal. Leaf size=58

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[a - a*Sec[e + f*x]])
```

Rubi [A] time = 0.154118, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3828, 3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (AppellF1[1/2, 1 - n, 1, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[e
 + f*x])/(f*Sqrt[a - a*Sec[e + f*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx &= \frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{\sqrt{1-\sec(e+fx)}}\,dx}{\sqrt{a-a\sec(e+fx)}}\\
&= \frac{\tan(e+fx)\,\mathrm{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)\sqrt{x}}\,dx, x, 1+\sec(e+fx)\right)}{f\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}\\
&= \frac{(2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{2-x^2}\,dx, x, \sqrt{1+\sec(e+fx)}\right)}{f\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}}\\
&= \frac{F_1\left(\frac{1}{2};1-n,1;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [F] time = 1.24638, size = 0, normalized size = 0.

$$\int \frac{(-\sec(e+fx))^n}{\sqrt{a-a\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] Integrate[(-Sec[e + f*x])^n/Sqrt[a - a*Sec[e + f*x]], x]
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \frac{1}{\sqrt{a-a\sec \left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\sqrt{-a\sec \left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a\sec \left(fx+e\right)+a}\left(-\sec \left(fx+e\right)\right)^{n}}{a\sec \left(fx+e\right)-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a*sec(f*x + e) - a),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec\left(e+fx\right)\right)^{n}}{\sqrt{-a\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/sqrt(-a*(sec(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec\left(fx+e\right)\right)^{n}}{\sqrt{-a\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n/sqrt(-a*sec(f*x + e) + a), x)
```

3.324 $\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{2af\sqrt{a-a\sec(e+fx)}}$$

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*a*f*Sqrt[a - a*Sec[e + f*x]])
```

Rubi [A] time = 0.167159, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3828, 3825, 130, 429}

$$\frac{\tan(e+fx)F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{2af\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (AppellF1[1/2, 1 - n, 2, 3/2, 1 + Sec[e + f*x], (1 + Sec[e + f*x])/2]*Tan[e
 + f*x])/(2*a*f*Sqrt[a - a*Sec[e + f*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
```

```
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 130

```
Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_
Symbol] :> With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)
*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x]] /; FreeQ[{a,
 b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx &= \frac{\sqrt{1-\sec(e+fx)}\int \frac{(-\sec(e+fx))^n}{(1-\sec(e+fx))^{3/2}}\,dx}{a\sqrt{a-a\sec(e+fx)}} \\
&= \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}}{(2-x)^2\sqrt{x}}\,dx, x, 1+\sec(e+fx)\right)}{af\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}} \\
&= \frac{(2\tan(e+fx))\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{-1+n}}{\left(2-x^2\right)^2}\,dx, x, \sqrt{1+\sec(e+fx)}\right)}{af\sqrt{1+\sec(e+fx)}\sqrt{a-a\sec(e+fx)}} \\
&= \frac{F_1\left(\frac{1}{2};1-n,2;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{2af\sqrt{a-a\sec(e+fx)}}
\end{aligned}$$

Mathematica [F] time = 1.8857, size = 0, normalized size = 0.

$$\int \frac{(-\sec(e+fx))^n}{(a-a\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(-Sec[e + f*x])^n/(a - a*Sec[e + f*x])^(3/2), x]
```

Maple [F] time = 0.171, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(a-a\sec \left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\left(-a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n/(-a*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a\sec \left(fx+e\right)+a}\left(-\sec \left(fx+e\right)\right)^{n}}{a^{2}\sec \left(fx+e\right)^{2}-2\,a^{2}\sec \left(fx+e\right)+a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(-sec(f*x + e))^n/(a^2*sec(f*x + e)^2 -
2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(e+fx\right)\right)^{n}}{\left(-a\left(\sec \left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n/(a-a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-sec(e + f*x))**n/(-a*(sec(e + f*x) - 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\sec \left(fx+e\right)\right)^{n}}{\left(-a\sec \left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n/(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n/(-a*sec(f*x + e) + a)^(3/2), x)
```

3.325 $\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\sin(e}{f(2n+}$$

```
[Out] (2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f
*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x
]]*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a
 - a*Sec[e + f*x]])
```

Rubi [A] time = 0.171405, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3814, 21, 3806, 67, 65}

$$\frac{2a^2(4n+1)\sin(e+fx)\sec^{n+1}(e+fx)(-\sec(e+fx))^{-n}\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\sin(e+fx)\sec^{n+1}(e+}{f(2n+1)\sqrt{a-a\sec(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f
*x]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x
]]*Sec[e + f*x]^(1 + n)*Sin[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a
 - a*Sec[e + f*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{\sec^n(e+fx)\left(-a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right)\sec(e+fx)\right)}{\sqrt{a-a\sec(e+fx)}}\,dx}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}}{1+2n}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{\left(a^3(1+4n)\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a+ax}}\,dx,\right.}{f(1+2n)\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec}}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{\left(a^3(1+4n)(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\right.}{f(1+2n)\sqrt{a-a\sec(e}}$$

$$= \frac{2a^2\sec^{1+n}(e+fx)\sin(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)}{f(1+2n)\sqrt{a-}}$$

Mathematica [C] time = 2.0676, size = 332, normalized size = 2.55

$$-\frac{2^{n-\frac{3}{2}}e^{-\frac{1}{2}i(2n+1)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{1}{2}}\csc^3\left(\frac{1}{2}(e+fx)\right)(a-a\sec(e+fx))^{3/2}\left(\left(n^3+6n^2+11n+6\right)e^{in(e+fx)}\text{Hypergeome}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*Csc[(
e + f*x)/2]^3*(E^(I*n*(e + f*x))*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1
[1, (-1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(2 + n)*(e + f*x)
)*n*(3 + 4*n + n^2)*Hypergeometric2F1[1, (1 - n)/2, (4 + n)/2, -E^((2*I)*(e
 + f*x))] - n*(2 + n)*(E^(I*(3 + n)*(e + f*x))*(1 + n)*Hypergeometric2F1[1,
 1 - n/2, (5 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(1 + n)*(e + f*x))*(3 +
 n)*Hypergeometric2F1[1, -n/2, (3 + n)/2, -E^((2*I)*(e + f*x))]))*(a - a*Se
c[e + f*x])^(3/2))/(E^((I/2)*(1 + 2*n)*(e + f*x))*f*n*(1 + n)*(2 + n)*(3 +
n)*Sec[e + f*x]^(3/2)))
```

Maple [F] time = 0.172, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{n}\left(a-a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a\sec\left(fx+e\right)-a\right)\sqrt{-a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*sec(f*x + e)^n, x)
```

3.326 $\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Sec[e + f*x]^(1 +
 n)*Sin[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Rubi [A] time = 0.0774771, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3806, 67, 65}

$$\frac{2a\sin(e+fx)(-\sec(e+fx))^{-n}\sec^{n+1}(e+fx)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*Sec[e + f*x]^(1 +
 n)*Sin[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \sec^n(e+fx)\sqrt{a-a\sec(e+fx)}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{\left(a^2(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+fx)\right)\text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{a+ax}}\,dx,x,\sec(e+\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$= \frac{2a\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}\sec^{1+n}(e+fx)\sin(e+fx)}{f\sqrt{a-a\sec(e+fx)}}$$

Mathematica [C] time = 0.384879, size = 185, normalized size = 2.68

$$-\frac{2^n e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n\cos(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sqrt{a-a\sec(e+fx)}\left(ne^{i(e+f(n+1)x)}\text{Hypergeometric2F1}\left(1,1-\right.\right.}{fn(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] -((2^n*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*
x))))^n*Cos[e + f*x]*Csc[(e + f*x)/2]*(-(E^(I*f*n*x)*(1 + n)*Hypergeometric
2F1[1, (1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))]) + E^(I*(e + f*(1 + n)*x
))*n*Hypergeometric2F1[1, 1 - n/2, (3 + n)/2, -E^((2*I)*(e + f*x))])*Sqrt[a
 - a*Sec[e + f*x]])/(f*n*(1 + n)))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^n\sqrt{a-a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)

[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(-a*(sec(e + f*x) - 1))*sec(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a \sec\left(fx+e\right)+a} \sec\left(fx+e\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*sec(f*x + e)^n, x)
```

3.327 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{2a^2(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan}{f(2n}$$

```
[Out] (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x
]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]
*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a - a
*Sec[e + f*x]])
```

Rubi [A] time = 0.179818, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3814, 21, 3806, 67, 65}

$$\frac{2a^2(4n+1)\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f(2n+1)\sqrt{a-a\sec(e+fx)}}+\frac{2a^2\tan(e+fx)(d\sec(e+}{f(2n+1)\sqrt{a-a\sec(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a^2*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*Sqrt[a - a*Sec[e + f*x
]]) + (2*a^2*(1 + 4*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]
*(d*Sec[e + f*x])^n*Tan[e + f*x])/(f*(1 + 2*n)*(-Sec[e + f*x])^n*Sqrt[a - a
*Sec[e + f*x]])
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
!IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^{3/2}\,dx = \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{(2a)\int \frac{(d\sec(e+fx))^n\left(-a\left(\frac{1}{2}+2n\right)+a\left(\frac{1}{2}+2n\right)\sec(e+\ldots}{\sqrt{a-a\sec(e+fx)}}}{1+2n}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{(a(1+4n))\int (d\sec(e+fx))^n\sqrt{a-a\sec(\ldots}{1+2n}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} - \frac{\left(a^3d(1+4n)\tan(e+fx)\right)\mathrm{Subst}\left(\int \frac{(dx)^{-\ldots}}{\sqrt{a+\ldots}}\ldots\right.}{f(1+2n)\sqrt{a-a\sec(e+fx)}\sqrt{a+\ldots}}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{\left(a^3(1+4n)(-\sec(e+fx))^{-n}(d\sec(e+f\ldots\right.}{f(1+2n)\sqrt{a-a\ldots}}$$

$$= \frac{2a^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+2n)\sqrt{a-a\sec(e+fx)}} + \frac{2a^2(1+4n)\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};1+\sec(e+\ldots\right.}{f(1+2n)\ldots}$$

Mathematica [C] time = 1.14138, size = 346, normalized size = 2.66

$$-\frac{2^{n-\frac{3}{2}}e^{-\frac{1}{2}i(2n+1)(e+fx)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n+\frac{1}{2}}\csc^3\left(\frac{1}{2}(e+fx)\right)(a-a\sec(e+fx))^{3/2}\sec^{-n-\frac{3}{2}}(e+fx)\left(\left(n^3+6n^2+11n+6\right)e^{in(e+f}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((2^(-3/2 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^(1/2 + n)*Csc[(
e + f*x)/2]^3*(E^(I*n*(e + f*x))*(6 + 11*n + 6*n^2 + n^3)*Hypergeometric2F1
[1, (-1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(2 + n)*(e + f*x)
)*n*(3 + 4*n + n^2)*Hypergeometric2F1[1, (1 - n)/2, (4 + n)/2, -E^((2*I)*(e
 + f*x))] - n*(2 + n)*(E^(I*(3 + n)*(e + f*x))*(1 + n)*Hypergeometric2F1[1,
 1 - n/2, (5 + n)/2, -E^((2*I)*(e + f*x))] + 3*E^(I*(1 + n)*(e + f*x))*(3 +
 n)*Hypergeometric2F1[1, -n/2, (3 + n)/2, -E^((2*I)*(e + f*x))]))*Sec[e + f
*x]^(-3/2 - n)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^(3/2))/(E^((I/2)*(1
+ 2*n)*(e + f*x))*f*n*(1 + n)*(2 + n)*(3 + n)))
```

Maple [F] time = 0.176, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a-a\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a\sec\left(fx+e\right)-a\right)\sqrt{-a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e) - a)*sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.328 $\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{2a\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*(d*Sec[e + f*x])^
n*Tan[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Rubi [A] time = 0.0808413, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3806, 67, 65}

$$\frac{2a\tan(e+fx)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},1-n;\frac{3}{2};\sec(e+fx)+1\right)}{f\sqrt{a-a\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Sec[e + f*x]]*(d*Sec[e + f*x])^
n*Tan[e + f*x])/(f*(-Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (d\sec(e+fx))^n\sqrt{a-a\sec(e+fx)}\,dx = -\frac{\left(a^2 d\tan(e+fx)\right)\text{Subst}\left(\int \frac{(dx)^{-1+n}}{\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(a^2(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+fx)\right)\text{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{a+ax}}\,dx, x, \sec(\right.}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a\,{}_2F_1\left(\frac{1}{2}, 1-n; \frac{3}{2}; 1+\sec(e+fx)\right)(-\sec(e+fx))^{-n}(d\sec(e+fx))^n\tan(e+}{f\sqrt{a-a\sec(e+fx)}}$$

Mathematica [C] time = 0.53145, size = 213, normalized size = 3.09

$$\frac{2^{n-\frac{1}{2}}e^{\frac{1}{2}i(e+f(1-2n)x)}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n-\frac{1}{2}}\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{a-a\sec(e+fx)}\sec^{-n-\frac{1}{2}}(e+fx)\left((n+1)e^{ifnx}\text{Hypergeometric2F1}\left(1,\right.\right.}{fn(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]],x]
```

```
[Out] (2^(-1/2 + n)*E^((I/2)*(e + f*(1 - 2*n)*x))*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^(-1/2 + n)*Csc[e/2 + (f*x)/2]*(E^(I*f*n*x)*(1 + n)*Hypergeomet
ric2F1[1, (1 - n)/2, (2 + n)/2, -E^((2*I)*(e + f*x))] - E^(I*(e + f*(1 + n)
*x))*n*Hypergeometric2F1[1, 1 - n/2, (3 + n)/2, -E^((2*I)*(e + f*x))])*Sec[
e + f*x]^(-1/2 - n)*(d*Sec[e + f*x])^n*Sqrt[a - a*Sec[e + f*x]])/(f*n*(1 +
n))
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^n\sqrt{a-a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\sqrt{-a\left(\sec\left(e+fx\right)-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*sqrt(-a*(sec(e + f*x) - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.329 $\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx$

Optimal. Leaf size=72

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rubi [A] time = 0.0640129, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3825, 133}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^n(e+fx)(1+\sec(e+fx))^m\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 14.2137, size = 2246, normalized size = 31.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*Sec[e + f*x]^n*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*Appell
F1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-
1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(Cos[
(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n,
2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2
, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF
1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n
)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos
[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-((1 - n)*AppellF1[
3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
```

```
f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m)*Ap
pellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Se
c[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e
 + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] +
3*(-((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3/2,
 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*(2 - n)
*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1 + m + n
, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 - n,
7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n)*App
ellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec
[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n)*Ta
n[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e
+ f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n
, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3
/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
 + f*x)/2]^2)))
```

Maple [F] time = 0.665, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{n} \left(1+\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^n*(1+sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sec\left(e+fx\right)+1\right)^{m}\sec^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(1+sec(f*x+e))**m,x)
```

```
[Out] Integral((sec(e + f*x) + 1)**m*sec(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

3.330 $\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{2}\tan(e+fx)(1-\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec[e
 + f*x]])
```

Rubi [A] time = 0.0701287, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3826, 133}

$$\frac{\sqrt{2}\tan(e+fx)(1-\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec[e
 + f*x]])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}x^{-\frac{1}{2}+m}}{\sqrt{2-x}}\,dx, x, 1-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m; 1-n, \frac{1}{2}; \frac{3}{2}+m; 1-\sec(e+fx), \frac{1}{2}(1-\sec(e+fx))\right)(1-\sec(e+f}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 2.20441, size = 255, normalized size = 2.87

$$\frac{(2m+3)\sin(e+fx)(1-\sec(e+fx))^m \sec}{f(2m+1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((n-1)F_1\left(m+\frac{3}{2}; m+n, 2-n; m+\frac{5}{2}; \tan^2\left(\frac{1}{2}(e+fx)\right), -\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m+n)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sec[e + f*x])^m*Sec[e + f*x]^n,x]
```

```
[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*Sec[e + f*x]^n*Sin[e + f*x])/(f*(1 +
 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5/2 +
 m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m, 1
+ m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
 f*x)/2]^2))
```

Maple [F] time = 0.702, size = 0, normalized size = 0.

$$\int \left(1-\sec\left(fx+e\right)\right)^m \left(\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

```
[Out] int((1-sec(f*x+e))^m*sec(f*x+e)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-\sec(fx+e)+1)^m \sec(fx+e)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((-\sec(fx+e)+1)^m \sec(fx+e)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (1-\sec(e+fx))^m \sec^n(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*sec(f*x+e)**n,x)
```

```
[Out] Integral((1 - sec(e + f*x))**m*sec(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)+1\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*sec(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e) + 1)^m*sec(f*x + e)^n, x)
```

3.331 $\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=88

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f
*x])/f
```

Rubi [A] time = 0.109332, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3828, 3825, 133}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f
*x])/f
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
```

```
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^n(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sec^n(e+fx)(1+\sec(e+fx))^m\,d$$

$$= \frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1-x)^{-1+n}(2-}{\sqrt{x}}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+f}{f}$$

Mathematica [B] time = 6.24224, size = 2248, normalized size = 25.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*Sec[e + f*x]^n*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(3*Ap
pellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1
[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
```

```
(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*App
ellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
 + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-((1 - n)*Appel
lF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m
)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Ta
n[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
] + 3*(-((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*(2
- n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1 + m
 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 -
 n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))
)/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n)
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n
)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos
[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m
 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appell
F1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2)))
```

Maple [F] time = 0.721, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{n} \left(a+a \sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(fx+e\right)+a\right)^{m} \sec \left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a \sec \left(fx+e\right)+a\right)^{m} \sec \left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \left(\sec \left(e+fx\right)+1\right)\right)^{m} \sec^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.332 $\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{2}\tan(e+fx)(a-a\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(a - a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec
[e + f*x]])
```

Rubi [A] time = 0.120068, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3828, 3826, 133}

$$\frac{\sqrt{2}\tan(e+fx)(a-a\sec(e+fx))^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)}{f(2m+1)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 - Sec[e + f*x], (1 - Sec[
e + f*x])/2]*(a - a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 + Sec
[e + f*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
```

```
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx))^m\sec^n(e+fx)\,dx$$

$$= \frac{\left((1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1-x)^{-1+n}x^{-\frac{1}{2}+}}{\sqrt{2-x}}\right.}{f\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1-\sec(e+fx),\frac{1}{2}(1-\sec(e+fx))\right)(a-a\sec(e+}{f(1+2m)\sqrt{1+\sec(e+fx)}}$$

Mathematica [F] time = 1.00283, size = 0, normalized size = 0.

$$\int \sec^n(e+fx)(a-a\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[Sec[e + f*x]^n*(a - a*Sec[e + f*x])^m, x]
```

Maple [F] time = 0.753, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^n\left(a-a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec \left(fx+e\right)+a\right)^{m} \sec \left(fx+e\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-a\sec \left(fx+e\right)+a\right)^{m} \sec \left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\left(\sec \left(e+fx\right)-1\right)\right)^{m} \sec^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a-a*sec(f*x+e))**m,x)
```

```
[Out] Integral((-a*(sec(e + f*x) - 1))**m*sec(e + f*x)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.333 $\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{2}\tan(e+fx)(\sec(e+fx)+1)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(1 + Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]])
```

Rubi [A] time = 0.0669408, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3826, 133}

$$\frac{\sqrt{2}\tan(e+fx)(\sec(e+fx)+1)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(1 + Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]])
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (-\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}x^{-\frac{1}{2}+m}}{\sqrt{2-x}}\,dx,x,1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e-}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 6.20321, size = 2248, normalized size = 26.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(-Sec[e + f*x])^n*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*App
ellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*
((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e
 + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m +
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*Appe
llF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
+ n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-((1 - n)*Appell
F1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
```

```
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f
*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m)
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan
[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
 + 3*(-((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3
/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*(2 -
 n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1 + m
+ n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 -
n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))
/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n)*
AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(
Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n)
*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[
(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 - n, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m
+ n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF
1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)))
```

Maple [F] time = 0.711, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(1+\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

```
[Out] int((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-\sec\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(e+fx\right)\right)^{n}\left(\sec\left(e+fx\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
```

```
[Out] Integral((-sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\left(\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

3.334 $\int (1-\sec(e+fx))^m (-\sec(e+fx))^n \, dx$

Optimal. Leaf size=70

$$\frac{2^{m+\frac{1}{2}} \tan(e+fx) F_1\left(\frac{1}{2}; 1-n, \frac{1}{2}-m; \frac{3}{2}; \sec(e+fx)+1, \frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{1-\sec(e+fx)}}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Rubi [A] time = 0.0654366, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3825, 133}

$$\frac{2^{m+\frac{1}{2}} \tan(e+fx) F_1\left(\frac{1}{2}; 1-n, \frac{1}{2}-m; \frac{3}{2}; \sec(e+fx)+1, \frac{1}{2}(\sec(e+fx)+1)\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*Tan[e + f*x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (1-\sec(e+fx))^m(-\sec(e+fx))^n\,dx = \frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(1-x)^{-1+n}(2-x)^{-\frac{1}{2}+m}}{\sqrt{x}}\,dx,x,1+\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{f\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 0.297664, size = 257, normalized size = 3.67

$$\frac{(2m+3)\sin(e+fx)(1-\sec(e+fx))^m(-\text{se}}{f(2m+1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((n-1)F_1\left(m+\frac{3}{2};m+n,2-n;m+\frac{5}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m+n)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n,x]
```

```
[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(-Sec[e + f*x])^n*Sin[e + f*x])/(f*(
1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5/
2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m,
 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2))
```

Maple [F] time = 0.724, size = 0, normalized size = 0.

$$\int \left(1-\sec\left(fx+e\right)\right)^m\left(-\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

```
[Out] int((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(e+fx\right)\right)^{n}\left(1-\sec\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*(-sec(f*x+e))**n,x)
```

```
[Out] Integral((-sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(-sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((-sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

3.335 $\int (-\sec(e+fx))^n (a+a\sec(e+fx))^m \, dx$

Optimal. Leaf size=87

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec
[e + f*x]])
```

Rubi [A] time = 0.117079, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3828, 3826, 133}

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};1-n,\frac{1}{2};m+\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1 - n, 1/2, 3/2 + m, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec
[e + f*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3826

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[((-((a*d)/b))^n*Cot[e + f*x])/(a^(n - 1)*f*Sq
rt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(x^(m - 1/2)*(a
 - x)^(n - 1))/Sqrt[2*a - x], x], x, a + b*Csc[e + f*x]], x] /; FreeQ[{a, b
```

```
, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
 !IntegerQ[n] && LtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (-\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (-\sec(e+fx))^n(1+\sec(e+f$$

$$= \frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1-x)^{-1+}}{\sqrt{2}}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;1-n,\frac{1}{2};\frac{3}{2}+m;1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(a+a\,\text{se}}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 6.20785, size = 2250, normalized size = 25.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(-Sec[e + f*x])^n*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(3
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n,
 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^
n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2,
m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
```

```
an[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Co
s[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2,
 m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*
AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 +
n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-((1 - n)*Ap
pellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3/2, 1 + m + n, 1 -
 n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1
+ m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)
*Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2] + 3*(-((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*Appell
F1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*
(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1
+ m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n,
2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5
))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m +
 n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m
+ n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2
, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*App
ellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.751, size = 0, normalized size = 0.

$$\int \left(-\sec \left(fx+e\right)\right)^{n} \left(a+a\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(fx+e\right)+a\right)^{m} \left(-\sec \left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec \left(fx+e\right)+a\right)^{m} \left(-\sec \left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec \left(e+fx\right)\right)^{n} \left(a\left(\sec \left(e+fx\right)+1\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((-sec(e + f*x))**n*(a*(sec(e + f*x) + 1))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

3.336 $\int (-\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$

Optimal. Leaf size=87

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f}$$

```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 - m)*(a - a*Sec[e + f*x])^m*Tan[e + f
*x])/f
```

Rubi [A] time = 0.121559, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3828, 3825, 133}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};\sec(e+fx)+1,\frac{1}{2}(\sec(e+fx)+1)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```


```
[Out] (2^(1/2 + m)*AppellF1[1/2, 1 - n, 1/2 - m, 3/2, 1 + Sec[e + f*x], (1 + Sec[
e + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 - m)*(a - a*Sec[e + f*x])^m*Tan[e + f
*x])/f
```


Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3825

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Dist[(((a*d)/b)^n*Cot[e + f*x])/(a^(n - 2)*f*Sqrt[
a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((a - x)^(n - 1)*(
2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a, b,
```

```
 d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
!IntegerQ[n] && GtQ[(a*d)/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx))^m(-\sec(e+fx$$

$$= \frac{\left((1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1-x)^{-1+n}(2}{\sqrt{x}}\right.}{f\sqrt{1+\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}F_1\left(\frac{1}{2};1-n,\frac{1}{2}-m;\frac{3}{2};1+\sec(e+fx),\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+}{f}$$

Mathematica [F] time = 0.2242, size = 0, normalized size = 0.

$$\int (-\sec(e+fx))^n (a-a\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(-Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

Maple [F] time = 0.79, size = 0, normalized size = 0.

$$\int \left(-\sec\left(fx+e\right)\right)^{n}\left(a-a\sec\left(fx+e\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

```
[Out] int((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{m}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(-a\sec\left(fx+e\right)+a\right)^{m}\left(-\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sec\left(e+fx\right)\right)^{n}\left(-a\left(\sec\left(e+fx\right)-1\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
```

```
[Out] Integral((-sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{m}\left(-\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*(-sec(f*x + e))^n, x)
```

3.337 $\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx$

Optimal. Leaf size=79

$$-\frac{\tan(e+fx)(d\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.0630893, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3827, 133}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```


```
[Out] -((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```


Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^n(1+\sec(e+fx))^m\,dx = -\frac{(d\tan(e+fx))\,\text{Subst}\left(\int \frac{(dx)^{-1+n}(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= -\frac{F_1\left(n;\frac{1}{2},\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+fx}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 6.2025, size = 2248, normalized size = 28.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*(1 + Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(d*Sec[e + f*x])^n*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(m + n)*(1 + Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(3*Ap
pellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n, 1
- n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*(
Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m +
 n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1
[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(
e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*App
ellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m
 + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 + n)*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-((1 - n)*Appel
lF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e +
f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,
```

```
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1 + m
)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Ta
n[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
] + 3*(-((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[
3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3*(2
- n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1 + m
 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n, 2 -
 n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))
)/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m + n)
*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m + n
)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos
[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 - n,
3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m
 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appell
F1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Ta
n[(e + f*x)/2]^2)))
```

Maple [F] time = 0.728, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(1+\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^n \left(\sec\left(fx+e\right)+1\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^n \left(\sec\left(fx+e\right)+1\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(e+fx\right)\right)^n \left(\sec\left(e+fx\right)+1\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(1+sec(f*x+e))**m,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(sec(e + f*x) + 1)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^n \left(\sec\left(fx+e\right)+1\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(1+sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*(sec(f*x + e) + 1)^m, x)
```

3.338 $\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx$

Optimal. Leaf size=79

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n F_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.0645752, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3827, 133}

$$-\frac{\tan(e+fx)(d\sec(e+fx))^n F_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

```
[Out] -((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]))
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (1-\sec(e+fx))^m (d\sec(e+fx))^n\,dx = -\frac{(d\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(1-x)^{-\frac{1}{2}+m}(dx)^{-1+n}}{\sqrt{1+x}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$
$$= -\frac{F_1\left(n;\frac{1}{2}-m,\frac{1}{2};1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n\tan(e+f}{fn\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 0.279261, size = 257, normalized size = 3.25

$$\frac{(2m+3)\sin(e+fx)(1-\sec(e+fx))^m(d\,\mathrm{se}}{f(2m+1)\left(2\tan^2\left(\frac{1}{2}(e+fx)\right)\left((n-1)F_1\left(m+\frac{3}{2};m+n,2-n;m+\frac{5}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(m+n)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n,x]
```

```
[Out] ((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*(1 - Sec[e + f*x])^m*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*
(1 + 2*m)*((3 + 2*m)*AppellF1[1/2 + m, m + n, 1 - n, 3/2 + m, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2 + m, m + n, 2 - n, 5
/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2 + m
, 1 + m + n, 1 - n, 5/2 + m, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[
(e + f*x)/2]^2))
```

Maple [F] time = 0.734, size = 0, normalized size = 0.

$$\int \left(1-\sec\left(fx+e\right)\right)^m \left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

```
[Out] int((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\left(1-\sec\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))**m*(d*sec(f*x+e))**n,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(1 - sec(e + f*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(-\sec\left(fx+e\right)+1\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sec(f*x+e))^m*(d*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n*(-sec(f*x + e) + 1)^m, x)
```

3.339 $\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=95

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}}$$

```
[Out] -((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])
/(f*n*Sqrt[1 - Sec[e + f*x]]))
```

Rubi [A] time = 0.115144, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3828, 3827, 133}

$$-\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(d\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((AppellF1[n, 1/2, 1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e +
 f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])
/(f*n*Sqrt[1 - Sec[e + f*x]]))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (d\sec(e+fx))^n(1+\sec(e+f$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(dx)^{-1}}{}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{F_1\left(n;\frac{1}{2},\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^n(1+\sec}{fn\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 6.20858, size = 2250, normalized size = 23.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (3*2^(1 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(d*Sec[e + f*x])^n*(Cos[(e + f*x)
/2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(
3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^m*AppellF1[1/2, m + n
, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)
^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n))/(3*AppellF1[1/2, m + n, 1 - n
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2,
 m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appe
llF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(-1 + n)*AppellF1[1/2, m + n, 1 - n, 3/2
```

```
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(C
os[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2
, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)
*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (3*2^(1 + m)*(Sec[(e + f*x)/2]^2)^(-1 +
 n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]*(-((1 - n)*A
ppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*AppellF1[3/2, 1 + m + n, 1
- n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2])/3))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*2^(1
 + m)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n
)*Tan[(e + f*x)/2]*(2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2] + 3*(-((1 - n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + ((m + n)*Appel
lF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + n)*((-3
*(2 - n)*AppellF1[5/2, m + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(m + n)*AppellF1[5/2, 1
 + m + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x
)/2]^2*Tan[(e + f*x)/2])/5) + (m + n)*((-3*(1 - n)*AppellF1[5/2, 1 + m + n,
 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2])/5 + (3*(1 + m + n)*AppellF1[5/2, 2 + m + n, 1 - n, 7/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/
5))))/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (3*2^(1 + m)*(m
+ n)*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(Sec[(e + f*x)/2]^2)^(-1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m
 + n)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
 Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(3*AppellF1[1/2, m + n, 1 -
 n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/
2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Ap
pellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
)*Tan[(e + f*x)/2]^2)))
```

Maple [F] time = 0.762, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n} \left(a+a\sec\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m} \left(d\sec\left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m} \left(d\sec\left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m} \left(d\sec\left(e+fx\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(d*sec(e + f*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(fx+e)+a)^m (d\sec(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.340 $\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$

Optimal. Leaf size=96

$$-\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m(d\sec(e+fx))^nF_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{\sec(e+fx)+1}}$$

```
[Out] -((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(1 - Sec[e
 + f*x])^(-1/2 - m)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m*Tan[e + f*x])
/(f*n*Sqrt[1 + Sec[e + f*x]]))
```

Rubi [A] time = 0.118964, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3828, 3827, 133}

$$-\frac{\tan(e+fx)(1-\sec(e+fx))^{-m-\frac{1}{2}}(a-a\sec(e+fx))^m(d\sec(e+fx))^nF_1\left(n;\frac{1}{2}-m,\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```



```
[Out] -((AppellF1[n, 1/2 - m, 1/2, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(1 - Sec[e
 + f*x])^(-1/2 - m)*(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m*Tan[e + f*x])
/(f*n*Sqrt[1 + Sec[e + f*x]]))
```


Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx = \left((1-\sec(e+fx))^{-m}(a-a\sec(e+fx))^m\right)\int (1-\sec(e+fx))^m(d\sec(e+fx)$$

$$= -\frac{\left(d(1-\sec(e+fx))^{-\frac{1}{2}-m}(a-a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int\frac{(1-x)^{-\frac{1}{2}}}{\sqrt{}}\right.}{f\sqrt{1+\sec(e+fx)}}$$

$$= -\frac{F_1\left(n;\frac{1}{2}-m,\frac{1}{2};1+n;\sec(e+fx),-\sec(e+fx)\right)(1-\sec(e+fx))^{-\frac{1}{2}-m}(d\,\text{s}}{fn\sqrt{1+\sec(e+fx)}}$$

Mathematica [F] time = 0.206287, size = 0, normalized size = 0.

$$\int (d\sec(e+fx))^n(a-a\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(d*Sec[e + f*x])^n*(a - a*Sec[e + f*x])^m, x]
```

Maple [F] time = 0.786, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(a-a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{m}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-a\sec\left(fx+e\right)+a\right)^{m}\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(e+fx\right)\right)^{n}\left(-a\left(\sec\left(e+fx\right)-1\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a-a*sec(f*x+e))**m,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(-a*(sec(e + f*x) - 1))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a\sec\left(fx+e\right)+a\right)^{m}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a-a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((-a*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.341 $\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=211

$$\frac{2^{m+\frac{1}{2}}m\left(m^2+3m+5\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\right.}{f(m+1)(m+2)(m+3)}$$

```
[Out] ((4 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)*(2 + m)*(3 + m)) +
 (Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + m)) + (2^(1/2
 + m)*m*(5 + 3*m + m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f
*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/
(f*(1 + m)*(2 + m)*(3 + m)) + (m*(a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])
/(a*f*(6 + 5*m + m^2))
```

Rubi [A] time = 0.352669, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3824, 4010, 4001, 3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}}m\left(m^2+3m+5\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)(m+2)(m+3)}+\frac{m}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((4 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)*(2 + m)*(3 + m)) +
 (Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + m)) + (2^(1/2
 + m)*m*(5 + 3*m + m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f
*x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/
(f*(1 + m)*(2 + m)*(3 + m)) + (m*(a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])
/(a*f*(6 + 5*m + m^2))
```

Rule 3824

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(m + n - 1)), x] + Dist[d^2/(b*(m + n - 1)), Int[(
a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*m*Csc[e + f*x
]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
```

```
] && NeQ[m + n - 1, 0] && IntegerQ[n]
```

Rule 4010

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(
a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Cs
c[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*Cs
c[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*B,
 0] &&  !LtQ[m, -1]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^4(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)} + \frac{\int \sec^2(e+fx)(a+a\sec(e+ \ldots}{a(3 \ldots}$$
$$= \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+m)} + \frac{m(a+a\sec(e+fx))^{1+m}\tan(\ldots}{af\left(6+5m+m^2\right)}$$
$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\,\mathrm{t} \ldots}{f(3+m)}$$
$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\,\mathrm{t} \ldots}{f(3+m)}$$
$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\,\mathrm{t} \ldots}{f(3+m)}$$
$$= \frac{(4+m)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)(2+m)(3+m)} + \frac{\sec^2(e+fx)(a+a\sec(e+fx))^m\,\mathrm{t} \ldots}{f(3+m)}$$

Mathematica [A] time = 1.27768, size = 154, normalized size = 0.73

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\left(2^{m+\frac{3}{2}}m\left(m^2+3m+5\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{3}{2},\frac{1}{2}(\ldots\right.\right.}{f(m+2)(m+3 \ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*(5 +
 3*m + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (
1 + Sec[e + f*x])^(1/2 + m)*(4 + m + m^2 + m*(1 + 2*m)*Sec[e + f*x] + (2 +
5*m + 2*m^2)*Sec[e + f*x]^2))*Tan[e + f*x])/(f*(2 + m)*(3 + m)*(1 + 2*m))
```

Maple [F] time = 0.31, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^4\left(a+a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

3.342 $\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=155

$$\frac{2^{m+\frac{1}{2}}\left(m^2+m+1\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e$$
$$f(m+1)(m+2)$$

```
[Out] -(((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(2 + 3*m + m^2))) + (2^(1/2 + m)
*(1 + m + m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m
)*(2 + m)) + ((a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(a*f*(2 + m))
```

Rubi [A] time = 0.194901, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3800, 4001, 3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}}\left(m^2+m+1\right)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)(m+2)}-\frac{\tan(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(2 + 3*m + m^2))) + (2^(1/2 + m)
*(1 + m + m^2)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m
)*(2 + m)) + ((a + a*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(a*f*(2 + m))
```

Rule 3800

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2
, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
 + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^3(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af(2+m)} + \frac{\int \sec(e+fx)(a(1+m)-a\sec(e+fx))(a}{a(2+m)}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)} + \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af(2+m)} + \frac{\left(1+\right.}{}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)} + \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af(2+m)} + \frac{\left(\left(1\right.\right.}{}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)} + \frac{(a+a\sec(e+fx))^{1+m}\tan(e+fx)}{af(2+m)} - \frac{\left(\left(1\right.\right.}{}$$

$$= -\frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(2+3m+m^2\right)} + \frac{2^{\frac{1}{2}+m}\left(1+m+m^2\right)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\right.}{}$$

Mathematica [A] time = 0.621114, size = 123, normalized size = 0.79

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\left(2^{m+\frac{3}{2}}\left(m^2+m+1\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\text{s}\right.\right.}{f(m+2)(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*(1 + m
 + m^2)*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1 +
Sec[e + f*x])^(1/2 + m)*(-1 + m + (1 + 2*m)*Sec[e + f*x]))*Tan[e + f*x])/(f
*(2 + m)*(1 + 2*m))
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^3\left(a+a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.343 $\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=107

$$\frac{2^{m+\frac{1}{2}} m \tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)} +$$

```
[Out] ((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)) + (2^(1/2 + m)*m*Hypergeo
metric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2
 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m))
```

Rubi [A] time = 0.112958, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3798, 3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}} m \tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m \,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(m+1)} + \frac{\tan(e+fx)(a\sec}{f(m+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)) + (2^(1/2 + m)*m*Hypergeo
metric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1 + Sec[e + f*x])^(-1/2
 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m))
```

Rule 3798

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[(a*m)/(b*(m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /;
FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^m\,dx = \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)} + \frac{m\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx}{1+m}$$

$$= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)} + \frac{\left(m(1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)}{1+m}$$

$$= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)} - \frac{\left(m(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))\right.}{f(1+m}$$

$$= \frac{(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+m)} + \frac{2^{\frac{1}{2}+m}m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{}$$

Mathematica [A] time = 0.209321, size = 95, normalized size = 0.89

$$\frac{\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\left(2^{m+\frac{3}{2}}m\text{Hypergeometric2F1}\left(\frac{1}{2},-m-\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)\right.}{2fm+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] ((1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(2^(3/2 + m)*m*Hype
rgeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sec[e + f*x])/2] + (1 + Sec[e + f*x]
```

```
)^(1/2 + m))*Tan[e + f*x])/(f + 2*f*m)
```

Maple [F] time = 0.242, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{2}\left(a+a\sec \left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec \left(fx+e\right)+a\right)^{m}\sec \left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec \left(fx+e\right)+a\right)^{m}\sec \left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.344 $\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=73

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
+ Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/f
```

Rubi [A] time = 0.0612911, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
+ Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/f
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sec(e+fx)(1+\sec(e+fx))^m\,dx$$

$$= -\frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}}\,dx \ldots\right)}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+f\ldots}{f}$$

Mathematica [A] time = 0.10297, size = 73, normalized size = 1.

$$\frac{2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a(\sec(e+fx)+1))^m\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sec(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sec[e + f*x])/2]*(1
+ Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*Tan[e + f*x])/f
```

Maple [F] time = 0.267, size = 0, normalized size = 0.

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.345 $\int (a + a\sec(e + fx))^m \, dx$

Optimal. Leaf size=83

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec[e
 + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e +
 f*x]])
```

Rubi [A] time = 0.056996, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3779, 3778, 136}

$$\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},1;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec[e
 + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e +
 f*x]])
```

Rule 3779

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3778

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot
[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1
+ (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int (a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (1+\sec(e+fx))^m\,dx$$

$$= -\frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}x}\,dx,x,\sec(e+\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},1;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(a+a\sec(e+fx))^m\tan(e+}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 6.57736, size = 711, normalized size = 8.57

$$\frac{}{f\left(45\cos^2\left(\frac{1}{2}(e+fx)\right)(-2m\cos(e+fx)+\cos(2(e+fx))+2m+1)F_1\left(\frac{1}{2};m,1;\frac{3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (30*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(
e + f*x)/2]^2*Cos[e + f*x]*(a*(1 + Sec[e + f*x]))^m*Sin[e + f*x]*(3*AppellF
1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/
2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1
+ m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
/(f*(45*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]^2
*Cos[(e + f*x)/2]^2*(1 + 2*m - 2*m*Cos[e + f*x] + Cos[2*(e + f*x)]) + 6*App
ellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[(e + f*x
)/2]^2*(-5*AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*(1 + 2*m - 2*(2 + m)*Cos[e + f*x] + Cos[2*(e + f*x)]) + 5*m*AppellF1[3/2,
 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + 2*m - 2*(2 +
```

```
m)*Cos[e + f*x] + Cos[2*(e + f*x)]) - 48*(2*AppellF1[5/2, m, 3, 7/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*m*AppellF1[5/2, 1 + m, 2, 7/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*(1 + m)*AppellF1[5/2, 2 + m, 1, 7/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Cot[e + f*x]*Csc[e + f*x]*Sin[(
e + f*x)/2]^4) + 40*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])^2*Cos[e + f*x]*Sin[(e + f*x)/2]^2*Tan[(e + f*x)/2]^2))
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \left(a + a\sec\left(fx + e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m,x)
```

```
[Out] int((a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx + e\right) + a\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx + e\right) + a\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(e+fx\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*sec(e + f*x) + a)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m, x)
```

3.346 $\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

```
[Out] -((Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]]))
```

Rubi [A] time = 0.0963728, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3828, 3827, 136}

$$-\frac{\sqrt{2}\tan(e+fx)(a\sec(e+fx)+a)^m F_1\left(m+\frac{1}{2};\frac{1}{2},2;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1),\sec(e+fx)+1\right)}{f(2m+1)\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] -((Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Sec[e + f*x])/2, 1 + Sec
[e + f*x]]*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[1 - Sec[e
 + f*x]]))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos(e+fx)(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \cos(e+fx)(1+\sec(e+fx))^m\,dx$$

$$= -\frac{\left((1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}x^2}\,dx\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{\sqrt{2}F_1\left(\frac{1}{2}+m;\frac{1}{2},2;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx)),1+\sec(e+fx)\right)(a+a\sec(e+f}{f(1+2m)\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 16.931, size = 3781, normalized size = 45.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1 + m)*Cos[(e + f*x)/2]^3*Cos[e + f*x]*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^m*(a*(1 + Sec[e + f*x]))^m*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1
/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2,
m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m
, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (
2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(Appel
lF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*Appel
lF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3
/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2)/3)))/(f*(2^m*Cos[(e + f*x)/2]^4*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(
```

```
(-3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2)/(3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2])*Tan[(e + f*x)/2]^2) + (2*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2])/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) - 3*2^m*Cos[(e + f*x)/2]^2*(Cos[(e +
f*x)/2]^2*Sec[e + f*x])^m*Sin[(e + f*x)/2]^2*((-3*AppellF1[1/2, m, 1, 3/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1/
2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m
, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m,
 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (2
*AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(Appell
F1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*Appell
F1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/
2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2)/3)) + 2^(1 + m)*Cos[(e + f*x)/2]^3*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m
*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3*AppellF1[1/2, m, 1, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m, 2, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3*Sec[(e +
f*x)/2]^2*(-(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/2, 1 + m, 1, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3))/(3*AppellF1[1/2, m, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] - 2*(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 m*AppellF1[3/2, 1 + m, 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) + (2*((-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/
2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/3))/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3) + (3*AppellF1[1/2, m, 1, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(-2*(AppellF1[3/2, m
, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m,
 1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(
e + f*x)/2] + 3*(-(AppellF1[3/2, m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/2, 1 + m,
1, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/3) - 2*Tan[(e + f*x)/2]^2*((-6*AppellF1[5/2, m, 3, 7/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (
```

```
3*m*AppellF1[5/2, 1 + m, 2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 - m*((-3*AppellF1[5/2, 1 + m, 2, 7/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/
2])/5 + (3*(1 + m)*AppellF1[5/2, 2 + m, 1, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, m
, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m, 2,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 - (2*A
ppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*((-2*Appel
lF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/
2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (2*(-2*Ap
pellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF
1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2])/3 + (2*Tan[(e + f*x)/2]^2*(-2*((-9*AppellF1[5/2,
m, 4, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/5 + (3*m*AppellF1[5/2, 1 + m, 3, 7/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + m*((-6*AppellF1
[5/2, 1 + m, 3, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m)*AppellF1[5/2, 2 + m, 2, 7/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))
/3))/(AppellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (
2*(-2*AppellF1[3/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m
*AppellF1[3/2, 1 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2)/3)^2) + 2^(1 + m)*m*Cos[(e + f*x)/2]^3*(Cos[(e + f*x)/2]^2
*Sec[e + f*x])^(-1 + m)*Sin[(e + f*x)/2]*((-3*AppellF1[1/2, m, 1, 3/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1/2, m
, 1, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(AppellF1[3/2, m, 2,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - m*AppellF1[3/2, 1 + m, 1,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (2*App
ellF1[1/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1
/2, m, 2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(-2*AppellF1[3
/2, m, 3, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1
 + m, 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
/3))*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^
2*Sec[e + f*x]*Tan[e + f*x])))
```

Maple [F] time = 0.373, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^m \cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(fx+e\right)+a\right)^m \cos\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^m \cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*cos(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(f x + e\right) + a\right)^m \cos \left(f x + e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

3.347 $\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx$

Optimal. Leaf size=98

$$-\frac{2\tan(e+fx)(d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}}$$

```
[Out] (-2*AppellF1[3/2, 1/2, 1/2 - m, 5/2, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^(3/2)*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e +
f*x])/(3*f*Sqrt[1 - Sec[e + f*x]])
```

Rubi [A] time = 0.133955, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3827, 133}

$$-\frac{2\tan(e+fx)(d\sec(e+fx))^{3/2}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```


```
[Out] (-2*AppellF1[3/2, 1/2, 1/2 - m, 5/2, Sec[e + f*x], -Sec[e + f*x]]*(d*Sec[e
+ f*x])^(3/2)*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e +
f*x])/(3*f*Sqrt[1 - Sec[e + f*x]])
```


Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int (d\sec(e+fx))^{3/2}(1+\sec(e+$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{dx}(1}{\sqrt{}}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{2F_1\left(\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;\frac{5}{2};\sec(e+fx),-\sec(e+fx)\right)(d\sec(e+fx))^{3/2}(1+\sec(}{3f\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 14.9408, size = 2529, normalized size = 25.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (-3*2^(1 + m)*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2)*(Cos[(e + f*x)/2]^2
*Sec[e + f*x])^m*(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*(-1 + Tan[(e
 + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((3*2^(1 + m)*AppellF1[
1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f
*x)/2]^2)/((-1 + Tan[(e + f*x)/2]^2)^2*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 +
 m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2
```

)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m)/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*2^m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Sin[e + f*x]*Tan[(e + f*x)/2])/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1 [3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (3*2^(1 + m)*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]*((AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/ 2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((3/2 + m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2] ^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/((-1 + Tan[(e + f*x)/2]^2)*(3* AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (3*2^(1 + m)*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]*((AppellF1[3/2, 3/2 + m, 1/ 2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x) /2]^2*Tan[(e + f*x)/2] + 3*((AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x) /2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((3/2 + m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*((-3*Appe llF1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(3/2 + m)*AppellF1[5/2, 5/2 + m, 1/ 2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3 + 2*m)*((3*AppellF1[5/2, 5/2 + m, 1/2, 7/2, Tan[(e + f*x) /2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(5 /2 + m)*AppellF1[5/2, 7/2 + m, -1/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((-1 + Tan[(e + f*x)/2]^ 2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/ 2]^2] + (AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, - Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2) - (3*2^(1 + m)*m*AppellF1[1/2, 3/2 + m, -1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[e + f *x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]* Tan[e + f*x]))/((-1 + Tan[(e + f*x)/2]^2)*(3*AppellF1[1/2, 3/2 + m, -1/2, 3 /2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (AppellF1[3/2, 3/2 + m, 1/2,

```
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)*AppellF1[3/2, 5/
2 + m, -1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2
]^2))))
```

Maple [F] time = 0.195, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a+a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sec\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m}d\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m*d*sec(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sec\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(a*sec(f*x + e) + a)^m, x)
```

3.348 $\int \sqrt{d \sec(e+fx)}(a+a\sec(e+fx))^m \, dx$

Optimal. Leaf size=96

$$-\frac{2\tan(e+fx)\sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

```
[Out] (-2*AppellF1[1/2, 1/2, 1/2 - m, 3/2, Sec[e + f*x], -Sec[e + f*x]]*Sqrt[d*Se
c[e + f*x]]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Rubi [A] time = 0.117808, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3827, 133}

$$-\frac{2\tan(e+fx)\sqrt{d\sec(e+fx)}(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (-2*AppellF1[1/2, 1/2, 1/2 - m, 3/2, Sec[e + f*x], -Sec[e + f*x]]*Sqrt[d*Se
c[e + f*x]]*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*Sqrt[1 - Sec[e + f*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
```

```
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}(a+a\sec(e+fx))^m\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \sqrt{d\sec(e+fx)}(1+\sec(e+fx$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{}}{\sqrt{1-x}}\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= -\frac{2F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\sec(e+fx),-\sec(e+fx)\right)\sqrt{d\sec(e+fx)}(1+\sec(e+}{f\sqrt{1-\sec(e+fx)}}$$

Mathematica [B] time = 14.5274, size = 2225, normalized size = 23.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + a*Sec[e + f*x])^m,x]
```

```
[Out] (2^(1 + m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sqrt[d*Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*
(a*(1 + Sec[e + f*x]))^m*Tan[(e + f*x)/2])/(f*Sqrt[Sec[(e + f*x)/2]^2]*(App
ellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((
AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
 (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)*((2^m*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sqrt[Sec[(e + f*x)/2]^2]*(Cos[(e + f
*x)/2]^2*Sec[e + f*x])^(1/2 + m))/(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3) - (
2^m*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
```

2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]^2)/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) + (2^(1 + m)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]*(-(AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((1/2 + m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)) - (2^(1 + m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*Tan[(e + f*x)/2]*(-(AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((1/2 + m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 - (Tan[(e + f*x)/2]^2*((-9*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(1/2 + m)*AppellF1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 - (1 + 2*m)*((-3*AppellF1[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(3/2 + m)*AppellF1[5/2, 5/2 + m, 1/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/3))/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)^2) + (2^(1 + m)*(1/2 + m)*AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(Sqrt[Sec[(e + f*x)/2]^2]*(AppellF1[1/2, 1/2 + m, 1/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - (1 + 2*m)*AppellF1[3/2, 3/2 + m, 1/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3))))

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int \sqrt{d\sec\left(fx+e\right)}\left(a+a\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}\sqrt{d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+a*sec(f*x+e))**m,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sqrt(d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \sec \left(fx+e\right)} \left(a \sec \left(fx+e\right)+a\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

3.349 $\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=96

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] (2*AppellF1[-1/2, 1/2, 1/2 - m, 1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec[
e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 - Sec[e
 + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.12527, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3827, 133}

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*AppellF1[-1/2, 1/2, 1/2 - m, 1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec[
e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 - Sec[e
 + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
```

```
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \frac{(1+\sec(e+fx))^m}{\sqrt{d\sec(e+fx)}}\,dx$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}(dx)^{3/2}}\,dx, x, \sec(e\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2F_1\left(-\frac{1}{2};\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m}{f\sqrt{1-\sec(e+fx)}\sqrt{d\sec(e+fx)}}$$

Mathematica [C] time = 14.804, size = 2424, normalized size = 25.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-3*2^(1 + m)*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sqrt[Sec[e + f*x]]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 +
m)*(a*(1 + Sec[e + f*x]))^m*(Cos[2*(e + f*x)]*((1 + Sec[e + f*x])^m/(2*Sqrt
[Sec[e + f*x]]) - (I/2)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m*Sin[e + f*x
]) + ((1 + Sec[e + f*x])^m/2 + (I/2)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)])
/Sqrt[Sec[e + f*x]] + Sqrt[Sec[e + f*x]]*Sin[e + f*x]*((-I/2)*(1 + Sec[e +
f*x])^m + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)])/2))*Tan[(e + f*x)/2])/(f*
(Sec[(e + f*x)/2]^2)^(3/2)*Sqrt[d*Sec[e + f*x]]*(1 + Sec[e + f*x])^m*(-3*Ap
pellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
(3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
```

+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)*((-3*2^m*AppellF1[1/2, -1/2 + m, 3/2, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^(-1/2 + m))/(Sqrt[Sec[(e + f*x)/2]^2]*(-3*AppellF1[1/2, -1/2 + m, 3/2, 3
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m, 5
/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2,
 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)
/2]^2)) + (9*2^m*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*x)
/2]^2)/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m, 5/2, 5/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 +
 m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
) - (3*2^(1 + m)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2 + m)*Tan[(e + f*x)
/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + ((-1/2 + m)*AppellF1[3/2, 1
/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/3))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2
 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2,
 -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*A
ppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2)) + (3*2^(1 + m)*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1/2
 + m)*Tan[(e + f*x)/2]*((3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] -
 3*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + ((-1/2 + m)*AppellF1[3/2, 1/
2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(3*((-3*AppellF1[5/2, -1/2 + m,
 7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/2 + (3*(-1/2 + m)*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (1
 - 2*m)*((-9*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/10 + (3*(1/2 + m)*AppellF1
[5/2, 3/2 + m, 3/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[
1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3*Appe
llF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1
 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2])*Tan[(e + f*x)/2]^2)^2) - (3*2^(1 + m)*(-1/2 + m)*AppellF1[1/2, -1/2
 + m, 3/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(-3/2 + m)*Tan[(e + f*x)/2]*(-(Cos[(e + f*x)/2]*Sec[e + f*x
]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/((Sec[
(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 3/2, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (3*AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f

```
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 3/2, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(fx+e\right)\right)^{m} \frac{1}{\sqrt{d\sec\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sec\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m}}{d\sec\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{m}}{\sqrt{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m/sqrt(d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m/sqrt(d*sec(f*x + e)), x)
```

3.350 $\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/2}}$$

```
[Out] (2*AppellF1[-3/2, 1/2, 1/2 - m, -1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec
[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(3*f*Sqrt[1 - Se
c[e + f*x]]*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.135567, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3828, 3827, 133}

$$\frac{2\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)}{3f\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*AppellF1[-3/2, 1/2, 1/2 - m, -1/2, Sec[e + f*x], -Sec[e + f*x]]*(1 + Sec
[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(3*f*Sqrt[1 - Se
c[e + f*x]]*(d*Sec[e + f*x])^(3/2))
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
```

```
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx = \left((1+\sec(e+fx))^{-m}(a+a\sec(e+fx))^m\right)\int \frac{(1+\sec(e+fx))^m}{(d\sec(e+fx))^{3/2}}\,dx$$

$$= -\frac{\left(d(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx))^m\tan(e+fx)\right)\text{Subst}\left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}(dx)^{5/2}}\,dx, x, \sec\right.}{f\sqrt{1-\sec(e+fx)}}$$

$$= \frac{2F_1\left(-\frac{3}{2};\frac{1}{2},\frac{1}{2}-m;-\frac{1}{2};\sec(e+fx),-\sec(e+fx)\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e+fx)}{3f\sqrt{1-\sec(e+fx)}(d\sec(e+fx))^{3/2}}$$

Mathematica [C] time = 19.3691, size = 3349, normalized size = 34.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[e + f*x])^m/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2^(1 + m)*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2 + m)*(
a*(1 + Sec[e + f*x]))^m*((Cos[2*(e + f*x)]^3*Sqrt[Sec[e + f*x]]*(1 + Sec[e
+ f*x])^m)/4 + Cos[2*(e + f*x)]^2*Sqrt[Sec[e + f*x]]*((1 + Sec[e + f*x])^m/
2 + (I/4)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]) + Cos[2*(e + f*x)]*Sqrt[Se
c[e + f*x]]*((1 + Sec[e + f*x])^m/4 + ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)
]^2)/4) + Sqrt[Sec[e + f*x]]*((-I/4)*(1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]
+ ((1 + Sec[e + f*x])^m*Sin[2*(e + f*x)]^2)/2 + (I/4)*(1 + Sec[e + f*x])^m*
Sin[2*(e + f*x)]^3))*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^
(1/2 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3/2
```

```
)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))))/(3*f*(d*Sec[e + f*x])^(3/2)*(
1 + Sec[e + f*x])^m*((2^m*Sec[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^(1/2 + m)*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2 + m)*Tan[(e + f*x)/
2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2
 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2,
 -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*A
ppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2))))/3 + (2^(1 + m)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(1/2
 + m)*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]
^2)^(1/2 + m)*Tan[(e + f*x)/2]) - (Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(1/2 +
m)*Tan[(e + f*x)/2]^2*((-3*AppellF1[5/2, -1/2 + m, 7/2, 7/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (3*(-1/
2 + m)*AppellF1[5/2, 1/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) - (1/2 + m)*AppellF1[3/2, -1/
2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec
[(e + f*x)/2]^2)^(-1/2 + m)*Tan[(e + f*x)/2]^2*(-(Sec[(e + f*x)/2]^2*Sin[e
+ f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (9*AppellF1[1
/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*
x])/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m,
 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) +
 (27*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*Cos[e + f*x]*Tan[(e + f*x)/2])/(2*(Sec[(e + f*x)/2]^2)^(3/2)*(-3*Appel
lF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*
AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2])*Tan[(e + f*x)/2]^2)) - (9*Cos[e + f*x]*((-5*AppellF1[3/2, -1/2
+ m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/6 + ((-1/2 + m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(
(Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + (9*A
ppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*C
os[e + f*x]*((5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] - 3*((-5*Ap
```

```
pellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2])/6 + ((-1/2 + m)*AppellF1[3/2, 1/2 + m, 5
/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2])/3) + Tan[(e + f*x)/2]^2*(5*((-21*AppellF1[5/2, -1/2 + m, 9/2, 7
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/10 + (3*(-1/2 + m)*AppellF1[5/2, 1/2 + m, 7/2, 7/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + (1 - 2*m
)*((-3*AppellF1[5/2, 1/2 + m, 7/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (3*(1/2 + m)*AppellF1[5/2, 3
/2 + m, 5/2, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]
^2*Tan[(e + f*x)/2])/5))))/((Sec[(e + f*x)/2]^2)^(3/2)*(-3*AppellF1[1/2, -1
/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (5*AppellF1[3/
2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 - 2*m)
*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2)^2)))/3 + (2^(1 + m)*(1/2 + m)*(Cos[(e + f*x)/2]^2*Sec[
e + f*x])^(-1/2 + m)*Tan[(e + f*x)/2]*(-(AppellF1[3/2, -1/2 + m, 5/2, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^
(1/2 + m)*Tan[(e + f*x)/2]^2) - (9*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/((Sec[(e + f*x)/2]^2)^(3/2
)*(-3*AppellF1[1/2, -1/2 + m, 5/2, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + (5*AppellF1[3/2, -1/2 + m, 7/2, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (1 - 2*m)*AppellF1[3/2, 1/2 + m, 5/2, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))*(-(Cos[(e + f*x)/2]*Sec[e + f*
x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/3))
```

Maple [F] time = 0.186, size = 0, normalized size = 0.

$$\int \left(a + a\sec\left(fx+e\right)\right)^{m} \left(d\sec\left(fx+e\right)\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

```
[Out] int((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right) + a\right)^{m}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\left(a\sec\left(fx+e\right)+a\right)^{m}}{d^{2}\sec\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(a*sec(f*x + e) + a)^m/(d^2*sec(f*x + e)^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))**m/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(f*x+e))^m/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m/(d*sec(f*x + e))^(3/2), x)
```

3.351 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{10a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin}{}$$

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.0902441, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2639, 2641}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin(c+dx)}{21}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= \int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx \\
&= a\int \cos^{\frac{5}{2}}(c+dx)\,dx + a\int \cos^{\frac{7}{2}}(c+dx)\,dx \\
&= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(3a)\int \sqrt{\cos(c+ \\
&= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{5d} \\
&= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a}{}
\end{aligned}$$

Mathematica [C] time = 6.17138, size = 490, normalized size = 4.41

$$a\left(-\frac{3\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos(\tan}}\right.}{10d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*Cot[c])/
(5*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[3
*d*x]*Sin[3*c])/(28*d) + (23*Cos[c]*Sin[d*x])/(84*d) + (Cos[2*c]*Sin[2*d*x]
)/(10*d) + (Cos[3*c]*Sin[3*d*x])/(28*d)) - (5*(1 + Cos[c + d*x])*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*(1 + Cos[c + d*x])*Csc[c]
*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [A] time = 1.414, size = 270, normalized size = 2.4

$$-\frac{2\,a}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-528\,\left(\sin\left(1/2\,dx+c/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+25*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos(dx+c)^3\sec(dx+c)+a\cos(dx+c)^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.352 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.080015, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```


```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```


Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx\\ &= a\int \cos^{\frac{3}{2}}(c+dx)\,dx + a\int \cos^{\frac{5}{2}}(c+dx)\,dx\\ &= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\\ &= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a\cos}{}\end{aligned}$$

Mathematica [C] time = 5.47604, size = 232, normalized size = 2.67

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-18\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hypergeo}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[
c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*
```

```
Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 1.461, size = 219, normalized size = 2.5

$$-\frac{2a}{15d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(24\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{7}-28\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{5}+5\,\sqrt{(\sin(1/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\cos(dx+c)^{2}\sec(dx+c)+a\cos(dx+c)^{2}\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec(dx + c) + a) \cos(dx + c)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.353 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0693064, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2639, 2635, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx &= \int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx\\
&= a\int \sqrt{\cos(c+dx)}\,dx + a\int \cos^{\frac{3}{2}}(c+dx)\,dx\\
&= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}
\end{aligned}
$$

Mathematica [C] time = 4.97961, size = 222, normalized size = 3.64

$$a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hypergeon}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos[c
 + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*S
in[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x + ArcTa
n[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d
```

```
*x]])
```

Maple [B] time = 1.333, size = 225, normalized size = 3.7

$$-\frac{2a}{3d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,(\sin(1/2\,dx+c/2))^{4}\cos(1/2\,dx+c/2)+\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\cos(dx+c)\sec(dx+c)+a\cos(dx+c))\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(cos(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec(dx + c) + a) \cos(dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.354 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

Rubi [A] time = 0.0587835, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4225, 2748, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))\,dx &= \int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\\ &= a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + a\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [C] time = 1.80084, size = 155, normalized size = 4.43

$$\frac{a\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+dx\right)\right)}{\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}} - \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d*
x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x + A
rcTan[Tan[c]]] - (HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(
2*d)
```

Maple [A] time = 1.16, size = 150, normalized size = 4.3

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\,a\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\,\left(\text{EllipticF}\left(\text{co}\right.\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\,\sin(1/2\,dx+c/2)\,\sqrt{2\,(\cos(1/}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)\sqrt{\cos (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a \sec (dx + c) + a)\sqrt{\cos (dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sqrt{\cos (c + dx)} \sec (c + dx)\, dx + \int \sqrt{\cos (c + dx)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c + d*x
)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.355 $\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0673231, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \sqrt{\cos(c+dx)}\,dx\\ &= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 4.77699, size = 209, normalized size = 3.67

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hypergeom}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c - d
*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[S
ec[c]^2] - 4*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x -
```

```
ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[S
in[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 1.641, size = 146, normalized size = 2.6

$$-2\frac{a\left(\text{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}+\text{EllipticE}\left(\cos\left(1/2\,dx+c\right.\right.}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*a*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\sec\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c + d
*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.356 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 0.0790063, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```


Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - a\int \sqrt{\cos(c+dx)}\,dx \\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 6.14859, size = 444, normalized size = 5.35

$$
a\left(\frac{\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+dx\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-1}(\tan\right.}}\right.}{2d}\right.
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((Csc[c]*Sec[
c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c + d*x]*(Sin[
c] + 3*Sin[d*x]))/(3*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*Sqrt[1 + Cot[c]^2]) + ((1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(2*d))
```

Maple [B] time = 2.303, size = 369, normalized size = 4.5

$$\frac{2a}{3d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(6\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\text{EllipticE}\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(6*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a \sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.357 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0907566, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```


```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3a)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(3a)\int \sqrt{\cos
\end{aligned}
$$

$$
= -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c}{5d\sqrt{\cos(c}}
$$

Mathematica [C] time = 6.17843, size = 477, normalized size = 4.3

$$a\left(\frac{3\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+dx\right)\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-1}(t\right.}}\right.}{10d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*Se
c[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*x]
^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*Sin
[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
t[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
10*d))
```

Maple [B] time = 2.505, size = 384, normalized size = 3.5

$$-4\,\frac{\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}a}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1}d}\left(-1/12\,\frac{\cos\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin(1/2\right.}}{\left(\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1/2\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/12*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.358 $\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{10a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a\sin(c}{5d\sqrt{\cos(}}$$

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*a*Sin[c + d*x])/(5
*d*Cos[c + d*x]^(5/2)) + (10*a*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (6
*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.101516, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2641, 2639}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*a*Sin[c + d*x])/(5
*d*Cos[c + d*x]^(5/2)) + (10*a*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)) + (6
*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \int \frac{a+a\cos(c+dx)}{\cos^{\frac{9}{2}}(c+dx)}\,dx\\ &= a\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}(3a)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{7}(5a)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{21}(5a)\int \frac{1}{\sqrt{\cos(c}}\\ &= -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{10a\sin(c}{21d\cos^{\frac{3}{2}}(c}\end{aligned}$$

Mathematica [C] time = 4.68017, size = 294, normalized size = 2.18

$$a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{126\sec(c)\cos^3(c+dx)\left(\csc(c)\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(3\cos\left(c-\tan^{-1}(\tan(c))-dx\right)+\cos\left(c+\tan^{-1}(\tan(c))+d\right.\right.\right.}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(ta\right.}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((189*Cos[c] + 85*Cos[d*x] - 85*Co
s[2*c + d*x] + 231*Cos[c + 2*d*x] + 21*Cos[3*c + 2*d*x] + 25*Cos[2*c + 3*d*
x] - 25*Cos[4*c + 3*d*x] + 63*Cos[3*c + 4*d*x])*Csc[c] - 200*Cos[c + d*x]^4
*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1
/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] -
 (126*Cos[c + d*x]^3*Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTa
n[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Ta
n[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(840*d*Cos
[c + d*x]^(7/2))
```

Maple [B] time = 2.555, size = 437, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/40*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+44/105*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/10*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/112*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/84*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a \sec(dx + c) + a}{\cos(dx + c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.359 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=147

$$\frac{20a^2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+$$

```
[Out] (32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2,
2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.174267, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2,
2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{s\right.$$
$$= \frac{4a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{7}\left(10a^2\sqrt{\cos(c\right.$$
$$= \frac{20a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4a^2\cos^{\frac{5}{2}}(c+}{}$$
$$= \frac{20a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4a^2\cos^{\frac{5}{2}}(c+}{}$$
$$= \frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}$$

Mathematica [C] time = 6.15495, size = 548, normalized size = 3.73

$$-\frac{4\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\text{co}\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{t}}}\right.}{15d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-8*Cot[c])
/(15*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (37*Cos[2*d*x]*Sin[2*c])/(360*d) +
(Cos[3*d*x]*Sin[3*c])/(28*d) + (Cos[4*d*x]*Sin[4*c])/(144*d) + (23*Cos[c]*S
in[d*x])/(84*d) + (37*Cos[2*c]*Sin[2*d*x])/(360*d) + (Cos[3*c]*Sin[3*d*x])/
(28*d) + (Cos[4*c]*Sin[4*d*x])/(144*d)) - (5*Cos[c + d*x]^2*Csc[c]*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2
]^4*(a + a*Sec[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (4*Cos[c +
 d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[
```

```
c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan
[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1
 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2
*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^
2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

Maple [A] time = 1.381, size = 260, normalized size = 1.8

$$-\frac{4\,a^2}{315\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(560\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{11}-960\,\left(\cos\left(1/2\,dx+c/2\right)\right)^9+608\,\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*cos
(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*cos
(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^4\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^4*sec(d*x
+ c) + a^2*cos(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2 \cos(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

3.360 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=121

$$\frac{8a^2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2}{}$$

```
[Out] (12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2]
)/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```

Rubi [A] time = 0.157682, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2]
)/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.$$

$$= \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}\left(6a^2\sqrt{\cos(c+\right.$$

$$= \frac{8a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{7d} + \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx}{7d}$$

$$= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{7d} + \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{8a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{7d} + \frac{4}{}$$

Mathematica [C] time = 6.12902, size = 516, normalized size = 4.26

$$-\frac{3\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan}}}\right.}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-3*Cot[c])
/(5*d) + (17*Cos[d*x]*Sin[c])/(56*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[
3*d*x]*Sin[3*c])/(56*d) + (17*Cos[c]*Sin[d*x])/(56*d) + (Cos[2*c]*Sin[2*d*x
])/(10*d) + (Cos[3*c]*Sin[3*d*x])/(56*d)) - (2*Cos[c + d*x]^2*Csc[c]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)
/2]^4*(a + a*Sec[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (3*Cos[c
+ d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Hypergeometri
cPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan
[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTa
```

```
n[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[
1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (
2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]
^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

Maple [A] time = 1.224, size = 272, normalized size = 2.3

$$-\frac{4a^2}{35d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(40\,\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^8-116\,(\sin(1/2\,dx+c/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
-21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2\cos(dx+c)^3\sec(dx+c)^2+2\,a^2\cos(dx+c)^3\sec(dx+c)+a^2\cos(dx+c)^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^3*sec(d*x
+ c) + a^2*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^2 \cos (dx + c)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

3.361 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=95

$$\frac{4a^2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.143299, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{\right.$$

$$= \frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2a^2\sqrt{\cos(c\right.$$

$$= \frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2a^2\right)\int \frac{}{\sqrt{\mathrm{co}}}$$

$$= \frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} +$$

Mathematica [C] time = 5.55007, size = 235, normalized size = 2.47

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-24\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyper}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20
*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[
c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c + d*x
)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan
[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 1.386, size = 250, normalized size = 2.6

$$-\frac{4a^2}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-12\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+32\left(\sin(1/2\,dx+c/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-13*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2\cos(dx+c)^2\sec(dx+c)^2+2\,a^2\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.362 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{8a^2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.128058, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3788, 3771, 2639, 4045, 2641}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{s\right.$$
$$= \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \left(2a^2\right)\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}\left(4a^2\sqrt{\cos(c+dx\right.$$
$$= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(4a^2\right)\int \frac{1}{\sqrt{\cos(c+dx}}$$
$$= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [C] time = 5.1414, size = 224, normalized size = 3.34

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyperg}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8*C
os[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]]*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d*x +
ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[
c + d*x]])
```

Maple [B] time = 1.22, size = 228, normalized size = 3.4

$$-\frac{4a^2}{3d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,(\sin(1/2\,dx+c/2))^4\cos(1/2\,dx+c/2)+2\,\mathrm{EllipticF}\left(\cos(1/2\,dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\cos\left(dx+c\right)\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a^2*cos(d*x + c)*sec(d*x + c)
 + a^2*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.363 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=44

$$\frac{4a^2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (4*a^2*EllipticF[(c + d*x)/2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rubi [A] time = 0.109488, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3788, 3771, 2641, 4043}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*a^2*EllipticF[(c + d*x)/2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx \\
&= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{\right. \\
&= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(2a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.177608, size = 39, normalized size = 0.89

$$
\frac{2a^2\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*(2*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Sqrt[Cos[c + d*x]]))/d
```

Maple [A] time = 1.662, size = 104, normalized size = 2.4

$$-4\,\frac{a^{2}\left(\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}-\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\cos\left(1/\right.}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -4*a^2*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(cos(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sqrt{\cos(c+dx)}\sec(c+dx)\,dx+\int\sqrt{\cos(c+dx)}\sec^2(c+dx)\,dx+\int\sqrt{\cos(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(sqrt(cos(c
+ d*x))*sec(c + d*x)**2, x) + Integral(sqrt(cos(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.364 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{8a^2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.138596, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\left(a^2+a^2\sec^2(c+dx)\right)dx + \left(2a^2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(4a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - \left(2a^2\right)\int \sqrt{\cos(c+dx)}\,dx$$

$$= -\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

<u>Mathematica</u> [C] time = 6.16723, size = 470, normalized size = 5.16

$$\frac{\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)}}}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((Csc[c]*Sec
[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(Sin
[c] + 6*Sin[d*x]))/(6*d)) - (2*Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec
[c + d*x])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*
Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d
*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (Cos[c + d*x]^2*Csc[c]*Se
c[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4},
 {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqr
t[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - (
(Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d)
```

Maple [B] time = 2.13, size = 371, normalized size = 4.1

$$\frac{4a^2}{3d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+
1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/sqrt(cos(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{2\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(2*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(sec(c + d*x
)**2/sqrt(cos(c + d*x)), x) + Integral(1/sqrt(cos(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.365 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{4a^2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]])
```

Rubi [A] time = 0.153581, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx + \left(2a^2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,d$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(2a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - \frac{1}{5}$$

$$= \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}\left(8a^2\right)\int$$

$$= -\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a^2}{5d\sqrt{}}$$

Mathematica [C] time = 6.22103, size = 503, normalized size = 4.16

$$\frac{2\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan}}}\right.}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((4*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d*
x]^2*(3*Sin[c] + 10*Sin[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 12
*Sin[d*x]))/(15*d)) - (Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x
])^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(S
qrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - Arc
Tan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (2*Cos[c + d*x]^2*Csc[c]*Sec[c/2
+ (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}
, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*
```

```
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d
*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

Maple [B] time = 2.259, size = 386, normalized size = 3.2

$$-8\,\frac{\sqrt{-\left(-2\,\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin(1/2\,dx+c/2)\right)^2}\,a^2}{\sin(1/2\,dx+c/2)\,\sqrt{2\,\left(\cos(1/2\,dx+c/2)\right)^2-1}\,d}\left(-1/12\,\frac{\cos(1/2\,dx+c/2)\,\sqrt{-2\,\left(\sin(1/2\,dx+c/2)\right)^4+(\sin(1/2}}{\left(\left(\cos(1/2\,dx+c/2)\right)^2-1/2\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/12*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-4/5*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)-2/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2 \sec(dx+c)^2 + 2a^2 \sec(dx+c) + a^2}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/cos(d*x + c)^(3/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.366 $\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{8a^2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7d}-\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{12a^2\sin(}{5d\sqrt{\cos(}}$$

```
[Out] (-12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Cos[c + d*x]^(5/2)) + (8*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(3/2
)) + (12*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.172944, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}-\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Cos[c + d*x]^(5/2)) + (8*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(3/2
)) + (12*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)dx + \left(2a^2\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(6a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)} + \frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{7}\left(4a^2\sqrt{\cos(c+\right.$$

$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\cos^{\frac{3}{2}}(c+dx)} + \frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{7}\left(4a^2\right)\int \frac{}{\sqrt{\text{co}}}$$

$$= -\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^2\,\text{s}}{7d\,\text{co}}$$

Mathematica [C] time = 6.22329, size = 531, normalized size = 3.61

$$\frac{3\csc(c)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2(\text{t}\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(}}}\right.}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] Cos[c + d*x]^(5/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((3*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(14*d) + (Sec[c]*Sec[c + d*
x]^2*(7*Sin[c] + 10*Sin[d*x]))/(35*d) + (Sec[c]*Sec[c + d*x]^3*(5*Sin[c] +
14*Sin[d*x]))/(70*d) + (Sec[c]*Sec[c + d*x]*(10*Sin[c] + 21*Sin[d*x]))/(35*
d)) - (2*Cos[c + d*x]^2*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x
 - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(7*d*Sqrt[1 + Cot[c]^2]) + (3*Cos[c + d*x]^2*Csc[c]*Sec[c/2 + (d*x)/2]^4*(
a + a*Sec[c + d*x])^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
```

```
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

Maple [B] time = 2.522, size = 439, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/14*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+31/70*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/224*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-1/40*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(
1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a \sec(dx + c) + a)^2}{\cos(dx + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)/cos(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.367 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=147

$$\frac{44a^3\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+$$

```
[Out] (68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.251881, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3769, 3771, 2639, 2641}

$$\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a^3\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)}\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{6a^3\cos^{\frac{5}{2}}(c+d}{7d}$$

$$= \frac{44a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{68a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{6a^3\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{44a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}$$

$$= \frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{44a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}$$

<u>Mathematica</u> [C] time = 6.1461, size = 548, normalized size = 3.73

$$-\frac{17\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},c\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)}\sqrt{}}\right.}{60d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((-17*Cot[c]
)/(30*d) + (97*Cos[d*x]*Sin[c])/(336*d) + (73*Cos[2*d*x]*Sin[2*c])/(720*d)
+ (3*Cos[3*d*x]*Sin[3*c])/(112*d) + (Cos[4*d*x]*Sin[4*c])/(288*d) + (97*Cos
[c]*Sin[d*x])/(336*d) + (73*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*Cos[3*c]*Sin[
3*d*x])/(112*d) + (Cos[4*c]*Sin[4*d*x])/(288*d)) - (11*Cos[c + d*x]^3*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
```

```
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) -
(17*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(60*d)
```

Maple [A] time = 1.34, size = 260, normalized size = 1.8

$$-\frac{4\,a^3}{315\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(560\,(\cos(1/2\,dx+c/2))^{11}-600\,(\cos(1/2\,dx+c/2))^{9}+212\,(\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(a^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)+a^3\cos\left(dx+\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^4*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^4*sec(d*x
+ c)^2 + 3*a^3*cos(d*x + c)^4*sec(d*x + c) + a^3*cos(d*x + c)^4)*sqrt(cos(d
*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

3.368 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=121

$$\frac{52a^3\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52}{}$$

```
[Out] (28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2
])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(7*d)
```

Rubi [A] time = 0.219671, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3769, 3771, 2641, 2639}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52a^3\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2
])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(7*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^3\cos^{\frac{5}{2}}(c+dx)}{7d}$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

Mathematica [C] time = 6.13782, size = 516, normalized size = 4.26

$$-\frac{7\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((-7*Cot[c])
/(10*d) + (107*Cos[d*x]*Sin[c])/(336*d) + (3*Cos[2*d*x]*Sin[2*c])/(40*d) +
(Cos[3*d*x]*Sin[3*c])/(112*d) + (107*Cos[c]*Sin[d*x])/(336*d) + (3*Cos[2*c]
*Sin[2*d*x])/(40*d) + (Cos[3*c]*Sin[3*d*x])/(112*d)) - (13*Cos[c + d*x]^3*C
sc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec
[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]
```

```
) - (7*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((
HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan
[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[
c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]
))/(20*d)
```

Maple [A] time = 1.633, size = 272, normalized size = 2.3

$$-\frac{4a^3}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8-432\left(\sin\left(1/2\,dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+65*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^3\cos\left(dx+c\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^3*sec(d*x
+ c)^2 + 3*a^3*cos(d*x + c)^3*sec(d*x + c) + a^3*cos(d*x + c)^3)*sqrt(cos(d
*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

3.369 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{4a^3\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

```
[Out] (36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2]
)/d + (2*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.185739, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3769, 3771, 2639, 2641}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2]
)/d + (2*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d}+\frac{2a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+a^3\int \frac{1}{\sqrt{\cos(c+}}$$

$$= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d}+\frac{2a}{}$$

$$= \frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d}+\frac{2}{}$$

Mathematica [C] time = 5.98857, size = 233, normalized size = 2.56

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-18\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyper}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*
Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c
]]]*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(40*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 1.434, size = 250, normalized size = 2.8

$$-\frac{4a^3}{5d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+14\left(\sin\left(1/2\,dx+c/2\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+5*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)-9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{3} \cos\left(dx+c\right)^{2} \sec\left(dx+c\right)^{3} + 3\,a^{3} \cos\left(dx+c\right)^{2} \sec\left(dx+c\right)^{2} + 3\,a^{3} \cos\left(dx+c\right)^{2} \sec\left(dx+c\right) + a^{3} \cos\left(dx+c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^2*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)^2*sec(d*x
+ c)^2 + 3*a^3*cos(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x + c)^2)*sqrt(cos(d
*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec\left(dx+c\right) + a\right)^{3} \cos\left(dx+c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

3.370 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{20a^3\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.19675, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+3a^3\sqrt{\sec(c+\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c-\right.$$

$$= \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\left(3a^3\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx+$$

$$= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+}}{3}$$

$$= \frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+}}{}$$

Mathematica [C] time = 4.84854, size = 240, normalized size = 2.64

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyperg}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-3*Cos[d*x]*Csc[c] - 9*Cos[2*
c + d*x]*Csc[c] + 9*Cos[c - d*x - ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] + 3
*Cos[c + d*x + ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt
[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Sin[
2*(c + d*x)] - 6*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Ar
cTan[Tan[c]]]^2]))/(24*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 1.664, size = 172, normalized size = 1.9

$$-\frac{4a^3}{3d}\left(2\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+5\,\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^3\cos(dx+c)\sec(dx+c)^3+3\,a^3\cos(dx+c)\sec(dx+c)^2+3\,a^3\cos(dx+c)\sec(dx+c)+a^3\cos(dx+c)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a^3*cos(d*x + c)*sec(d*x + c)
^2 + 3*a^3*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c))*sqrt(cos(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.371 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{20a^3\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (6*a^3*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.194153, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3771, 2639, 2641, 3768}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (6*a^3*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx \\
&= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(\frac{a^3}{\sqrt{\sec(c+dx)}}+3a^3\sqrt{\sec(c+dx)}+3a^3\sec^{\frac{3}{2}}\right. \\
&= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c\right. \\
&= \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+a^3\int \sqrt{\cos(c+dx)}\,dx+\left(3a^3\right)\int \frac{1}{\sqrt{\cos(}} \\
&= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+d}{d\sqrt{\cos(c+d}} \\
&= -\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c}{d\sqrt{\cos(c}}
\end{aligned}$$

Mathematica [C] time = 6.20862, size = 479, normalized size = 5.26

$$\frac{\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)}}}\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(-((-5 + Cos
[2*c])*Csc[c]*Sec[c])/(8*d) + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*d) + (Se
c[c]*Sec[c + d*x]*(Sin[c] + 9*Sin[d*x]))/(12*d)) - (5*Cos[c + d*x]^3*Csc[c]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + (C
os[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((Hypergeo
metricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTa
n[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x +
ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*
Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2
] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + S
in[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [B] time = 2.535, size = 371, normalized size = 4.1

$$\frac{4a^3}{3d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(10\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(10*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
```

```
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^2-18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.372 $\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=117

$$\frac{4a^3\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2
])/d + (2*a^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a^3*Sin[c + d*x])
/(d*Cos[c + d*x]^(3/2)) + (36*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.223226, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3771, 2641, 3768, 2639}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2
])/d + (2*a^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a^3*Sin[c + d*x])
/(d*Cos[c + d*x]^(3/2)) + (36*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(a^3\sqrt{\sec(c+dx)} + 3a^3\sec^{\frac{3}{2}}(c+dx) + 3a^3\sec^{\frac{5}{2}}(c+dx) + a\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,dx + \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec$$

$$= \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + a^3\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}\left(3a^3\sqrt{c\right.$$

$$= \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + a^3\int \frac{}{\sqrt{\cos}}$$

$$= -\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{36a^3\,\mathrm{s}}{5d\sqrt{\mathrm{c}}}$$

$$= -\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{36a^3}{5d\sqrt{}}$$

Mathematica [C] time = 6.22254, size = 501, normalized size = 4.28

$$\frac{9\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\mathrm{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((9*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(20*d) + (Sec[c]*Sec[c + d
*x]^2*(Sin[c] + 5*Sin[d*x]))/(20*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 18*S
in[d*x]))/(20*d)) - (Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(2*d*Sqrt[1 + Cot[c]^2]) + (9*Cos[c + d*x]^3*Csc[c]*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
```

```
Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] time = 2.706, size = 386, normalized size = 3.3

$$-16\,\frac{\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}a^{3}}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(\frac{7\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{Ellip}}{10\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-9/10*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x
+1/2*c)^2)^(1/2)-9/20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{3}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a\sec(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.373 $\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{52a^3\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^
(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.238677, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3791, 3768, 3771, 2639, 2641}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^
(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
```

```
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \left(a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+a^3\right.$$

$$= \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \left(a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{9}{2}}($$

$$= \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{7}\left(5a^3\sqrt{\cos(c+\right.$$

$$= \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{28a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + a^3\int \frac{1}{\sqrt{\cos(c}}$$

$$= -\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{52a^3\mathrm{s}}{21d\cos}$$

$$= -\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{52a}{21d}$$

Mathematica [C] time = 6.25741, size = 531, normalized size = 3.61

$$\frac{7\csc(c)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2(t\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*((7*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[c]*Sec[c + d
*x]^3*(5*Sin[c] + 21*Sin[d*x]))/(140*d) + (Sec[c]*Sec[c + d*x]^2*(63*Sin[c]
 + 130*Sin[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(65*Sin[c] + 147*Sin[d*x])
)/(210*d)) - (13*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) + (7*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*
```

```
x)/2]^6*(a + a*Sec[c + d*x])^3*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos
[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [B] time = 2.587, size = 439, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-1/448*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^4-13/168*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/160*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^3-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-
2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-7/20*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3 \sec(dx+c)^3 + 3a^3 \sec(dx+c)^2 + 3a^3 \sec(dx+c) + a^3}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.374 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{5\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+d}{d(a\sec}$$

```
[Out] (21*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + (7*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*a*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c +
d*x]))
```

Rubi [A] time = 0.175728, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3819, 3787, 3769, 3771, 2639, 2641}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(}{d(a\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (21*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + (7*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*a*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c +
d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
```

```
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{\left(7\sqrt{\cos(c+dx)}\right.}{}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(5\sqrt{c\right.}{}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{5\int}{}$$

$$= \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5ad}$$

Mathematica [C] time = 2.15215, size = 314, normalized size = 2.45

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{-20\sin(c)\cos(dx)+6\sin(2c)\cos(2dx)-20\cos(c)\sin(dx)+6\cos(2c)\sin(2dx)-30\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)-96\cot(c)-30\csc(c)}{d\sqrt{\cos(c+dx)}}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
)*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c
 + d*x)))/E^(I*(c + d*x))]) + (-96*Cot[c] - 30*Csc[c] - 20*Cos[d*x]*Sin[c]
+ 6*Cos[2*d*x]*Sin[2*c] - 30*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 20*Co
s[c]*Sin[d*x] + 6*Cos[2*c]*Sin[2*d*x])/(d*Sqrt[Cos[c + d*x]])))/(15*a*(1 +
Sec[c + d*x]))
```

Maple [A] time = 1.533, size = 229, normalized size = 1.8

$$-\frac{1}{15\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4
+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

3.375 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{5\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\sec(c+dx)+a)}$$

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d
) + (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) - (Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.161366, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3819, 3787, 3769, 3771, 2641, 2639}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d
) + (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) - (Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\right.}{}$$

$$= \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sqrt{\cos(c+dx)}\,dx}{2a} + \frac{\left(5\sqrt{\cos(c+d\right.}{}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{6a}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\sqrt{\cos(c+dx)}\sin(c+d}{d(a+a\sec(c+dx))}$$

Mathematica [C] time = 2.43629, size = 292, normalized size = 2.92

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin(c)\cos(dx)+4\cos(c)\sin(dx)+6\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+12\cot(c)+6\csc(c)}{d\sqrt{\cos(c+dx)}} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sec(c+dx)\left(9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+d}}\right.}{}\right.}{3a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*
Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c +
 d*x)))/E^(I*(c + d*x))]) + (12*Cot[c] + 6*Csc[c] + 4*Cos[d*x]*Sin[c] + 6*S
ec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 4*Cos[c]*Sin[d*x])/(d*Sqrt[Cos[c +
d*x]])))/(3*a*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.281, size = 215, normalized size = 2.2

$$-\frac{1}{3\,ad}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(5\,\mathrm{E}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/3/a*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)-8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

3.376 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=72

$$-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)}$$

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) - Sin
[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.147076, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3819, 3787, 3771, 2639, 2641}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```


```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) - Sin
[c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}}{a+a\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx\\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}\\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,dx}{2a} + \left(3\sqrt{\text{co}\cdots}\right.\\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{3\int \sqrt{\cos(c+dx)}\,dx}{2a}\\
&= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}
\end{aligned}
$$

Mathematica [C] time = 2.08641, size = 270, normalized size = 3.75

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+2\cot(c)+\csc(c)\right)}{d\sqrt{\cos(c+dx)}} + \frac{2i\sqrt{2}e^{-i(c+dx)}\sec(c+dx)\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\cdots\right.\right.}{\left(-1+e^{2i}\cdots\right.}\right.}{a(\sec(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec
[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*
x)))/E^(I*(c + d*x))]) - (2*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*
Sin[(d*x)/2]))/(d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.555, size = 199, normalized size = 2.8

$$\frac{1}{da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\left(\mathrm{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sqrt{\cos\left(c+dx\right)}}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\cos\left(dx+c\right)}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.377 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} \, dx$

Optimal. Leaf size=70

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)}$$

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a*d)) + EllipticF[(c + d*x)/2, 2]/(a*d) + Sin[
c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.141377, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3820, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a*d)) + EllipticF[(c + d*x)/2, 2]/(a*d) + Sin[
c + d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx \\
&= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,d}{2a^2} \\
&= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} \\
&= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{2a} \\
&= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}
\end{aligned}
$$

Mathematica [C] time = 1.43361, size = 262, normalized size = 3.74

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c)\right)}{d\sqrt{\cos(c+dx)}} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sec(c+dx)\left(\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+(\ldots}{\left(-1+e^{2ic}\right)d\sqrt{e^{-i(c+dx)}\ldots}}\right.}{a(\sec(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^
((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c +
d*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/
E^(I*(c + d*x))]) + (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]))/(
d*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.665, size = 198, normalized size = 2.8

$$-\frac{1}{da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\left(\mathrm{Elli}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)\sec\left(dx+c\right)+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c)),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.378 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=70

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)}$$

```
[Out] EllipticE[(c + d*x)/2, 2]/(a*d) + EllipticF[(c + d*x)/2, 2]/(a*d) - Sin[c +
 d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.143734, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3818, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a*d) + EllipticF[(c + d*x)/2, 2]/(a*d) - Sin[c +
 d*x]/(d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx \\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a}{2}-\frac{1}{2}a\sec(c+}{\sqrt{\sec(c+dx}}}{a^2} \\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,d}{2a} \\
&= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\cos(c+dx)}\,dx}{2a} \\
&= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))}
\end{aligned}
$$

Mathematica [C] time = 1.45026, size = 263, normalized size = 3.76

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c)\right)}{d\sqrt{\cos(c+dx)}} + \frac{2i\sqrt{2}e^{-i(c+dx)}\sec(c+dx)\left(\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\left(-1+e^{2ic}\right)d\sqrt{e^{-i(c+d}}}\right.}{a(\sec(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(
(2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d
*x])/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - (2*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]))/(d
*Sqrt[Cos[c + d*x]])))/(a*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.293, size = 200, normalized size = 2.9

$$\frac{1}{da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+a\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.379 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=96

$$-\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) + (3
*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(3/2
)*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.15654, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3818, 3787, 3771, 2641, 3768, 2639}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) + (3
*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(3/2
)*(a + a*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
```

```
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+}}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+}}{2a}$$

$$= \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} - \frac{\left(3\sqrt{c}\right.}{}$$

$$= -\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{3}{}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+}{d\cos^{\frac{3}{2}}(c+dx)(a+}$$

Mathematica [C] time = 2.01631, size = 303, normalized size = 3.16

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(2\cos\left(\frac{1}{2}(c-dx)\right)+\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{2d\cos^{\frac{3}{2}}(c+dx)} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sec(c+dx)\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\right.}{}\right.}{a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c +
 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Cos[c + d*x]^(3/2)) -
((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyper
geometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E^(I*(c
 + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]
)))/(a*(1 + Sec[c + d*x]))
```

Maple [A] time = 1.639, size = 253, normalized size = 2.6

$$-\frac{1}{da}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\,(\sin(1/2\,dx+c/2))^{2}-1}\sqrt{-2\,(\sin(1/2\,dx+c/2))^{4}+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\mathrm{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/cos
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^{3}\sec(dx+c)+a\cos(dx+c)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)
^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a \sec \left(dx+c\right)+a\right) \cos \left(dx+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.380 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=124

$$\frac{5\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}$$

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d)
 + (5*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (3*Sin[c + d*x])/(a*d*Sqrt
[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.17489, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3818, 3787, 3768, 3771, 2639, 2641}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d)
 + (5*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (3*Sin[c + d*x])/(a*d*Sqrt
[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
```

```
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{(5\sqrt{}}{}$$

$$= \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{5\int}{}$$

$$= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} -$$

Mathematica [C] time = 3.93872, size = 338, normalized size = 2.73

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(10\cos\left(\frac{1}{2}(c-dx)\right)+8\cos\left(\frac{1}{2}(3c+dx)\right)+4\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+9\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{4d\cos^{\frac{5}{2}}(c+dx)} + \frac{2i\sqrt{2}e}{}\right.}{3a(s}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(-((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4*Cos[
(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc[c/2]*
Sec[c/2]*Sec[(c + d*x)/2])/(4*d*Cos[c + d*x]^(5/2)) + ((2*I)*Sqrt[2]*(9*(1
+ E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x)
)*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1
/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a*(1 + Sec[c
 + d*x]))
```

Maple [B] time = 2.473, size = 413, normalized size = 3.3

$$\frac{1}{3\,da}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(10\,\sqrt{2\,\left(\sin\left(1/2\,d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/sin(1/2*d*x
+1/2*c)^3/cos(1/2*d*x+1/2*c)/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)+9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d
*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)+a\cos\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^4*sec(d*x + c) + a*cos(d*x + c)
^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.381 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=160

$$-\frac{5\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d}-\frac{3\sin(c}{a^2d(s}$$

```
[Out] (56*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(a
^2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d) + (56*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(15*a^2*d) - (3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a^2*d*(1
+ Sec[c + d*x])) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*
x])^2)
```

Rubi [A] time = 0.283813, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d}-\frac{3\sin(c+dx)\,\mathrm{co}}{a^2d(\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (56*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(a
^2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d) + (56*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(15*a^2*d) - (3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a^2*d*(1
+ Sec[c + d*x])) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*
x])^2)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
```

```
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{11a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{3a^4}$$

$$= -\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(15\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{2a^2}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} -$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\sec(c+dx))} -$$

$$= \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin}{15a^2d}$$

Mathematica [C] time = 2.75431, size = 366, normalized size = 2.29

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\left(-40\sin(c)\cos(dx)+6\sin(2c)\cos(2dx)-40\cos(c)\sin(dx)+6\cos(2c)\sin(2dx)+5\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)+5\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{1}{2}(c+\right.\right.}{3d\cos^{\frac{3}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(56*(1 + E^((2*I)*(c + d*x))) + 56*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
)*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]) + (2*(-216*Cot[c] - 120*Csc[c] - 40*Cos[d*x]*
Sin[c] + 6*Cos[2*d*x]*Sin[2*c] - 120*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2]
```

```
 + 5*Sec[c/2]*Sec[(c + d*x)/2]^3*Sin[(d*x)/2] - 40*Cos[c]*Sin[d*x] + 6*Cos[
2*c]*Sin[2*d*x] + 5*Sec[(c + d*x)/2]^2*Tan[c/2]))/(3*d*Cos[c + d*x]^(3/2)))
)/(5*a^2*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 1.534, size = 283, normalized size = 1.8

$$-\frac{1}{30\,a^2d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(96\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{10}-352\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+120\,\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d
*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.382 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{10\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d(a\sec(c}$$

```
[Out] (-7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*
a^2*d) + (10*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) - (7*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.267216, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c}}{3d(a\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*
a^2*d) + (10*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) - (7*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
```

f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{9a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{3a^4}$$

$$= -\frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2a^2}$$

$$= \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - 7\int$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\cos(c+}{3d(a+a}$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\sqrt{\cos(c+dx)}\sin}{3a^2d(1+\sec(c}$$

<u>Mathematica</u> [C] time = 2.04052, size = 341, normalized size = 2.47

$$\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{8\sin(c)\cos(dx)+8\cos(c)\sin(dx)-2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)-2\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)+36\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+48\cot(c}{d\cos^{\frac{3}{2}}(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)
*(c + d*x)))/E^(I*(c + d*x))]) + (48*Cot[c] + 36*Csc[c] + 8*Cos[d*x]*Sin[c]
 + 36*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 2*Sec[c/2]*Sec[(c + d*x)/2]^
3*Sin[(d*x)/2] + 8*Cos[c]*Sin[d*x] - 2*Sec[(c + d*x)/2]^2*Tan[c/2])/(d*Cos[
```

```
c + d*x]^(3/2))))/(3*a^2*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 1.419, size = 270, normalized size = 2.

$$-\frac{1}{6\,a^2d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(16\,(\cos(1/2\,dx+c/2))^8+12\,(\cos(1/2\,dx+c/2))^6+20\sqrt{(\sin(1/$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d*
x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x
+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*d*x+
1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{3}{2}}}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.383 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{5\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^2}$$

```
[Out] (4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a^
2*d) - (5*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - S
in[c + d*x]/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.242225, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a^
2*d) - (5*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - S
in[c + d*x]/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{7a}{2}+\frac{3}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+a}}{3a^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+a\right.}}{}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{\left(5\sqrt{\cos(c+\right.}}{}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{5\int \frac{1}{\sqrt{\cos(c+dx}}}{6a^2}$$

$$= \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{}{3d\sqrt{\cos(c+}}$$

Mathematica [C] time = 6.65402, size = 374, normalized size = 3.34

$$\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}+\frac{2\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}-\frac{8\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{8\cot\left(\frac{c}{2}\right)}{d}\right)}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2} + \frac{4i\sqrt{2}e^{-i(c+dx)}\cos^4\left(\frac{c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (((4*I)/3)*Sqrt[2]*Cos[c/2 + (d*x)/2]^4*(12*(1 + E^((2*I)*(c + d*x))) + 12*
(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/
2]^4*((-8*Cot[c/2])/d - (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d + (2
*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) + (2*Sec[c/2 + (d*x)/2]^
2*Tan[c/2])/(3*d)))/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 1.599, size = 257, normalized size = 2.3

$$\frac{1}{6\,a^2d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\,\left(\cos\left(1/2\,dx+c/2\right)\right)^6+10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x
+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d*x+1
/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\cos(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.384 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} \, dx$

Optimal. Leaf size=109

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a^2*d)) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2
*d) + Sin[c + d*x]/(a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c +
d*x]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.240585, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3817, 4019, 3787, 3771, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a^2*d)) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2
*d) + Sin[c + d*x]/(a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c +
d*x]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
```

```
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+}}{}}{3a^2}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{(\sqrt{}}{}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{(\sqrt{}}{}$$

$$= \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\int}{}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} -$$

Mathematica [C] time = 6.3173, size = 656, normalized size = 6.02

$$-\frac{4\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2(c+dx)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\cot\right.}}{3d\sqrt{\cot^2(c)+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((-I/2)*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*((2*E^((2*I)*
d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2
)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/
E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3
*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqr
t[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d
*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(
1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Sec[c +
 d*x])^2 - (4*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {
```

```
5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*S
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*((4*Csc[c
])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2*Sec[c/2]*Sec[c/2
 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) - (2*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)
))/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Maple [A] time = 1.439, size = 257, normalized size = 2.4

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\left(\cos\left(1/2\,dx+c/2\right)\right)^6+4\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\left(\cos\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/
2*d*x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x
+1/2*c)^2-1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)\sec\left(dx+c\right)+a^{2}\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a^2*cos(d*
x + c)*sec(d*x + c) + a^2*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{\sqrt{\cos\left(c+dx\right)}\sec^{2}\left(c+dx\right)+2\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(c
 + d*x) + sqrt(cos(c + d*x))), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.385 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

```
[Out] EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + Sin[c + d*x]/(3*d*Cos[c + d*x]^(3/2)*
(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.0992939, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3815, 21, 3771, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + Sin[c + d*x]/(3*d*Cos[c + d*x]^(3/2)*
(a + a*Sec[c + d*x])^2)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{a+a\ldots}}{3a^2}$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+\ldots}}{6a^2}$$

$$= \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{6a^2}$$

$$= \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

Mathematica [A] time = 0.22277, size = 63, normalized size = 1.11

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 1.449, size = 188, normalized size = 3.3

$$-\frac{1}{6\,a^2 d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}\mathrm{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)^2\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a^2*cos(
d*x + c)^2*sec(d*x + c) + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.386 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

```
[Out] EllipticE[(c + d*x)/2, 2]/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2*d)
 - Sin[c + d*x]/(a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x
]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.242495, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3816, 4019, 3787, 3771, 2639, 2641}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(\sec(c+dx)+1)}-\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2*d)
 - Sin[c + d*x]/(a^2*d*Sqrt[Cos[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x
]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
```

```
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+}}{}}{3a^2}$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} +$$

$$= -\frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} +$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}(1+\sec(c+dx))} -$$

Mathematica [C] time = 5.06204, size = 312, normalized size = 2.86

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(7\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4i\sqrt{2}e^{-i(c+dx)}\sec^2(c+dx)\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i}}\right.}{}\right.}{3a^2(\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(-((7*Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 3*Cos[(
c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(2*d*Cos[c + d*x]^(3/2
)) + ((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqr
t[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
 d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^2)/(
d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c
 + d*x))])))/(3*a^2*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 1.493, size = 257, normalized size = 2.4

$$\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\left(\cos\left(1/2\,dx+c/2\right)\right)^6-4\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\left(\cos\left(1/2\,dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)^3\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a^2*cos(
d*x + c)^3*sec(d*x + c) + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{2}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.387 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$-\frac{5\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}-\frac{s}{3d\cos^{\frac{5}{2}}(c+}$$

```
[Out] (-4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (4*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (5*Sin[c + d*x])/(3*a
^2*d*Cos[c + d*x]^(3/2)*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x
]^(5/2)*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.262784, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}-\frac{\sin(c+}{3d\cos^{\frac{5}{2}}(c+dx)(a\,\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (4*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (5*Sin[c + d*x])/(3*a
^2*d*Cos[c + d*x]^(3/2)*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x
]^(5/2)*(a + a*Sec[c + d*x])^2)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
```

```
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+d\ldots}{a\ldots}}{3a^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} - \ldots$$

$$= -\frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2} - \ldots$$

$$= \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+d\ldots}{3d\cos^{\frac{5}{2}}(c+dx)(a+\ldots}$$

$$= -\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \ldots$$

$$= -\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(\ldots}{3a^2d\cos^{\frac{3}{2}}(c+d\ldots}$$

Mathematica [C] time = 6.78334, size = 393, normalized size = 2.89

$$\frac{\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} + \frac{2\tan\left(\frac{c}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} + \frac{8\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} + \frac{8\sec(c)\sin(dx)\sec(c+dx)}{d} + \frac{8\cot\left(\frac{c}{2}\right)\sec(c)\ldots}{d}\ldots\right.}{\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (((-4*I)/3)*Sqrt[2]*Cos[c/2 + (d*x)/2]^4*(12*(1 + E^((2*I)*(c + d*x))) + 12
*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1
/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2
```

```
*I)*(c + d*x)))/E^(I*(c + d*x))]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)
/2]^4*((8*Cot[c/2]*Sec[c])/d + (8*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])
/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(3*d) + (8*Sec[c]*Sec[c
 + d*x]*Sin[d*x])/d + (2*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Cos[c + d*
x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Maple [B] time = 1.683, size = 405, normalized size = 3.

$$-\frac{1}{6\,a^2d}\left(2\,\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\left(5\,\mathrm{Elliptic}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^4\sec(dx+c)^2+2\,a^2\cos(dx+c)^4\sec(dx+c)+a^2\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a^2*cos(
d*x + c)^4*sec(d*x + c) + a^2*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

3.388 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{10\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(\sec(c+d$$

```
[Out] (7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) - (7*Sin[c + d*x])/(
a^2*d*Sqrt[Cos[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(5/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x
])^2)
```

Rubi [A] time = 0.288213, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) - (7*Sin[c + d*x])/(
a^2*d*Sqrt[Cos[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(5/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x
])^2)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+}{}}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} -$$

$$= \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\cos^{\frac{5}{2}}(c+dx)(1+\sec(c+dx))} -$$

$$= \frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx}{a^2d\sqrt{\cos(c+}}$$

Mathematica [C] time = 2.36898, size = 372, normalized size = 2.3

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(82\cos\left(\frac{1}{2}(c-dx)\right)+65\cos\left(\frac{1}{2}(3c+dx)\right)+68\cos\left(\frac{1}{2}(c+3dx)\right)+37\cos\left(\frac{1}{2}(5c+3dx)\right)+53\cos\left(\frac{1}{2}(3c+5dx)\right)+10\cos\left(\frac{1}{2}(7c+5dx\right.\right.}{8d\cos^{\frac{7}{2}}(c+dx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(-((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] + 68*Co
s[(c + 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2] + 10*C
```

```
os[(7*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d
*x)/2]^3)/(8*d*Cos[c + d*x]^(7/2)) + ((4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c +
d*x))) + 21*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((
2*I)*(c + d*x))])*Sec[c + d*x]^2)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqr
t[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(3*a^2*(1 + Sec[c + d*x])^2
)
```

Maple [B] time = 2.685, size = 413, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(-2/3*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2-22/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)+14*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^
2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^5\sec(dx+c)^2+2\,a^2\cos(dx+c)^5\sec(dx+c)+a^2\cos(dx+c)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^5*sec(d*x + c)^2 + 2*a^2*cos(
d*x + c)^5*sec(d*x + c) + a^2*cos(d*x + c)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2)), x)
```

3.389 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$-\frac{21\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{63\sin}{10d\,(}$$

```
[Out] (231*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (21*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) - (21*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) + (77*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(10*a^3*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d
*(a + a*Sec[c + d*x])^3) - (4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a +
a*Sec[c + d*x])^2) - (63*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Sec[c + d*x]))
```

Rubi [A] time = 0.428436, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{63\sin(c+dx)}{10d\left(a^3\sec(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (231*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (21*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) - (21*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) + (77*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(10*a^3*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d
*(a + a*Sec[c + d*x])^3) - (4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a +
a*Sec[c + d*x])^2) - (63*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{15a}{2}+\frac{9}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{15a^4}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{63\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)} - \frac{(\sqrt{}}{}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{63\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)} - \frac{(63}{}$$

$$= -\frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - 4$$

$$= -\frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - 4$$

$$= \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{10a^3d}$$

Mathematica [C] time = 3.03894, size = 391, normalized size = 1.89

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\frac{1}{16}\sec\left(\frac{c}{2}\right)\left(770\sin\left(c+\frac{dx}{2}\right)-840\sin\left(c+\frac{3dx}{2}\right)+150\sin\left(2c+\frac{3dx}{2}\right)-238\sin\left(2c+\frac{5dx}{2}\right)-40\sin\left(3c+\frac{5dx}{2}\right)-5\sin\left(3c+\frac{7dx}{2}\right)-5\sin\left(4c+\frac{7dx}{2}\right)+\sin}{\cos^{\frac{5}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((42*I)*Sqrt[2]*(11*(1 + E^((2*I)*(c + d*x))) + 11*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)])*Sec[c + d*x]^3)/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*
(c + d*x)))/E^(I*(c + d*x))]) + (-264*Cot[c] - 198*Csc[c] + (Sec[c/2]*Sec[(
c + d*x)/2]^5*(-1210*Sin[(d*x)/2] + 770*Sin[c + (d*x)/2] - 840*Sin[c + (3*d
*x)/2] + 150*Sin[2*c + (3*d*x)/2] - 238*Sin[2*c + (5*d*x)/2] - 40*Sin[3*c +
 (5*d*x)/2] - 5*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + Sin[4*c + (
9*d*x)/2] + Sin[5*c + (9*d*x)/2]))/16)/Cos[c + d*x]^(5/2)))/(5*a^3*d*(1 + S
ec[c + d*x])^3)
```

Maple [A] time = 1.726, size = 296, normalized size = 1.4

$$-\frac{1}{20\,a^3d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(64\,(\cos(1/2\,dx+c/2))^{12}-288\,(\cos(1/2\,dx+c/2))^{10}-76\,(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*d
*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos(1/2
*d*x+1/2*c)^2-1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a^3\sec\left(dx+c\right)^3+3\,a^3\sec\left(dx+c\right)^2+3\,a^3\sec\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x)
```

3.390 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$\frac{11\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\cos(c+dx)}}{30d\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin\cdots}{3ad\cdots}$$

```
[Out] (-119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2]
)/(2*a^3*d) + (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) - (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (2*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) - (119*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(30*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.396354, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{119\sin(c+dx)\sqrt{\cos(c+dx)}}{30d\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+dx)\cdots}{3ad(a\sec(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2]
)/(2*a^3*d) + (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) - (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (2*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*x])^2) - (119*Sqrt[Cos[c + d*x]]*Sin[
c + d*x])/(30*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{13a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{15a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sqrt{\cos(c+dx)}\sin(c+dx)}{30d\left(a^3+a^3\sec(c+dx)\right)} -$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{119\sqrt{\cos(c+dx)}\sin(c+dx)}{30d\left(a^3+a^3\sec(c+dx)\right)} -$$

$$= \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad(a+a\sec(c+dx))^2} - \frac{11}{3}$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sqrt{\cos(}{3ad(a}$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\sqrt{\cos(c+dx)}\,s}{5d(a+a\sec(}$$

Mathematica [C] time = 2.48146, size = 375, normalized size = 2.07

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\frac{1}{4}\sec\left(\frac{c}{2}\right)\left(-709\sin\left(c+\frac{dx}{2}\right)+715\sin\left(c+\frac{3dx}{2}\right)-170\sin\left(2c+\frac{3dx}{2}\right)+202\sin\left(2c+\frac{5dx}{2}\right)+25\sin\left(3c+\frac{5dx}{2}\right)+5\sin\left(3c+\frac{7dx}{2}\right)+5\sin\left(4c+\frac{7dx}{2}\right)+10\right.}{3d\cos^{\frac{5}{2}}(c+dx)}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x))) + 119*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*
I)*(c + d*x)))/E^(I*(c + d*x))]) + (720*Cot[c] + 708*Csc[c] + (Sec[c/2]*Sec
[(c + d*x)/2]^5*(1061*Sin[(d*x)/2] - 709*Sin[c + (d*x)/2] + 715*Sin[c + (3*
d*x)/2] - 170*Sin[2*c + (3*d*x)/2] + 202*Sin[2*c + (5*d*x)/2] + 25*Sin[3*c
+ (5*d*x)/2] + 5*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (7*d*x)/2]))/4)/(3*d*Co
s[c + d*x]^(5/2))))/(5*a^3*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.446, size = 283, normalized size = 1.6

$$-\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{10}+468\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+330\,\sqrt{(\mathrm{si}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*
d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(
1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{3}{2}}}{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x)
```

3.391 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{13\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a\sec(c+$$

```
[Out] (49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/
(6*a^3*d) - Sin[c + d*x]/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3) -
(8*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) - (13*S
in[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.36957, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a\sec(c+dx)+a$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/
(6*a^3*d) - Sin[c + d*x]/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3) -
(8*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) - (13*S
in[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
```

```
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{11a}{2}+\frac{5}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx)}}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+\right.}{}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{}{6d\sqrt{\cos(c}}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{}{6d\sqrt{\cos(c}}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2} - \frac{}{6d\sqrt{\cos(c}}$$

$$= \frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3} - \frac{}{15ad\sqrt{\cos}}$$

Mathematica [C] time = 2.16161, size = 357, normalized size = 2.3

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(806\cos\left(\frac{1}{2}(c-dx)\right)+664\cos\left(\frac{1}{2}(3c+dx)\right)+470\cos\left(\frac{1}{2}(c+3dx)\right)+265\cos\left(\frac{1}{2}(5c+3dx)\right)+117\cos\left(\frac{1}{2}(3c+5dx)\right)+30\cos\left(\frac{1}{2}(7c+\right.\right.}{8d\cos^{\frac{5}{2}}(c+dx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-((806*Cos[(c - d*x)/2] + 664*Cos[(3*c + d*x)/2] + 470
*Cos[(c + 3*d*x)/2] + 265*Cos[(5*c + 3*d*x)/2] + 117*Cos[(3*c + 5*d*x)/2] +
 30*Cos[(7*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d*Cos[c
+ d*x]^(5/2)) + ((4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E
^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*S
ec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c
```

```
+ d*x)))/E^(I*(c + d*x))]))))/(15*a^3*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 1.601, size = 270, normalized size = 1.7

$$\frac{1}{60\,a^{3}d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(348\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{8}+130\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,(\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d
*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2
*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^{3}\sec\left(dx+c\right)^{3}+3\,a^{3}\sec\left(dx+c\right)^{2}+3\,a^{3}\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\cos(c+dx)}}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x)
```

3.392 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+$$

```
[Out] (-9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^
3*d) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) + (2*Si
n[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x
]/(2*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.384733, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3817, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] (-9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^
3*d) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) + (2*Si
n[c + d*x])/(5*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x
]/(2*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
```

f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

Rule 4019

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0]

Rule 4020

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m + 1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c}}{}}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{2\sin(c+dx)}{5ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}$$

Mathematica [C] time = 6.39968, size = 721, normalized size = 4.65

$$-\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(c\right.}}{d\sqrt{\cot^2(c)+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (((-9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^3*((2*E^((
2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[
```

```
c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]
)/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) -
(2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I
)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Se
c[c + d*x])^3 - (2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^3*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
d*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*((36*C
sc[c])/(5*d) + (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (12*Se
c[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*
x)/2]^5*Sin[(d*x)/2])/(5*d) - (12*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) + (2
*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(5/2)*(a + a*Sec[c +
d*x])^3)
```

Maple [A] time = 1.454, size = 270, normalized size = 1.7

$$-\frac{1}{20\,a^3 d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(36\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos(1/\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/20/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1
/2*d*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)
^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2
*d*x+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)\sec\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)\sec\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)\sec\left(dx+c\right)+a^{3}\cos\left(dx+c\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a^3*cos(d*
x + c)*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.393 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+$$

```
[Out] -EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
 + Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - Sin[c + d
*x]/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x]/(6*d*
Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.378032, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3815, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}-\frac{}{15}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] -EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
 + Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - Sin[c + d
*x]/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x]/(6*d*
Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
```

```
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+d}}{(a+}}{5a^2}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} -$$

Mathematica [C] time = 2.01075, size = 342, normalized size = 2.21

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(14\cos\left(\frac{1}{2}(c-dx)\right)+16\cos\left(\frac{1}{2}(3c+dx)\right)+20\cos\left(\frac{1}{2}(c+3dx)\right)-5\cos\left(\frac{1}{2}(5c+3dx)\right)+3\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{8d\cos^{\frac{5}{2}}(c+dx)} - \frac{4i}{}\right.}{15a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((14*Cos[(c - d*x)/2] + 16*Cos[(3*c + d*x)/2] + 20*Cos
[(c + 3*d*x)/2] - 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)/2])*Csc[c/2]
```

```
*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d*Cos[c + d*x]^(5/2)) - ((4*I)*Sqrt[2]*(3*
(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d
*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4
, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 +
 E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]))))/(15*a^3*(1
 + Sec[c + d*x])^3)
```

Maple [A] time = 1.563, size = 270, normalized size = 1.7

$$-\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^2\sec(dx+c)^3+3a^3\cos(dx+c)^2\sec(dx+c)^2+3a^3\cos(dx+c)^2\sec(dx+c)+a^3\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^2*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^2*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x
 + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

3.394 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}$$

```
[Out] EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
- Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - (4*Sin[c +
 d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x]/(6
*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.38195, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}-\frac{}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
- Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3) - (4*Sin[c +
 d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2) + Sin[c + d*x]/(6
*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
```

sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+d}}{(a+}}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{4\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^2}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{}{15}$$

Mathematica [C] time = 1.98815, size = 342, normalized size = 2.21

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(4\cos\left(\frac{1}{2}(c-dx)\right)+26\cos\left(\frac{1}{2}(3c+dx)\right)+10\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+3\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{4i\sqrt{}}{}\right.}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-((4*Cos[(c - d*x)/2] + 26*Cos[(3*c + d*x)/2] + 10*Cos
[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)/2])*Csc[c/2]
```

```
*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d*Cos[c + d*x]^(5/2)) + ((4*I)*Sqrt[2]*(3*
(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d
*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4
, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 +
 E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]))))/(15*a^3*(1
 + Sec[c + d*x])^3)
```

Maple [A] time = 1.445, size = 270, normalized size = 1.7

$$\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8-10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos(1/\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^3\sec(dx+c)^3+3\,a^3\cos(dx+c)^3\sec(dx+c)^2+3\,a^3\cos(dx+c)^3\sec(dx+c)+a^3\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^3*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^3*sec(d*x + c) + a^3*cos(d*x
 + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.395 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)}{10d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)$$

```
[Out] (9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3
*d) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3) - (2*Sin
[c + d*x])/(5*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) - (9*Sin[c + d
*x])/(10*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.390425, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3816, 4019, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)}{10d\sqrt{\cos(c+dx)}\left(a^3\sec(c+dx)+a^3\right)}-\frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3
*d) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3) - (2*Sin
[c + d*x])/(5*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2) - (9*Sin[c + d
*x])/(10*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
```

```
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+}{(}}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{5ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= \frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3} -$$

Mathematica [C] time = 6.34794, size = 721, normalized size = 4.65

$$-\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^3(c+dx)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(c\right.}}{d\sqrt{\cot^2(c)+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (((9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^3*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
```

```
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Sec
[c + d*x])^3 - (2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2
}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^3*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*((-36*C
sc[c])/(5*d) - (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (8*Sec
[c/2]*Sec[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x
)/2]^5*Sin[(d*x)/2])/(5*d) - (8*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) - (2*S
ec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*
x])^3)
```

Maple [A] time = 1.578, size = 268, normalized size = 1.7

$$\frac{1}{20\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(36\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8-10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)^4\sec\left(dx+c\right)+a^3\cos\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^4*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^4*sec(d*x + c) + a^3*cos(d*x
 + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^3\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

3.396 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$-\frac{13\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}-\frac{\cdots}{15ad\cdots}$$

```
[Out] (-49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])
/(6*a^3*d) + (49*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]
/(5*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d
*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2) - (13*Sin[c + d*x])/(6*d*Cos[c
+ d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.406274, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}-\frac{8\cdots}{15ad\cos^{\frac{5}{2}}(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (-49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])
/(6*a^3*d) + (49*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]
/(5*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d
*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2) - (13*Sin[c + d*x])/(6*d*Cos[c
+ d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+d}{(a}}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}$$

$$= \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{8\sin(}{15ad\cos^{\frac{5}{2}}(c+dx)}$$

$$= -\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{}{5d\cos^{\frac{7}{2}}(c+}$$

Mathematica [C] time = 2.63599, size = 372, normalized size = 2.06

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}(c+3dx)\right)+374\cos\left(\frac{1}{2}(5c+3dx)\right)+670\cos\left(\frac{1}{2}(3c+5dx)\right)+65\cos\left(\frac{1}{2}(7c+\right.\right.}{16d\cos^{\frac{7}{2}}(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 124
3*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2]
+ 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec
[(c + d*x)/2]^5)/(16*d*Cos[c + d*x]^(7/2)) - ((4*I)*Sqrt[2]*(147*(1 + E^((2
*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hype
rgeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 65*E^(I*(c + d*x))*(-
1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2,
5/4, -E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*
I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(15*a^3*(1 + Sec[
c + d*x])^3)
```

Maple [B] time = 1.779, size = 555, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^5\sec(dx+c)^3+3\,a^3\cos(dx+c)^5\sec(dx+c)^2+3\,a^3\cos(dx+c)^5\sec(dx+c)+a^3\cos(dx+c)^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^5*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^5*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^5*sec(d*x + c) + a^3*cos(d*x
 + c)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

3.397 $\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$\frac{11\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{119\sin(c+dx)}{30d\cos^{\frac{5}{2}}(c+dx)\left(a^3\sec(\right.}$$

```
[Out] (119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Cos[c + d*x]^(3/2)) - (119*Sin[c +
d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(9/2)*
(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(7/2)*(a + a
*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Cos[c + d*x]^(5/2)*(a^3 + a^3*
Sec[c + d*x]))
```

Rubi [A] time = 0.434142, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{119\sin(c+dx)}{30d\cos^{\frac{5}{2}}(c+dx)\left(a^3\sec(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```


```
[Out] (119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Cos[c + d*x]^(3/2)) - (119*Sin[c +
d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(9/2)*
(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(7/2)*(a + a
*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Cos[c + d*x]^(5/2)*(a^3 + a^3*
Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{11}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+}{(}}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2} -$$

$$= \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}$$

$$= \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^3}$$

$$= \frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+}{10a^3d\sqrt{\cos(c}}$$

Mathematica [C] time = 3.55563, size = 402, normalized size = 1.94

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(5134\cos\left(\frac{1}{2}(c-dx)\right)+4148\cos\left(\frac{1}{2}(3c+dx)\right)+4664\cos\left(\frac{1}{2}(c+3dx)\right)+2476\cos\left(\frac{1}{2}(5c+3dx)\right)+3340\cos\left(\frac{1}{2}(3c+5dx)\right)+944\cos\right.}{96d\cos^{\frac{9}{2}}(c+dx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(-((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/2] + 4
664*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c + 5*d*x)
/2] + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Cos[(9*c +
 7*d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5
)/(96*d*Cos[c + d*x]^(9/2)) + ((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x)))
 + 119*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-
1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c)
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*
(c + d*x))])*Sec[c + d*x]^3)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1
+ E^((2*I)*(c + d*x)))/E^(I*(c + d*x))])))/(5*a^3*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 2.977, size = 453, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(32/15*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3+11
8/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+238/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4/3*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2-1/2)^2+1/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)^5+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2
*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^3\cos(dx+c)^6\sec(dx+c)^3+3\,a^3\cos(dx+c)^6\sec(dx+c)^2+3\,a^3\cos(dx+c)^6\sec(dx+c)+a^3\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^6*sec(d*x + c)^3 + 3*a^3*cos(
d*x + c)^6*sec(d*x + c)^2 + 3*a^3*cos(d*x + c)^6*sec(d*x + c) + a^3*cos(d*x
 + c)^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(11/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^3\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(11/2)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(11/2)), x)
```

3.398 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\cos(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (32*a*Sin[c + d*x])/(35*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (1
6*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (12*
a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C
os[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.297772, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4264, 3805, 3804}

$$\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{12a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\cos(c+dx)}}{35d\sqrt{a\sec(c+dx)+a}}+\frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (32*a*Sin[c + d*x])/(35*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (1
6*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (12*
a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*C
os[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
```

```
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\left(6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+}}{\sec^{\frac{5}{2}}(c+dx}$$

$$= \frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{35}\left(24\sqrt{\cos(c+d}\right.$$

$$= \frac{16a\sqrt{\cos(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{12a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\,s}{7d\sqrt{a+a\sec(}}$$

$$= \frac{32a\sin(c+dx)}{35d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a\sqrt{\cos(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{12a\cos^{\frac{3}{2}}(}{35d\sqrt{a}}$$

Mathematica [A] time = 0.222206, size = 80, normalized size = 0.52

$$\frac{(140\sin(c+dx)+42\sin(2(c+dx))+12\sin(3(c+dx))+5\sin(4(c+dx)))\sqrt{\cos(c+dx)}\sqrt{a(\sec(c+dx)+1)}}{140d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(140*Sin[c + d*x] + 42*Sin[2
*(c + d*x)] + 12*Sin[3*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(140*d*(1 + Cos[c
+ d*x]))
```

Maple [A] time = 0.199, size = 80, normalized size = 0.5

$$-\frac{10\left(\cos\left(dx+c\right)\right)^{4}+2\left(\cos\left(dx+c\right)\right)^{3}+4\left(\cos\left(dx+c\right)\right)^{2}+16\cos\left(dx+c\right)-32}{35\,d\sin\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/35/d*(5*cos(d*x+c)^4+cos(d*x+c)^3+2*cos(d*x+c)^2+8*cos(d*x+c)-16)*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 3.12502, size = 396, normalized size = 2.59

$$\frac{\sqrt{2}\left(105\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+35\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/280*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*
c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2*c)
*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7/2*
d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.65524, size = 215, normalized size = 1.41

$$\frac{2\left(5\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+16\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec (dx + c) + a} \cos (dx + c)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.399 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (16*a*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (8
*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.232277, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4264, 3805, 3804}

$$\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}+\frac{8a\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (16*a*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (8
*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{1}{5}\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+}}{\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{8a\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{1}{15}\left(8\sqrt{\cos(c+dx)}\right.$$

$$= \frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{8a\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a\cos^{\frac{3}{2}}(c+}{5d\sqrt{a+}}$$

Mathematica [A] time = 0.174736, size = 61, normalized size = 0.53

$$\frac{\sqrt{\cos(c+dx)}(8\cos(c+dx)+3\cos(2(c+dx))+19)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(19 + 8*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*(1 +
Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.186, size = 70, normalized size = 0.6

$$-\frac{6\,(\cos(dx+c))^{3}+2\,(\cos(dx+c))^{2}+8\,\cos(dx+c)-16}{15\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d*(3*cos(d*x+c)^3+cos(d*x+c)^2+4*cos(d*x+c)-8)*(a*(cos(d*x+c)+1)/cos(
d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.68739, size = 274, normalized size = 2.38

$$\frac{\sqrt{2}\left(30\,\cos\left(\frac{4}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\cos\left(\frac{2}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x +
 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.63023, size = 188, normalized size = 1.63

$$\frac{2\left(3\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+8\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.400 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.173543, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4264, 3805, 3804}

$$\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*a*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
```

```
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+}}{\sqrt{\sec(c+d}}$$
$$= \frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.116315, size = 49, normalized size = 0.64

$$\frac{2\sqrt{\cos(c+dx)}(\cos(c+dx)+2)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(2 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c
+ d*x)/2])/(3*d)
```

Maple [A] time = 0.178, size = 58, normalized size = 0.8

$$-\frac{2\,(\cos(dx+c))^2+2\,\cos(dx+c)-4}{3\,d\sin(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(cos(d*x+c)^2+cos(d*x+c)-2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(
d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [A] time = 2.80224, size = 153, normalized size = 1.99

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx\right.\right.\right.\right.}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.62224, size = 158, normalized size = 2.05

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)}{3(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) + 2)*sqrt(cos(d*x
 + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec (dx + c) + a} \cos (dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.401 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.109959, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {4264, 3804}

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.102854, size = 39, normalized size = 1.08

$$\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [A] time = 0.142, size = 50, normalized size = 1.4

$$-2\,\frac{\sqrt{\cos(dx+c)}\,(-1+\cos(dx+c))}{d\sin(dx+c)}\sqrt{\frac{a\,(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))/s
in(d*x+c)
```

Maxima [A] time = 2.61384, size = 27, normalized size = 0.75

$$\frac{2\,\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [A] time = 1.645, size = 130, normalized size = 3.61

$$\frac{2\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\sec(c+dx)+1)}\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sec(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.402 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.116187, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4264, 3801, 215}

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.16116, size = 74, normalized size = 1.3

$$-\frac{2\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] time = 0.201, size = 139, normalized size = 2.4

$$\frac{\sqrt{2}(-1+\cos(dx+c))}{d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\left(\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1-\sin(dx+c))}{4}\sqrt{-2\,(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*
x+c))*(arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))-arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.7798, size = 325, normalized size = 5.7

$$\frac{\sqrt{a}\left(\log\left(2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)-\log\left(2\,\cos\left(\frac{1}{2}\,d\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))/d
```

Fricas [A] time = 1.7912, size = 475, normalized size = 8.33

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-4\,\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7\,a\cos(dx+c)^{2}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{2\,d},\ \frac{\sqrt{-a}\arctan\left(\frac{2\,\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\ldots}{a\cos(dx+c)^{2}-a\cos(dx+c)-\ldots}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*
x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*sqr
```

```
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.403 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.174606, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3803, 3801, 215}

$$\frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[
c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
```

```
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+a}$$

$$= \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx, x, -\right.}{d}$$

$$= \frac{\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.264592, size = 90, normalized size = 0.98

$$\frac{2a\sin(c+dx)\left(\frac{1}{2}\cos(c+dx)+\frac{\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\right)}{d\cos^{\frac{5}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(Cos[c + d*x]/2 + ArcSin[Sqrt[1 - Sec[c + d*x]]]/(2*Sqrt[1 - Sec[c + d
*x]]*Sec[c + d*x]^(3/2)))*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2)*Sqrt[a*(1 + S
ec[c + d*x])])
```

Maple [B] time = 0.204, size = 178, normalized size = 1.9

$$-\frac{-1+\cos(dx+c)}{2\,d\,(\sin(dx+c))^2}\left(\arctan\left(\frac{\sqrt{2}\,(\cos(dx+c)+1+\sin(dx+c))}{4}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{2}\cos(dx+c)-\arctan\left(\frac{\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)-arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)+2*sin(d*x+c)*(-2/(c
os(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/(-2/(cos(d*x+c)+1)
)^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [B] time = 3.01921, size = 894, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
```

```
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)
/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.80677, size = 869, normalized size = 9.45

$$\left[\frac{\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^{2}+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\sqrt{\frac{a\cos\ldots}{\cos\ldots}}}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*((cos(d*x + c)^2 + cos(
d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.404 $\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=136

$$\frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a\,\mathrm{se}}}\right.}{4d}$$

```
[Out] (3*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*Sin[c + d*x])/(2*d*Cos[c + d*x]^
(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (3*a*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.229755, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3803, 3801, 215}

$$\frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{3\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}}{\sqrt{a\,\mathrm{se}}}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```


```
[Out] (3*Sqrt[a]*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*Sin[c + d*x])/(2*d*Cos[c + d*x]^
(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (3*a*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
```

```
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}\left(3\sqrt{\cos(c+dx$$

$$= \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{3a\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{}\right.}{}$$

$$= \frac{3\sqrt{a}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}$$

Mathematica [A] time = 0.482874, size = 100, normalized size = 0.74

$$\frac{2a\sin(c+dx)\left(\frac{1}{8}\cos(c+dx)(3\cos(c+dx)+2)+\frac{3\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{8\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\right)}{d\cos^{\frac{7}{2}}(c+dx)\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*((Cos[c + d*x]*(2 + 3*Cos[c + d*x]))/8 + (3*ArcSin[Sqrt[1 - Sec[c + d*
x]]])/(8*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)))*Sin[c + d*x])/(d*Cos[c
 + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.211, size = 213, normalized size = 1.6

$$\frac{(\cos(dx+c))^2-1}{16\,d\,(\sin(dx+c))^2}\left(3\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^2-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/16/d*(3*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))*cos(d*x+c)^2-3*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+6*(-2/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+
c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)/sin(d*x+
c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 3.17533, size = 1706, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*a
rctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(
2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
```

```
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1
)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2)
- 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(
sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*co
s(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*
c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2
*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.84813, size = 944, normalized size = 6.94

$$\left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(3\cos(dx+c)+2)\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\cdots}}{\cdots}\right.}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt(
cos(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(co
s(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2),
 1/8*(2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(d*x + c) + 2)*sqrt(c
os(d*x + c))*sin(d*x + c) + 3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3
+ d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.405 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{26a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (208*a^2*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
+ (104*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]
) + (26*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]
) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.311456, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3813, 21, 3805, 3804}

$$\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}+\frac{26a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (208*a^2*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
+ (104*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]
) + (26*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]
) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
```

```
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{13a}{2}+}{\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{7}\left(13a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{s}}}{\sec^{\frac{5}{2}}(}$$

$$= \frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{1}{35}\left(52a\sqrt{\cos}\right.$$

$$= \frac{104a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{26a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{5}{2}}(c}{7d\sqrt{a+}}$$

$$= \frac{208a^2\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{104a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{26a}{}$$

Mathematica [A] time = 0.288549, size = 72, normalized size = 0.45

$$\frac{a\sqrt{\cos(c+dx)}(253\cos(c+dx)+78\cos(2(c+dx))+15\cos(3(c+dx))+494)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(494 + 253*Cos[c + d*x] + 78*Cos[2*(c + d*x)] + 15*Co
s[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(210*d)
```

Maple [A] time = 0.169, size = 83, normalized size = 0.5

$$-\frac{2\,a\left(15\,(\cos(dx+c))^4+24\,(\cos(dx+c))^3+13\,(\cos(dx+c))^2+52\,\cos(dx+c)-104\right)}{105\,d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)}{\cos(dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/105/d*a*(15*cos(d*x+c)^4+24*cos(d*x+c)^3+13*cos(d*x+c)^2+52*cos(d*x+c)-1
04)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.67839, size = 409, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/840*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), c
os(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d*
x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 1
75*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arctan
2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.67582, size = 232, normalized size = 1.44

$$\frac{2\left(15\,a\cos\left(dx+c\right)^{3}+39\,a\cos\left(dx+c\right)^{2}+52\,a\cos\left(dx+c\right)+104\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104*
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

3.406 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a}}{5d}$$

```
[Out] (8*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*Cos
[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.242447, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3812, 3809, 3804}

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*Cos
[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+}{5d}$$

$$= \frac{8a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c}{5d}$$

Mathematica [A] time = 0.236338, size = 60, normalized size = 0.52

$$\frac{a\sqrt{\cos(c+dx)}(6\cos(c+dx)+\cos(2(c+dx))+13)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(13 + 6*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[a*(1 +
Sec[c + d*x])]*Tan[(c + d*x)/2])/(5*d)
```

Maple [A] time = 0.168, size = 71, normalized size = 0.6

$$-\frac{2\,a\left(\left(\cos\left(dx+c\right)\right)^{3}+2\,\left(\cos\left(dx+c\right)\right)^{2}+3\,\cos\left(dx+c\right)-6\right)}{5\,d\sin\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d*a*(cos(d*x+c)^3+2*cos(d*x+c)^2+3*cos(d*x+c)-6)*cos(d*x+c)^(1/2)*(a*(
cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.85546, size = 284, normalized size = 2.45

$$\frac{\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\cos\left(\frac{2}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/
2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin
(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)))*sqrt(a)/d
```

Fricas [A] time = 1.62227, size = 192, normalized size = 1.66

$$\frac{2\left(a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+6\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 6*a)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

3.407 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=79

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

```
[Out] (8*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.176176, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4264, 3809, 3804}

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(4a\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$
$$= \frac{8a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c}{3d}$$

Mathematica [A] time = 0.164584, size = 50, normalized size = 0.63

$$\frac{2a\sqrt{\cos(c+dx)}(\cos(c+dx)+5)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(5 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(
c + d*x)/2])/(3*d)
```

Maple [A] time = 0.15, size = 61, normalized size = 0.8

$$-\frac{2a\left((\cos(dx+c))^2+4\cos(dx+c)-5\right)}{3d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*a*(cos(d*x+c)^2+4*cos(d*x+c)-5)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/c
os(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 2.86343, size = 51, normalized size = 0.65

$$\frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

Fricas [A] time = 1.6357, size = 163, normalized size = 2.06

$$\frac{2\,(a\cos\left(dx+c\right)+5\,a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c) + 5*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

3.408 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.182958, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3813, 21, 3801, 215}

$$\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(}}{}$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c}$$

$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int\right.}{d}$$

$$= \frac{2a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+}{d\sqrt{\cos(c+dx)}\sqrt{a+}}$$

Mathematica [A] time = 0.137769, size = 81, normalized size = 0.84

$$\frac{2a^2\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}+\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*(Sqrt[1 - Sec[c + d*x]] + ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c
+ d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])
])
```

Maple [B] time = 0.138, size = 172, normalized size = 1.8

$$\frac{a}{2\,d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1-\sin(dx+c))}{4}\sqrt{-2\,(\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*a*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(2^(1/2)*arcta
n(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(-2/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*co
s(d*x+c)+4)/sin(d*x+c)
```

Maxima [B] time = 2.90611, size = 370, normalized size = 3.85

$$\frac{\sqrt{2}\left(\sqrt{2}a\log\left(2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)-\sqrt{2}a\log\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos(
```

```
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*sqrt(
a)/d
```

Fricas [A] time = 1.73587, size = 799, normalized size = 8.32

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\,\mathrm{s}}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right.}{2\left(d\cos\left(dx+c\right)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
)/(d*cos(d*x + c) + d), (2*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + (a*cos(d*x + c) + a)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.409 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

```
[Out] (3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.180846, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3814, 21, 3801, 215}

$$\frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{3a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (3*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2
)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
```

```
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{3a}{2}+\right.}{\sqrt{a+a\sec}}$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a}$$

$$= \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,\right.}{d}$$

$$= \frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^2\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+d}}$$

Mathematica [A] time = 0.318187, size = 92, normalized size = 0.97

$$-\frac{a^2 \sin(c+dx)\left(\frac{3\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}-\sqrt{1-\sec(c+dx)}\right)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((a^2*(-Sqrt[1 - Sec[c + d*x]] + (3*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec[c
 + d*x]])*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)*Sqrt[1 - Sec[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])]))
```

Maple [B] time = 0.19, size = 182, normalized size = 1.9

$$\frac{a\left((\cos(dx+c))^2-1\right)}{4d(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(1/4*2^(1/2)*(-2/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)-3*arctan(1/4
*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d
*x+c)+2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)/cos
(d*x+c)^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.78546, size = 1543, normalized size = 16.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos
```

```
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x + 3/
2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 4
*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(2*d*
x + 2*c))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*d)
```

Fricas [A] time = 1.76234, size = 891, normalized size = 9.38

$$\left[\frac{4\,a\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(\ldots}{\cos(d\ldots}\right.}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + 3*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^
3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sq
rt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(2*a*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a*cos(d*
x + c)^2 + a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*co
s(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.410 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\cdots}{\sqrt{a\,s\cdots}}\right.}{4d}$$

```
[Out] (7*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*Sin[c + d*x])/(2*d*Cos[c + d*x
]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (7*a^2*Sin[c + d*x])/(4*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.242585, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3814, 21, 3803, 3801, 215}

$$\frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{7a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\cdots}{\sqrt{a\,s\cdots}}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*Sin[c + d*x])/(2*d*Cos[c + d*x
]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (7*a^2*Sin[c + d*x])/(4*d*Cos[c + d*x]^
(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{7}{2}\right.}{\sqrt{a+a}}$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}\left(7a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{}$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}\left(7a\sqrt{\cos(c}\right.$$

$$= \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{7a^2\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(7a\sqrt{\cos(c+}\right.}{}$$

$$= \frac{7a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a^2\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 0.431454, size = 99, normalized size = 0.71

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)+7\sqrt{2}\cos^2(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+\right.\right.\right.}{8d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(7*Sqrt[2]*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7*Sin[(3*(c + d*x))/
2]))/(8*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.2, size = 212, normalized size = 1.5

$$-\frac{a(-1+\cos(dx+c))}{8d(\sin(dx+c))^2}\left(7\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/8/d*a*(-1+cos(d*x+c))*(7*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-7*2^(1/2)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+14*(-2/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(3/2)/(-
2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 3.4779, size = 3029, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*sqr
t(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin(3/
2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(7/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(5/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a*co
s(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a
)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*si
```

```
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*
log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*si
n(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*lo
g(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(
2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*
cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + sqrt(2)*a)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
```

```
*x + 3/2*c))) + 8*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) - 7*sqrt(2)*a*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/((2*(2*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 4*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*d)
```

Fricas [A] time = 1.86983, size = 965, normalized size = 6.89

$$\left[\frac{4\,(7\,a\cos\left(dx+c\right)+2\,a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+7\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}\cdots}{\cdots}\right.}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sq
rt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^
2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x
+ c)^2), 1/8*(2*(7*a*cos(d*x + c) + 2*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(a*cos(d*x + c)^3 + a*cos(d*x + c
)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d
*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.411 $\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{11a^2\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{11a^{3/2}}{}$$

```
[Out] (11*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*Sin[c + d*x])/(3*d*Cos[c + d*
x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Sin[c + d*x])/(12*d*Cos[c + d*
x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Sin[c + d*x])/(8*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.302473, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3814, 21, 3803, 3801, 215}

$$\frac{11a^2\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{11a^{3/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```


```
[Out] (11*a^(3/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*Sin[c + d*x])/(3*d*Cos[c + d*
x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Sin[c + d*x])/(12*d*Cos[c + d*
x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (11*a^2*Sin[c + d*x])/(8*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{11a}{2}\right.}{\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}\left(11a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a}$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}\left(11a\sqrt{\cos(c\right.$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2}{8d\cos^{\frac{3}{2}}(c+d}$$

$$= \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{11a^2}{8d\cos^{\frac{3}{2}}(c+d}$$

$$= \frac{11a^{3/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{a^2\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

Mathematica [A] time = 0.537936, size = 112, normalized size = 0.62

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(54\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)+66\sqrt{2}\cos^3(c+dx)\right.}{96d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(66*Sqrt[2]*ArcTanh[Sqrt[2]*
Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] + 11*(Sin[(3*(c + d*
x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.214, size = 244, normalized size = 1.4

$$\frac{a\left((\cos(dx+c))^2-1\right)}{96\,d\,(\sin(dx+c))^2}\left(33\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^3\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/96/d*a*(33*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin
(d*x+c)))*cos(d*x+c)^3*2^(1/2)-33*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+66*(-2/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)*cos(d*x+c)^2+44*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d
*x+c)+16*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c)
)^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^2*(cos(d*x+c)
^2-1)
```

Maxima [B] time = 3.46769, size = 3187, normalized size = 17.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/96*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*s
qrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) + 3*sq
rt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2
*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*
a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos
(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d
*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x
```

```
 + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c)
+ a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a
*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*c
os(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin
(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x +
2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*
c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) +
 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 +
 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 1
8*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos
(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d
*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4
*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqr
t(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x +
4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*
cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c) +
3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*si
n(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(6*d*
x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sq
rt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sqrt(2
)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x
 + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sqrt(a)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(
4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) +
 1)*d)
```

Fricas [A] time = 1.83971, size = 1031, normalized size = 5.73

$$\left[\frac{4\left(33\,a\cos\left(dx+c\right)^{2}+22\,a\cos\left(dx+c\right)+8\,a\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+33\left(a\cos\left(dx+c\right)^{4}+a\cos\left(dx\right.\right.}{96\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(33*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 33*(a*cos(d*x + c)^
4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos
(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(33*a*cos(d*x + c)^2 + 22*a*cos(d*
x + c) + 8*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqr
t(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*
x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

3.412 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=201

$$\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{9d}+\frac{38a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{146a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{58}{}$$

```
[Out] (1168*a^3*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
 + (584*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]
]) + (146*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*
x]]) + (38*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a^2*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*
d)
```

Rubi [A] time = 0.406101, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3813, 4015, 3805, 3804}

$$\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{9d}+\frac{38a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{146a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{58}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (1168*a^3*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
 + (584*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]
]) + (146*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*
x]]) + (38*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a^2*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*
d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3805

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a
 + b*Csc[e + f*x]]), x] + Dist[(a*(2*n + 1))/(2*b*d*n), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{9d} + \frac{1}{9}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{38a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{9d}$$

$$= \frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\cos^{\frac{7}{2}}(}{}$$

$$= \frac{584a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{146a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{38a^3\cos}{63d\sqrt{}}$$

$$= \frac{1168a^3\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{584a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{14}{}$$

Mathematica [A] time = 0.239832, size = 90, normalized size = 0.45

$$\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\left(35\cos^4(c+dx)+130\cos^3(c+dx)+219\cos^2(c+dx)+292\cos(c+dx)+584\right)\sqrt{a(\sec(c}{315d(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(584 + 292*Cos[c + d*x] + 219*Cos[c + d*x]^2 + 13
0*Cos[c + d*x]^3 + 35*Cos[c + d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*
x])/(315*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.183, size = 95, normalized size = 0.5

$$-\frac{2a^2\left(35(\cos(dx+c))^5+95(\cos(dx+c))^4+89(\cos(dx+c))^3+73(\cos(dx+c))^2+292\cos(dx+c)-584\right)}{315d\sin(dx+c)}\sqrt{\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/315/d*a^2*(35*cos(d*x+c)^5+95*cos(d*x+c)^4+89*cos(d*x+c)^3+73*cos(d*x+c)
^2+292*cos(d*x+c)-584)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)
/sin(d*x+c)
```

Maxima [B] time = 2.8915, size = 570, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/5040*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/
2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^
2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
+ 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)
*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos
(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos
(9/2*d*x + 9/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.66525, size = 279, normalized size = 1.39

$$\frac{2\left(35\,a^2\cos\left(dx+c\right)^4+130\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+292\,a^2\cos\left(dx+c\right)+584\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx\right.}}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x + c
)^2 + 292*a^2*cos(d*x + c) + 584*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.413 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=156

$$\frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\ \ldots}{7d}$$

```
[Out] (64*a^3*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) +
(2*a*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2
*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.299475, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3812, 3809, 3804}

$$\frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\ \ldots}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (64*a^3*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) +
(2*a*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*d) + (2
*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
```

```
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c-\right.$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2\cos^{\frac{5}{2}}(c+dx)(a+a\sec(}{7d}$$

$$= \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec}{7a}$$

$$= \frac{64a^3\sin(c+dx)}{21d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,s}{21d}$$

Mathematica [A] time = 0.255969, size = 74, normalized size = 0.47

$$\frac{a^2\sqrt{\cos(c+dx)}(101\cos(c+dx)+24\cos(2(c+dx))+3\cos(3(c+dx))+208)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{42d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(208 + 101*Cos[c + d*x] + 24*Cos[2*(c + d*x)] + 3*C
os[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(42*d)
```

Maple [A] time = 0.168, size = 85, normalized size = 0.5

$$-\frac{2\,a^{2}\left(3\,\left(\cos\left(dx+c\right)\right)^{4}+9\,\left(\cos\left(dx+c\right)\right)^{3}+11\,\left(\cos\left(dx+c\right)\right)^{2}+23\,\cos\left(dx+c\right)-46\right)}{21\,d\sin\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d*a^2*(3*cos(d*x+c)^4+9*cos(d*x+c)^3+11*cos(d*x+c)^2+23*cos(d*x+c)-46
)*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.80294, size = 436, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/168*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c)
, cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*cos
(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^2*s
in(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin(3/
7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(a)/d
```

Fricas [A] time = 1.61995, size = 239, normalized size = 1.53

$$\frac{2\left(3 a^{2} \cos\left(dx+c\right)^{3}+12 a^{2} \cos\left(dx+c\right)^{2}+23 a^{2} \cos\left(dx+c\right)+46 a^{2}\right)\sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{21\left(d \cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) +
46*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec\left(dx+c\right)+a\right)^{\frac{5}{2}} \cos\left(dx+c\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

3.414 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=119

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\text{s}}{5d}$$

```
[Out] (64*a^3*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) +
(2*a*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.231381, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4264, 3809, 3804}

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\text{s}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(16*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) +
(2*a*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3809

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*
(d*Csc[e + f*x])^n)/(f*m), x] + Dist[(b*(2*m - 1))/(d*m), Int[(a + b*Csc[e
+ f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m
, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ[2
*m]
```

Rule 3804

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[(-2*a*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*
Sqrt[d*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(8a\sqrt{\cos(c+dx)}\sqrt{\sec(\right.$$

$$= \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)(a+a\sec}{5d}$$

$$= \frac{64a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{16a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}{15d}$$

Mathematica [A] time = 0.235726, size = 64, normalized size = 0.54

$$\frac{a^2\sqrt{\cos(c+dx)}(28\cos(c+dx)+3\cos(2(c+dx))+89)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]]*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sqrt[a*
(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.149, size = 75, normalized size = 0.6

$$-\frac{2\,a^2\left(3\,(\cos(dx+c))^3+11\,(\cos(dx+c))^2+29\,\cos(dx+c)-43\right)}{15\,d\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d*a^2*(3*cos(d*x+c)^3+11*cos(d*x+c)^2+29*cos(d*x+c)-43)*cos(d*x+c)^(1
/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 2.51076, size = 81, normalized size = 0.68

$$\frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.64596, size = 207, normalized size = 1.74

$$\frac{2\left(3\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+43\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

3.415 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=138

$$\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{s}}{d}$$

```
[Out] (2*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (14*a^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.282126, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3813, 4015, 3801, 215}

$$\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{s}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (14*a^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3813

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[a/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f*
```

```
x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
&& (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin}{3d}$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin}{3d}$$

$$= \frac{2a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{14a^3\sin(}{3d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Mathematica [A] time = 0.23641, size = 93, normalized size = 0.67

$$\frac{2a^3\sin(c+dx)\left((\cos(c+dx)+8)\sqrt{1-\sec(c+dx)}+3\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^3*((8 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*ArcSin[Sqrt[1 - Sec[c
 + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.194, size = 185, normalized size = 1.3

$$\frac{a^2}{6\,d\sin(dx+c)}\left(3\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1-\sin(dx+c))\right)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/6/d*a^2*(3*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*2^(1/2)*arctan(1/4
*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-4*cos(d*x+c)^2-28*cos(d*x+c)+32)*(a*(cos(d*x+c)+1)/
cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 3.12526, size = 801, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(s
```

```
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*x
 + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))))*sqrt(a)/d
```

Fricas [A] time = 1.82495, size = 888, normalized size = 6.43

$$\left[\frac{4\left(a^2\cos(dx+c)+8a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+3\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos}{\cos}}}{}\right.}{6\left(d\cos(dx+c)+d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a
*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(a^2*cos(d*x
+ c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 3*(a^2*cos(d*x + c) + a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

3.416 $\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=132

$$\frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{a\sec(}}\right.}{d}$$

```
[Out] (5*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.2807, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3814, 4015, 3801, 215}

$$\frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d\sqrt{\cos(c+dx)}}+\frac{5a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{a\sec(}}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (5*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
```

```
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4015

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b^2*C
ot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n*Sqrt[a + b*Csc[e + f*x]]), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{s}}}{}$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(5a\right.$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(5a^2\right.}{}$$

$$= \frac{5a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^3\sin(c+}{d\sqrt{\cos(c+dx)}\sqrt{a+}}$$

Mathematica [A] time = 0.283634, size = 90, normalized size = 0.68

$$\frac{a^3\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}(\sec(c+dx)+2)+5\sqrt{\sec(c+dx)}\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\right)}{d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^3*(5*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqrt[1 - Sec[c
+ d*x]]*(2 + Sec[c + d*x]))*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.204, size = 197, normalized size = 1.5

$$-\frac{a^2}{4\,d\sin(dx+c)}\left(5\,\sin(dx+c)\cos(dx+c)\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*a^2*(5*sin(d*x+c)*cos(d*x+c)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*(-2/(cos(d*x+c)+1))^(1/2)*2^(1/2)-5*sin(d*
x+c)*cos(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)+8*cos(d*x+c)^2-4*cos(d*x+c)-
4)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.85778, size = 969, normalized size = 7.34

$$\left[\frac{4\left(2a^2\cos(dx+c)+a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+5\left(a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+\right.}{}\right.}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))
*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d
*x + c)), 1/2*(2*(2*a^2*cos(d*x + c) + a^2)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(a^2*cos(d*x + c)^2 + a^2*cos(
d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)
))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

3.417 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{19a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{a}\sec}\right.}{4d}$$

```
[Out] (19*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (9*a^3*Sin[c + d*x])/(4*d*Cos[c +
d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.279887, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3814, 4016, 3801, 215}

$$\frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{19a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{a}\sec}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (19*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (9*a^3*Sin[c + d*x])/(4*d*Cos[c +
d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d*Cos[c + d*x]^(3/2))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
```

```
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+a$$

$$= \frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{8}\left(19a^2\sqrt{\cos(c+\right.$$

$$= \frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(19a^2\sqrt{\cos(c+d\right.}{}$$

$$= \frac{19a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{9a^3\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

Mathematica [A] time = 0.633854, size = 95, normalized size = 0.68

$$\frac{a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(\sqrt{1-\sec(c+dx)}(2\sec(c+dx)+11)-\frac{19\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)}{\sqrt{\sec(c+dx)}}\right)}{4d\sqrt{\cos(c+dx)-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sec[c + d*x])]*((-19*ArcSin[Sqrt[Sec[c + d*x]]])/Sqrt[Sec[
c + d*x]] + Sqrt[1 - Sec[c + d*x]]*(11 + 2*Sec[c + d*x]))*Tan[(c + d*x)/2])
/(4*d*Sqrt[-1 + Cos[c + d*x]])
```

Maple [A] time = 0.208, size = 216, normalized size = 1.5

$$\frac{a^2\left((\cos(dx+c))^2-1\right)}{16\,d\,(\sin(dx+c))^2}\left(19\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

[Out] 1/16/d*a^2*(19*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-19*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+22*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)/cos(d*x+c)^(3/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)

Maxima [B] time = 23.9252, size = 3815, normalized size = 27.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/16*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si

n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
- a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)
^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)
- 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
 - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2
*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22
*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*
(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) +
 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) -
 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)

```
*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) - 44*(2*sqrt(2)*a^2*cos(2*d*x
 + 2*c) + sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + 28*(2*sqrt(2)*a^2*cos(2*d*x +
 2*c) + sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c) + 8*(7*sqrt(2)*a^2*cos(3/2*d*x +
3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sqrt(a)/((2
*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d
*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*
sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.79501, size = 992, normalized size = 7.09

$$\left[\frac{4\left(11\,a^2\cos\left(dx+c\right)+2\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+19\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a}{-}\right.}{16\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x
+ c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos
(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 +
d*cos(d*x + c)^2), 1/8*(2*(11*a^2*cos(d*x + c) + 2*a^2)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 19*(a^2*cos(d*x + c)
^3 + a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(
d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

3.418 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\cos^{\frac{5}{2}}(c+dx)}+\frac{25a^{5/2}}{}$$

```
[Out] (25*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (13*a^3*Sin[c + d*x])/(12*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (25*a^3*Sin[c + d*x])/(8*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(3*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.342548, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3814, 4016, 3803, 3801, 215}

$$\frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\cos^{\frac{5}{2}}(c+dx)}+\frac{25a^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```


```
[Out] (25*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (13*a^3*Sin[c + d*x])/(12*d*Cos[c
+ d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (25*a^3*Sin[c + d*x])/(8*d*Cos[c +
 d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(3*d*Cos[c + d*x]^(5/2))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{8}\left(25a^2\sqrt{\cos(\right.$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\,\mathrm{se}}}{3d\,\mathrm{c}}$$

$$= \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{25a^3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\,\mathrm{se}}}{3d\,\mathrm{c}}$$

$$= \frac{25a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{13a^3\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c}$$

Mathematica [C] time = 5.47527, size = 180, normalized size = 1.

$$\frac{a^2(\cos(c+dx)+1)^2\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(-75ie^{\frac{1}{2}i(c+dx)}\cos^3(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-e^{2i}\right.\right.}{96}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^5*Sqrt[a*(1 + Sec[c + d*x])]*((-
75*I)*E^((I/2)*(c + d*x))*Cos[c + d*x]^3*Hypergeometric2F1[1/4, 1, 5/4, -E^
((2*I)*(c + d*x))] - (25*I)*E^(((3*I)/2)*(c + d*x))*Cos[c + d*x]^3*Hypergeo
metric2F1[3/4, 1, 7/4, -E^((2*I)*(c + d*x))] + (8 + 34*Cos[c + d*x] + 75*Co
s[c + d*x]^2)*Sin[(c + d*x)/2]))/(96*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.224, size = 244, normalized size = 1.4

$$-\frac{a^2(-1+\cos(dx+c))}{48\,d\,(\sin(dx+c))^2}\left(75\arctan\left(1/4\sqrt{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^3\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/48/d*a^2*(-1+cos(d*x+c))*(75*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)-75*arctan(1/4*2^(1/2)*(-2
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*2^(1/2)+150*
(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^2+68*(-2/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)*sin(d*x+c)+16*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(
d*x+c)+1)/cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1
))^(1/2)
```

Maxima [B] time = 3.02727, size = 4683, normalized size = 26.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)
*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*
a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6
*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12
*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -
 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 4
56*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*
d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^
2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*co
s(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*
```

x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*
a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*
x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*
sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sq
rt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7
5*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^
2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2
+ 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*
a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) +
a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*
sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*l
og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(
6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +

3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*
cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2) + 28*(sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c))*sin(6*d*x + 6*c) + 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*
sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ sqrt(2)*a^2)*sin(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 12*(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x + 3/
2*c) - 114*sqrt(2)*a^2*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 114*sqrt(2)*a^2*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 456*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*sin(7/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*cos(6*d*x + 6*
c) + 3*sqrt(2)*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + sqrt(2)*a^2)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
c))) + 12(7*sqrt(2)*a^2*cos(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*cos(3/2*d*x
+ 3/2*c) + 75*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
 300*(sqrt(2)*a^2*cos(6*d*x + 6*c) + sqrt(2)*a^2)*sin(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/((cos(6*d*x + 6*c)^2 + 6*(cos(
6*d*x + 6*c) + 3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*co
s(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*(cos(6*d*x
 + 6*c) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 9*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(6*d
*x + 6*c)^2 + 6*(sin(6*d*x + 6*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 9*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 6*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 9*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(6*d*x + 6*c) + 1)*d)

Fricas [A] time = 1.86816, size = 1058, normalized size = 5.88

$$\left[\frac{4\left(75\,a^2\cos\left(dx+c\right)^2+34\,a^2\cos\left(dx+c\right)+8\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+75\left(a^2\cos\left(dx+c\right)^4+a^2\cos\right.}{96\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(4*(75*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos(d
*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*(2*(75*a^2*cos(d*x + c)^2 +
 34*a^2*cos(d*x + c) + 8*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + 75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*s
qrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d
*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a \sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

3.419 $\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{163a^3\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2}{}$$

```
[Out] (163*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (17*a^3*Sin[c + d*x])/(24*d*Cos[
c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Sin[c + d*x])/(96*d*Cos
[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Sin[c + d*x])/(64*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.403076, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3814, 4016, 3803, 3801, 215}

$$\frac{163a^3\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```


```
[Out] (163*a^(5/2)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (17*a^3*Sin[c + d*x])/(24*d*Cos[
c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Sin[c + d*x])/(96*d*Cos
[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (163*a^3*Sin[c + d*x])/(64*d*Co
s[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*Sqrt[a + a*Sec[c + d*x]]*
Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 4016

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-2*b*B*
Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]]), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 3803

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/
(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[(2*a*d*(n - 1))/(b*(2*n -
 1)), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{4}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{48}\left(163a^2\sqrt{\text{co}$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2\sqrt{a+a\,\text{s}}}{4a}$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{64d\cos^{\frac{3}{2}}(c}$$

$$= \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{163a^3\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{64d\cos^{\frac{3}{2}}(c}$$

$$= \frac{163a^{5/2}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{17a^3\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(}$$

Mathematica [C] time = 5.56629, size = 190, normalized size = 0.86

$$\frac{a^2(\cos(c+dx)+1)^2\sec^5\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\sec(c+dx)+1)}\left(-489ie^{\frac{1}{2}i(c+dx)}\cos^4(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{4},1,\frac{5}{4},-e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^5*Sqrt[a*(1 + Sec[c + d*x])]*((-
489*I)*E^((I/2)*(c + d*x))*Cos[c + d*x]^4*Hypergeometric2F1[1/4, 1, 5/4, -E
^((2*I)*(c + d*x))] - (163*I)*E^(((3*I)/2)*(c + d*x))*Cos[c + d*x]^4*Hyperg
eometric2F1[3/4, 1, 7/4, -E^((2*I)*(c + d*x))] + (48 + 184*Cos[c + d*x] + 3
26*Cos[c + d*x]^2 + 489*Cos[c + d*x]^3)*Sin[(c + d*x)/2]))/(768*d*Cos[c + d
*x]^(7/2))
```

Maple [A] time = 0.235, size = 276, normalized size = 1.3

$$\frac{a^2\left((\cos(dx+c))^2-1\right)}{768\,d\,(\sin(dx+c))^2}\left(489\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)\sqrt{2}(\cos(dx+c))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/768/d*a^2*(489*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1
+sin(d*x+c)))*2^(1/2)*cos(d*x+c)^4-489*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)^4+978*(-2/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^3+652*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)*cos(d*x+c)^2+368*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+96*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)/cos(d*x+c)^(7/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 3.74836, size = 5211, normalized size = 23.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/768*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c)
 + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(15/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(8*d*x
 + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) +
 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x
+ 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*c
os(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a^2*si
n(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x +
 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(
6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2
*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*
d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*
```

sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4

```
*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^
2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*
d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(
4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*c
os(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2
*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*si
n(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*
x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ sqrt(2)*a^2)*sin(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*
(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*
a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(13
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*cos(8*d
*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c)
 + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2
)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos
(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 2060*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c
) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(
2)*a^2)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6204*(sqrt(2
)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*cos(8*d*x + 8*c)
 + 4*sqrt(2)*a^2*cos(6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt
(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(3/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1956*(sqrt(2)*a^2*cos(8*d*x + 8*c) + 4*sqrt(2)*a^2*cos(
6*d*x + 6*c) + 6*sqrt(2)*a^2*cos(4*d*x + 4*c) + 4*sqrt(2)*a^2*cos(2*d*x + 2
*c) + sqrt(2)*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sq
rt(a)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1
)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2
 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d
*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c
) + 1)*d)
```

Fricas [A] time = 1.96465, size = 1139, normalized size = 5.18

$$\left[\frac{4\left(489\,a^2\cos\left(dx+c\right)^3+326\,a^2\cos\left(dx+c\right)^2+184\,a^2\cos\left(dx+c\right)+48\,a^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+4}{768\left(d\cos\left(dx+c\right)^5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(4*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*
x + c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*log((
a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d
*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(c
os(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/
384*(2*(489*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x +
 c) + 48*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + 489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(
2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*c
os(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

3.420 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=189

$$\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
 (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.413719, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3823, 4022, 4013, 3808, 206}

$$\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
 (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-4a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{5a}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{15a^2}$$

$$= \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5d\sqrt{a+a\sec(c+dx}}$$

$$= \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5d\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\,\mathrm{se}}}$$

Mathematica [A] time = 0.303028, size = 136, normalized size = 0.72

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(13\sec^2(c+dx)-\sec(c+dx)+3\right)+15\sqrt{2}\sec^{\frac{5}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+d}}{\sqrt{1-\sec(c+dx}}\right.\right.}{15d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(15*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1
- Sec[c + d*x]]]*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3 - Sec[c +
 d*x] + 13*Sec[c + d*x]^2))*Sin[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt
[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.193, size = 120, normalized size = 0.6

$$\frac{1}{15\,ad\sin(dx+c)}\left(15\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\sin(dx+c)-6\,(\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d/a*(15*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-6*cos(d*x+c)^3+8*cos(d*x+c)^2-28*cos(d*x+c)+26)*(a*
(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.22697, size = 482, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d
*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin
(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(si
n(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*
x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))))/(sqrt(a)*d)
```

Fricas [A] time = 1.77105, size = 886, normalized size = 4.69

$$\left[\frac{4\left(3\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+13\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^{2}+\frac{2\sqrt{2}\sqrt{\frac{a\cos\ldots}{c\ldots}}}{\ldots}}{c\ldots}\right.}{\sqrt{a}}}{30\left(ad\cos\left(dx+c\right)+ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(4*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c) +
 a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x +
c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqr
t(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*cos(d*x
 + c)^2 - cos(d*x + c) + 13)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a), x)
```

3.421 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\mathrm{se}}}{\sqrt{2}\sqrt{a\sec(c+}}\right.}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.282625, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3823, 4013, 3808, 206}

$$\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\mathrm{se}}}{\sqrt{2}\sqrt{a\sec(c+}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3823

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n*Sqrt[a +
 b*Csc[e + f*x]]), x] + Dist[1/(2*b*d*n), Int[((d*Csc[e + f*x])^(n + 1)*(a
```

```
+ b*(2*n + 1)*Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-2a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx}{3a}$$

$$= -\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= -\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.}{}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c}}$$

Mathematica [A] time = 0.19632, size = 116, normalized size = 0.77

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\left(2(1-\sec(c+dx))^{3/2}-3\sqrt{2}\sec^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{3d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(1 - Sec[c + d*x])^(3/2) - 3*Sqrt[2]*ArcTan[(Sqrt[2]
*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2))*Sin[c + d*
x])/(3*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.182, size = 110, normalized size = 0.7

$$-\frac{1}{3\,ad\sin\left(dx+c\right)}\left(3\,\arctan\left(1/2\,\sin\left(dx+c\right)\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\right)\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\sin\left(dx+c\right)+2\,(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/3/d/a*(3*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c)^2-4*cos(d*x+c)+2)*cos(d*x+c)^(1/2)*(a*(
cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.21675, size = 381, normalized size = 2.52

$$-\frac{3\sqrt{2}\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3\sqrt{2}\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin
```

```
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Fricas [A] time = 1.75197, size = 828, normalized size = 5.48

$$\left[\frac{4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)+\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{\sqrt{a}}}{6(ad\cos(dx+c)+ad)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 1)*sqrt(cos
(d*x + c))*sin(d*x + c) + 3*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)
^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sqrt(2)*(a*cos(d*x + c) +
a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-
1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c
) + a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a), x)
```

3.422 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
 (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.170166, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3812, 3808, 206}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
 (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3812

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(m + 1)), x] + Dist[(a*m)/(b*d*(m + 1)), Int[(a + b*Csc[e +
f*x])^m*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x]
 && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a}{}\right.}{d}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(}}$$

Mathematica [A] time = 0.0913849, size = 100, normalized size = 0.88

$$\frac{\sin(c+dx)\left(2\sqrt{1-\sec(c+dx)}+\sqrt{2}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{d\sqrt{\cos(c+dx)-1}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(d*Sqrt[-1 + Cos[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.138, size = 98, normalized size = 0.9

$$\frac{1}{ad\sin(dx+c)}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx+c)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(arctan(1/2*sin(
d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*co
s(d*x+c)+2)/sin(d*x+c)
```

Maxima [A] time = 2.17774, size = 140, normalized size = 1.24

$$-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}{2\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))/(sq
rt(a)*d)
```

Fricas [A] time = 1.69679, size = 764, normalized size = 6.76

$$\left[\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{2(ad\cos(dx+c)+ad)},\ \frac{\sqrt{2}(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*c
os(d*x + c) + a*d), (sqrt(2)*(a*cos(d*x + c) + a)*sqrt(-1/a)*arctan(sqrt(2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(
d*x + c)) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(a*sec(d*x + c) + a), x)
```

3.423 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.11586, antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4264, 3808, 206}

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a
+ a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx\\ &= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] time = 0.0706392, size = 95, normalized size = 1.7

$$-\frac{\sqrt{2}\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt
[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*S
qrt[a*(1 + Sec[c + d*x])]))
```

Maple [A] time = 0.161, size = 91, normalized size = 1.6

$$\frac{(\cos(dx+c))^2-1}{ad(\sin(dx+c))^2}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^2*(co
s(d*x+c)^2-1)
```

Maxima [A] time = 1.98756, size = 122, normalized size = 2.18

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}{2\,\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Fricas [A] time = 1.74101, size = 439, normalized size = 7.84

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\,\cos(dx+c)-3}{\cos(dx+c)^2+2\,\cos(dx+c)+1}\right)}{2\,\sqrt{a}d},\ -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
 c))/sin(d*x + c))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a(\sec(c+dx)+1)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.424 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]
]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rubi [A] time = 0.238314, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3821, 3801, 215, 3808, 206}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]
]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3821

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e
 + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
```

```
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= -\left(\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx\right) + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{s\right.}}{}$$

$$= \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} - \frac{(2}{}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a+a\sec}}\right.}{}$$

Mathematica [A] time = 0.0848619, size = 109, normalized size = 0.81

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)-2\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((-2*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]])*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d
*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.187, size = 171, normalized size = 1.3

$$\frac{-1+\cos(dx+c)}{ad(\sin(dx+c))^2}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1-\sin(dx+c))}{4}\sqrt{-2(\cos(dx+c)+1)^{-1}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*(2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))-2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c)))+2*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)))*(a*(co
s(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^(1/2)/(-2/(cos(d*x
+c)+1))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.08876, size = 643, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), co
```

```
s(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c))) + 2))/(sqrt(a)*d)
```

<u>Fricas</u> [A] time = 1.88105, size = 915, normalized size = 6.78

$$\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(}}{\cos(dx+c)^3+\cos(dx+}\right.}{2ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c
) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + sqrt(a)*log((a*cos(d*x + c)
^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*s
qrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)))/(a*d), (sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) +
sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.425 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=168

$$\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c}{}$$

```
[Out] -((ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*Sqr
t[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.340052, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4264, 3822, 4023, 3808, 206, 3801, 215}

$$\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*Sqr
t[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(a}{\sqrt{a+a\,\mathrm{se}}}}{2a}$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(}}{\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int \frac{}{2}\right.}{d}$$

$$= -\frac{\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a+a\,\mathrm{sec}}}\right.}{}$$

Mathematica [A] time = 0.232818, size = 145, normalized size = 0.86

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}+\sin^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+2\sin^{-1}\left(\sqrt{\sec(c+\right.\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*
ArcSin[Sqrt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c +
d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.188, size = 212, normalized size = 1.3

$$\frac{-1+\cos(dx+c)}{2\,ad\,(\sin(dx+c))^2}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(\arctan\left(\frac{\sqrt{2}(\cos(dx+c)+1+\sin(dx+c))}{4}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{2}\,\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d/a*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(arctan(1/4*2^(
1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c
)-arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*2
^(1/2)*cos(d*x+c)-4*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*
x+c)-2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))/cos(d*x+c)^(1/2)/sin(d*x+c)^2/
(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.21239, size = 1183, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*
c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c
) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(
2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) -
 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(
2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1)
+ 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(
d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(
```

```
3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/((cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

Fricas [A] time = 1.98838, size = 1405, normalized size = 8.36

$$\left[\frac{\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}+4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^{2}+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+\frac{2\sqrt{2}(a\,\mathrm{co}}{}}{4\left(ad\cos\left(dx+c\right)^{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)) + 2*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*log(-(cos(d*x + c)
^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1))/sqrt(a) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/2*(2*sqrt(2)*(a
*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + (cos
(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.426 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=211

$$-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\ldots}{\ldots}\right.}{\sqrt{a}d}$$

```
[Out] (7*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(2*d*Cos[c + d*x]^(5/2)*
Sqrt[a + a*Sec[c + d*x]]) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a
*Sec[c + d*x]])
```

Rubi [A] time = 0.475004, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4264, 3822, 4021, 4023, 3808, 206, 3801, 215}

$$-\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\ldots}{\ldots}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (7*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(4*Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(2*d*Cos[c + d*x]^(5/2)*
Sqrt[a + a*Sec[c + d*x]]) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a
*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3822

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/
(f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]]), x] + Dist[d^2/(b*(2*n - 3)), Int[((
d*Csc[e + f*x])^(n - 2)*(2*b*(n - 2) - a*Csc[e + f*x]))/Sqrt[a + b*Csc[e +
f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 2
] && IntegerQ[2*n]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a}}}{4a}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(\sqrt{}}{}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - (\sqrt{}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(2}{}$$

$$= \frac{7\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a+a\sec}}\right.}{}$$

Mathematica [A] time = 0.380735, size = 178, normalized size = 0.84

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(2\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}-\sin^{-1}\left(\sqrt{1-}\right.\right.}{4d\sqrt{1-\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(-ArcSin[Sqrt[1 - Sec[c + d*x]]] - 8
*ArcSin[Sqrt[Sec[c + d*x]]] + 4*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])
/Sqrt[1 - Sec[c + d*x]]] + 2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) - Sq
rt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.204, size = 247, normalized size = 1.2

$$\frac{-1+\cos(dx+c)}{8\,ad\,(\sin(dx+c))^2}\left(-7\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c))\right)(\cos(dx+c))^2+7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d/a*(-1+cos(d*x+c))*(-7*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+7*2^(1/2)*arctan(1/4*2^(1/2)*
(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+16*arctan
(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+2*(-2/(cos(d*x+c)+1
))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(a*(
cos(d*x+c)+1)/cos(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(cos(d*x+c)+1))^(1/2)/cos(
d*x+c)^(3/2)
```

Maxima [B] time = 2.26965, size = 2222, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2
)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt
(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x
+ c), cos(d*x + c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
```

```
 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*co
s(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))
)^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*si
n(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^
2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
 + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(
sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d
*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2
*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) +
 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(4*d*x + 4*c) +
2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 20*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c),
cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) +
 sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))))/((2*(2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + si
n(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(a)*d)
```

Fricas [A] time = 1.93779, size = 1480, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(c
os(d*x + c))*sin(d*x + c) - 7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log
((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos
(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/
(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*
x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*
x + c)^2), 1/8*(8*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(-1/a)*
arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(
d*x + c))/sin(d*x + c)) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 7*(cos(d*x + c)^3 + cos(d*x +
 c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/
(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.427 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{15\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{9\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-$$

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d
) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (4
9*Sin[c + d*x])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (13*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) + (9*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.594095, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3817, 4022, 4013, 3808, 206}

$$-\frac{15\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{9\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d
) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (4
9*Sin[c + d*x])/(10*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (13*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]]) + (9*Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{9a}{2}+3a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{9\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{5a^3}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} + \frac{9\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\sec(c+dx)}} - \frac{(}{}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{\cos(c+dx)}\sin(c}{10ad\sqrt{a+a\sec(c+}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{49\sin(c+dx)}{10ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{13\sqrt{\cos(c+dx)}\sin(c}{10ad\sqrt{a+a\sec(c+}}$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} +$$

Mathematica [A] time = 0.916746, size = 152, normalized size = 0.64

$$\frac{75\sqrt{2}\sin(c+dx)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right) + \sqrt{1-\sec(c+dx)}\left(-2\sin(2(c+dx)) + 49\tan(c+}{10d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (75*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos
[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(4
*(9 + Cos[c + d*x]^2)*Sin[c + d*x] - 2*Sin[2*(c + d*x)] + 49*Tan[c + d*x]))
/(10*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.194, size = 193, normalized size = 0.8

$$-\frac{1}{20\,da^{2}\left(\sin\left(dx+c\right)\right)^{3}}\left(75\,\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)\arctan\left(1/2\,\sin\left(dx+c\right)\sqrt{-2\,\left(\cos\left(dx+c\right)+1\right)^{-1}}\right)\sqrt{-2\,\left(\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/20/d/a^2*(75*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+
c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)-8*cos(d*x+c)^5+24*cos(d*x+c)^4-75*a
rctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)-96*cos(d*x+c)^3+54*cos(d*x+c)^2+124*cos(d*x+c)-98)*(a*(cos(d*x+c)
+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.88524, size = 1077, normalized size = 4.54

$$\left[\frac{75\,\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\,\left(\right.}{40\,\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x +
c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/20*(75*sqrt
(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))
) + 2*(4*cos(d*x + c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x + c) + 49)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos
(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.428 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+}$$

```
[Out] (11*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - (19
*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (7*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.44625, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3817, 4022, 4013, 3808, 206}

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (11*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - (19
*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (7*Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{7a}{2}+2a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{3a^3}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{\cos(c+dx)}\sin(c}{6ad\sqrt{a+a\sec(c+}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{19\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{7\sqrt{\cos(c+dx)}\sin(c}{6ad\sqrt{a+a\sec(c+}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} -$$

Mathematica [A] time = 0.611882, size = 133, normalized size = 0.68

$$\frac{\sin(c+dx)\left(\sqrt{1-\sec(c+dx)}(4\cos(c+dx)-19\sec(c+dx)-12)-33\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(}}{\sqrt{1-\sec(}}\right.\right.}{6d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-12 + 4*Cos[c + d*x] - 19*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 33*Sqrt
[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*
x)/2]^2*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(6*d*Sqrt[-1 + Cos[c + d*x]]*(a*(
1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.184, size = 183, normalized size = 0.9

$$\frac{1}{12\,da^2\,(\sin(dx+c))^3}\left(33\,(\cos(dx+c))^2\sin(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/12/d/a^2*(33*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c
)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+8*cos(d*x+c)^4-33*arctan(1/2*sin(d*x
+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-40*cos(
d*x+c)^3+18*cos(d*x+c)^2+52*cos(d*x+c)-38)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1
/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.79999, size = 1027, normalized size = 5.21

$$\left[\frac{33 \sqrt{2}\left(\cos \left(dx+c\right)^2+2 \cos \left(dx+c\right)+1\right)\sqrt{a} \log \left(-\frac{a \cos(dx+c)^2-2 \sqrt{2}\sqrt{a}\sqrt{\frac{a \cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)} \sin(dx+c)-2 a \cos(dx+c)-3 a}{\cos(dx+c)^2+2 \cos(dx+c)+1}\right)+4\left(4\right.}{24\left(a^2 d \cos \left(dx+c\right)^2+2 a^2 d \cos \left(dx+c\right)+a^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos
(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*
cos(d*x + c) + 1)) + 4*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d
*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(33*sqrt(2)*(cos(d*x + c)^
```

```
2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(4*cos(d*x +
 c)^2 - 12*cos(d*x + c) - 19)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) +
a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2), x)
```

3.429 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+}{2d\sqrt{\cos(c+dx)}(a\,s}$$

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
 - Sin[c + d*x]/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (5*Si
n[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.306625, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3817, 4013, 3808, 206}

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{\sin(c+}{2d\sqrt{\cos(c+dx)}(a\,s}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-7*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
 - Sin[c + d*x]/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (5*Si
n[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
```

```
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx \\
&= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5a}{2}+a\sec(c+d}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec}}}{2a^2} \\
&= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{\left(7\sqrt{\cos(}\right.}{} \\
&= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{\left(7\sqrt{\cos}\right.}{} \\
&= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\sec(}
\end{aligned}
$$

Mathematica [A] time = 0.880909, size = 138, normalized size = 0.88

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left((4\cos(c+dx)+5)\sqrt{(\cos(c+dx)-1)\sec^2(c+dx)}+7\sqrt{2}\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tan^{-1}\left(\right.\right.}{2d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(7*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 -
 Sec[c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x] + (5 + 4*Cos[c + d*x])*Sqrt
[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(2*d*Sqrt[-1 + Cos[c +
d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.175, size = 173, normalized size = 1.1

$$-\frac{1}{4\,da^2\left(\sin\left(dx+c\right)\right)^3}\sqrt{\cos\left(dx+c\right)}\sqrt{\frac{a\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)}}\left(7\,\left(\cos\left(dx+c\right)\right)^2\sin\left(dx+c\right)\arctan\left(1/2\,\sin\left(dx+c\right)\sqrt{-2\,(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(7*cos(d*x+
c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d
*x+c)+1))^(1/2)-7*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-8*cos(d*x+c)^3+6*cos(d*x+c)^2+12*cos(d*x+c)-10
)/sin(d*x+c)^3
```

Maxima [B] time = 3.35644, size = 9688, normalized size = 61.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] -1/4*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin(1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2

*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*s
in(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/2
*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c
))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (
7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
 + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x +
 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)
 - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x +
1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + (8*(7*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 259*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(1/2*d*x + 1/2*c)^2 + 91*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^
2 - 104*sin(1/2*d*x + 1/2*c)^3 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*
c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 8*
(37*cos(1/2*d*x + 1/2*c)^2 + 21)*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)
^2 + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*
c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^3 + 7*(log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*

d*x + 1/2*c)^3 + 13*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d
*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + (2*(7*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*
c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 7*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*sin(3
/2*d*x + 3/2*c)^2 + 2*(84*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 7*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 - 16*(6*cos
(1/2*d*x + 1/2*c)^2 + 1)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*
(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d
*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x
 + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2
*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*cos(3/2*d*x + 3/2*c) - 8*cos(1/2*d*x +
1/2*c))*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^3 + 2*cos(1/2*d*x
+ 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c) + 2*(147*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
 1))*cos(1/2*d*x + 1/2*c)^3 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)*sin
(1/2*d*x + 1/2*c)^2 - 40*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^3 - 56*(
3*cos(1/2*d*x + 1/2*c)^3 + cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c))*cos(
3/2*d*x + 3/2*c) + 2*(2*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(
3/2*d*x + 3/2*c)^3 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*
x + 1/2*c) + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^3 - 8*sin(1/2*d*x + 1
/2*c)^4 + (7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c

```
)^2 - 4)*cos(3/2*d*x + 3/2*c)^2 + (35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c
) - 40*sin(1/2*d*x + 1/2*c)^2 - 36)*sin(3/2*d*x + 3/2*c)^2 - 4*(18*cos(1/2*
d*x + 1/2*c)^2 + 5)*sin(1/2*d*x + 1/2*c)^2 + 6*(7*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2
*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1
/2*c)^2 - 4*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 36*cos(1/2*d*x + 1
/2*c)^2 + 2*((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3
/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 14*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*sin(1/2*d*x + 1/2*c)^2 - 16*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(
3/2*d*x + 3/2*c) - 4*(18*cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c))*
sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(133*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1
/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c) + 21*(log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x +
 1/2*c)^3 - 24*sin(1/2*d*x + 1/2*c)^4 + 2*(21*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x
 + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*cos(3/2*d*x + 3/2*c)^2 - 8*(19*
cos(1/2*d*x + 1/2*c)^2 + 7)*sin(1/2*d*x + 1/2*c)^2 + 16*(7*(log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/
2*d*x + 1/2*c)^2 - 5*cos(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 80*cos(1/
2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^4 + 11*c
os(1/2*d*x + 1/2*c)^2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/((4*sqrt(2)*a^2*cos(3/
2*d*x + 3/2*c)^4 + 28*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3*cos(1/2*d*x + 1/2*
c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^4 + 4*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c)^4 + 12*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3*sin(1/2*d*x + 1/2*c) + 10*sqr
t(2)*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/
2*d*x + 1/2*c)^4 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(
3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^
```

```
2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*cos(5/2*d*x + 5
/2*c)^2 + (61*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 5*sqrt(2)*a^2*sin(1/2*d*
x + 1/2*c)^2)*cos(3/2*d*x + 3/2*c)^2 + (sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2
+ 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*c
os(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2 + 2*sqrt(2)*a^2*
sin(3/2*d*x + 3/2*c)*sin(1/2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c
)^2)*sin(5/2*d*x + 5/2*c)^2 + (8*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^2 + 28*sq
rt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c) + 37*sqrt(2)*a^2*cos(1/
2*d*x + 1/2*c)^2 + 13*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^2)*sin(3/2*d*x + 3/2
*c)^2 + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)^3 + 13*sqrt(2)*a^2*cos(3/2*d*
x + 3/2*c)^2*cos(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^3 +
sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2 + (2*sqrt(2)*a^2*co
s(3/2*d*x + 3/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)
^2 + 2*(12*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + sqrt(2)*a^2*sin(1/2*d*x + 1
/2*c)^2)*cos(3/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*sin(1
/2*d*x + 1/2*c) + sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*si
n(3/2*d*x + 3/2*c))*cos(5/2*d*x + 5/2*c) + 2*(21*sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^3 + 5*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c)^2)*cos(3
/2*d*x + 3/2*c) + 2*(2*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^3 + sqrt(2)*a^2*cos
(3/2*d*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*
c)*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 9*sqrt(2)*a^2*cos(1/2*d*x +
1/2*c)^2*sin(1/2*d*x + 1/2*c) + 5*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)^2*sin(1/
2*d*x + 1/2*c) + sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3 + 2*(sqrt(2)*a^2*cos(3/
2*d*x + 3/2*c)^2 + 6*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*cos(1/2*d*x + 1/2*c)
+ 9*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)
^2)*sin(3/2*d*x + 3/2*c))*sin(5/2*d*x + 5/2*c) + 2*(6*sqrt(2)*a^2*cos(3/2*d
*x + 3/2*c)^2*sin(1/2*d*x + 1/2*c) + 16*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c)*co
s(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 19*sqrt(2)*a^2*cos(1/2*d*x + 1/2*
c)^2*sin(1/2*d*x + 1/2*c) + 3*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)^3)*sin(3/2*d
*x + 3/2*c))*d)
```

Fricas [A] time = 1.78981, size = 964, normalized size = 6.14

$$\left[\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+4\sqrt{\cdots}}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(4*cos(d*x + c
) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d), 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sq
rt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(4*cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos{(c+dx)}}}{(a(\sec{(c+dx)}+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos{(dx+c)}}}{(a\sec{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(3/2), x)
```

3.430 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.187229, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3811, 3808, 206}

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
```

```
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{}}{\sqrt{a}}}{4a}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subs}}{2}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin}{2d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.453637, size = 131, normalized size = 1.12

$$-\frac{\sin(c+dx)\left(2\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}+3\sqrt{2}(\sec(c+dx)+1)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)}{4ad\sqrt{\cos(c+dx)-1}(\cos(c+dx)+1)\sqrt{\sec(c+dx)}\sqrt{a(\sec(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -((2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 3*Sqrt[2]*ArcTan[(Sqrt[2]*
Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x]))*Sin[c + d*x
```

```
])/(4*a*d*Sqrt[-1 + Cos[c + d*x]]*(1 + Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sqr
t[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.172, size = 138, normalized size = 1.2

$$\frac{(\cos(dx+c))^2-1}{4\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\left(\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\cos(dx+c)+3\arctan\left(1/2\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*((-2/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)+3*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2)
)*sin(d*x+c)-(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c
)^3*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.988, size = 1392, normalized size = 11.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*si
n(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)^
2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
+ c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(
d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2
```

```
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2*
sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c)
*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) +
 4*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(d
*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*sin
(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2*sq
rt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a)*d)
```

Fricas [A] time = 1.83717, size = 909, normalized size = 7.77

$$\left[\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\sqrt{\frac{a\cdots}{\cdots}}\cdots}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -
1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2
)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*si
n(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.431 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

```
[Out] (ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) +
Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.187742, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3810, 3808, 206}

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[
c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) +
Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
```

```
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\mathrm{se}}}{\sqrt{a+a}}}{4a}$$

$$= \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}}{2a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} + \frac{\sin}{2d\cos^{\frac{3}{2}}(c+dx)(}$$

Mathematica [B] time = 0.860841, size = 248, normalized size = 2.12

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+\cos(2(c+dx))\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-\right.}{4d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-(Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[
c + d*x]])/Sqrt[1 - Sec[c + d*x]]]) - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c + d*x
]]]*(1 + Cos[c + d*x]) + 2*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x]) +
Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*Sqrt[1 - Sec[c
 + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*(
1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.17, size = 136, normalized size = 1.2

$$\frac{-1+\cos(dx+c)}{2\,da^2(\sin(dx+c))^3}\left(\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\cos(dx+c)-\arctan\left(\frac{\sin(dx+c)}{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sin(dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d/a^2*((-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-arctan(1/2*sin(d*x+c)*(-2/(
cos(d*x+c)+1))^(1/2))*sin(d*x+c)-(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+
1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*cos(d*x+c)^(1/2)/(-2/(cos(d*x+c)+1))^(
1/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.72901, size = 903, normalized size = 7.72

$$\left[\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\,\sqrt{\frac{a\cos}{}}}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
 + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/
4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*
x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.432 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{}{2d}$$

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2
)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.355804, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4264, 3816, 4023, 3808, 206, 3801, 215}

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2
)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\text{se}}}{}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\text{s}}}{a^2}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Sub}}{a^2}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a+a\sec}}\right.}{}$$

Mathematica [A] time = 0.851286, size = 248, normalized size = 1.43

$$-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+\cos(2(c+dx))\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)-\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-5*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Se
c[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] - 5*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c
 + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 2*ArcSin[Sqrt[1 - Sec[c +
d*x]]]*(1 + Cos[c + d*x]) + 10*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x]
) + Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + Cos[2*(c + d*x)]*Sqrt[1 - S
ec[c + d*x]]*Sec[c + d*x]^(3/2))*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*
(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.18, size = 229, normalized size = 1.3

$$-\frac{(\cos(dx+c))^2-1}{4\,da^2(\sin(dx+c))^3}\left(2\sqrt{2}\arctan\left(1/4\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1-\sin(dx+c))\right)\sin(dx+c)-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*sin(d*x+c)-2*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+5*arctan(1/2*sin(d*x+c)*(-2
/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+(-2
/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2
)*(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.84466, size = 2865, normalized size = 16.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
+ 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
```

os(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d
*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*s
in(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*s
in(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sq
rt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*
a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)

Fricas [A] time = 2.00089, size = 1477, normalized size = 8.49

$$\left[\frac{5\,\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\,\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\left(\cos\right.}{8\left(a^{2}d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d
*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*c
os(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
+ c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c
) + a^2*d), 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)*a
rctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))/(a*sin(d*x + c))) + 4*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(-a)
*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
 + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.433 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{}{2ad\cos}$$

```
[Out] (-3*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + (9*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(5/
2)*(a + a*Sec[c + d*x])^(3/2)) + (3*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)
*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.491832, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4264, 3816, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{}{2ad\cos}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (-3*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + (9*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(5/
2)*(a + a*Sec[c + d*x])^(3/2)) + (3*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)
*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(\ldots}{\ldots}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\ldots}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\ldots}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{3\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\ldots}$$

$$= -\frac{3\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\ldots}{\sqrt{2}\sqrt{a+a\sec\ldots}}\right.\ldots}{\ldots}$$

Mathematica [A] time = 0.959853, size = 242, normalized size = 1.13

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(6\cos(c+dx)\sqrt{(\cos(c+dx)-1)\sec^2(c+dx)}+4\sqrt{(\cos(c+dx)-1)\sec^2(c+dx)}-\ldots\right.}{4d\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(-9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec
[c + d*x]])/Sqrt[1 - Sec[c + d*x]]] - 9*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c
+ d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d
*x]]]*(1 + Cos[c + d*x]) + 18*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Cos[c + d*x])
 + 4*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] + 6*Cos[c + d*x]*Sqrt[(-1 + C
os[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*(a*
(1 + Sec[c + d*x]))^(3/2))
```

Maple [A] time = 0.197, size = 273, normalized size = 1.3

$$-\frac{(\cos(dx+c))^2-1}{4\,da^2(\sin(dx+c))^3}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(3\,\arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c)+1+\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(3*arctan(1/4*2^(1/2)*(-2/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)*sin(d*x+c
)-3*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
*2^(1/2)*cos(d*x+c)*sin(d*x+c)-9*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^
(1/2))*cos(d*x+c)*sin(d*x+c)+3*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-(-2/(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c
)+1))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [B] time = 4.92724, size = 6661, normalized size = 31.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sin(4
```

```
*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x
+ 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqr
t(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sq
rt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*s
in(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(sqrt(
2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(
2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqr
t(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*
x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt
(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*
c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(
2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(
2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
```

```
)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x +
 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
 + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c)
+ 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(
2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c
)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2
*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x +
4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*s
in(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x +
 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 9*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4
*c) + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*
```

x + 4*c) + 2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 9*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((sqrt(2)*a*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a*cos(2*d*x + 2*c) + 2*(2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(4*d*x + 4*c) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + 2*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*cos(4*d*x + 4*c) + 2*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos

```
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*a*sin(4*d*x
+ 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c) + 2*sqrt(2)*a*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*(sqrt(2)*a*sin(4*d*x + 4*c) + 2*sqrt(2)*a*sin(2*d*x + 2*c))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)*d)
```

Fricas [A] time = 1.99548, size = 1646, normalized size = 7.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(
3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x + c)^3 + 2
*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c)
)*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2
)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -
1/4*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*
arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*co
s(d*x + c) + 2)*sqrt(cos(d*x + c))*sin(d*x + c) + 6*(cos(d*x + c)^3 + 2*cos
(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*c
os(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d
*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.434 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{95\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{a\,s}}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] (163*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (
17*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - (
299*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(95*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.595581, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4264, 3817, 4020, 4022, 4013, 3808, 206}

$$\frac{95\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+}{\sqrt{2}\sqrt{a\,s}}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (163*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - (
17*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - (
299*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) +
(95*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4022

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{11a}{2}+3a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{8a}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{95\sqrt{\cos(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\,\mathrm{se}}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}$$

Mathematica [A] time = 1.1622, size = 144, normalized size = 0.61

$$-\frac{\sin(c+dx)\left(2\sqrt{1-\sec(c+dx)}\left(-32\cos(c+dx)+299\sec^2(c+dx)+503\sec(c+dx)+160\right)+1956\sqrt{2}\cos^4\left(\frac{1}{2}(c+d\right.\right.}{96d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -((1956*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]
*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(160 - 32
```

```
*Cos[c + d*x] + 503*Sec[c + d*x] + 299*Sec[c + d*x]^2))*Sin[c + d*x])/(96*d
*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.188, size = 244, normalized size = 1.

$$-\frac{(-1+\cos(dx+c))^2}{96\,da^3(\sin(dx+c))^5}\left(489\,(\cos(dx+c))^2\sin(dx+c)\arctan\left(1/2\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\sqrt{-2\,(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/96/d/a^3*(-1+cos(d*x+c))^2*(489*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+978*cos(d*x+c)*s
in(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+
1))^(1/2)+64*cos(d*x+c)^4+489*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/
2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-384*cos(d*x+c)^3-686*cos(d*x+c)^2+
408*cos(d*x+c)+598)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/si
n(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.83439, size = 1215, normalized size = 5.13

$$\left[\frac{489\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\,c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{192\left(a^3d\cos(dx+c)^3+3a^3d\cos(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^3 - 160*cos(d*
x + c)^2 - 503*cos(d*x + c) - 299)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(489*sqrt(2)*(cos(d*x + c)^3 +
3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) -
 2*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 - 503*cos(d*x + c) - 299)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2), x)
```

3.435 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=197

$$\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin\cdots}{16ad\sqrt{\cos(c+dx)}\cdots}$$

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - Sin[c + d*x]/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - (13
*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (49
*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.458366, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3817, 4020, 4013, 3808, 206}

$$\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin\cdots}{16ad\sqrt{\cos(c+dx)}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-75*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*
d) - Sin[c + d*x]/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - (13
*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + (49
*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
```

```
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4013

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{9a}{2}+2a\sec(c+d}{\sqrt{\sec(c+dx)}(a+a\sec(c+}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} - \frac{\left(\sqrt{\cos\right.}{}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{}{16a^2d}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{3/2}} + \frac{}{16a^2d}$$

$$= -\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\sec(}$$

Mathematica [A] time = 0.930474, size = 141, normalized size = 0.72

$$\frac{\sqrt{1-\sec(c+dx)}(32\sin(c+dx)+\tan(c+dx)(49\sec(c+dx)+85))+150\sqrt{2}\sin(c+dx)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\,\mathrm{t}}{16d\sqrt{\cos(c+dx)-1}(a(\sec(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (150*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(
32*Sin[c + d*x] + (85 + 49*Sec[c + d*x])*Tan[c + d*x]))/(16*d*Sqrt[-1 + Cos
[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.18, size = 234, normalized size = 1.2

$$\frac{(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\sqrt{\cos(dx+c)}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\left(75\,(\cos(dx+c))^2\sin(dx+c)\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\mathrm{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/32/d/a^3*cos(d*x+c)^(1/2)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x
+c))^2*(75*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1)
)^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+150*cos(d*x+c)*sin(d*x+c)*arctan(1/2*sin
(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)+75*arctan(1/2*
sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*(-2/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
64*cos(d*x+c)^3-106*cos(d*x+c)^2+72*cos(d*x+c)+98)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.843, size = 1148, normalized size = 5.83

$$\left[\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.\cdots}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(32*cos(d*x + c)^2 + 85*cos(d*x +
 c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d), 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*
```

```
x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*cos(d*x + c)^2 + 85*
cos(d*x + c) + 49)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(5/2), x)
```

3.436 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{19\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\text{si}}{4d\cos^{\frac{3}{2}}(c+dx}$$

```
[Out] (19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - (9*S
in[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.315036, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3817, 4012, 3808, 206}

$$\frac{19\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\text{si}}{4d\cos^{\frac{3}{2}}(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*S
ec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - (9*S
in[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
```

```
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4012

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x
] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m,
-1]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\text{se}}}{}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+d}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{9\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+d}$$

$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{s}{4d\cos^{\frac{3}{2}}(c+}$$

Mathematica [A] time = 1.21376, size = 168, normalized size = 1.07

$$-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\left(9\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+13\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}+38\sqrt{2}\,\text{c}\right.}{16d\sqrt{1-\sec(c+dx)}(a(\sec(c+dx)+1))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(9*Sqrt[1 - Sec[c + d*x]]*Sec[c + d
*x]^(3/2) + 38*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
 d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2 + 13*Sqrt[-((-1 + Sec[c + d*x])*S
ec[c + d*x])])*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d
*x]))^(5/2))
```

Maple [A] time = 0.182, size = 200, normalized size = 1.3

$$\frac{(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\sqrt{\frac{a(\cos(dx+c)+1)}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\left(13\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c))^2+19\,\arctan\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/16/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x
+c))^2*(13*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+19*arctan(1/2*sin(d*x+c)*
(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-4*(-2/(cos(d*x+c)+1))^(1/2
)*cos(d*x+c)+19*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)
-9*(-2/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 4.27831, size = 4116, normalized size = 26.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*(19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3
*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 +
 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(
1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1
/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x +
 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*
```

c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d
*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
+ 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4
c) + 104(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7/
2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10*
sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*si
n(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
+ 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*s
in(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(1
/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*
c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) +
 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
- 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*co
s(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2
*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + 5*cos(5/2*
d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1/2*d*x + 1/2*c))*sin(3*d*x
+ 3*c) - 20*(6*cos(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c)
+ 24*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*

```
sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) - 5*cos(3/2*d*x + 3/2*c) - 13*cos(1
/2*d*x + 1/2*c))*sin(2*d*x + 2*c) + 20*(4*cos(d*x + c) + 1)*sin(3/2*d*x + 3
/2*c) - 80*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 208*cos(1/2*d*x + 1/2*c)*sin
(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c
)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 52*sin(1/2*d*x + 1/2*c))/((sqrt(2)*a^2*
cos(4*d*x + 4*c)^2 + 16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(d*x + c)^2 + sqrt(2)*a^2*sin(4*d*x + 4
*c)^2 + 16*sqrt(2)*a^2*sin(3*d*x + 3*c)^2 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)
^2 + 48*sqrt(2)*a^2*sin(2*d*x + 2*c)*sin(d*x + c) + 16*sqrt(2)*a^2*sin(d*x
+ c)^2 + 8*sqrt(2)*a^2*cos(d*x + c) + sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*
d*x + 3*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(d*x + c) +
sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(
2)*a^2*cos(d*x + c) + sqrt(2)*a^2)*cos(3*d*x + 3*c) + 12*(4*sqrt(2)*a^2*cos
(d*x + c) + sqrt(2)*a^2)*cos(2*d*x + 2*c) + 4*(2*sqrt(2)*a^2*sin(3*d*x + 3*
c) + 3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*sin(4*d*x
 + 4*c) + 16*(3*sqrt(2)*a^2*sin(2*d*x + 2*c) + 2*sqrt(2)*a^2*sin(d*x + c))*
sin(3*d*x + 3*c))*sqrt(a)*d)
```

Fricas [A] time = 1.80102, size = 1092, normalized size = 6.96

$$\left[\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(13*cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(1
9*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) + 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(13*
cos(d*x + c) + 9)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
```

```
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.437 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c}{4d\cos^{\frac{5}{2}}(c+dx)(a}$$

```
[Out] (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 - Sin[c + d*x]/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + (5*Si
n[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.259746, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4264, 3811, 3810, 3808, 206}

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{\sin(c}{4d\cos^{\frac{5}{2}}(c+dx)(a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 - Sin[c + d*x]/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + (5*Si
n[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3811

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
```

```
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[m/(a*(2*m + 1)), Int[(a + b*Csc[e + f*
x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LtQ[m, -2^(-1)]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\mathrm{s}}{(a+a}}{8a}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin}{4d\cos^{\frac{5}{2}}(c+dx)(}$$

Mathematica [A] time = 3.59808, size = 224, normalized size = 1.43

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-8\sin(c+dx)\sec^{\frac{5}{2}}(c+dx) - \frac{5(\sec(c+dx)+1)\left(2\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)-\tan(c+dx)(\sec(c+dx)+1)\right(}{}\right.}{32d(a(\sec(c+dx)+1))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-8*Sec[c + d*x]^(5/2)*Sin[c + d*x]
- (5*(1 + Sec[c + d*x])*(2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c
+ d*x] - (1 + Sec[c + d*x])*(2*ArcSin[Sqrt[1 - Sec[c + d*x]]] + 2*ArcSin[Sq
rt[Sec[c + d*x]]] - Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]] + 2*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])*Tan[c + d*x]))/
Sqrt[1 - Sec[c + d*x]]))/(32*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A] time = 0.173, size = 198, normalized size = 1.3

$$\frac{(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\left(5\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right)\cos(dx+c)\sin(dx+c) - 5\sqrt{-2\,(\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/16/d/a^3*(-1+cos(d*x+c))^2*(5*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)*sin(d*x+c)-5*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+5*arct
an(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+4*(-2/(cos(d*x+c)+1
))^(1/2)*cos(d*x+c)+(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x+c)
)^(1/2)*cos(d*x+c)^(1/2)/(-2/(cos(d*x+c)+1))^(1/2)/sin(d*x+c)^5
```

Maxima [B] time = 3.68109, size = 3881, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*sin
(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x +
 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*c
) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x +
 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*c
os(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*d
*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 1
2*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*
(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin
```

(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c) + 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(sin(3/2

```
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) - 16*(3*cos(3/2*d*x + 3/2*c) - 5*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 20*(4*cos(3*d*x + 3*c) + 1)*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*sin(3/2*d*x + 3/2*c))/(
(16*sqrt(2)*a^2*cos(3*d*x + 3*c)^2 + sqrt(2)*a^2*cos(8/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 36*sqrt(2)*a^2*cos(4/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*cos(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 16*sqrt(2)*a^2*sin(3*d*x +
 3*c)^2 + sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 36*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 32*sqrt(2)*a^2*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8*sqrt(2)*a^2*cos(3*d*x + 3*c) +
sqrt(2)*a^2 + 2*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + 6*sqrt(2)*a^2*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a^2*cos(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 12*(4*sqrt(2)*a^2*co
s(3*d*x + 3*c) + 4*sqrt(2)*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + sqrt(2)*a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 8*(4*sqrt(2)*a^2*cos(3*d*x + 3*c) + sqrt(2)*a^2)*cos(2/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*(2*sqrt(2)*a^2*s
in(3*d*x + 3*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 2*sqrt(2)*a^2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 48*(sqrt(2)*a^2*sin(3*d*x + 3*c) + sqrt(2)*a^2*sin(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*d)
```

Fricas [A] time = 1.7637, size = 1087, normalized size = 6.92

$$\left[\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos\cdots}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(d\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*(5*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(5*s
qrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(5*cos(
d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.438 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{\sin}{4d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 + Sin[c + d*x]/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + (3*Si
n[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.259496, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3810, 3808, 206}

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{\sin}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Se
c[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 + Sin[c + d*x]/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + (3*Si
n[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3810

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
```

```
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[(d*(m + 1))/(b*(2*m + 1)),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\text{sec}}{(a+a\,\text{s}}}{8a}$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3}}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \frac{\sin}{4d\cos^{\frac{5}{2}}(c+dx)(}$$

Mathematica [B] time = 1.36278, size = 341, normalized size = 2.17

$$14\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) + 6\sin(c+dx)\sqrt{-(\sec(c+dx)-1)\sec(c+dx)} - 3\sqrt{2}\tan(c+dx)\sec(c+d$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sin
[c + d*x] + 14*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 6*S
qrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - 6*Sqrt[2]*ArcTan[(S
qrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 3*Sqrt[2]
*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*T
an[c + d*x] + 6*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Sec[c +
 d*x])*Tan[c + d*x]) + 6*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Se
c[c + d*x])*Tan[c + d*x]))/(32*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^
2*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A] time = 0.176, size = 200, normalized size = 1.3

$$-\frac{(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\left(3\sqrt{-2\,(\cos(dx+c)+1)^{-1}}(\cos(dx+c))^2-3\,\arctan\left(1/2\,\sin(dx+c)\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d/a^3*(-1+cos(d*x+c))^2*(3*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-3*a
rctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+4*(-2
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1
))^(1/2))*sin(d*x+c)-7*(-2/(cos(d*x+c)+1))^(1/2))*(a*(cos(d*x+c)+1)/cos(d*x
+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.83264, size = 1087, normalized size = 6.92

$$\left[\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*(3*cos(d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(3*s
qrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*ar
ctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))/(a*sin(d*x + c))) - 2*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(3*cos(
d*x + c) + 7)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.439 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{}{16a}$$

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - (43*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(5
/2)*(a + a*Sec[c + d*x])^(5/2)) - (11*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3
/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.502705, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4264, 3816, 4019, 4023, 3808, 206, 3801, 215}

$$-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - (43*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(5
/2)*(a + a*Sec[c + d*x])^(5/2)) - (11*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3
/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(}{}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= \frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a+a\sec}}\right.}{}$$

Mathematica [A] time = 1.22271, size = 328, normalized size = 1.53

$$\frac{-30\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx) - 22\sin(c+dx)\sqrt{-(\sec(c+dx)-1)\sec(c+dx)} + 43\sqrt{2}\tan(c+dx)\sec(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (43*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sin
[c + d*x] - 30*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x] - 22*
Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] + 86*Sqrt[2]*ArcTan[
(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 43*Sqrt
[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x
]*Tan[c + d*x] - 22*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + (2 + Sec
[c + d*x])*Tan[c + d*x]) - 86*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c + d*x] + (2
 + Sec[c + d*x])*Tan[c + d*x]))/(32*d*Cos[c + d*x]^(9/2)*Sqrt[1 - Sec[c + d
*x]]*Sec[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.199, size = 396, normalized size = 1.9

$$-\frac{(-1+\cos(dx+c))^2}{16\,da^3(\sin(dx+c))^5}\left(16\ \arctan\left(1/4\,\sqrt{2}\sqrt{-2\,(\cos(dx+c)+1)^{-1}}\,(\cos(dx+c)+1-\sin(dx+c))\right)\sqrt{2}\cos(dx+c)\mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d/a^3*(-1+cos(d*x+c))^2*(16*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))*2^(1/2)*cos(d*x+c)*sin(d*x+c)-16*arctan(1/4*
2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*
x+c)*sin(d*x+c)+16*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)-16*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)-11*(-2/(cos(d*x+c)+1)
)^(1/2)*cos(d*x+c)^2+43*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)*sin(d*x+c)-4*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+43*arctan(1/2*si
n(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+15*(-2/(cos(d*x+c)+1))^(1/2)
)*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)^5/(-2/(co
s(d*x+c)+1))^(1/2)
```

Maxima [B] time = 4.76528, size = 6734, normalized size = 31.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(19*
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2
)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*
d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*s
qrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4
*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x +
 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x +
4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 +
 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqr
t(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4
*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*s
in(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2
)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(
```

1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 43*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4

```
*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x +
 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) +
 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 12*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x + 4*c) +
6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*sin(2*d*x
 + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*c) + 1)*l
og(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) + 43*(2*(6*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 36*cos(2*d*x + 2*c)^2 + 8*(cos(4*d*x + 4*c) + 6*co
s(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(4*d*x + 4*c) + 6*cos(2*d*x
 + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(4*d*x + 4*c)^2 + 1
2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sin(2*d*x + 2*c)^2 + 8*(sin(4*d*x
+ 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(4*d*x + 4*c) + 6*
sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*cos(2*d*x + 2*
c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x +
2*c) + 4*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*(19*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - 19*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
1*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 76*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*
c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(5/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 76*(cos(4*d*x + 4*c) + 6*cos(2
*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*s
in(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 176*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 44*(cos(4*d*x + 4*c) + 6*cos(2*d*x + 2*c) + 1)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 176*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))/((sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 +
```

```
16*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*
sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2
)*a^2*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 36*sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 12*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*
a^2 + 2*(6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) + 8
*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)
*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(4*d
*x + 4*c) + 6*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*s
qrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 8*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 6*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*d)
```

Fricas [A] time = 1.97716, size = 1717, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(d*x + c) + 32*(cos(d*x
 + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c
)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*
sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3
 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d), 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(11*cos(d*x + c) + 15)*sqrt(cos(d*x + c))*sin(
d*x + c) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt
(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

3.440 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{35\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{115\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\sqrt{\cos(c+dx)}}{}$$

```
[Out] (-5*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + (115*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*S
qrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^
(7/2)*(a + a*Sec[c + d*x])^(5/2)) - (15*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^
(5/2)*(a + a*Sec[c + d*x])^(3/2)) + (35*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.645607, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4264, 3816, 4019, 4021, 4023, 3808, 206, 3801, 215}

$$\frac{35\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{115\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{5\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (-5*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + (115*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*S
qrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^
(7/2)*(a + a*Sec[c + d*x])^(5/2)) - (15*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^
(5/2)*(a + a*Sec[c + d*x])^(3/2)) + (35*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x
]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4021

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*d*C
ot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(f*(m + n)), x
] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)
*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &&
GtQ[n, 1]
```

Rule 4023

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3808

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*b*d)/(a*f), Subst[Int[1/(2*b - d*x^2), x], x
, (b*Cot[e + f*x])/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]])], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3801

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(}{}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{15\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}$$

$$= -\frac{5\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+}}{\sqrt{2}\sqrt{a+a}}\right.}{}$$

Mathematica [A] time = 1.50192, size = 348, normalized size = 1.37

$$\frac{\sqrt{\sec(c+dx)}\left(32\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)+110\sin(c+dx)\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)+70\sin(c+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[
1 - Sec[c + d*x]]]*Sin[c + d*x] + 110*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(
3/2)*Sin[c + d*x] + 32*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*
x] + 70*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - 230*Sqrt[2
```

```
]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x]
- 115*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*S
ec[c + d*x]*Tan[c + d*x] + 70*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x]
+ (2 + Sec[c + d*x])*Tan[c + d*x]) + 230*ArcSin[Sqrt[Sec[c + d*x]]]*(Sin[c
+ d*x] + (2 + Sec[c + d*x])*Tan[c + d*x])))/(32*d*Sqrt[-1 + Cos[c + d*x]]*(
a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] time = 0.212, size = 444, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/16/d/a^3*(a*(cos(d*x+c)+1)/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(40*2^(1/
2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*
cos(d*x+c)^2*sin(d*x+c)-40*2^(1/2)*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)+40*arctan(1/4*2^(1/
2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*2^(1/2)*cos(d*x+c)*
sin(d*x+c)-40*arctan(1/4*2^(1/2)*(-2/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))*2^(1/2)*cos(d*x+c)*sin(d*x+c)+35*(-2/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)^3-115*arctan(1/2*sin(d*x+c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*s
in(d*x+c)+20*(-2/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-115*arctan(1/2*sin(d*x+
c)*(-2/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-39*(-2/(cos(d*x+c)+1))^
(1/2)*cos(d*x+c)-16*(-2/(cos(d*x+c)+1))^(1/2))/cos(d*x+c)^(1/2)/sin(d*x+c)^
5/(-2/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 23.5264, size = 12215, normalized size = 48.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/32*(140*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), co
```

s(2*d*x + 2*c))))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
6*(75*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 24*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*sin(5/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 75*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(sin(6*d*x + 6*c) + 7*si
n(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 96*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*(24*sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 75*sin(3/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 35*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 96*(
sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 300*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*
d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 140*(sin(6*d*x + 6*c) + 7*
sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x +
4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c
)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x +
 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*s
qrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c)
 + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*
cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c)
+ 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c)
+ 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt
(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqr
t(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*s
qrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d

*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(
sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x
 + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*
cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt(2)*cos
(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*
c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*
d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4
*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*s
qrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2
)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*sin(6*d*
x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt
(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(
2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) + 40*(sqrt(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4
*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*

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c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*
x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(
2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 64*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7
*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x
 + 6*c) + 14*(7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(s
qrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x
+ 2*c) + 8*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*co
s(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x
+ 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) +
 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c
) + sqrt(2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c
) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 16*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4
*c) + 7*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*si
n(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 40*(sqrt
(2)*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*cos(2*d
*x + 2*c)^2 + 16*sqrt(2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 64*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)
*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*sin(4*d*x + 4*c)^2 + 98*sqrt(2)*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(5/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 64*sqrt(2)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(7*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt
(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(6*d*x + 6*c) + 14*(7*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2
)*cos(4*d*x + 4*c) + 7*sqrt(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 16*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c
) + 7*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c)
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, cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 8*(sqrt(2)*cos(6*d*x + 6*c) + 7*sqrt(2)*cos(4*d*x + 4*c) + 7*sq
rt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 14*(sqrt(2)*sin(4*d*x + 4*c) + sqrt(2)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 8*(sqrt(2)*sin(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7
*sqrt(2)*sin(2*d*x + 2*c) + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*s
in(6*d*x + 6*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c) +
 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(6*d*x + 6*c) +
7*sqrt(2)*sin(4*d*x + 4*c) + 7*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) - 115*(2*(7*cos(4*d*x + 4*c) + 7*cos(2*d*x +
 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 14*(7*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 49*cos(4*d*x + 4*c)^2 + 49*cos(2*d*x + 2*c)^2 + 8*(c
os(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 16*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(cos(
6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 64*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*(sin(4*d*x + 4*c) + sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 49*sin(4*d*x + 4*c)^2 + 98*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sin(2*d*x + 2*c)^2 + 8*(sin(6*d*x + 6*c
) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 8*sin(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*(sin(6*d*x + 6*c) + 7*s
in(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
64*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(sin(6*d*x +
6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 14*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 115*(2*
(7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x
+ 6*c)^2 + 14*(7*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 49*cos(4*d*x + 4*
```

c)^2 + 49*cos(2*d*x + 2*c)^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7
*cos(2*d*x + 2*c) + 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(5/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*cos(5/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 16*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*co
s(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*cos(3/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(cos(6*d*x + 6*c) + 7*cos(4*d*x
 + 4*c) + 7*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 16*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 1
4*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)
^2 + 49*sin(4*d*x + 4*c)^2 + 98*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sin(
2*d*x + 2*c)^2 + 8*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2
*c) + 8*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 16*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 16*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x + 2*c) + 4
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 8*(sin(6*d*x + 6*c) + 7*sin(4*d*x + 4*c) + 7*sin(2*d*x
+ 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 14*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 140*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*
cos(2*d*x + 2*c) + 4*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(11/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 16*(75*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 24*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 24*cos
(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*cos(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 35*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 300
*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 8*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 96*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) +
8*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(7/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 32*(24*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 75*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 35*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 96*(cos(6*d*x + 6*c) + 7*cos(4*d*x + 4*c) + 7*
cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 300*(cos(6*d*x +

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6*c) + 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 4*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 560*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 140*(cos(6*d*x + 6*c) +
 7*cos(4*d*x + 4*c) + 7*cos(2*d*x + 2*c) + 1)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 560*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((sqrt(2)*a^2
*cos(6*d*x + 6*c)^2 + 49*sqrt(2)*a^2*cos(4*d*x + 4*c)^2 + 49*sqrt(2)*a^2*co
s(2*d*x + 2*c)^2 + 16*sqrt(2)*a^2*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 64*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 16*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sqrt(2)*a^2*sin(6*d*x + 6*c)^2 + 49*sqrt(2)*a^2*sin(4*d*x + 4*c)^
2 + 98*sqrt(2)*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 49*sqrt(2)*a^2*sin(2
*d*x + 2*c)^2 + 16*sqrt(2)*a^2*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 64*sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 16*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 14*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2 + 2*(7*sqrt(2)*a^2*cos
(4*d*x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(6*d*x + 6
*c) + 14*(7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(4*d*x + 4*c) +
8*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7*sqrt(2
)*a^2*cos(2*d*x + 2*c) + 8*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sqrt(2)*a^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 16*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*x + 4*c) + 7
*sqrt(2)*a^2*cos(2*d*x + 2*c) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*cos(6*d*x + 6*c) + 7*sqrt(2)*a^2*cos(4*d*
x + 4*c) + 7*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 14*(sqrt(2)*a^2*sin(4*d*x + 4*c) + sqr
t(2)*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*(sqrt(2)*a^2*sin(6*d*x + 6*
c) + 7*sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c) + 8*sq
rt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*
a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*
sqrt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c) + 4*sqrt(2)*a
^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 7*sq
rt(2)*a^2*sin(4*d*x + 4*c) + 7*sqrt(2)*a^2*sin(2*d*x + 2*c))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*d)
```

Fricas [A] time = 2.05072, size = 1886, normalized size = 7.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x
 + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(35*cos(d*x + c)^2
+ 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x +
c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x +
 c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^3*d*
cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*co
s(d*x + c)), -1/32*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(35*cos(d
*x + c)^2 + 55*cos(d*x + c) + 16)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*
cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*
a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2)), x)
```

3.441 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$

Optimal. Leaf size=244

$$-\frac{a^3(7-4n)\sin(e+fx)(d\cos(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a^3(1-4n)\sin(e+fx)\cos(e+}{}$$

```
[Out] -((a^3*(7 - 4*n)*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 - n)*n*Sqrt[Sin[e + f*x]^2])) - (a^3*(1
 - 4*n)*Cos[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (
3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x
]^2]) + (a^3*(5 - 2*n)*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n)*(2 - n))
 + ((d*Cos[e + f*x])^n*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(f*(2 - n))
```

Rubi [A] time = 0.399734, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3814, 3997, 3787, 3772, 2643}

$$-\frac{a^3(7-4n)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a^3(1-4n)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n}{f(1-n)(n+1)\sqrt{\sin^2(e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```


```
[Out] -((a^3*(7 - 4*n)*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 - n)*n*Sqrt[Sin[e + f*x]^2])) - (a^3*(1
 - 4*n)*Cos[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (
3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x
]^2]) + (a^3*(5 - 2*n)*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n)*(2 - n))
 + ((d*Cos[e + f*x])^n*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(f*(2 - n))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3814

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[b/(m + n - 1), Int[(a + b*
Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n -
4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2,
0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}(a+a\sec(e+fx))^3\,dx$$
$$= \frac{(d\cos(e+fx))^n\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{f(2-n)} + \frac{\left(a(d\cos(e+fx))^n(d\sec\right.}{}$$
$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\left(a^3+a^3\sec(e+f\right.}{f(2-n)}$$
$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\left(a^3+a^3\sec(e+f\right.}{f(2-n)}$$
$$= \frac{a^3(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{(d\cos(e+fx))^n\left(a^3+a^3\sec(e+f\right.}{f(2-n)}$$
$$= -\frac{a^3(7-4n)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{f(2-n)n\sqrt{\sin^2(e+fx)}} - \frac{a^3(1}{}$$

Mathematica [F] time = 2.51072, size = 0, normalized size = 0.

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3,x]
```

```
[Out] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^3, x]
```

Maple [F] time = 2.663, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^n\left(a+a\sec\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3\sec(fx+e)^3+3\,a^3\sec(fx+e)^2+3\,a^3\sec(fx+e)+a^3\right)(d\cos(fx+e))^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) +
a^3)*(d*cos(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(fx+e)+a)^3(d\cos(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

3.442 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$

Optimal. Leaf size=179

$$-\frac{2a^2\sin(e+fx)(d\cos(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^2(1-2n)\sin(e+fx)\cos(e+fx)(d}{f}$$

```
[Out] (-2*a^2*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f
*x]^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 - 2*n)*Cos[e + f*
x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e +
f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (a^2*(d*Co
s[e + f*x])^n*Tan[e + f*x])/(f*(1 - n))
```

Rubi [A] time = 0.226341, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3788, 3772, 2643, 4046}

$$-\frac{2a^2\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a^2(1-2n)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1}{f(1-n)(n+1)\sqrt{\sin^2(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-2*a^2*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f
*x]^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) - (a^2*(1 - 2*n)*Cos[e + f*
x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e +
f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (a^2*(d*Co
s[e + f*x])^n*Tan[e + f*x])/(f*(1 - n))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}(a+a\sec(e+fx))^2\,dx$$

$$= \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}\left(a^2+a^2\sec^2(e+fx)\right)$$

$$= \frac{a^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)} + \frac{\left(2a^2\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+fx))^n\right)\int\left(\frac{\cos(}{}\right.}{d}$$

$$= -\frac{2a^2(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} + \frac{a^2(d\cos(e+}{f}$$

$$= -\frac{2a^2(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{a^2(1-2n)\,\mathrm{c}}{}$$

Mathematica [F] time = 0.973964, size = 0, normalized size = 0.

$$\int (d\cos(e+fx))^n(a+a\sec(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x])^2, x]
```

Maple [F] time = 1.253, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\left(a+a\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{2}\left(d\cos\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}\right)\left(d\cos\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)*(d*cos(f*x + e))^n
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)^{2}\left(d\cos\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

3.443 $\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx$

<u>**Optimal**</u>. Leaf size=132

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\text{Hyper}}{df(n+1)}$$

```
[Out] -((a*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])
/(d*f*(1 + n)*Sqrt[Sin[e + f*x]^2])
```

<u>**Rubi**</u> **[A]** time = 0.11783, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4225, 16, 2748, 2643}

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+\right.}{df(n+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] -((a*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])
/(d*f*(1 + n)*Sqrt[Sin[e + f*x]^2])
```

<u>Rule 4225</u>

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

<u>Rule 16</u>

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (d\cos(e+fx))^n(a+a\sec(e+fx))\,dx &= \int (d\cos(e+fx))^n(a+a\cos(e+fx))\sec(e+fx)\,dx\\ &= d\int (d\cos(e+fx))^{-1+n}(a+a\cos(e+fx))\,dx\\ &= a\int (d\cos(e+fx))^n\,dx+(ad)\int (d\cos(e+fx))^{-1+n}\,dx\\ &= -\frac{a(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a(d\cos(e+fx)}{}\end{aligned}$$

Mathematica [A] time = 0.152142, size = 105, normalized size = 0.8

$$-\frac{a\sqrt{\sin^2(e+fx)}(d\cos(e+fx))^n\left(n\cot(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)+(n+1)\csc(e+fx)\text{H}\right.}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + a*Sec[e + f*x]),x]
```

```
[Out] -((a*(d*Cos[e + f*x])^n*((1 + n)*Csc[e + f*x]*Hypergeometric2F1[1/2, n/2, (
2 + n)/2, Cos[e + f*x]^2] + n*Cot[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2
, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

Maple [F] time = 0.628, size = 0, normalized size = 0.

$$\int \left(d \cos \left(f x +e \right)\right)^{n} \left(a +a \sec \left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(f x + e\right) + a\right) \left(d \cos \left(f x + e\right)\right)^{n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(a \sec\left(f x + e\right) + a\right)} \left(d \cos\left(f x + e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \left(\int \left(d \cos{\left(e + f x \right)}\right)^{n}\, dx + \int \left(d \cos{\left(e + f x \right)}\right)^{n} \sec{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+a*sec(f*x+e)),x)
```

```
[Out] a*(Integral((d*cos(e + f*x))**n, x) + Integral((d*cos(e + f*x))**n*sec(e +
f*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

3.444 $\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=178

$$-\frac{\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e+fx)}}+\frac{(n+1)\sin(e+fx)\cos^2(e$$

```
[Out] ((d*Cos[e + f*x])^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e + f*x]*
(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x
]^2]*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 + n)*Cos[e + f*x]^2*(d*
Cos[e + f*x])^n*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[e + f*x]^2
]*Sin[e + f*x])/(a*f*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.244499, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3820, 3787, 3772, 2643}

$$-\frac{\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e+fx)}}+\frac{(n+1)\sin(e+fx)\cos^2(e+fx)(d\cos(e+f}{af(n+2)\sqrt{\sin^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((d*Cos[e + f*x])^n*Sin[e + f*x])/(f*(a + a*Sec[e + f*x])) - (Cos[e + f*x]*
(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x
]^2]*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) + ((1 + n)*Cos[e + f*x]^2*(d*
Cos[e + f*x])^n*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[e + f*x]^2
]*Sin[e + f*x])/(a*f*(2 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}}{a+a\sec(e+fx)}\,dx$$
$$= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\left(d(1+n)(d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int(d\sec(e+fx))^{-1-n}}{a^2}$$
$$= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1+n)(d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int(d\sec(e+fx))^{-n}\,dx}{a}$$
$$= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left((1+n)\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+fx))^n\right)\int\left(\frac{\cos(e+fx)}{d}\right)^n dx}{a} - \frac{\Big(d(1}{}$$
$$= \frac{(d\cos(e+fx))^n\sin(e+fx)}{f(a+a\sec(e+fx))} - \frac{\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin(e}{af\sqrt{\sin^2(e+fx)}}$$

Mathematica [F] time = 0.969104, size = 0, normalized size = 0.

$$\int \frac{(d\cos(e+fx))^n}{a+a\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]),x]
```

```
[Out] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x]), x]
```

Maple [F] time = 0.848, size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{a+a\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^{n}}{a\sec\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(d\cos\left(e+fx\right)\right)^n}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral((d*cos(e + f*x))**n/(sec(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^n}{a\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a), x)
```

3.445 $\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=215

$$\frac{2(n+2)\sin(e+fx)(d\cos(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(2n+3)\sin(e+fx)\cos(e+fx)(}{}$$

```
[Out] (2*(2 + n)*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e
+ f*x]^2]*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - ((3 + 2*n)*Cos[e +
 f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (2*(2 + n)*(d*Cos
[e + f*x])^n*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - ((d*Cos[e + f*x])
^n*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rubi [A] time = 0.411779, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3817, 4020, 3787, 3772, 2643}

$$\frac{2(n+2)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{(2n+3)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F}{3a^2f\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```


```
[Out] (2*(2 + n)*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e
+ f*x]^2]*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - ((3 + 2*n)*Cos[e +
 f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e
 + f*x]^2]*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (2*(2 + n)*(d*Cos
[e + f*x])^n*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - ((d*Cos[e + f*x])
^n*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}}{(a+a\sec(e+fx))^2}\,dx$$

$$= -\frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}(a(-3-n)-a}{a+a\sec(e+}}{3a^2}$$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left((d\cos(e+fx))^n\right.}{}$$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left(2n(2+n)(d\cos(\right.}{}$$

$$= -\frac{2(2+n)(d\cos(e+fx))^n\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{(d\cos(e+fx))^n\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\left(2n(2+n)\left(\frac{\cos(e+}{d}\right.\right.}{}$$

$$= \frac{2(2+n)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{(3+2n)\cos(e+fx)(a}{}$$

Mathematica [F] time = 1.67316, size = 0, normalized size = 0.

$$\int \frac{(d\cos(e+fx))^n}{(a+a\sec(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] Integrate[(d*Cos[e + f*x])^n/(a + a*Sec[e + f*x])^2, x]
```

Maple [F] time = 0.299, size = 0, normalized size = 0.

$$\int \frac{(d\cos(fx+e))^n}{(a+a\sec(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{\left(a\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^{n}}{a^{2}\sec\left(fx+e\right)^{2}+2\,a^{2}\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(d\cos\left(e+fx\right)\right)^{n}}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*cos(e + f*x))**n/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)/a**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d\cos(fx+e))^n}{(a\sec(fx+e)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+a*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(a*sec(f*x + e) + a)^2, x)
```

3.446 $\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*b*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rubi [A] time = 0.0665066, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 3767, 3768, 3770}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*b*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^4(c+dx)\,dx + b\int \sec^5(c+dx)\,dx$$
$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3b)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx\right.}{d}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a\,\text{ta}}{}$$
$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(}{}$$

Mathematica [A] time = 0.287118, size = 76, normalized size = 0.89

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{b\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3b\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*b*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.023, size = 92, normalized size = 1.1

$$\frac{2a\tan(dx+c)}{3d} + \frac{a\tan(dx+c)(\sec(dx+c))^2}{3d} + \frac{b(\sec(dx+c))^3\tan(dx+c)}{4d} + \frac{3b\sec(dx+c)\tan(dx+c)}{8d} + \frac{3b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/4*b*sec(d*x+c)^3*tan(d
*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+3/8/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.20689, size = 128, normalized size = 1.51

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a-3\,b\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.82608, size = 266, normalized size = 3.13

$$\frac{9\,b\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,b\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,a\cos\left(dx+c\right)^{3}+9\,b\cos\left(dx+c\right)^{2}\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(9*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*b*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*b*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + 6*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\sec^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Giac [B] time = 1.3283, size = 221, normalized size = 2.6

$$\frac{9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-40\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\cdots\right.}{\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*b*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(24*a*tan(1/2*d*x + 1/2*c)^7 - 15*b*tan(1/2*d*x + 1/2*c)^7 -
 40*a*tan(1/2*d*x + 1/2*c)^5 - 9*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*tan(1/2*d*
x + 1/2*c)^3 - 9*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c) - 15*
b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.447 $\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=63

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (b*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.052903, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 3768, 3770, 3767}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (b*Tan[c + d*x]^3)/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^3(c+dx)\,dx + b\int \sec^4(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int\left(1+x^2\right)\,dx, x, -\right.}{d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{b\tan^3(c}{3d}$$

Mathematica [A] time = 0.168503, size = 60, normalized size = 0.95

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d} + \frac{b\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.024, size = 72, normalized size = 1.1

$$\frac{a\sec(dx+c)\tan(dx+c)}{2d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{2b\tan(dx+c)}{3d} + \frac{b\tan(dx+c)(\sec(dx+c))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*b*tan(d
*x+c)/d+1/3/d*b*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.19734, size = 95, normalized size = 1.51

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)b-3\,a\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.75695, size = 236, normalized size = 3.75

$$\frac{3\,a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,b\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*b*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*b)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] time = 1.28724, size = 165, normalized size = 2.62

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a\,t}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^5 + 4*
b*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c) - 6*b*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.448 $\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rubi [A] time = 0.0492891, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3787, 3767, 8, 3768, 3770}

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))\,dx &= a\int \sec^2(c+dx)\,dx + b\int \sec^3(c+dx)\,dx\\ &= \frac{b\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}b\int \sec(c+dx)\,dx - \frac{a\,\text{Subst}(\int 1\,dx, x, -\tan(c+}{d}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{b\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0213023, size = 47, normalized size = 1.

$$\frac{a\tan(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Maple [A] time = 0.019, size = 51, normalized size = 1.1

$$\frac{a\tan(dx+c)}{d} + \frac{b\sec(dx+c)\tan(dx+c)}{2\,d} + \frac{b\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

```
[Out] a*tan(d*x+c)/d+1/2*b*sec(d*x+c)*tan(d*x+c)/d+1/2/d*b*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 1.05301, size = 78, normalized size = 1.66

$$-\frac{b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,a\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

Fricas [A] time = 1.93901, size = 198, normalized size = 4.21

$$\frac{b\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-b\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,a\cos\left(dx+c\right)+b\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*cos(d*x + c)^2*log(sin(d*x + c) + 1) - b*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*a*cos(d*x + c) + b)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.30635, size = 144, normalized size = 3.06

$$\frac{b \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - b \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2} - 1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/
d
```

3.449 $\int \sec(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Rubi [A] time = 0.0260089, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3787, 3770, 3767, 8}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)(a+b\sec(c+dx))\,dx &= a\int \sec(c+dx)\,dx + b\int \sec^2(c+dx)\,dx \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0122902, size = 24, normalized size = 1.

$$
\frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\tan(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Tan[c + d*x])/d
```

Maple [A] time = 0.018, size = 32, normalized size = 1.3

$$
\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b\tan(dx+c)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*a*ln(sec(d*x+c)+tan(d*x+c))+b*tan(d*x+c)/d
```

Maxima [A] time = 1.06224, size = 39, normalized size = 1.62

$$
\frac{a\log(\sec(dx+c)+\tan(dx+c)) + b\tan(dx+c)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + b*tan(d*x + c))/d
```

Fricas [B] time = 1.97673, size = 162, normalized size = 6.75

$$\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2b\sin(dx+c)}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [A] time = 3.75222, size = 37, normalized size = 1.54

$$\begin{cases} \frac{a\log(\tan(c+dx)+\sec(c+dx))+b\tan(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\sec(c))\sec(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*tan(c + d*x))/d, Ne(d, 0
)), (x*(a + b*sec(c))*sec(c), True))
```

Giac [B] time = 1.25528, size = 85, normalized size = 3.54

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.450 $\int (a+b\sec(c+dx))\,dx$

Optimal. Leaf size=16

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*x + (b*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0081961, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3770}

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sec[c + d*x],x]
```

```
[Out] a*x + (b*ArcTanh[Sin[c + d*x]])/d
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\,dx &= ax+b\int \sec(c+dx)\,dx\\ &= ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0015626, size = 16, normalized size = 1.

$$ax+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sec[c + d*x],x]
```

```
[Out] a*x + (b*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] time = 0.006, size = 24, normalized size = 1.5

$$ax + \frac{b \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sec(d*x+c),x)
```

```
[Out] a*x+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.20236, size = 31, normalized size = 1.94

$$ax + \frac{b \log(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + b*log(sec(d*x + c) + tan(d*x + c))/d
```

Fricas [B] time = 2.03651, size = 95, normalized size = 5.94

$$\frac{2\,adx + b\log(\sin(dx + c) + 1) - b\log(-\sin(dx + c) + 1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1))/d
```

Sympy [A] time = 1.67693, size = 41, normalized size = 2.56

$$ax + b\left(\begin{cases} \frac{\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{for } d \neq 0 \\ \frac{x\left(\tan\left(c\right)\sec\left(c\right)+\sec^{2}\left(c\right)\right)}{\tan\left(c\right)+\sec\left(c\right)} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x)
```

```
[Out] a*x + b*Piecewise((log(tan(c + d*x) + sec(c + d*x))/d, Ne(d, 0)), (x*(tan(c
)*sec(c) + sec(c)**2)/(tan(c) + sec(c)), True))
```

Giac [B] time = 1.27521, size = 66, normalized size = 4.12

$$ax + \frac{b\left(\log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx + c\right) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(dx+c)} + \sin\left(dx + c\right) - 2\right|\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sec(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + 1/4*b*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*
x + c) + sin(d*x + c) - 2)))/d
```

3.451 $\int \cos(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=15

$$\frac{a\sin(c+dx)}{d}+bx$$

```
[Out] b*x + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0231894, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3787, 2637, 8}

$$\frac{a\sin(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] b*x + (a*Sin[c + d*x])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))\,dx = a\int \cos(c+dx)\,dx + b\int 1\,dx$$
$$= bx + \frac{a\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0092314, size = 26, normalized size = 1.73

$$\frac{a\sin(c)\cos(dx)}{d} + \frac{a\cos(c)\sin(dx)}{d} + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] b*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.034, size = 21, normalized size = 1.4

$$\frac{\sin(dx+c)\,a + (dx+c)\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(sin(d*x+c)*a+(d*x+c)*b)
```

Maxima [A] time = 1.16739, size = 27, normalized size = 1.8

$$\frac{(dx+c)b + a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*b + a*sin(d*x + c))/d
```

Fricas [A] time = 1.7471, size = 38, normalized size = 2.53

$$\frac{bdx + a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b*d*x + a*sin(d*x + c))/d
```

Sympy [A] time = 2.44176, size = 15, normalized size = 1.

$$a\left(\begin{cases}\sin(c) & \text{for } d = 0\\ \frac{\sin(c+dx)}{d} & \text{otherwise}\end{cases}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] a*Piecewise((sin(c), Eq(d, 0)), (sin(c + d*x)/d, True)) + b*x
```

Giac [B] time = 1.23696, size = 53, normalized size = 3.53

$$\frac{(dx + c)b + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*b + 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.452 $\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin(c+dx)}{d}$$

```
[Out] (a*x)/2 + (b*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0377009, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 2635, 8, 2637}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*x)/2 + (b*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)(a+b\sec(c+dx))\,dx &= a\int \cos^2(c+dx)\,dx + b\int \cos(c+dx)\,dx \\
&= \frac{b\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx \\
&= \frac{ax}{2} + \frac{b\sin(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.0661786, size = 35, normalized size = 0.92

$$\frac{a(2(c+dx)+\sin(2(c+dx)))+4b\sin(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (4*b*Sin[c + d*x] + a*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.045, size = 38, normalized size = 1.

$$\frac{1}{d}\left(a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+\sin(dx+c)\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+sin(d*x+c)*b)
```

Maxima [A] time = 1.07731, size = 46, normalized size = 1.21

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a+4\,b\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*b*sin(d*x + c))/d
```

Fricas [A] time = 1.60059, size = 72, normalized size = 1.89

$$\frac{adx + (a\cos(dx + c) + 2\,b)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*b)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))\cos^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [B] time = 1.31119, size = 111, normalized size = 2.92

$$\frac{(dx + c)a - \frac{2\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 2\,b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 2\,b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^3
 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 + 1)^2)/d
```

3.453 $\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

```
[Out] (b*x)/2 + (a*Sin[c + d*x])/d + (b*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0458303, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 2633, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*x)/2 + (a*Sin[c + d*x])/d + (b*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sec(c+dx))\,dx &= a\int \cos^3(c+dx)\,dx+b\int \cos^2(c+dx)\,dx\\ &= \frac{b\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}b\int 1\,dx-\frac{a\,\mathrm{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx\right.}{d}\\ &= \frac{bx}{2}+\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0721349, size = 57, normalized size = 1.06

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b(c+dx)}{2d}+\frac{b\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.049, size = 49, normalized size = 0.9

$$\frac{1}{d}\left(\frac{a\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}+b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*(cos(d*x+c)^2+2)*sin(d*x+c)+b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.22448, size = 62, normalized size = 1.15

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)b}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*b)/d
```

Fricas [A] time = 1.71877, size = 105, normalized size = 1.94

$$\frac{3\,bdx+\left(2\,a\cos\left(dx+c\right)^{2}+3\,b\cos\left(dx+c\right)+4\,a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*b*d*x + (2*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 4*a)*sin(d*x + c))/
d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\cos^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*cos(c + d*x)**3, x)
```

Giac [B] time = 1.29038, size = 132, normalized size = 2.44

$$\frac{3\,(dx+c)b+\frac{2\left(6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*b + 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*
c)^5 + 4*a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.454 $\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=76

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] (3*a*x)/8 + (b*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (b*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.056168, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 2635, 8, 2633}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (b*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (b*Sin[c + d*x]^3)/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + b\int \cos^3(c+dx)\,dx$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{b\,\mathrm{Subst}\left(\int\left(1-x^2\right)dx\right.}{d}$$
$$= \frac{b\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{b\sin^3}{}$$
$$= \frac{3ax}{8} + \frac{b\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

Mathematica [A] time = 0.125481, size = 73, normalized size = 0.96

$$\frac{3a(c+dx)}{8d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d} - \frac{b\sin^3(c+dx)}{3d} + \frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a*
Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.048, size = 60, normalized size = 0.8

$$\frac{1}{d}\left(a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{3dx}{8} + \frac{3c}{8}\right) + \frac{b\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b*(
cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A] time = 1.03257, size = 77, normalized size = 1.01

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))a-32\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a - 32*(sin
(d*x + c)^3 - 3*sin(d*x + c))*b)/d
```

Fricas [A] time = 1.68452, size = 136, normalized size = 1.79

$$\frac{9\,adx+\left(6\,a\cos\left(dx+c\right)^{3}+8\,b\cos\left(dx+c\right)^{2}+9\,a\cos\left(dx+c\right)+16\,b\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 + 9*a*cos(d*x + c)
 + 16*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24692, size = 189, normalized size = 2.49

$$\frac{9\,(dx+c)a-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x + 1
/2*c)^7 - 9*a*tan(1/2*d*x + 1/2*c)^5 - 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*ta
n(1/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2
*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.455 $\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=92

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] (3*b*x)/8 + (a*Sin[c + d*x])/d + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c
 + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0582848, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3787, 2633, 2635, 8}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*x)/8 + (a*Sin[c + d*x])/d + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c
 + d*x]^5)/(5*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))\,dx = a\int \cos^5(c+dx)\,dx + b\int \cos^4(c+dx)\,dx$$
$$= \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3b)\int \cos^2(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int\left(1-2x^2+x\right.\right.}{a}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{2a\sin}{}$$
$$= \frac{3bx}{8} + \frac{a\sin(c+dx)}{d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+dx)}{4d}$$

Mathematica [A] time = 0.116753, size = 89, normalized size = 0.97

$$\frac{a\sin^5(c+dx)}{5d} - \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} + \frac{3b(c+dx)}{8d} + \frac{b\sin(2(c+dx))}{4d} + \frac{b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d) + (
a*Sin[c + d*x]^5)/(5*d) + (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A] time = 0.047, size = 70, normalized size = 0.8

$$\frac{1}{d}\left(\frac{\sin(dx+c)\,a}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)+b\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\,\cos(dx+c)}{2}\right)+\frac{3\,dx}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.28043, size = 93, normalized size = 1.01

$$\frac{32\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+c\right)^{3}+15\sin\left(dx+c\right)\right)a+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)b}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d
```

Fricas [A] time = 1.70366, size = 173, normalized size = 1.88

$$\frac{45\,bdx+\left(24\,a\cos\left(dx+c\right)^{4}+30\,b\cos\left(dx+c\right)^{3}+32\,a\cos\left(dx+c\right)^{2}+45\,b\cos\left(dx+c\right)+64\,a\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(45*b*d*x + (24*a*cos(d*x + c)^4 + 30*b*cos(d*x + c)^3 + 32*a*cos(d*x
 + c)^2 + 45*b*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20211, size = 208, normalized size = 2.26

$$\frac{45\,(dx+c)b+\frac{2\left(120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-30\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+464\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(45*(d*x + c)*b + 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x
+ 1/2*c)^9 + 160*a*tan(1/2*d*x + 1/2*c)^7 - 30*b*tan(1/2*d*x + 1/2*c)^7 + 4
64*a*tan(1/2*d*x + 1/2*c)^5 + 160*a*tan(1/2*d*x + 1/2*c)^3 + 30*b*tan(1/2*d
*x + 1/2*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.456 $\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{(5a^2+4b^2)\tan^3(c+dx)}{15d}+\frac{(5a^2+4b^2)\tan(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan(c+dx)\sec^3(c+dx)}{2d}+\frac{3ab\tan$$

```
[Out] (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) + ((5*a^2 + 4*b^2)*Tan[c + d*x])/(5*d)
+ (3*a*b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a*b*Sec[c + d*x]^3*Tan[c + d*x
])/(2*d) + (b^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((5*a^2 + 4*b^2)*Tan[c
 + d*x]^3)/(15*d)
```

Rubi [A] time = 0.106355, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 3768, 3770, 4046, 3767}

$$\frac{(5a^2+4b^2)\tan^3(c+dx)}{15d}+\frac{(5a^2+4b^2)\tan(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan(c+dx)\sec^3(c+dx)}{2d}+\frac{3ab\tan$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```



```
[Out] (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) + ((5*a^2 + 4*b^2)*Tan[c + d*x])/(5*d)
+ (3*a*b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a*b*Sec[c + d*x]^3*Tan[c + d*x
])/(2*d) + (b^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((5*a^2 + 4*b^2)*Tan[c
 + d*x]^3)/(15*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \sec^5(c+dx)\,dx+\int \sec^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx\\ &= \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{b^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{2}(3ab)\int \sec^3(c\\ &= \frac{3ab\sec(c+dx)\tan(c+dx)}{4d}+\frac{ab\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{b^2\sec^4(c+dx)\,\mathrm{ta}}{5d}\\ &= \frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{\left(5a^2+4b^2\right)\tan(c+dx)}{5d}+\frac{3ab\sec(c+dx)\tan(c+}{4d}\end{aligned}$$

Mathematica [A] time = 0.599066, size = 118, normalized size = 0.87

$$\frac{a^2\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{ab\tan(c+dx)\sec^3(c+dx)}{2d}+\frac{3ab\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) + (3*a*b*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x]))/(4*d) + (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/
```

```
d + (b^2*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A] time = 0.031, size = 157, normalized size = 1.2

$$\frac{2 a^2 \tan(dx+c)}{3d} + \frac{a^2 (\sec(dx+c))^2 \tan(dx+c)}{3d} + \frac{ab (\sec(dx+c))^3 \tan(dx+c)}{2d} + \frac{3 ab \sec(dx+c) \tan(dx+c)}{4d} + \frac{3 a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/3*a^2*tan(d*x+c)/d+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3
*tan(d*x+c)/d+3/4*a*b*sec(d*x+c)*tan(d*x+c)/d+3/4/d*a*b*ln(sec(d*x+c)+tan(d
*x+c))+8/15*b^2*tan(d*x+c)/d+1/5*b^2*sec(d*x+c)^4*tan(d*x+c)/d+4/15*b^2*sec
(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.20733, size = 178, normalized size = 1.32

$$\frac{40 \left(\tan(dx+c)^3 + 3 \tan(dx+c)\right) a^2 + 8 \left(3 \tan(dx+c)^5 + 10 \tan(dx+c)^3 + 15 \tan(dx+c)\right) b^2 - 15 ab \left(\frac{2 \left(3 \sin(dx+c)^3 - 5\right.}{\sin(dx+c)^4 - 2 \sin}\right.}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/120*(40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 8*(3*tan(d*x + c)^5 + 10*
tan(d*x + c)^3 + 15*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.82317, size = 352, normalized size = 2.61

$$\frac{45 ab \cos(dx+c)^5 \log(\sin(dx+c)+1) - 45 ab \cos(dx+c)^5 \log(-\sin(dx+c)+1) + 2 \left(45 ab \cos(dx+c)^3 + 8 \left(5 a^2 + \right.\right.}{120 d \cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/120*(45*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*b*cos(d*x + c)^5*
log(-sin(d*x + c) + 1) + 2*(45*a*b*cos(d*x + c)^3 + 8*(5*a^2 + 4*b^2)*cos(d
*x + c)^4 + 30*a*b*cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 12*b^2
)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^2 \sec^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**4, x)
```

Giac [B] time = 1.39754, size = 367, normalized size = 2.72

$$45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^9 - 75\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^9 + 60\,b^2\tan\left(\frac{1}{2}\,d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x +
1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 - 160*a^2*tan(1/2*d*x + 1/2*c)^7 +
 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 80*b^2*tan(1/2*d*x + 1/2*c)^7 + 200*a^2*ta
n(1/2*d*x + 1/2*c)^5 + 232*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 30*a*b*tan(1/2*d*x + 1/2*c)^3 - 80*b^2*tan(1/2*d*x + 1/2*c)^3
 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^2*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.457 $\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=110

$$\frac{(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2+3b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(}{}$$

```
[Out] ((4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*Tan[c + d*x])/d + ((
4*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (2*a*b*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0941703, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 3767, 4046, 3768, 3770}

$$\frac{(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2+3b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\tan^3(c+dx)}{3d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*Tan[c + d*x])/d + ((
4*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (2*a*b*Tan[c + d*x]^3)/(3*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^4(c+dx)\,dx + \int \sec^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(4a^2+3b^2\right)\int \sec^3(c+dx)\,dx - \frac{(2ab)\,\text{Subst}\,(\ldots}{\ldots}$$

$$= \frac{2ab\tan(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b^2\sec^3(c+dx)\tan(\ldots}{4d}$$

$$= \frac{\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2ab\tan(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\sec(c+dx)\ldots}{8d}$$

Mathematica [A] time = 0.277427, size = 82, normalized size = 0.75

$$\frac{3\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(3\left(4a^2+3b^2\right)\sec(c+dx) + 16ab\left(\tan^2(c+dx)+3\right) + 6b^2\sec^3(c+dx\ldots\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(4*a^2 + 3*b^2)*
Sec[c + d*x] + 6*b^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.028, size = 142, normalized size = 1.3

$$\frac{a^2 \sec(dx+c)\tan(dx+c)}{2d}+\frac{a^2 \ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{4ab\tan(dx+c)}{3d}+\frac{2ab\tan(dx+c)(\sec(dx+c))^2}{3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+4/3*a*b
*tan(d*x+c)/d+2/3/d*a*b*tan(d*x+c)*sec(d*x+c)^2+1/4*b^2*sec(d*x+c)^3*tan(d*
x+c)/d+3/8*b^2*sec(d*x+c)*tan(d*x+c)/d+3/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.10035, size = 194, normalized size = 1.76

$$\frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab-3b^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*b^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.76404, size = 332, normalized size = 3.02

$$\frac{3\left(4a^2+3b^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(4a^2+3b^2\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(32ab\cos(dx\right.}{48d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*a^2 + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 + 3
*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 + 16
*a*b*cos(d*x + c) + 3*(4*a^2 + 3*b^2)*cos(d*x + c)^2 + 6*b^2)*sin(d*x + c))
/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^2 \sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [B] time = 1.35778, size = 348, normalized size = 3.16

$$3\left(4\,a^2+3\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,a^2+3\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\tan\left(\frac{1}{2}\,d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 + 3*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + 3*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 -
 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 15*b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*tan
(1/2*d*x + 1/2*c)^5 + 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1
/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 + 9
*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*tan(1/2*d*x + 1/2*c) + 48*a*b*tan(1/2*
d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)
/d
```

3.458 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=80

$$\frac{(3a^2+2b^2)\tan(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + ((3*a^2 + 2*b^2)*Tan[c + d*x])/(3*d) + (a*b
*Sec[c + d*x]*Tan[c + d*x])/d + (b^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0901502, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3788, 3768, 3770, 4046, 3767, 8}

$$\frac{(3a^2+2b^2)\tan(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + ((3*a^2 + 2*b^2)*Tan[c + d*x])/(3*d) + (a*b
*Sec[c + d*x]*Tan[c + d*x])/d + (b^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \sec^3(c+dx)\,dx+\int \sec^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx\\
&= \frac{ab\sec(c+dx)\tan(c+dx)}{d}+\frac{b^2\sec^2(c+dx)\tan(c+dx)}{3d}+(ab)\int \sec(c+dx)\\
&= \frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\sec(c+dx)\tan(c+dx)}{d}+\frac{b^2\sec^2(c+dx)\tan(c+\\}{3d}\\
&= \frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan(c+dx}{d}\end{aligned}$$

Mathematica [A] time = 0.22306, size = 71, normalized size = 0.89

$$\frac{a^2\tan(c+dx)}{d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}+\frac{b^2\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*Ta
n[c + d*x])/d + (b^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.027, size = 89, normalized size = 1.1

$$\frac{a^2 \tan(dx+c)}{d} + \frac{ab\sec(dx+c)\tan(dx+c)}{d} + \frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{2b^2\tan(dx+c)}{3d} + \frac{b^2(\sec(dx+c)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

```
[Out] a^2*tan(d*x+c)/d+a*b*sec(d*x+c)*tan(d*x+c)/d+1/d*a*b*ln(sec(d*x+c)+tan(d*x+
c))+2/3*b^2*tan(d*x+c)/d+1/3*b^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.25119, size = 113, normalized size = 1.41

$$\frac{2\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)b^2 - 3ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 6a^2\tan(dx+c}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 3*a*b*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan
(d*x + c))/d
```

Fricas [A] time = 1.74968, size = 259, normalized size = 3.24

$$\frac{3ab\cos(dx+c)^3\log(\sin(dx+c)+1) - 3ab\cos(dx+c)^3\log(-\sin(dx+c)+1) + 2\left(3ab\cos(dx+c) + \left(3a^2+2b^2\right)c\right.}{6d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (3*a^2 + 2*b^2)*cos(d*x + c)^2
 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sec (c + dx))^2 \sec^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [B] time = 1.37486, size = 240, normalized size = 3.

$$\frac{3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2*c
)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^2*t
an(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/
2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.459 $\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (b^2
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0536586, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3788, 3767, 8, 4046, 3770}

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\tan(c+dx)}{d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (b^2
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^2(c+dx)\,dx + \int \sec(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{b^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(2a^2+b^2\right)\int \sec(c+dx)\,dx - \frac{(2ab)\,\mathrm{Subst}(\int 1\,d}{}$$
$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.104026, size = 45, normalized size = 0.76

$$\frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))+b\tan(c+dx)(4a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + b*(4*a + b*Sec[c + d*x])*Tan[c + d*x
])/(2*d)
```

Maple [A] time = 0.025, size = 78, normalized size = 1.3

$$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\,\frac{ab\tan(dx+c)}{d} + \frac{b^2\sec(dx+c)\tan(dx+c)}{2\,d} + \frac{b^2\ln(\sec(dx+c)+\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b*tan(d*x+c)/d+1/2*b^2*sec(d*x+c)*tan
(d*x+c)/d+1/2/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.16169, size = 108, normalized size = 1.83

$$-\frac{b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 4\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right) - 8\,ab\tan\left(dx+c\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b*tan(d
*x + c))/d
```

Fricas [A] time = 1.68205, size = 236, normalized size = 4.

$$\frac{\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(4\,ab\cos\left(dx+c\right)+b\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 + b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 + b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + b^2)*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(c+dx\right)\right)^2\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] time = 1.2196, size = 174, normalized size = 2.95

$$\frac{\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 + b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*tan(1
/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.460 $\int (a + b\sec(c + dx))^2\,dx$

Optimal. Leaf size=33

$$a^2x + \frac{2ab\tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2\tan(c + dx)}{d}$$

```
[Out] a^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.0256904, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3773, 3770, 3767, 8}

$$a^2x + \frac{2ab\tanh^{-1}(\sin(c + dx))}{d} + \frac{b^2\tan(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 3773

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\sec(c+dx))^2\,dx &= a^2x+(2ab)\int \sec(c+dx)\,dx+b^2\int \sec^2(c+dx)\,dx\\
&= a^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\,\mathrm{Subst}(\int 1\,dx,x,-\tan(c+dx))}{d}\\
&= a^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0750027, size = 32, normalized size = 0.97

$$\frac{a^2dx+2ab\tanh^{-1}(\sin(c+dx))+b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*d*x + 2*a*b*ArcTanh[Sin[c + d*x]] + b^2*Tan[c + d*x])/d
```

Maple [A] time = 0.023, size = 49, normalized size = 1.5

$$a^2x+2\,\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\tan(dx+c)}{d}+\frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2,x)
```

```
[Out] a^2*x+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+b^2*tan(d*x+c)/d+1/d*a^2*c
```

Maxima [A] time = 1.16769, size = 54, normalized size = 1.64

$$a^2x+\frac{2\,ab\log(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 2*a*b*log(sec(d*x + c) + tan(d*x + c))/d + b^2*tan(d*x + c)/d
```

Fricas [B] time = 1.73746, size = 193, normalized size = 5.85

$$\frac{a^2dx\cos\left(dx+c\right)+ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+b^2\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*
x + c)*log(-sin(d*x + c) + 1) + b^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.31261, size = 104, normalized size = 3.15

$$\frac{(dx+c)a^2+2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

3.461 $\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=33

$$\frac{a^2\sin(c+dx)}{d}+2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] 2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.0554316, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3788, 8, 4045, 3770}

$$\frac{a^2\sin(c+dx)}{d}+2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] 2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int 1\,dx+\int \cos(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx\\ &= 2abx+\frac{a^2\sin(c+dx)}{d}+b^2\int \sec(c+dx)\,dx\\ &= 2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0159462, size = 46, normalized size = 1.39

$$\frac{a^2\sin(c)\cos(dx)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}+2abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] 2*a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Co
s[c]*Sin[d*x])/d
```

Maple [A] time = 0.043, size = 49, normalized size = 1.5

$$2\,abx+\frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\sin(dx+c)}{d}+2\,\frac{abc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2*a*b*x+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*sin(d*x+c)/d+2/d*a*b*c
```

Maxima [A] time = 1.17769, size = 69, normalized size = 2.09

$$\frac{4\,(dx+c)ab+b^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,a^2\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(d*x + c)*a*b + b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.68157, size = 131, normalized size = 3.97

$$\frac{4\,abdx + b^2\log\left(\sin\left(dx+c\right)+1\right) - b^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*d*x + b^2*log(sin(d*x + c) + 1) - b^2*log(-sin(d*x + c) + 1) + 2
*a^2*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(c+dx\right)\right)^2\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x), x)
```

Giac [B] time = 1.36628, size = 105, normalized size = 3.18

$$\frac{2\left(dx+c\right)ab + b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a*b + b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

3.462 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

```
[Out] ((a^2 + 2*b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*
x])/(2*d)
```

Rubi [A] time = 0.0657883, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3788, 2637, 4045, 8}

$$\frac{1}{2}x\left(a^2+2b^2\right)+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*
x])/(2*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \cos(c+dx)\,dx + \int \cos^2(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx\\ &= \frac{2ab\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(a^2+2b^2\right)\int 1\,dx\\ &= \frac{1}{2}\left(a^2+2b^2\right)x + \frac{2ab\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0718922, size = 46, normalized size = 0.92

$$\frac{2\left(a^2+2b^2\right)(c+dx)+a^2\sin(2(c+dx))+8ab\sin(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + 2*b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)])/(4*
d)
```

Maple [A] time = 0.046, size = 51, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,ab\sin(dx+c)+b^2(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*sin(d*x+c)+b^2*(d*
x+c))
```

Maxima [A] time = 1.15161, size = 63, normalized size = 1.26

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))a^2 + 4\,(dx + c)b^2 + 8\,ab\sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 + 4*(d*x + c)*b^2 + 8*a*b*sin(d*x
 + c))/d
```

Fricas [A] time = 1.60525, size = 93, normalized size = 1.86

$$\frac{\left(a^2 + 2\,b^2\right)dx + \left(a^2\cos(dx + c) + 4\,ab\right)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + 2*b^2)*d*x + (a^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^2\cos^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*cos(c + d*x)**2, x)
```

Giac [B] time = 1.26894, size = 130, normalized size = 2.6

$$\frac{\left(a^2 + 2\,b^2\right)(dx + c) - \frac{2\left(a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 4\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 4\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*b^2)*(d*x + c) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/
2*d*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))/(
tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.463 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=58

$$\frac{(a^2+b^2)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{d}+abx$$

```
[Out] a*b*x + ((a^2 + b^2)*Sin[c + d*x])/d + (a*b*Cos[c + d*x]*Sin[c + d*x])/d -
(a^2*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.089153, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 2635, 8, 4044, 3013}

$$\frac{(a^2+b^2)\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{d}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```


```
[Out] a*b*x + ((a^2 + b^2)*Sin[c + d*x])/d + (a*b*Cos[c + d*x]*Sin[c + d*x])/d -
(a^2*Sin[c + d*x]^3)/(3*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \cos^2(c+dx)\,dx + \int \cos^3(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$
$$= \frac{ab\cos(c+dx)\sin(c+dx)}{d} + (ab)\int 1\,dx + \int \cos(c+dx)\left(b^2+a^2\cos^2(c+dx\right.$$
$$= abx + \frac{ab\cos(c+dx)\sin(c+dx)}{d} - \frac{\text{Subst}\left(\int \left(a^2+b^2-a^2x^2\right)\,dx, x, -\sin(c+\right.}{d}$$
$$= abx + \frac{\left(a^2+b^2\right)\sin(c+dx)}{d} + \frac{ab\cos(c+dx)\sin(c+dx)}{d} - \frac{a^2\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.168356, size = 59, normalized size = 1.02

$$\frac{3\left(3a^2+4b^2\right)\sin(c+dx)+a(a\sin(3(c+dx))+12b(c+dx)+6b\sin(2(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3*(3*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*b*(c + d*x) + 6*b*Sin[2*(c + d*x)]
+ a*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.051, size = 63, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a^2\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3} + 2\,ab\,(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2) + b^2\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*a^2*(cos(d*x+c)^2+2)*sin(d*x+c)+2*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+b^2*sin(d*x+c))
```

Maxima [A] time = 1.0886, size = 81, normalized size = 1.4

$$-\frac{2\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)a^{2}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab-6\,b^{2}\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a*b - 6*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.6223, size = 124, normalized size = 2.14

$$\frac{3\,abdx+\left(a^{2}\cos\left(dx+c\right)^{2}+3\,ab\cos\left(dx+c\right)+2\,a^{2}+3\,b^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*d*x + (a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 2*a^2 + 3*b^2)*
sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24984, size = 207, normalized size = 3.57

$$\frac{3\,(dx+c)ab+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a*b + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x
+ 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/2*d*x + 1/2*c)^3 +
6*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d
*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.464 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a^2+4b^2)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}$$

```
[Out] ((3*a^2 + 4*b^2)*x)/8 + (2*a*b*Sin[c + d*x])/d + ((3*a^2 + 4*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*b*
Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0878394, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3788, 2633, 4045, 2635, 8}

$$\frac{(3a^2+4b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a^2+4b^2)+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + 4*b^2)*x)/8 + (2*a*b*Sin[c + d*x])/d + ((3*a^2 + 4*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*b*
Sin[c + d*x]^3)/(3*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \cos^3(c+dx)\,dx+\int \cos^4(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx\\ &= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\left(3a^2+4b^2\right)\int \cos^2(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\,(\ldots}{\ldots}\\ &= \frac{2ab\sin(c+dx)}{d}+\frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c\ldots}{4d}\\ &= \frac{1}{8}\left(3a^2+4b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\ldots}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.169573, size = 86, normalized size = 0.85

$$\frac{24\left(a^2+b^2\right)\sin(2(c+dx))+3a^2\sin(4(c+dx))+36a^2c+36a^2dx-64ab\sin^3(c+dx)+192ab\sin(c+dx)+48b^2c+48\ldots}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (36*a^2*c + 48*b^2*c + 36*a^2*d*x + 48*b^2*d*x + 192*a*b*Sin[c + d*x] - 64*
a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*a^2*Sin[4*(c + d*x
)])/(96*d)
```

Maple [A] time = 0.056, size = 89, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}+b^2\left(\frac{\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
*b*(cos(d*x+c)^2+2)*sin(d*x+c)+b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.0472, size = 111, normalized size = 1.1

$$\frac{3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^2-64\left(\sin(dx+c)^3-3\sin(dx+c)\right)ab+24(2dx+2c+\sin(2dx}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 64*(s
in(d*x + c)^3 - 3*sin(d*x + c))*a*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2)/d
```

Fricas [A] time = 1.64619, size = 184, normalized size = 1.82

$$\frac{3\left(3a^2+4b^2\right)dx+\left(6a^2\cos(dx+c)^3+16ab\cos(dx+c)^2+32ab+3\left(3a^2+4b^2\right)\cos(dx+c)\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*d*x + (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2
 + 32*a*b + 3*(3*a^2 + 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3467, size = 302, normalized size = 2.99

$$\frac{3\left(3a^2+4b^2\right)(dx+c)-\frac{2\left(15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-48ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-9a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-80ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12b^2\,\mathrm{ta}\right.}{}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*(d*x + c) - 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a
*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*a^2*tan(1/2*d
*x + 1/2*c)^5 - 80*a*b*tan(1/2*d*x + 1/2*c)^5 + 12*b^2*tan(1/2*d*x + 1/2*c)
^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*b^2*
tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*d*x +
 1/2*c) - 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.465 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=111

$$-\frac{(2a^2+b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{a^2\sin^5(c+dx)}{5d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c$$

```
[Out] (3*a*b*x)/4 + ((a^2 + b^2)*Sin[c + d*x])/d + (3*a*b*Cos[c + d*x]*Sin[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - ((2*a^2 + b^2)*Sin[c
+ d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.122443, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3788, 2635, 8, 4044, 3013, 373}

$$-\frac{(2a^2+b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{a^2\sin^5(c+dx)}{5d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (3*a*b*x)/4 + ((a^2 + b^2)*Sin[c + d*x])/d + (3*a*b*Cos[c + d*x]*Sin[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - ((2*a^2 + b^2)*Sin[c
+ d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sec(c+dx))^2\,dx &= (2ab)\int \cos^4(c+dx)\,dx+\int \cos^5(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx\\ &= \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}(3ab)\int \cos^2(c+dx)\,dx+\int \cos^3(c+dx)\left(b^2\right.\\ &= \frac{3ab\cos(c+dx)\sin(c+dx)}{4d}+\frac{ab\cos^3(c+dx)\sin(c+dx)}{2d}+\frac{1}{4}(3ab)\int 1\,dx-\\ &= \frac{3abx}{4}+\frac{3ab\cos(c+dx)\sin(c+dx)}{4d}+\frac{ab\cos^3(c+dx)\sin(c+dx)}{2d}-\frac{\text{Subst}\left(\int\right.}{}\\ &= \frac{3abx}{4}+\frac{\left(a^2+b^2\right)\sin(c+dx)}{d}+\frac{3ab\cos(c+dx)\sin(c+dx)}{4d}+\frac{ab\cos^3(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.163394, size = 85, normalized size = 0.77

$$\frac{-80\left(2a^2+b^2\right)\sin^3(c+dx)+240\left(a^2+b^2\right)\sin(c+dx)+48a^2\sin^5(c+dx)+15ab(12(c+dx)+8\sin(2(c+dx))+\sin(}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (240*(a^2 + b^2)*Sin[c + d*x] - 80*(2*a^2 + b^2)*Sin[c + d*x]^3 + 48*a^2*Si
n[c + d*x]^5 + 15*a*b*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/(240*d)
```

Maple [A] time = 0.054, size = 95, normalized size = 0.9

$$\frac{1}{d}\left(\frac{a^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+2ab\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b^2*(cos(d*x+c)^2+2
)*sin(d*x+c))
```

Maxima [A] time = 1.03851, size = 127, normalized size = 1.14

$$\frac{16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2+15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))ab-8}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 80*(sin(d*x
+ c)^3 - 3*sin(d*x + c))*b^2)/d
```

Fricas [A] time = 1.70132, size = 215, normalized size = 1.94

$$\frac{45abdx+\left(12a^2\cos(dx+c)^4+30ab\cos(dx+c)^3+45ab\cos(dx+c)+4\left(4a^2+5b^2\right)\cos(dx+c)^2+32a^2+40b^2\right)\mathrm{si}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(45*a*b*d*x + (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b*
cos(d*x + c) + 4*(4*a^2 + 5*b^2)*cos(d*x + c)^2 + 32*a^2 + 40*b^2)*sin(d*x
+ c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30324, size = 333, normalized size = 3.

$$45\,(dx+c)ab + \frac{2\left(60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 160\,b^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a*b + 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*
d*x + 1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 + 80*a^2*tan(1/2*d*x + 1/2*c
)^7 - 30*a*b*tan(1/2*d*x + 1/2*c)^7 + 160*b^2*tan(1/2*d*x + 1/2*c)^7 + 232*
a^2*tan(1/2*d*x + 1/2*c)^5 + 200*b^2*tan(1/2*d*x + 1/2*c)^5 + 80*a^2*tan(1/
2*d*x + 1/2*c)^3 + 30*a*b*tan(1/2*d*x + 1/2*c)^3 + 160*b^2*tan(1/2*d*x + 1/
2*c)^3 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

3.466 $\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=189

$$-\frac{(-52a^2b^2+3a^4-16b^4)\tan(c+dx)}{30bd}+\frac{a(4a^2+9b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(3a^2-16b^2)\tan(c+dx)(a+b\sec(c+dx))}{60bd}$$

```
[Out] (a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - ((3*a^4 - 52*a^2*b^2 - 16
*b^4)*Tan[c + d*x])/(30*b*d) - (a*(6*a^2 - 71*b^2)*Sec[c + d*x]*Tan[c + d*x
])/(120*d) - ((3*a^2 - 16*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*b*d
) - (a*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) + ((a + b*Sec[c + d*x]
)^4*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.312282, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3840, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{(-52a^2b^2+3a^4-16b^4)\tan(c+dx)}{30bd}+\frac{a(4a^2+9b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{(3a^2-16b^2)\tan(c+dx)(a+b\sec(c+dx))}{60bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - ((3*a^4 - 52*a^2*b^2 - 16
*b^4)*Tan[c + d*x])/(30*b*d) - (a*(6*a^2 - 71*b^2)*Sec[c + d*x]*Tan[c + d*x
])/(120*d) - ((3*a^2 - 16*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*b*d
) - (a*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(20*b*d) + ((a + b*Sec[c + d*x]
)^4*Tan[c + d*x])/(5*b*d)
```


Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5bd} + \frac{\int \sec(c+dx)(4b-a\sec(c+dx))(a+b\sec(c}{5b}$$

$$= -\frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{20bd} + \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5bd} + \frac{\int \sec(c}{}$$

$$= -\frac{\left(3a^2-16b^2\right)(a+b\sec(c+dx))^2\tan(c+dx)}{60bd} - \frac{a(a+b\sec(c+dx))^3\tan(c+d}{20bd}$$

$$= -\frac{a\left(6a^2-71b^2\right)\sec(c+dx)\tan(c+dx)}{120d} - \frac{\left(3a^2-16b^2\right)(a+b\sec(c+dx))^2\tan(}{60bd}$$

$$= -\frac{a\left(6a^2-71b^2\right)\sec(c+dx)\tan(c+dx)}{120d} - \frac{\left(3a^2-16b^2\right)(a+b\sec(c+dx))^2\tan(}{60bd}$$

$$= \frac{a\left(4a^2+9b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\left(6a^2-71b^2\right)\sec(c+dx)\tan(c+dx)}{120d} - \frac{(3}{}$$

$$= \frac{a\left(4a^2+9b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{\left(3a^4-52a^2b^2-16b^4\right)\tan(c+dx)}{30bd} - \frac{a\left(6a^2\right.}{}$$

Mathematica [A] time = 0.856563, size = 120, normalized size = 0.63

$$\frac{15a\left(4a^2+9b^2\right)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(8b\left(5\left(3a^2+2b^2\right)\tan^2(c+dx)+15\left(3a^2+b^2\right)+3b^2\tan^4(c+dx)\right)+\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (15*a*(4*a^2 + 9*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*a*(4*a^2 + 9
*b^2)*Sec[c + d*x] + 90*a*b^2*Sec[c + d*x]^3 + 8*b*(15*(3*a^2 + b^2) + 5*(3
*a^2 + 2*b^2)*Tan[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.033, size = 206, normalized size = 1.1

$$\frac{a^3\sec(dx+c)\tan(dx+c)}{2d} + \frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 2\frac{a^2b\tan(dx+c)}{d} + \frac{a^2b\tan(dx+c)(\sec(dx+c))^2}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2
*b*tan(d*x+c)+1/d*a^2*b*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a*b^2*tan(d*x+c)*sec(
d*x+c)^3+9/8*a*b^2*sec(d*x+c)*tan(d*x+c)/d+9/8/d*a*b^2*ln(sec(d*x+c)+tan(d*
x+c))+8/15/d*b^3*tan(d*x+c)+1/5/d*b^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*b^3*ta
n(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.18682, size = 244, normalized size = 1.29

$$\frac{240\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{2}b+16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)b^{3}-45\,ab^{2}\left(\frac{2\left(3\,\sin(\ldots}{\sin(dx+\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^3 - 45*a*b^2*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.75416, size = 421, normalized size = 2.23

$$\frac{15\left(4\,a^{3}+9\,ab^{2}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,a^{3}+9\,ab^{2}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(15\,\ldots\right.\right.}{240\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*a^3 + 9*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^
3 + 9*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(15*a^2*b + 4*b^
3)*cos(d*x + c)^4 + 90*a*b^2*cos(d*x + c) + 15*(4*a^3 + 9*a*b^2)*cos(d*x +
c)^3 + 24*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^3 \sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**3, x)
```

Giac [B] time = 1.33344, size = 495, normalized size = 2.62

$$15\left(4a^3 + 9ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15\left(4a^3 + 9ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2\left(60a^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^9 - 360a^2b\tan \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*a^3 + 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
+ 9*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2
*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 225*a*b^2*tan(1/2*d*x + 1/2*c)^9
 - 120*b^3*tan(1/2*d*x + 1/2*c)^9 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 960*a^
2*b*tan(1/2*d*x + 1/2*c)^7 - 90*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 160*b^3*tan(
1/2*d*x + 1/2*c)^7 - 1200*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 464*b^3*tan(1/2*d*
x + 1/2*c)^5 + 120*a^3*tan(1/2*d*x + 1/2*c)^3 + 960*a^2*b*tan(1/2*d*x + 1/2
*c)^3 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*b^3*tan(1/2*d*x + 1/2*c)^3 -
60*a^3*tan(1/2*d*x + 1/2*c) - 360*a^2*b*tan(1/2*d*x + 1/2*c) - 225*a*b^2*ta
n(1/2*d*x + 1/2*c) - 120*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

3.467 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=130

$$\frac{a\left(a^2+4b^2\right)\tan(c+dx)}{2d}+\frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{\tan(c+dx)(a}{}$$

```
[Out] (3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(a^2 + 4*b^2)*Tan[c +
d*x])/(2*d) + (b*(2*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(a +
 b*Sec[c + d*x])^2*Tan[c + d*x])/(4*d) + ((a + b*Sec[c + d*x])^3*Tan[c + d*
x])/(4*d)
```

Rubi [A] time = 0.198026, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3835, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{a\left(a^2+4b^2\right)\tan(c+dx)}{2d}+\frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2+3b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{\tan(c+dx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(a^2 + 4*b^2)*Tan[c +
d*x])/(2*d) + (b*(2*a^2 + 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(a +
 b*Sec[c + d*x])^2*Tan[c + d*x])/(4*d) + ((a + b*Sec[c + d*x])^3*Tan[c + d*
x])/(4*d)
```


Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
```

```
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{3}{4}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec$$
$$= \frac{a(a+b\sec(c+dx))^2\tan(c+dx)}{4d} + \frac{(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{4}\int \sec$$
$$= \frac{b\left(2a^2+3b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a(a+b\sec(c+dx))^2\tan(c+dx)}{4d} + \frac{(a}{}$$
$$= \frac{b\left(2a^2+3b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a(a+b\sec(c+dx))^2\tan(c+dx)}{4d} + \frac{(a}{}$$
$$= \frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\left(2a^2+3b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a}{}$$
$$= \frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\left(a^2+4b^2\right)\tan(c+dx)}{2d} + \frac{b\left(2a^2+3b^2\right)}{}$$

Mathematica [A] time = 0.444184, size = 90, normalized size = 0.69

$$\frac{3b\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(8a\left(a^2+b^2\tan^2(c+dx)+3b^2\right)+3b\left(4a^2+b^2\right)\sec(c+dx)+2b^3\sec^3\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(4*a^2 + b^2)*
Sec[c + d*x] + 2*b^3*Sec[c + d*x]^3 + 8*a*(a^2 + 3*b^2 + b^2*Tan[c + d*x]^2
)))/(8*d)
```

Maple [A] time = 0.029, size = 160, normalized size = 1.2

$$\frac{a^3\tan(dx+c)}{d} + \frac{3a^2b\sec(dx+c)\tan(dx+c)}{2d} + \frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 2\frac{ab^2\tan(dx+c)}{d} + \frac{ab^2\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x)
```

```
[Out] a^3*tan(d*x+c)/d+3/2/d*a^2*b*sec(d*x+c)*tan(d*x+c)+3/2/d*a^2*b*ln(sec(d*x+c
)+tan(d*x+c))+2*a*b^2*tan(d*x+c)/d+1/d*a*b^2*tan(d*x+c)*sec(d*x+c)^2+1/4/d*
```

```
b^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*b^3*sec(d*x+c)*tan(d*x+c)+3/8/d*b^3*ln(se
c(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.12123, size = 213, normalized size = 1.64

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)ab^{2}-b^{3}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2 - b^3*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*b*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x
 + c))/d
```

Fricas [A] time = 1.70749, size = 348, normalized size = 2.68

$$\frac{3\left(4\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,ab^{2}\cos\left(dx+\right.\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(4*a^2*b + b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2*b +
 b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a*b^2*cos(d*x + c) + 8*(
a^3 + 2*a*b^2)*cos(d*x + c)^3 + 2*b^3 + 3*(4*a^2*b + b^3)*cos(d*x + c)^2)*s
in(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^{3}\sec^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [B] time = 1.34871, size = 446, normalized size = 3.43

$$3\left(4\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-12\,a^2b\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(4*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b + b^
3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a^3*tan(1/2*d*x + 1/2*c)^7 - 1
2*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 5*b^3*ta
n(1/2*d*x + 1/2*c)^7 - 24*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 - 40*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^
5 + 24*a^3*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 40*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*tan(1/2*d
*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 24*a*b^2*tan(1/2*d*x + 1/2*c)
 - 5*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.468 $\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{2b(4a^2+b^2)\tan(c+dx)}{3d}+\frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{6d}+\frac{b\tan(c+dx)(a+b\sec}{3d}$$

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*b*(4*a^2 + b^2)*Tan[c
+ d*x])/(3*d) + (5*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*(a + b*Sec[c
 + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.131079, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3830, 3997, 3787, 3770, 3767, 8}

$$\frac{2b(4a^2+b^2)\tan(c+dx)}{3d}+\frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)\sec(c+dx)}{6d}+\frac{b\tan(c+dx)(a+b\sec}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*b*(4*a^2 + b^2)*Tan[c
+ d*x])/(3*d) + (5*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*(a + b*Sec[c
 + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 3830

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*
m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^
2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
```

```
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int \sec(c+dx)(a+b\sec(c+dx))\left(3a^2+2\right.\\ &= \frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{6}\int \sec(c+\\ &= \frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\left(2b\left(4a^2-\right.\right.\\ &= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\sec(c+dx)\tan(c+dx)}{6d}+\frac{b(a+b\sec}{}\\ &= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2b\left(4a^2+b^2\right)\tan(c+dx)}{3d}+\frac{5ab^2\sec(c+}{6}\end{aligned}$$

Mathematica [A] time = 0.240821, size = 70, normalized size = 0.71

$$\frac{\left(6a^3+9ab^2\right)\tanh^{-1}(\sin(c+dx))+b\tan(c+dx)\left(18a^2+9ab\sec(c+dx)+2b^2\tan^2(c+dx)+6b^2\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((6*a^3 + 9*a*b^2)*ArcTanh[Sin[c + d*x]] + b*Tan[c + d*x]*(18*a^2 + 6*b^2 +
 9*a*b*Sec[c + d*x] + 2*b^2*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.029, size = 118, normalized size = 1.2

$$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{d}+3\,\frac{a^2 b\tan(dx+c)}{d}+\frac{3\,ab^2\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{3\,ab^2\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b*tan(d*x+c)+3/2*a*b^2*sec(d*x+c)
*tan(d*x+c)/d+3/2/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*b^3*tan(d*x+c)+1/
3/d*b^3*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.08217, size = 143, normalized size = 1.44

$$\frac{4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)b^3-9\,ab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12\,a^3\log(\sec(}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^3 - 9*a*b^2*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3
*log(sec(d*x + c) + tan(d*x + c)) + 36*a^2*b*tan(d*x + c))/d
```

Fricas [A] time = 1.73864, size = 309, normalized size = 3.12

$$\frac{3\left(2\,a^3+3\,ab^2\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(2\,a^3+3\,ab^2\right)\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(9\,ab^2\cos(\right.}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 +
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a*b^2*cos(d*x + c) +
 2*b^3 + 2*(9*a^2*b + 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec (c + dx))^3 \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] time = 1.38141, size = 277, normalized size = 2.8

$$\frac{3\left(2\,a^3 + 3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\left(2\,a^3 + 3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(18\,a^2b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 - 9\,ab^2\tan\cdots\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 + 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b*tan(1/2*d*x + 1/2*c
)^5 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*b^3*tan(1/2*d*x + 1/2*c)^5 - 36*a^
2*b*tan(1/2*d*x + 1/2*c)^3 - 4*b^3*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*b*tan(1/
2*d*x + 1/2*c) + 9*a*b^2*tan(1/2*d*x + 1/2*c) + 6*b^3*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.469 $\int (a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=73

$$\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3x+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a*b^2*Tan[c + d*
x])/(2*d) + (b^2*(a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0485526, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3782, 3770, 3767, 8}

$$\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+a^3x+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3,x]
```

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a*b^2*Tan[c + d*
x])/(2*d) + (b^2*(a + b*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\,dx = \frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}\int \left(2a^3+b\left(6a^2+b^2\right)\sec(c+dx)+5ab^2\sec^2(c+dx\right.$$
$$= a^3x+\frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}\left(5ab^2\right)\int \sec^2(c+dx)\,dx+\frac{1}{2}\left(b\left(6a^2+b^2\right)\right)\int$$
$$= a^3x+\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2(a+b\sec(c+dx))\tan(c+dx)}{2d}-\frac{\left(5ab^2\right)\text{Subst}}{}$$
$$= a^3x+\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5ab^2\tan(c+dx)}{2d}+\frac{b^2(a+b\sec(c+dx))\tan(c+}{2d}$$

Mathematica [A] time = 0.154597, size = 55, normalized size = 0.75

$$\frac{b\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))+2a^3dx+b^2\tan(c+dx)(6a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a^3*d*x + b*(6*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + b^2*(6*a + b*Sec[c + d
*x])*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.027, size = 95, normalized size = 1.3

$$a^3x+\frac{a^3c}{d}+3\,\frac{a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\,\frac{ab^2\tan(dx+c)}{d}+\frac{b^3\sec(dx+c)\tan(dx+c)}{2\,d}+\frac{b^3\ln(\sec(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3,x)
```

```
[Out] a^3*x+1/d*a^3*c+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3*a*b^2*tan(d*x+c)/d+1/
2/d*b^3*sec(d*x+c)*tan(d*x+c)+1/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.20637, size = 126, normalized size = 1.73

$$a^3x - \frac{b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right)}{4\,d} + \frac{3\,a^2b\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d} + \frac{3\,ab^2\tan\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/4*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1
) + log(sin(d*x + c) - 1))/d + 3*a^2*b*log(sec(d*x + c) + tan(d*x + c))/d +
 3*a*b^2*tan(d*x + c)/d
```

Fricas [A] time = 1.71792, size = 281, normalized size = 3.85

$$\frac{4\,a^3dx\cos\left(dx+c\right)^2 + \left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left(6\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right) \cdots}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + (6*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (6*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a*
b^2*cos(d*x + c) + b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\sec\left(c+dx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.26545, size = 196, normalized size = 2.68

$$\frac{2\,(dx+c)a^3 + \left(6\,a^2b + b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - \left(6\,a^2b + b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - \cdots}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 + (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 (6*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a*b^2*tan(1/2*d*
x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c) -
b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.470 $\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{a\left(a^2-b^2\right)\sin(c+dx)}{d}+3a^2bx+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

```
[Out] 3*a^2*b*x + (3*a*b^2*ArcTanh[Sin[c + d*x]])/d + (a*(a^2 - b^2)*Sin[c + d*x]
)/d + (b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/d
```

Rubi [A] time = 0.111861, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3842, 4047, 8, 4045, 3770}

$$\frac{a\left(a^2-b^2\right)\sin(c+dx)}{d}+3a^2bx+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*ArcTanh[Sin[c + d*x]])/d + (a*(a^2 - b^2)*Sin[c + d*x]
)/d + (b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/d
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} + \int \cos(c+dx)\left(a\left(a^2-b^2\right)+3a^2b\sec(c+d$$

$$= \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} + \left(3a^2b\right)\int 1\,dx + \int \cos(c+dx)\left(a\left(a^2-b^2\right)\right.$$

$$= 3a^2bx + \frac{a\left(a^2-b^2\right)\sin(c+dx)}{d} + \frac{b^2(a+b\sec(c+dx))\sin(c+dx)}{d} + \left(3ab^2\right)\int$$

$$= 3a^2bx + \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a\left(a^2-b^2\right)\sin(c+dx)}{d} + \frac{b^2(a+b\sec(c+}{d}$$

Mathematica [A] time = 0.331382, size = 88, normalized size = 1.31

$$\frac{a^3\sin(c+dx)+3ab\left(ac+adx-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*b*(a*c + a*d*x - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[c + d*x] + b^3*Tan[c + d*x])/
d
```

Maple [A] time = 0.039, size = 68, normalized size = 1.

$$3a^2bx+3\frac{ab^2\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{b^3\tan\left(dx+c\right)}{d}+\frac{a^3\sin\left(dx+c\right)}{d}+3\frac{a^2bc}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 3*a^2*b*x+3/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^3*tan(d*x+c)+a^3*sin(d*
x+c)/d+3/d*a^2*b*c
```

Maxima [A] time = 1.14064, size = 89, normalized size = 1.33

$$\frac{6\left(dx+c\right)a^2b+3ab^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+2a^3\sin\left(dx+c\right)+2b^3\tan\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a^2*b + 3*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 2*a^3*sin(d*x + c) + 2*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.70274, size = 246, normalized size = 3.67

$$\frac{6a^2bdx\cos\left(dx+c\right)+3ab^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3ab^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^3\cos\left(dx\right.\right.}{2d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^2*b*d*x*cos(d*x + c) + 3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a*b^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + b^3)*
sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^3 \cos(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*cos(c + d*x), x)
```

Giac [A] time = 1.36235, size = 177, normalized size = 2.64

$$\frac{3\,(dx + c)a^2 b + 3\,ab^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,ab^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - b^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 \cdots}{\tan\left(\frac{1}{2}\,dx + \cdots\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a^2*b + 3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*tan
(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.471 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*(a^2 + 6*b^2)*x)/2 + (b^3*ArcTanh[Sin[c + d*x]])/d + (5*a^2*b*Sin[c + d*
x])/(2*d) + (a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.119398, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3841, 4047, 8, 4045, 3770}

$$\frac{1}{2}ax\left(a^2+6b^2\right)+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))}{2d}+\frac{b^3\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*x)/2 + (b^3*ArcTanh[Sin[c + d*x]])/d + (5*a^2*b*Sin[c + d*
x])/(2*d) + (a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int \cos(c+dx)\left(5a^2b+a\left(a^2\right.\right.\\
&= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int \cos(c+dx)\left(5a^2b+2b^3\,\text{se}\right.\\
&= \frac{1}{2}a\left(a^2+6b^2\right)x+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\cos(c+dx)(a+b\sec(c+dx))\sin(c+}{2d}\\
&= \frac{1}{2}a\left(a^2+6b^2\right)x+\frac{b^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^2b\sin(c+dx)}{2d}+\frac{a^2\cos(c+dx)(}{}\end{aligned}$$

Mathematica [A] time = 0.150798, size = 105, normalized size = 1.33

$$\frac{2a\left(a^2+6b^2\right)(c+dx)+12a^2b\sin(c+dx)+a^3\sin(2(c+dx))-4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4b^3\log\left(\text{s}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*a*(a^2 + 6*b^2)*(c + d*x) - 4*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 4*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a^2*b*Sin[c + d*x]
 + a^3*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.048, size = 90, normalized size = 1.1

$$\frac{a^{3}\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{a^{3}x}{2}+\frac{a^{3}c}{2d}+3\,\frac{a^{2}b\sin\left(dx+c\right)}{d}+3\,ab^{2}x+3\,\frac{ab^{2}c}{d}+\frac{b^{3}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3*c+3*a^2*b*sin(d*x+c)/d+
3*a*b^2*x+3/d*a*b^2*c+1/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.21987, size = 103, normalized size = 1.3

$$\frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^{3}+12\,(dx+c)ab^{2}+2\,b^{3}(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+12\,a^{2}b\sin\left(dx+c\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a*b^2 + 2*b^3*(log
(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*a^2*b*sin(d*x + c))/d
```

Fricas [A] time = 1.75536, size = 176, normalized size = 2.23

$$\frac{b^{3}\log\left(\sin\left(dx+c\right)+1\right)-b^{3}\log\left(-\sin\left(dx+c\right)+1\right)+\left(a^{3}+6\,ab^{2}\right)dx+\left(a^{3}\cos\left(dx+c\right)+6\,a^{2}b\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*log(sin(d*x + c) + 1) - b^3*log(-sin(d*x + c) + 1) + (a^3 + 6*a*b^
2)*d*x + (a^3*cos(d*x + c) + 6*a^2*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33009, size = 185, normalized size = 2.34

$$\frac{2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,b^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(a^3+6\,ab^2\right)(dx+c)-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^2b\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b^3*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (a^3 + 6*a*b^2)*(d*x + c) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 -
 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.472 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=100

$$\frac{a\left(2a^2+9b^2\right)\sin(c+dx)}{3d}+\frac{1}{2}bx\left(3a^2+2b^2\right)+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{6d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+d}{3d}$$

```
[Out] (b*(3*a^2 + 2*b^2)*x)/2 + (a*(2*a^2 + 9*b^2)*Sin[c + d*x])/(3*d) + (7*a^2*b
*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x]
)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.149988, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3841, 4047, 2637, 4045, 8}

$$\frac{a\left(2a^2+9b^2\right)\sin(c+dx)}{3d}+\frac{1}{2}bx\left(3a^2+2b^2\right)+\frac{7a^2b\sin(c+dx)\cos(c+dx)}{6d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+d}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (b*(3*a^2 + 2*b^2)*x)/2 + (a*(2*a^2 + 9*b^2)*Sin[c + d*x])/(3*d) + (7*a^2*b
*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x]
)*Sin[c + d*x])/(3*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
```

```
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\int \cos^2(c+dx)\left(7a^2b+a\left(2\right.\right.$$
$$= \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\int \cos^2(c+dx)\left(7a^2b+b\left(a\right.\right.$$
$$= \frac{a\left(2a^2+9b^2\right)\sin(c+dx)}{3d} + \frac{7a^2b\cos(c+dx)\sin(c+dx)}{6d} + \frac{a^2\cos^2(c+dx)(a+}{}$$
$$= \frac{1}{2}b\left(3a^2+2b^2\right)x + \frac{a\left(2a^2+9b^2\right)\sin(c+dx)}{3d} + \frac{7a^2b\cos(c+dx)\sin(c+dx)}{6d} +$$

Mathematica [A] time = 0.120722, size = 80, normalized size = 0.8

$$\frac{9a\left(a^2+4b^2\right)\sin(c+dx)+9a^2b\sin(2(c+dx))+18a^2bc+18a^2bdx+a^3\sin(3(c+dx))+12b^3c+12b^3dx}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (18*a^2*b*c + 12*b^3*c + 18*a^2*b*d*x + 12*b^3*d*x + 9*a*(a^2 + 4*b^2)*Sin[
c + d*x] + 9*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.049, size = 76, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a^3\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}+3a^2b\left(1/2\cos(dx+c)\sin(dx+c)+1/2dx+c/2\right)+3ab^2\sin(dx+c)+b^3(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*a^3*(cos(d*x+c)^2+2)*sin(d*x+c)+3*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+3*a*b^2*sin(d*x+c)+b^3*(d*x+c))
```

Maxima [A] time = 1.17445, size = 99, normalized size = 0.99

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3-9\left(2dx+2c+\sin(2dx+2c)\right)a^2b-12(dx+c)b^3-36ab^2\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 9*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^2*b - 12*(d*x + c)*b^3 - 36*a*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.67069, size = 153, normalized size = 1.53

$$\frac{3\left(3a^2b+2b^3\right)dx+\left(2a^3\cos(dx+c)^2+9a^2b\cos(dx+c)+4a^3+18ab^2\right)\sin(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^2*b*cos(d*x + c)
 + 4*a^3 + 18*a*b^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28825, size = 230, normalized size = 2.3

$$\frac{3\left(3\,a^{2}b+2\,b^{3}\right)(dx+c)+\frac{2\left(6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+18\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+36\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,a^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*(d*x + c) + 2*(6*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^
2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*a^3*tan(1/
2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2*d*x + 1/
2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.473 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=123

$$\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{4d}+\frac{3a\left(a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}ax\left(a^2+4b^2\right)-\frac{3a^2b\sin^3(c+dx)}{4d}+\frac{a^2\sin(c+dx)}{}$$

```
[Out] (3*a*(a^2 + 4*b^2)*x)/8 + (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/(4*d) + (3*a*(a
^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*(a + b*S
ec[c + d*x])*Sin[c + d*x])/(4*d) - (3*a^2*b*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.183282, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4047, 2635, 8, 4044, 3013}

$$\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{4d}+\frac{3a\left(a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}ax\left(a^2+4b^2\right)-\frac{3a^2b\sin^3(c+dx)}{4d}+\frac{a^2\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 + 4*b^2)*x)/8 + (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/(4*d) + (3*a*(a
^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*(a + b*S
ec[c + d*x])*Sin[c + d*x])/(4*d) - (3*a^2*b*Sin[c + d*x]^3)/(4*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
```

```
x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3(c+dx)\left(9a^2b+3a\left(\right.\right.$$

$$=\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3(c+dx)\left(9a^2b+2b\left(\right.\right.$$

$$=\frac{3a\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))\sin(}{4d}$$

$$=\frac{3}{8}a\left(a^2+4b^2\right)x+\frac{3a\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)(a+}{}$$

$$=\frac{3}{8}a\left(a^2+4b^2\right)x+\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{4d}+\frac{3a\left(a^2+4b^2\right)\cos(c+dx)\sin(c}{8d}$$

Mathematica [A] time = 0.272784, size = 100, normalized size = 0.81

$$\frac{8b\left(9a^2+4b^2\right)\sin(c+dx)+a\left(8\left(a^2+3b^2\right)\sin(2(c+dx))+a^2\sin(4(c+dx))+12a^2c+12a^2dx+8ab\sin(3(c+dx))+\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (8*b*(9*a^2 + 4*b^2)*Sin[c + d*x] + a*(12*a^2*c + 48*b^2*c + 12*a^2*d*x + 4
8*b^2*d*x + 8*(a^2 + 3*b^2)*Sin[2*(c + d*x)] + 8*a*b*Sin[3*(c + d*x)] + a^2
*Sin[4*(c + d*x)]))/(32*d)
```

Maple [A] time = 0.058, size = 102, normalized size = 0.8

$$\frac{1}{d}\left(a^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+a^2b\left((\cos(dx+c))^2+2\right)\sin(dx+c)+3ab^2(1/2\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^2*b
*(cos(d*x+c)^2+2)*sin(d*x+c)+3*a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+b^3*sin(d*x+c))
```

Maxima [A] time = 1.20503, size = 128, normalized size = 1.04

$$\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^2b+24\,(2\,dx+2\,c+\sin(2\,dx}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 32*(sin
(d*x + c)^3 - 3*sin(d*x + c))*a^2*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a
*b^2 + 32*b^3*sin(d*x + c))/d
```

Fricas [A] time = 1.67574, size = 197, normalized size = 1.6

$$\frac{3\left(a^3+4\,ab^2\right)dx+\left(2\,a^3\cos(dx+c)^3+8\,a^2b\cos(dx+c)^2+16\,a^2b+8\,b^3+3\left(a^3+4\,ab^2\right)\cos(dx+c)\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a^3 + 4*a*b^2)*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^2*b*cos(d*x + c)^2
 + 16*a^2*b + 8*b^3 + 3*(a^3 + 4*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36587, size = 401, normalized size = 3.26

$$3\left(a^3+4ab^2\right)(dx+c)-\frac{2\left(5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-8b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-40a^2b\,\mathrm{ta}\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 + 4*a*b^2)*(d*x + c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^2
*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*b^3*tan(1/2
*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^2*b*tan(1/2*d*x + 1/2
*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 24*b^3*tan(1/2*d*x + 1/2*c)^5 + 3
*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 24*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*tan(1/2*d*x +
1/2*c) - 24*a^2*b*tan(1/2*d*x + 1/2*c) - 12*a*b^2*tan(1/2*d*x + 1/2*c) - 8*
b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.474 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=160

$$-\frac{a\left(4a^2+15b^2\right)\sin^3(c+dx)}{15d}+\frac{a\left(4a^2+15b^2\right)\sin(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(9a^2+4b^2\right)+\frac{1}{}$$

```
[Out] (b*(9*a^2 + 4*b^2)*x)/8 + (a*(4*a^2 + 15*b^2)*Sin[c + d*x])/(5*d) + (b*(9*a
^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (11*a^2*b*Cos[c + d*x]^3*Sin
[c + d*x])/(20*d) + (a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])*Sin[c + d*x])/
(5*d) - (a*(4*a^2 + 15*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.191936, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4047, 2633, 4045, 2635, 8}

$$-\frac{a\left(4a^2+15b^2\right)\sin^3(c+dx)}{15d}+\frac{a\left(4a^2+15b^2\right)\sin(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(9a^2+4b^2\right)+\frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (b*(9*a^2 + 4*b^2)*x)/8 + (a*(4*a^2 + 15*b^2)*Sin[c + d*x])/(5*d) + (b*(9*a
^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (11*a^2*b*Cos[c + d*x]^3*Sin
[c + d*x])/(20*d) + (a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])*Sin[c + d*x])/
(5*d) - (a*(4*a^2 + 15*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
```

```
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+b\sec(c+dx))^3\,dx &= \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4(c+dx)\left(11a^2b+a\left(\right.\right.\\ &= \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}+\frac{1}{5}\int \cos^4(c+dx)\left(11a^2b+b\left(\right.\right.\\ &= \frac{11a^2b\cos^3(c+dx)\sin(c+dx)}{20d}+\frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}\\ &= \frac{a\left(4a^2+15b^2\right)\sin(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{11a^2b\cos}{}\\ &= \frac{1}{8}b\left(9a^2+4b^2\right)x+\frac{a\left(4a^2+15b^2\right)\sin(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\cos(c+dx)\sin(}{8d}\end{aligned}$$

Mathematica [A] time = 0.29678, size = 130, normalized size = 0.81

$$\frac{60a\left(5a^2+18b^2\right)\sin(c+dx)+120\left(3a^2b+b^3\right)\sin(2(c+dx))+45a^2b\sin(4(c+dx))+540a^2bc+540a^2bdx+50a^3\sin(3}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (540*a^2*b*c + 240*b^3*c + 540*a^2*b*d*x + 240*b^3*d*x + 60*a*(5*a^2 + 18*b
^2)*Sin[c + d*x] + 120*(3*a^2*b + b^3)*Sin[2*(c + d*x)] + 50*a^3*Sin[3*(c +
 d*x)] + 120*a*b^2*Sin[3*(c + d*x)] + 45*a^2*b*Sin[4*(c + d*x)] + 6*a^3*Sin
[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.057, size = 123, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)+3\,a^2b\left(1/4\,\left((\cos(dx+c))^3+3/2\,\cos(dx+c)\right)\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^2*b*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*b^2*(cos(d*x+c)^2+2
)*sin(d*x+c)+b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.20398, size = 161, normalized size = 1.01

$$\frac{32\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)a^3+45\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^2b-}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 + 45
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 480*(sin(d
*x + c)^3 - 3*sin(d*x + c))*a*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*b^
```

```
3)/d
```

Fricas [A] time = 1.69645, size = 265, normalized size = 1.66

$$\frac{15\left(9\,a^2b+4\,b^3\right)dx+\left(24\,a^3\cos\left(dx+c\right)^4+90\,a^2b\cos\left(dx+c\right)^3+64\,a^3+240\,ab^2+8\left(4\,a^3+15\,ab^2\right)\cos\left(dx+c\right)^2+15\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(9*a^2*b + 4*b^3)*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^2*b*cos(d*x
 + c)^3 + 64*a^3 + 240*a*b^2 + 8*(4*a^3 + 15*a*b^2)*cos(d*x + c)^2 + 15*(9*
a^2*b + 4*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29605, size = 448, normalized size = 2.8

$$15\left(9\,a^2b+4\,b^3\right)(dx+c)+\frac{2\left(120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-225\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+160\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(9*a^2*b + 4*b^3)*(d*x + c) + 2*(120*a^3*tan(1/2*d*x + 1/2*c)^9 -
 225*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*b
^3*tan(1/2*d*x + 1/2*c)^9 + 160*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*a^2*b*tan(1
/2*d*x + 1/2*c)^7 + 960*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*b^3*tan(1/2*d*x
+ 1/2*c)^7 + 464*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*a*b^2*tan(1/2*d*x + 1/2*
c)^5 + 160*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 9
60*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*
tan(1/2*d*x + 1/2*c) + 225*a^2*b*tan(1/2*d*x + 1/2*c) + 360*a*b^2*tan(1/2*d
*x + 1/2*c) + 60*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/
d
```

3.475 $\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=185

$$-\frac{b\left(5a^2+b^2\right)\sin^3(c+dx)}{3d}+\frac{b\left(17a^2+6b^2\right)\sin(c+dx)}{6d}+\frac{a\left(5a^2+18b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\left(5a^2+18b^2\right)\sin(}{1}$$

```
[Out] (a*(5*a^2 + 18*b^2)*x)/16 + (b*(17*a^2 + 6*b^2)*Sin[c + d*x])/(6*d) + (a*(5
*a^2 + 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(5*a^2 + 18*b^2)*Cos[
c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])*
Sin[c + d*x])/(6*d) - (b*(5*a^2 + b^2)*Sin[c + d*x]^3)/(3*d) + (13*a^2*b*Si
n[c + d*x]^5)/(30*d)
```

Rubi [A] time = 0.232106, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3841, 4047, 2635, 8, 4044, 3013, 373}

$$-\frac{b\left(5a^2+b^2\right)\sin^3(c+dx)}{3d}+\frac{b\left(17a^2+6b^2\right)\sin(c+dx)}{6d}+\frac{a\left(5a^2+18b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\left(5a^2+18b^2\right)\sin(}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(5*a^2 + 18*b^2)*x)/16 + (b*(17*a^2 + 6*b^2)*Sin[c + d*x])/(6*d) + (a*(5
*a^2 + 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a*(5*a^2 + 18*b^2)*Cos[
c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])*
Sin[c + d*x])/(6*d) - (b*(5*a^2 + b^2)*Sin[c + d*x]^3)/(3*d) + (13*a^2*b*Si
n[c + d*x]^5)/(30*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^5(c+dx)\left(13a^2b+a\left(\right.\right.$$

$$= \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^5(c+dx)\left(13a^2b+2b\right.$$

$$= \frac{a\left(5a^2+18b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))\sin}{6d}$$

$$= \frac{a\left(5a^2+18b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\left(5a^2+18b^2\right)\cos^3(c+dx)\sin(c+d}{24d}$$

$$= \frac{1}{16}a\left(5a^2+18b^2\right)x + \frac{a\left(5a^2+18b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\left(5a^2+18b^2\right)c}{}$$

$$= \frac{1}{16}a\left(5a^2+18b^2\right)x + \frac{b\left(17a^2+6b^2\right)\sin(c+dx)}{6d} + \frac{a\left(5a^2+18b^2\right)\cos(c+dx)s}{16d}$$

Mathematica [A] time = 0.336222, size = 159, normalized size = 0.86

$$\frac{360b\left(5a^2+2b^2\right)\sin(c+dx)+45\left(5a^3+16ab^2\right)\sin(2(c+dx))+300a^2b\sin(3(c+dx))+36a^2b\sin(5(c+dx))+45a^3\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (300*a^3*c + 1080*a*b^2*c + 300*a^3*d*x + 1080*a*b^2*d*x + 360*b*(5*a^2 + 2
*b^2)*Sin[c + d*x] + 45*(5*a^3 + 16*a*b^2)*Sin[2*(c + d*x)] + 300*a^2*b*Sin
[3*(c + d*x)] + 80*b^3*Sin[3*(c + d*x)] + 45*a^3*Sin[4*(c + d*x)] + 90*a*b^
2*Sin[4*(c + d*x)] + 36*a^2*b*Sin[5*(c + d*x)] + 5*a^3*Sin[6*(c + d*x)])/(9
60*d)
```

Maple [A] time = 0.058, size = 145, normalized size = 0.8

$$\frac{1}{d}\left(a^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^2b\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*a^2*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a
*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*b^3*(
cos(d*x+c)^2+2)*sin(d*x+c))
```

Maxima [A] time = 1.1925, size = 196, normalized size = 1.06

$$-\frac{5\left(4\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^{3}-192\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3} \cdots\right.}{9 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a^3 - 192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d
*x + c))*a^2*b - 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*a*b^2 + 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*b^3)/d
```

Fricas [A] time = 1.75999, size = 324, normalized size = 1.75

$$\frac{15\left(5\,a^{3}+18\,ab^{2}\right)dx+\left(40\,a^{3}\cos\left(dx+c\right)^{5}+144\,a^{2}b\cos\left(dx+c\right)^{4}+10\left(5\,a^{3}+18\,ab^{2}\right)\cos\left(dx+c\right)^{3}+384\,a^{2}b+160\,b^{3}+\cdots\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(5*a^3 + 18*a*b^2)*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^2*b*cos(d
*x + c)^4 + 10*(5*a^3 + 18*a*b^2)*cos(d*x + c)^3 + 384*a^2*b + 160*b^3 + 16
*(12*a^2*b + 5*b^3)*cos(d*x + c)^2 + 15*(5*a^3 + 18*a*b^2)*cos(d*x + c))*si
n(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28646, size = 582, normalized size = 3.15

$$15\left(5\,a^{3}+18\,ab^{2}\right)(dx+c)-\frac{2\left(165\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-720\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+450\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-240\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-25\,a^{3}\tan\left(\frac{1}{2}\,d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^3 + 18*a*b^2)*(d*x + c) - 2*(165*a^3*tan(1/2*d*x + 1/2*c)^11
 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^11 + 450*a*b^2*tan(1/2*d*x + 1/2*c)^11 -
240*b^3*tan(1/2*d*x + 1/2*c)^11 - 25*a^3*tan(1/2*d*x + 1/2*c)^9 - 1680*a^2*
b*tan(1/2*d*x + 1/2*c)^9 + 630*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 880*b^3*tan(1
/2*d*x + 1/2*c)^9 + 450*a^3*tan(1/2*d*x + 1/2*c)^7 - 3744*a^2*b*tan(1/2*d*x
 + 1/2*c)^7 + 180*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 1440*b^3*tan(1/2*d*x + 1/2
*c)^7 - 450*a^3*tan(1/2*d*x + 1/2*c)^5 - 3744*a^2*b*tan(1/2*d*x + 1/2*c)^5
- 180*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 1440*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*a
^3*tan(1/2*d*x + 1/2*c)^3 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 630*a*b^2*t
an(1/2*d*x + 1/2*c)^3 - 880*b^3*tan(1/2*d*x + 1/2*c)^3 - 165*a^3*tan(1/2*d*
x + 1/2*c) - 720*a^2*b*tan(1/2*d*x + 1/2*c) - 450*a*b^2*tan(1/2*d*x + 1/2*c
) - 240*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.476 $\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=244

$$-\frac{a\left(-121a^2b^2+4a^4-128b^4\right)\tan(c+dx)}{60bd}+\frac{\left(36a^2b^2+8a^4+5b^4\right)\tanh^{-1}(\sin(c+dx))}{16d}-\frac{\left(4a^2-25b^2\right)\tan(c+dx)(a+b}{120bd}$$

```
[Out] ((8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]])/(16*d) - (a*(4*a^4 - 1
21*a^2*b^2 - 128*b^4)*Tan[c + d*x])/(60*b*d) - ((8*a^4 - 178*a^2*b^2 - 75*b
^4)*Sec[c + d*x]*Tan[c + d*x])/(240*d) - (a*(4*a^2 - 53*b^2)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(120*b*d) - ((4*a^2 - 25*b^2)*(a + b*Sec[c + d*x])^3
*Tan[c + d*x])/(120*b*d) - (a*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(30*b*d)
 + ((a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.450268, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3840, 4002, 3997, 3787, 3770, 3767, 8}

$$-\frac{a\left(-121a^2b^2+4a^4-128b^4\right)\tan(c+dx)}{60bd}+\frac{\left(36a^2b^2+8a^4+5b^4\right)\tanh^{-1}(\sin(c+dx))}{16d}-\frac{\left(4a^2-25b^2\right)\tan(c+dx)(a+b}{120bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```


```
[Out] ((8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]])/(16*d) - (a*(4*a^4 - 1
21*a^2*b^2 - 128*b^4)*Tan[c + d*x])/(60*b*d) - ((8*a^4 - 178*a^2*b^2 - 75*b
^4)*Sec[c + d*x]*Tan[c + d*x])/(240*d) - (a*(4*a^2 - 53*b^2)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(120*b*d) - ((4*a^2 - 25*b^2)*(a + b*Sec[c + d*x])^3
*Tan[c + d*x])/(120*b*d) - (a*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(30*b*d)
 + ((a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```


Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{(a+b\sec(c+dx))^5\tan(c+dx)}{6bd} + \frac{\int \sec(c+dx)(5b-a\sec(c+dx))(a+b\sec(c}{6b}$$
$$= -\frac{a(a+b\sec(c+dx))^4\tan(c+dx)}{30bd} + \frac{(a+b\sec(c+dx))^5\tan(c+dx)}{6bd} + \frac{\int \sec(c}{}$$
$$= -\frac{\left(4a^2-25b^2\right)(a+b\sec(c+dx))^3\tan(c+dx)}{120bd} - \frac{a(a+b\sec(c+dx))^4\tan(c+a}{30bd}$$
$$= -\frac{a\left(4a^2-53b^2\right)(a+b\sec(c+dx))^2\tan(c+dx)}{120bd} - \frac{\left(4a^2-25b^2\right)(a+b\sec(c+d}{120bd}$$
$$= -\frac{\left(8a^4-178a^2b^2-75b^4\right)\sec(c+dx)\tan(c+dx)}{240d} - \frac{a\left(4a^2-53b^2\right)(a+b\sec(c+}{120bd}$$
$$= -\frac{\left(8a^4-178a^2b^2-75b^4\right)\sec(c+dx)\tan(c+dx)}{240d} - \frac{a\left(4a^2-53b^2\right)(a+b\sec(c+}{120bd}$$
$$= \frac{\left(8a^4+36a^2b^2+5b^4\right)\tanh^{-1}(\sin(c+dx))}{16d} - \frac{\left(8a^4-178a^2b^2-75b^4\right)\sec(c+dx)}{240d}$$
$$= \frac{\left(8a^4+36a^2b^2+5b^4\right)\tanh^{-1}(\sin(c+dx))}{16d} - \frac{a\left(4a^4-121a^2b^2-128b^4\right)\tan(c+}{60bd}$$

Mathematica [A] time = 0.901929, size = 154, normalized size = 0.63

$$\frac{15\left(36a^2b^2+8a^4+5b^4\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(64ab\left(5\left(a^2+2b^2\right)\tan^2(c+dx)+15\left(a^2+b^2\right)+3b^2\tan^4(c\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(
8*a^4 + 36*a^2*b^2 + 5*b^4)*Sec[c + d*x] + 10*b^2*(36*a^2 + 5*b^2)*Sec[c +
d*x]^3 + 40*b^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(a^2 + 2*b^2)*T
an[c + d*x]^2 + 3*b^2*Tan[c + d*x]^4)))/(240*d)
```

Maple [A] time = 0.036, size = 302, normalized size = 1.2

$$\frac{a^4\sec(dx+c)\tan(dx+c)}{2d} + \frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{8a^3b\tan(dx+c)}{3d} + \frac{4a^3b\tan(dx+c)(\sec(dx+c))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+8/3/d*a
^3*b*tan(d*x+c)+4/3/d*a^3*b*tan(d*x+c)*sec(d*x+c)^2+3/2/d*a^2*b^2*tan(d*x+c
)*sec(d*x+c)^3+9/4/d*a^2*b^2*sec(d*x+c)*tan(d*x+c)+9/4/d*a^2*b^2*ln(sec(d*x
+c)+tan(d*x+c))+32/15*a*b^3*tan(d*x+c)/d+4/5/d*a*b^3*tan(d*x+c)*sec(d*x+c)^
4+16/15/d*a*b^3*tan(d*x+c)*sec(d*x+c)^2+1/6/d*b^4*tan(d*x+c)*sec(d*x+c)^5+5
/24/d*b^4*tan(d*x+c)*sec(d*x+c)^3+5/16/d*b^4*sec(d*x+c)*tan(d*x+c)+5/16/d*b
^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.1976, size = 371, normalized size = 1.52

$$\frac{640\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{3}b+128\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)ab^{3}-5\,b^{4}\left(\frac{2\left(15\sin}{\sin(dx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b + 128*(3*tan(d*x + c)^5
+ 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a*b^3 - 5*b^4*(2*(15*sin(d*x + c)^5
- 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 180*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2
*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) -
 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.83403, size = 528, normalized size = 2.16

$$\frac{15\left(8\,a^{4}+36\,a^{2}b^{2}+5\,b^{4}\right)\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,a^{4}+36\,a^{2}b^{2}+5\,b^{4}\right)\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1)
 - 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) +
```

```
2*(128*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^5 + 192*a*b^3*cos(d*x + c) + 15*(8*
a^4 + 36*a^2*b^2 + 5*b^4)*cos(d*x + c)^4 + 40*b^4 + 64*(5*a^3*b + 4*a*b^3)*
cos(d*x + c)^3 + 10*(36*a^2*b^2 + 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*c
os(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^4 \sec^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**3, x)
```

Giac [B] time = 1.3573, size = 799, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12
0*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2
*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*b^4*
tan(1/2*d*x + 1/2*c)^11 - 360*a^4*tan(1/2*d*x + 1/2*c)^9 + 3520*a^3*b*tan(1
/2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 2240*a*b^3*tan(1/
2*d*x + 1/2*c)^9 + 25*b^4*tan(1/2*d*x + 1/2*c)^9 + 240*a^4*tan(1/2*d*x + 1/
2*c)^7 - 5760*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*
c)^7 - 4992*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^4*tan(1/2*d*x + 1/2*c)^7 +
 240*a^4*tan(1/2*d*x + 1/2*c)^5 + 5760*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*a
^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4992*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 450*b^4
*tan(1/2*d*x + 1/2*c)^5 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 - 3520*a^3*b*tan(1
/2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2240*a*b^3*tan(1/
2*d*x + 1/2*c)^3 + 25*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^4*tan(1/2*d*x + 1/
2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*d*x + 1/2*c) +
960*a*b^3*tan(1/2*d*x + 1/2*c) + 165*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
```

```
+ 1/2*c)^2 - 1)^6)/d
```

3.477 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=179

$$\frac{2(28a^2b^2+3a^4+4b^4)\tan(c+dx)}{15d}+\frac{ab(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3a^2+4b^2)\tan(c+dx)(a+b\sec(c+dx))^2}{15d}$$

```
[Out] (a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a^4 + 28*a^2*b^2
+ 4*b^4)*Tan[c + d*x])/(15*d) + (a*b*(6*a^2 + 29*b^2)*Sec[c + d*x]*Tan[c +
d*x])/(30*d) + ((3*a^2 + 4*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(15*d)
 + (a*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(5*d) + ((a + b*Sec[c + d*x])^4*
Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.301871, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3835, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{2(28a^2b^2+3a^4+4b^4)\tan(c+dx)}{15d}+\frac{ab(4a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3a^2+4b^2)\tan(c+dx)(a+b\sec(c+dx))^2}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a^4 + 28*a^2*b^2
+ 4*b^4)*Tan[c + d*x])/(15*d) + (a*b*(6*a^2 + 29*b^2)*Sec[c + d*x]*Tan[c +
d*x])/(30*d) + ((3*a^2 + 4*b^2)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(15*d)
 + (a*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(5*d) + ((a + b*Sec[c + d*x])^4*
Tan[c + d*x])/(5*d)
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
```

```
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5d} + \frac{4}{5}\int \sec(c+dx)(b+a\sec(c+dx))(a+b\sec($$
$$= \frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{5d} + \frac{(a+b\sec(c+dx))^4\tan(c+dx)}{5d} + \frac{1}{5}\int \sec($$
$$= \frac{\left(3a^2+4b^2\right)(a+b\sec(c+dx))^2\tan(c+dx)}{15d} + \frac{a(a+b\sec(c+dx))^3\tan(c+dx)}{5d}$$
$$= \frac{ab\left(6a^2+29b^2\right)\sec(c+dx)\tan(c+dx)}{30d} + \frac{\left(3a^2+4b^2\right)(a+b\sec(c+dx))^2\tan(}{15d}$$
$$= \frac{ab\left(6a^2+29b^2\right)\sec(c+dx)\tan(c+dx)}{30d} + \frac{\left(3a^2+4b^2\right)(a+b\sec(c+dx))^2\tan(}{15d}$$
$$= \frac{ab\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{ab\left(6a^2+29b^2\right)\sec(c+dx)\tan(c+dx)}{30d} +$$
$$= \frac{ab\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2\left(3a^4+28a^2b^2+4b^4\right)\tan(c+dx)}{15d} + \frac{ab(}{}$$

Mathematica [A] time = 0.735435, size = 125, normalized size = 0.7

$$\frac{15ab\left(4a^2+3b^2\right)\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(20b^2\left(3a^2+b^2\right)\tan^2(c+dx) + 15ab\left(4a^2+3b^2\right)\sec(c+dx) + 30\left(6\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (15*a*b*(4*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(30*(a^4 + 6*a
^2*b^2 + b^4) + 15*a*b*(4*a^2 + 3*b^2)*Sec[c + d*x] + 30*a*b^3*Sec[c + d*x]
^3 + 20*b^2*(3*a^2 + b^2)*Tan[c + d*x]^2 + 6*b^4*Tan[c + d*x]^4))/(30*d)
```

Maple [A] time = 0.032, size = 225, normalized size = 1.3

$$\frac{a^4\tan(dx+c)}{d} + 2\frac{a^3b\sec(dx+c)\tan(dx+c)}{d} + 2\frac{a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d} + 4\frac{a^2b^2\tan(dx+c)}{d} + 2\frac{a^2b^2\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x)
```

```
[Out] a^4*tan(d*x+c)/d+2/d*a^3*b*sec(d*x+c)*tan(d*x+c)+2/d*a^3*b*ln(sec(d*x+c)+ta
n(d*x+c))+4/d*a^2*b^2*tan(d*x+c)+2/d*a^2*b^2*tan(d*x+c)*sec(d*x+c)^2+1/d*a*
b^3*tan(d*x+c)*sec(d*x+c)^3+3/2*a*b^3*sec(d*x+c)*tan(d*x+c)/d+3/2/d*a*b^3*l
n(sec(d*x+c)+tan(d*x+c))+8/15/d*b^4*tan(d*x+c)+1/5/d*b^4*tan(d*x+c)*sec(d*x
+c)^4+4/15/d*b^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.21169, size = 263, normalized size = 1.47

$$\frac{120\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{2}b^{2}+4\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)b^{4}-15\,ab^{3}\left(\frac{2\left(3\,\sin(d\ldots}{\sin(dx+c\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/60*(120*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 + 4*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*b^4 - 15*a*b^3*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x +
 c))/d
```

Fricas [A] time = 1.7504, size = 443, normalized size = 2.47

$$\frac{15\left(4\,a^{3}b+3\,ab^{3}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,a^{3}b+3\,ab^{3}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(30\,ab\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(15*(4*a^3*b + 3*a*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a
^3*b + 3*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*a*b^3*cos(d*x
 + c) + 2*(15*a^4 + 60*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 6*b^4 + 15*(4*a^3*
b + 3*a*b^3)*cos(d*x + c)^3 + 4*(15*a^2*b^2 + 2*b^4)*cos(d*x + c)^2)*sin(d*
x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^4\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x)**2, x)
```

Giac [B] time = 1.39396, size = 622, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(15*(4*a^3*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
*b + 3*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x +
1/2*c)^9 - 60*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 75*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 - 12
0*a^4*tan(1/2*d*x + 1/2*c)^7 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*a^2*b
^2*tan(1/2*d*x + 1/2*c)^7 + 30*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 40*b^4*tan(1/
2*d*x + 1/2*c)^7 + 180*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 116*b^4*tan(1/2*d*x + 1/2*c)^5 - 120*a^4*tan(1/2*d*x + 1/2*c)
^3 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3
- 30*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 40*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*
tan(1/2*d*x + 1/2*c) + 60*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 75*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.478 $\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=146

$$\frac{ab\left(19a^2+16b^2\right)\tan(c+dx)}{6d}+\frac{\left(24a^2b^2+8a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\left(26a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{24d}+$$

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*b*(19*a^2 +
 16*b^2)*Tan[c + d*x])/(6*d) + (b^2*(26*a^2 + 9*b^2)*Sec[c + d*x]*Tan[c + d
*x])/(24*d) + (7*a*b*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (b*(a +
b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.242574, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3830, 4002, 3997, 3787, 3770, 3767, 8}

$$\frac{ab\left(19a^2+16b^2\right)\tan(c+dx)}{6d}+\frac{\left(24a^2b^2+8a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\left(26a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{24d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*b*(19*a^2 +
 16*b^2)*Tan[c + d*x])/(6*d) + (b^2*(26*a^2 + 9*b^2)*Sec[c + d*x]*Tan[c + d
*x])/(24*d) + (7*a*b*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (b*(a +
b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 3830

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*
m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^
2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
```

```
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 3997

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(b*B*Cot[e
 + f*x]*(d*Csc[e + f*x])^n)/(f*(n + 1)), x] + Dist[1/(n + 1), Int[(d*Csc[e
+ f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x],
x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n,
-1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{b(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{4}\int \sec(c+dx)(a+b\sec(c+dx))^2\left(4a^2+\right.$$
$$= \frac{7ab(a+b\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{b(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{12}\int$$
$$= \frac{b^2\left(26a^2+9b^2\right)\sec(c+dx)\tan(c+dx)}{24d} + \frac{7ab(a+b\sec(c+dx))^2\tan(c+dx)}{12d}$$
$$= \frac{b^2\left(26a^2+9b^2\right)\sec(c+dx)\tan(c+dx)}{24d} + \frac{7ab(a+b\sec(c+dx))^2\tan(c+dx)}{12d}$$
$$= \frac{\left(8a^4+24a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b^2\left(26a^2+9b^2\right)\sec(c+dx)\tan(c+}{24d}$$
$$= \frac{\left(8a^4+24a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{ab\left(19a^2+16b^2\right)\tan(c+dx)}{6d} + \frac{b^2}{}$$

Mathematica [A] time = 0.515846, size = 101, normalized size = 0.69

$$\frac{3\left(24a^2b^2+8a^4+3b^4\right)\tanh^{-1}(\sin(c+dx)) + b\tan(c+dx)\left(32a\left(3\left(a^2+b^2\right)+b^2\tan^2(c+dx)\right)+9b\left(8a^2+b^2\right)\sec(c+\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]] + b*Tan[c + d*x]*(9*b
*(8*a^2 + b^2)*Sec[c + d*x] + 6*b^3*Sec[c + d*x]^3 + 32*a*(3*(a^2 + b^2) +
b^2*Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.034, size = 188, normalized size = 1.3

$$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d} + 4\,\frac{a^3b\tan(dx+c)}{d} + 3\,\frac{a^2b^2\sec(dx+c)\tan(dx+c)}{d} + 3\,\frac{a^2b^2\ln(\sec(dx+c)+\tan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b*tan(d*x+c)+3/d*a^2*b^2*sec(d*x+
c)*tan(d*x+c)+3/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/3*a*b^3*tan(d*x+c)/d+
```

```
4/3/d*a*b^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*b^4*tan(d*x+c)*sec(d*x+c)^3+3/8/d
*b^4*sec(d*x+c)*tan(d*x+c)+3/8/d*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.19612, size = 243, normalized size = 1.66

$$64\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)ab^{3}-3\,b^{4}\left(\frac{2\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 3*b^4*(2*(3*sin(d*x + c)
^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*a^4*log
(sec(d*x + c) + tan(d*x + c)) + 192*a^3*b*tan(d*x + c))/d
```

Fricas [A] time = 1.72338, size = 394, normalized size = 2.7

$$\frac{3\left(8\,a^{4}+24\,a^{2}b^{2}+3\,b^{4}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,a^{4}+24\,a^{2}b^{2}+3\,b^{4}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right.}{48\,d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(
32*a*b^3*cos(d*x + c) + 6*b^4 + 32*(3*a^3*b + 2*a*b^3)*cos(d*x + c)^3 + 9*(
8*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^{4}\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4*sec(c + d*x), x)
```

Giac [B] time = 1.3343, size = 486, normalized size = 3.33

$$3\left(8a^4 + 24a^2b^2 + 3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 3\left(8a^4 + 24a^2b^2 + 3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(96a^3b\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(8*a^4 + 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(96*a^
3*b*tan(1/2*d*x + 1/2*c)^7 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 96*a*b^3*t
an(1/2*d*x + 1/2*c)^7 - 15*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*a^3*b*tan(1/2*d
*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 160*a*b^3*tan(1/2*d*x +
 1/2*c)^5 - 9*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/2*c)^3
 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 9
*b^4*tan(1/2*d*x + 1/2*c)^3 - 96*a^3*b*tan(1/2*d*x + 1/2*c) - 72*a^2*b^2*ta
n(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.479 $\int (a + b\sec(c + dx))^4 \, dx$

Optimal. Leaf size=107

$$\frac{b^2 \left(17a^2 + 2b^2\right) \tan(c + dx)}{3d} + \frac{2ab \left(2a^2 + b^2\right) \tanh^{-1}(\sin(c + dx))}{d} + a^4x + \frac{4ab^3 \tan(c + dx) \sec(c + dx)}{3d} + \frac{b^2 \tan(c + dx}{}$$

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tan[c + d*x])/(3*d) + (4*a*b^3*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (b^2*(
a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.115748, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3782, 4048, 3770, 3767, 8}

$$\frac{b^2 \left(17a^2 + 2b^2\right) \tan(c + dx)}{3d} + \frac{2ab \left(2a^2 + b^2\right) \tanh^{-1}(\sin(c + dx))}{d} + a^4x + \frac{4ab^3 \tan(c + dx) \sec(c + dx)}{3d} + \frac{b^2 \tan(c + dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4,x]
```

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tan[c + d*x])/(3*d) + (4*a*b^3*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (b^2*(
a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^4\,dx = \frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\sec(c+dx))\left(3a^3+b\left(9a^2+2b^2\right)\sec(c+dx\right.$$
$$= \frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d}+\frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{6}\int \left(6a^4+12ab\left(2a^2\right.\right.$$
$$= a^4x+\frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d}+\frac{b^2(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\left(2ab\left(2a^2+b^2\right)\right)$$
$$= a^4x+\frac{2ab\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\sec(c+dx)\tan(c+dx)}{3d}+\frac{b^2(a+b\sec(c\right.}{}$$
$$= a^4x+\frac{2ab\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\left(17a^2+2b^2\right)\tan(c+dx)}{3d}+\frac{4ab^3\sec(c+dx)}{3d}$$

Mathematica [A] time = 0.284188, size = 77, normalized size = 0.72

$$\frac{6ab\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))+3b^2\tan(c+dx)\left(6a^2+2ab\sec(c+dx)+b^2\right)+3a^4dx+b^4\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (3*a^4*d*x + 6*a*b*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + 3*b^2*(6*a^2 + b^2
 + 2*a*b*Sec[c + d*x])*Tan[c + d*x] + b^4*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.031, size = 135, normalized size = 1.3

$$a^4x+\frac{a^4c}{d}+4\,\frac{a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d}+6\,\frac{a^2b^2\tan(dx+c)}{d}+2\,\frac{ab^3\sec(dx+c)\tan(dx+c)}{d}+2\,\frac{ab^3\ln(\sec(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4,x)
```

```
[Out] a^4*x+1/d*a^4*c+4/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*tan(d*x+c)+
2*a*b^3*sec(d*x+c)*tan(d*x+c)/d+2/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*b
^4*tan(d*x+c)+1/3/d*b^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.20451, size = 163, normalized size = 1.52

$$a^4x+\frac{\left(\tan(dx+c)^3+3\tan(dx+c)\right)b^4}{3\,d}-\frac{ab^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{d}+\frac{4\,a^3b\log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*b^4/d - a*b^3*(2*sin(d*x + c)
/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d +
4*a^3*b*log(sec(d*x + c) + tan(d*x + c))/d + 6*a^2*b^2*tan(d*x + c)/d
```

Fricas [A] time = 1.70564, size = 339, normalized size = 3.17

$$\frac{3\,a^4dx\cos(dx+c)^3+3\left(2\,a^3b+ab^3\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(2\,a^3b+ab^3\right)\cos(dx+c)^3\log(-\sin(dx+c}{3\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(sin(
d*x + c) + 1) - 3*(2*a^3*b + a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 (6*a*b^3*cos(d*x + c) + b^4 + 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x
```

```
+ c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.29683, size = 298, normalized size = 2.79

$$3(dx + c)a^4 + 6\left(2a^3b + ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 6\left(2a^3b + ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) - \frac{2\left(18a^2b^2\tan\left(\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 6*(2*a^3*b + a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*a^2*b^2
*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*
x + 1/2*c)^5 - 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^4*tan(1/2*d*x + 1/2*
c)^3 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c) + 6*a*b^3*tan(1/2*d*x + 1/2*c) + 3*b
^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.480 $\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=104

$$\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{2d}+\frac{b^2\left(12a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+4a^3bx+\frac{3ab^3\tan(c+dx)}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c}{2d}$$

```
[Out] 4*a^3*b*x + (b^2*(12*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*a^2
- b^2)*Sin[c + d*x])/(2*d) + (b^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d
) + (3*a*b^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.21183, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3842, 4076, 4047, 8, 4045, 3770}

$$\frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{2d}+\frac{b^2\left(12a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+4a^3bx+\frac{3ab^3\tan(c+dx)}{d}+\frac{b^2\sin(c+dx)(a+b\sec(c}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] 4*a^3*b*x + (b^2*(12*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*a^2
- b^2)*Sin[c + d*x])/(2*d) + (b^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d
) + (3*a*b^3*Tan[c + d*x])/d
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
 !(IGtQ[n, 2] && !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
```

```
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c+dx)(a+b\sec(c+dx))\left(a\left(2a^2\right.\right.$$

$$= \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{3ab^3\tan(c+dx)}{d} + \frac{1}{2}\int \cos(c+dx)\left(a^2\left(2\right.\right.$$

$$= \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{3ab^3\tan(c+dx)}{d} + \frac{1}{2}\int \cos(c+dx)\left(a^2\left(2\right.\right.$$

$$= 4a^3bx + \frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{2d} + \frac{b^2(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{3ab^3\,\mathrm{t}}{}$$

$$= 4a^3bx + \frac{b^2\left(12a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\left(2a^2-b^2\right)\sin(c+dx)}{2d} + \frac{b^2(a}{}$$

Mathematica [B] time = 0.520243, size = 280, normalized size = 2.69

$$\frac{\sec^2(c+dx)\left(\left(a^4+2b^4\right)\sin(c+dx)-12a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^2b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^2*(8*a^3*b*c + 8*a^3*b*d*x - 12*a^2*b^2*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]] - b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*a^2
*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^4*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + b*Cos[2*(c + d*x)]*(8*a^3*(c + d*x) - b*(12*a^2 + b^2)*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*(12*a^2 + b^2)*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + (a^4 + 2*b^4)*Sin[c + d*x] + 8*a*b^3*Sin[2*(c
+ d*x)] + a^4*Sin[3*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.048, size = 114, normalized size = 1.1

$$\frac{a^4\sin(dx+c)}{d}+4a^3bx+4\frac{a^3bc}{d}+6\frac{a^2b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+4\frac{ab^3\tan(dx+c)}{d}+\frac{b^4\sec(dx+c)\tan(dx}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^4,x)
```

```
[Out] a^4*sin(d*x+c)/d+4*a^3*b*x+4/d*a^3*b*c+6/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c)
)+4*a*b^3*tan(d*x+c)/d+1/2/d*b^4*sec(d*x+c)*tan(d*x+c)+1/2/d*b^4*ln(sec(d*x
+c)+tan(d*x+c))
```

Maxima [A] time = 1.34408, size = 155, normalized size = 1.49

$$\frac{16(dx+c)a^3b-b^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12a^2b^2(\log(\sin(dx+c)+1)-\log(\sin}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^3*b - b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.71756, size = 324, normalized size = 3.12

$$\frac{16\,a^3bdx\cos\left(dx+c\right)^2+\left(12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^3*b*d*x*cos(d*x + c)^2 + (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - (12*a^2*b^2 + b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*(2*a^4*cos(d*x + c)^2 + 8*a*b^3*cos(d*x + c) + b^4)*sin(d*x + c))/(d*co
s(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28211, size = 242, normalized size = 2.33

$$\frac{8\left(dx+c\right)a^3b+\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\left(12\,a^2b^2+b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(12\,a^2b^2+b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a^3*b + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2
 + 1) + (12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (12*a^2*b^2
 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(8*a*b^3*tan(1/2*d*x + 1/2*c
)^3 - b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b^3*tan(1/2*d*x + 1/2*c) - b^4*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.481 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=108

$$-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{2d}+\frac{1}{2}a^2x\left(a^2+12b^2\right)+\frac{3a^3b\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{4\ldots}{}$$

```
[Out] (a^2*(a^2 + 12*b^2)*x)/2 + (4*a*b^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*b*Sin
[c + d*x])/d + (a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d)
 - (b^2*(a^2 - 2*b^2)*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.217404, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4076, 4047, 8, 4045, 3770}

$$-\frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{2d}+\frac{1}{2}a^2x\left(a^2+12b^2\right)+\frac{3a^3b\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{4\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a^2*(a^2 + 12*b^2)*x)/2 + (4*a*b^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*b*Sin
[c + d*x])/d + (a^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2*d)
 - (b^2*(a^2 - 2*b^2)*Tan[c + d*x])/(2*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
```

```
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} + \frac{1}{2}\int \cos(c+dx)(a+b\sec(c+
$$

$$= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{2d} + \frac{1}{2}\int
$$

$$= \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d} - \frac{b^2\left(a^2-2b^2\right)\tan(c+dx)}{2d} + \frac{1}{2}\int
$$

$$= \frac{1}{2}a^2\left(a^2+12b^2\right)x + \frac{3a^3b\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)(a+b\sec(c+dx))^2\sin(c
$$

$$= \frac{1}{2}a^2\left(a^2+12b^2\right)x + \frac{4ab^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^3b\sin(c+dx)}{d} + \frac{a^2\cos(c+
$$

Mathematica [A] time = 0.670839, size = 119, normalized size = 1.1

$$\frac{2a\left(a\left(a^2+12b^2\right)(c+dx)-8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (2*a*(a*(a^2 + 12*b^2)*(c + d*x) - 8*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 8*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a^3*b*Sin[c +
 d*x] + a^4*Sin[2*(c + d*x)] + 4*b^4*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.05, size = 109, normalized size = 1.

$$\frac{a^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^4x}{2}+\frac{a^4c}{2d}+4\frac{a^3b\sin(dx+c)}{d}+6a^2b^2x+6\frac{a^2b^2c}{d}+4\frac{ab^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*cos(d*x+c)*sin(d*x+c)/d+1/2*a^4*x+1/2/d*a^4*c+4*a^3*b*sin(d*x+c)/d+
6*a^2*b^2*x+6/d*a^2*b^2*c+4/d*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^4*tan(d
*x+c)
```

Maxima [A] time = 1.4042, size = 122, normalized size = 1.13

$$\frac{(2dx+2c+\sin(2dx+2c))a^4+24(dx+c)a^2b^2+8ab^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+16a^3b\sin(dx}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^2*b^2 + 8*a*b^3*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^3*b*sin(d*x + c) + 4
*b^4*tan(d*x + c))/d
```

Fricas [A] time = 1.7507, size = 294, normalized size = 2.72

$$\frac{4\,ab^{3}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-4\,ab^{3}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(a^{4}+12\,a^{2}b^{2}\right)dx\cos\left(dx+c\right)+\left(a^{4}\cos\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a*b^3*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (a^4 + 12*a^2*b^2)*d*x*cos(d*x + c) + (a^4*cos(d*x + c
)^2 + 8*a^3*b*cos(d*x + c) + 2*b^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32051, size = 230, normalized size = 2.13

$$\frac{8\,ab^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,ab^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\left(a^{4}+12\,a^{2}b^{2}\right)\left(dx+c\right)-\frac{2\left(a^{4}\tan\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a*b^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
 + (a^4 + 12*a^2*b^2)*(d*x + c) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b*t
```

```
an(1/2*d*x + 1/2*c)^3 - a^4*tan(1/2*d*x + 1/2*c) - 8*a^3*b*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.482 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=115

$$\frac{a^2(2a^2+17b^2)\sin(c+dx)}{3d}+2abx(a^2+2b^2)+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+$$

```
[Out] 2*a*b*(a^2 + 2*b^2)*x + (b^4*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^
2)*Sin[c + d*x])/(3*d) + (4*a^3*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (a^2*C
os[c + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.239785, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4074, 4047, 8, 4045, 3770}

$$\frac{a^2(2a^2+17b^2)\sin(c+dx)}{3d}+2abx(a^2+2b^2)+\frac{4a^3b\sin(c+dx)\cos(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] 2*a*b*(a^2 + 2*b^2)*x + (b^4*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^
2)*Sin[c + d*x])/(3*d) + (4*a^3*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (a^2*C
os[c + d*x]^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
```

```
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int \cos^2(c+dx)(a+b\sec(c$$

$$= \frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}$$

$$= \frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d}$$

$$= 2ab\left(a^2+2b^2\right)x+\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{3d}+\frac{4a^3b\cos(c+dx)\sin(c+dx)}{3d}$$

$$= 2ab\left(a^2+2b^2\right)x+\frac{b^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\left(2a^2+17b^2\right)\sin(c+dx)}{3d}+\frac{4a^3$$

Mathematica [A] time = 0.157782, size = 128, normalized size = 1.11

$$\frac{24ab\left(a^2+2b^2\right)(c+dx)+9a^2\left(a^2+8b^2\right)\sin(c+dx)+12a^3b\sin(2(c+dx))+a^4\sin(3(c+dx))-12b^4\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (24*a*b*(a^2 + 2*b^2)*(c + d*x) - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*b^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*a^2*(a^2 + 8*b
^2)*Sin[c + d*x] + 12*a^3*b*Sin[2*(c + d*x)] + a^4*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.053, size = 131, normalized size = 1.1

$$\frac{\sin(dx+c)(\cos(dx+c))^2a^4}{3d}+\frac{2a^4\sin(dx+c)}{3d}+2\frac{a^3b\cos(dx+c)\sin(dx+c)}{d}+2a^3bx+2\frac{a^3bc}{d}+6\frac{a^2b^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^4+2/3*a^4*sin(d*x+c)/d+2*a^3*b*cos(d*x+c)*s
in(d*x+c)/d+2*a^3*b*x+2/d*a^3*b*c+6/d*a^2*b^2*sin(d*x+c)+4*a*b^3*x+4/d*a*b^
3*c+1/d*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.3234, size = 138, normalized size = 1.2

$$-\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4-6(2dx+2c+\sin(2dx+2c))a^3b-24(dx+c)ab^3-3b^4(\log(\sin(dx+c)+1)-}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 6*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*a^3*b - 24*(d*x + c)*a*b^3 - 3*b^4*(log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1)) - 36*a^2*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.7455, size = 239, normalized size = 2.08

$$\frac{3\,b^4\log\left(\sin\left(dx+c\right)+1\right)-3\,b^4\log\left(-\sin\left(dx+c\right)+1\right)+12\left(a^3b+2\,ab^3\right)dx+2\left(a^4\cos\left(dx+c\right)^2+6\,a^3b\cos\left(dx+c\right)+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*log(sin(d*x + c) + 1) - 3*b^4*log(-sin(d*x + c) + 1) + 12*(a^3*b
 + 2*a*b^3)*d*x + 2*(a^4*cos(d*x + c)^2 + 6*a^3*b*cos(d*x + c) + 2*a^4 + 18
*a^2*b^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33233, size = 286, normalized size = 2.49

$$\frac{3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+6\left(a^3b+2\,ab^3\right)(dx+c)+\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^3b\tan\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 6*(a^3*b + 2*a*b^3)*(d*x + c) + 2*(3*a^4*tan(1/2*d*x + 1/2*
c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 +
```

```
 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^4*t
an(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.483 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=145

$$\frac{4ab(2a^2+3b^2)\sin(c+dx)}{3d}+\frac{a^2(3a^2+22b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(24a^2b^2+3a^4+8b^4)+\frac{5a^3b\sin(c+dx)\mathrm{cc}}{6d}$$

```
[Out] ((3*a^4 + 24*a^2*b^2 + 8*b^4)*x)/8 + (4*a*b*(2*a^2 + 3*b^2)*Sin[c + d*x])/(
3*d) + (a^2*(3*a^2 + 22*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a^3*b*Co
s[c + d*x]^2*Sin[c + d*x])/(6*d) + (a^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.312865, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4074, 4047, 2637, 4045, 8}

$$\frac{4ab(2a^2+3b^2)\sin(c+dx)}{3d}+\frac{a^2(3a^2+22b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(24a^2b^2+3a^4+8b^4)+\frac{5a^3b\sin(c+dx)\mathrm{cc}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((3*a^4 + 24*a^2*b^2 + 8*b^4)*x)/8 + (4*a*b*(2*a^2 + 3*b^2)*Sin[c + d*x])/(
3*d) + (a^2*(3*a^2 + 22*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a^3*b*Co
s[c + d*x]^2*Sin[c + d*x])/(6*d) + (a^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^2*Sin[c + d*x])/(4*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sec(c+dx))^4\,dx &= \frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}+\frac{1}{4}\int \cos^3(c+dx)(a+b\sec(c+\\ &= \frac{5a^3b\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}\\ &= \frac{5a^3b\cos^2(c+dx)\sin(c+dx)}{6d}+\frac{a^2\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}\\ &= \frac{4ab\left(2a^2+3b^2\right)\sin(c+dx)}{3d}+\frac{a^2\left(3a^2+22b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{5a^3b\,\mathrm{co}}{}\\ &= \frac{1}{8}\left(3a^4+24a^2b^2+8b^4\right)x+\frac{4ab\left(2a^2+3b^2\right)\sin(c+dx)}{3d}+\frac{a^2\left(3a^2+22b^2\right)\cos(c}{8d}\end{aligned}$$

Mathematica [A] time = 0.22748, size = 104, normalized size = 0.72

$$\frac{12\left(24a^2b^2+3a^4+8b^4\right)(c+dx)+24a^2\left(a^2+6b^2\right)\sin(2(c+dx))+96ab\left(3a^2+4b^2\right)\sin(c+dx)+32a^3b\sin(3(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (12*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(c + d*x) + 96*a*b*(3*a^2 + 4*b^2)*Sin[c +
 d*x] + 24*a^2*(a^2 + 6*b^2)*Sin[2*(c + d*x)] + 32*a^3*b*Sin[3*(c + d*x)] +
 3*a^4*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.063, size = 116, normalized size = 0.8

$$\frac{1}{d}\left(a^4\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^3b\left((\cos(dx+c))^2+2\right)\sin(dx+c)}{3}+6a^2b^2(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a
^3*b*(cos(d*x+c)^2+2)*sin(d*x+c)+6*a^2*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d
*x+1/2*c)+4*a*b^3*sin(d*x+c)+b^4*(d*x+c))
```

Maxima [A] time = 1.19136, size = 147, normalized size = 1.01

$$\frac{3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^4-128\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3b+144(2dx+2c+\sin}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 - 128*(
sin(d*x + c)^3 - 3*sin(d*x + c))*a^3*b + 144*(2*d*x + 2*c + sin(2*d*x + 2*c
))*a^2*b^2 + 96*(d*x + c)*b^4 + 384*a*b^3*sin(d*x + c))/d
```

Fricas [A] time = 1.66033, size = 224, normalized size = 1.54

$$\frac{3\left(3a^4+24a^2b^2+8b^4\right)dx+\left(6a^4\cos(dx+c)^3+32a^3b\cos(dx+c)^2+64a^3b+96ab^3+9\left(a^4+8a^2b^2\right)\cos(dx+c)\right)\mathrm{si}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^3*b
*cos(d*x + c)^2 + 64*a^3*b + 96*a*b^3 + 9*(a^4 + 8*a^2*b^2)*cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31884, size = 429, normalized size = 2.96

$$\frac{3\left(3a^4+24a^2b^2+8b^4\right)(dx+c)-\frac{2\left(15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-96a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+72a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-96ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-9a^4\tan\left(\frac{1}{2}dx\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(15*a^4*tan(1/2*d*x + 1/
2*c)^7 - 96*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^
7 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^4*tan(1/2*d*x + 1/2*c)^5 - 160*a^
```

```
3*b*tan(1/2*d*x + 1/2*c)^5 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*a*b^3*
tan(1/2*d*x + 1/2*c)^5 + 9*a^4*tan(1/2*d*x + 1/2*c)^3 - 160*a^3*b*tan(1/2*d
*x + 1/2*c)^3 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*a*b^3*tan(1/2*d*x +
 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c) - 96*a^3*b*tan(1/2*d*x + 1/2*c) - 7
2*a^2*b^2*tan(1/2*d*x + 1/2*c) - 96*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^4)/d
```

3.484 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=173

$$-\frac{a^2\left(4a^2+27b^2\right)\sin^3(c+dx)}{15d}+\frac{\left(29a^2b^2+4a^4+5b^4\right)\sin(c+dx)}{5d}+\frac{ab\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}abx\left(3a^2\right.$$

```
[Out] (a*b*(3*a^2 + 4*b^2)*x)/2 + ((4*a^4 + 29*a^2*b^2 + 5*b^4)*Sin[c + d*x])/(5*
d) + (a*b*(3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*b*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d) + (a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*
Sin[c + d*x])/(5*d) - (a^2*(4*a^2 + 27*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.351732, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3841, 4074, 4047, 2635, 8, 4044, 3013}

$$-\frac{a^2\left(4a^2+27b^2\right)\sin^3(c+dx)}{15d}+\frac{\left(29a^2b^2+4a^4+5b^4\right)\sin(c+dx)}{5d}+\frac{ab\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}abx\left(3a^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*b*(3*a^2 + 4*b^2)*x)/2 + ((4*a^4 + 29*a^2*b^2 + 5*b^4)*Sin[c + d*x])/(5*
d) + (a*b*(3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*b*Cos[c
 + d*x]^3*Sin[c + d*x])/(5*d) + (a^2*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*
Sin[c + d*x])/(5*d) - (a^2*(4*a^2 + 27*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{1}{5}\int \cos^4(c+dx)(a+b\sec(c+$$
$$= \frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d} + \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}$$
$$= \frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d} + \frac{a^2\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}$$
$$= \frac{ab\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3b\cos^3(c+dx)\sin(c+dx)}{5d} + \frac{a^2\cos}{}$$
$$= \frac{1}{2}ab\left(3a^2+4b^2\right)x + \frac{ab\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3b\cos^3(c+dx)}{5d}$$
$$= \frac{1}{2}ab\left(3a^2+4b^2\right)x + \frac{\left(4a^4+29a^2b^2+5b^4\right)\sin(c+dx)}{5d} + \frac{ab\left(3a^2+4b^2\right)\cos(c+}{2d}$$

Mathematica [A] time = 0.497746, size = 133, normalized size = 0.77

$$\frac{30\left(36a^2b^2+5a^4+8b^4\right)\sin(c+dx)+a\left(240b\left(a^2+b^2\right)\sin(2(c+dx))+5\left(5a^3+24ab^2\right)\sin(3(c+dx))+30a^2b\sin(4(c+\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (30*(5*a^4 + 36*a^2*b^2 + 8*b^4)*Sin[c + d*x] + a*(360*a^2*b*c + 480*b^3*c
+ 360*a^2*b*d*x + 480*b^3*d*x + 240*b*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(5*a
^3 + 24*a*b^2)*Sin[3*(c + d*x)] + 30*a^2*b*Sin[4*(c + d*x)] + 3*a^3*Sin[5*(
c + d*x)]))/(240*d)
```

Maple [A] time = 0.062, size = 138, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+4a^3b\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^3*b*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*b^2*(cos(d*x+c)
```

```
^2+2)*sin(d*x+c)+4*a*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^4*sin(
d*x+c))
```

Maxima [A] time = 1.17717, size = 180, normalized size = 1.04

$$\frac{8\left(3\sin\left(dx+c\right)^{5}-10\sin\left(dx+c\right)^{3}+15\sin\left(dx+c\right)\right)a^{4}+15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{3}b-}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 15*
(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3*b - 240*(sin(d*
x + c)^3 - 3*sin(d*x + c))*a^2*b^2 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a
*b^3 + 120*b^4*sin(d*x + c))/d
```

Fricas [A] time = 1.74751, size = 285, normalized size = 1.65

$$\frac{15\left(3\,a^{3}b+4\,ab^{3}\right)dx+\left(6\,a^{4}\cos\left(dx+c\right)^{4}+30\,a^{3}b\cos\left(dx+c\right)^{3}+16\,a^{4}+120\,a^{2}b^{2}+30\,b^{4}+4\left(2\,a^{4}+15\,a^{2}b^{2}\right)\cos\left(dx\right.\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^3*b*cos(d*x
 + c)^3 + 16*a^4 + 120*a^2*b^2 + 30*b^4 + 4*(2*a^4 + 15*a^2*b^2)*cos(d*x +
c)^2 + 15*(3*a^3*b + 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37699, size = 574, normalized size = 3.32

$$\frac{15\left(3\,a^{3}b+4\,ab^{3}\right)(dx+c)+\frac{2\left(30\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+180\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-60\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+30\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*(d*x + c) + 2*(30*a^4*tan(1/2*d*x + 1/2*c)^9 -
 75*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*
a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 + 40*a^4*tan(1
/2*d*x + 1/2*c)^7 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 480*a^2*b^2*tan(1/2*d
*x + 1/2*c)^7 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 120*b^4*tan(1/2*d*x + 1/
2*c)^7 + 116*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x + 1/2*c)^
5 + 180*b^4*tan(1/2*d*x + 1/2*c)^5 + 40*a^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^3
*b*tan(1/2*d*x + 1/2*c)^3 + 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^3*
tan(1/2*d*x + 1/2*c)^3 + 120*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*tan(1/2*d*
x + 1/2*c) + 75*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*d*x + 1/2*
c) + 60*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^5)/d
```

3.485 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=213

$$-\frac{4ab\left(4a^2+5b^2\right)\sin^3(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\sin(c+dx)}{5d}+\frac{a^2\left(5a^2+32b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{\left(36a^2b^2+5a\right.}{}$$

```
[Out] ((5*a^4 + 36*a^2*b^2 + 8*b^4)*x)/16 + (4*a*b*(4*a^2 + 5*b^2)*Sin[c + d*x])/
(5*d) + ((5*a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (
a^2*(5*a^2 + 32*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (7*a^3*b*Cos[c +
 d*x]^4*Sin[c + d*x])/(15*d) + (a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*S
in[c + d*x])/(6*d) - (4*a*b*(4*a^2 + 5*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.380011, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3841, 4074, 4047, 2633, 4045, 2635, 8}

$$-\frac{4ab\left(4a^2+5b^2\right)\sin^3(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\sin(c+dx)}{5d}+\frac{a^2\left(5a^2+32b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{\left(36a^2b^2+5a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```


```
[Out] ((5*a^4 + 36*a^2*b^2 + 8*b^4)*x)/16 + (4*a*b*(4*a^2 + 5*b^2)*Sin[c + d*x])/
(5*d) + ((5*a^4 + 36*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (
a^2*(5*a^2 + 32*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (7*a^3*b*Cos[c +
 d*x]^4*Sin[c + d*x])/(15*d) + (a^2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*S
in[c + d*x])/(6*d) - (4*a*b*(4*a^2 + 5*b^2)*Sin[c + d*x]^3)/(15*d)
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^5(c+dx)(a+b\sec(c$$
$$= \frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d} + \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{6d}$$
$$= \frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d} + \frac{a^2\cos^5(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{6d}$$
$$= \frac{a^2\left(5a^2+32b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{7a^3b\cos^4(c+dx)\sin(c+dx)}{15d} + \frac{a^2}{}$$
$$= \frac{4ab\left(4a^2+5b^2\right)\sin(c+dx)}{5d} + \frac{\left(5a^4+36a^2b^2+8b^4\right)\cos(c+dx)\sin(c+dx)}{16d} +$$
$$= \frac{1}{16}\left(5a^4+36a^2b^2+8b^4\right)x + \frac{4ab\left(4a^2+5b^2\right)\sin(c+dx)}{5d} + \frac{\left(5a^4+36a^2b^2+8b\right.}{}$$

Mathematica [A] time = 0.470141, size = 156, normalized size = 0.73

$$\frac{60\left(36a^2b^2+5a^4+8b^4\right)(c+dx)+45a^2\left(a^2+4b^2\right)\sin(4(c+dx))+480ab\left(5a^2+6b^2\right)\sin(c+dx)+80ab\left(5a^2+4b^2\right)\mathrm{s}}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (60*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(c + d*x) + 480*a*b*(5*a^2 + 6*b^2)*Sin[c
+ d*x] + 15*(15*a^4 + 96*a^2*b^2 + 16*b^4)*Sin[2*(c + d*x)] + 80*a*b*(5*a^2
 + 4*b^2)*Sin[3*(c + d*x)] + 45*a^2*(a^2 + 4*b^2)*Sin[4*(c + d*x)] + 48*a^3
*b*Sin[5*(c + d*x)] + 5*a^4*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.064, size = 174, normalized size = 0.8

$$\frac{1}{d}\left(a^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^3b\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a^3*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a
```

```
^2*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a*b
^3*(cos(d*x+c)^2+2)*sin(d*x+c)+b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.22446, size = 230, normalized size = 1.08

$$-\frac{5\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^{4}-256\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d
*x + c))*a^3*b - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c)
)*a^2*b^2 + 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a*b^3 - 240*(2*d*x + 2*c
 + sin(2*d*x + 2*c))*b^4)/d
```

Fricas [A] time = 1.68636, size = 358, normalized size = 1.68

$$\frac{15\left(5\,a^{4}+36\,a^{2}b^{2}+8\,b^{4}\right)dx+\left(40\,a^{4}\cos\left(dx+c\right)^{5}+192\,a^{3}b\cos\left(dx+c\right)^{4}+512\,a^{3}b+640\,ab^{3}+10\left(5\,a^{4}+36\,a^{2}b^{2}\right)\cos\left(\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*d*x + (40*a^4*cos(d*x + c)^5 + 192*a
^3*b*cos(d*x + c)^4 + 512*a^3*b + 640*a*b^3 + 10*(5*a^4 + 36*a^2*b^2)*cos(d
*x + c)^3 + 64*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^2 + 15*(5*a^4 + 36*a^2*b^2
+ 8*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28985, size = 743, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*(d*x + c) - 2*(165*a^4*tan(1/2*d*x +
 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2*b^2*tan(1/2*d*x +
1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*b^4*tan(1/2*d*x + 1/2*c
)^11 - 25*a^4*tan(1/2*d*x + 1/2*c)^9 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^9 +
1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 3
60*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d*x + 1/2*c)^7 - 4992*a^3*b
*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a*b^3*t
an(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1/2*c)^7 - 450*a^4*tan(1/2*d*
x + 1/2*c)^5 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 360*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*b^4*tan(1/2*d*x + 1/2*
c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*a^3*b*tan(1/2*d*x + 1/2*c)^3 -
1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 3
60*b^4*tan(1/2*d*x + 1/2*c)^3 - 165*a^4*tan(1/2*d*x + 1/2*c) - 960*a^3*b*ta
n(1/2*d*x + 1/2*c) - 900*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*a*b^3*tan(1/2*d
*x + 1/2*c) - 120*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)
/d
```

3.486 $\int (a + b\sec(c + dx))^5 \, dx$

Optimal. Leaf size=158

$$\frac{ab^2\left(53a^2+20b^2\right)\tan(c+dx)}{6d}+\frac{b\left(40a^2b^2+40a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^3\left(58a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{24d}$$

```
[Out] a^5*x + (b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*
b^2*(53*a^2 + 20*b^2)*Tan[c + d*x])/(6*d) + (b^3*(58*a^2 + 9*b^2)*Sec[c + d
*x]*Tan[c + d*x])/(24*d) + (11*a*b^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(
12*d) + (b^2*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.236012, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3782, 4056, 4048, 3770, 3767, 8}

$$\frac{ab^2\left(53a^2+20b^2\right)\tan(c+dx)}{6d}+\frac{b\left(40a^2b^2+40a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^3\left(58a^2+9b^2\right)\tan(c+dx)\sec(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^5,x]
```

```
[Out] a^5*x + (b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*
b^2*(53*a^2 + 20*b^2)*Tan[c + d*x])/(6*d) + (b^3*(58*a^2 + 9*b^2)*Sec[c + d
*x]*Tan[c + d*x])/(24*d) + (11*a*b^2*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(
12*d) + (b^2*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
```

```
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4048

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(b*C*Csc[e +
f*x]*Cot[e + f*x])/(2*f), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A +
 C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, e, f, A, B, C}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^5\,dx = \frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{4}\int (a+b\sec(c+dx))^2\left(4a^3+3b\left(4a^2+b^2\right)\sec(c+
$$

$$= \frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{b^2(a+b\sec(c+dx))^3\tan(c+dx)}{4d} + \frac{1}{12}\int (a+b\,\mathrm{se}
$$

$$= \frac{b^3\left(58a^2+9b^2\right)\sec(c+dx)\tan(c+dx)}{24d} + \frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{b^2(a+b
$$

$$= a^5x + \frac{b^3\left(58a^2+9b^2\right)\sec(c+dx)\tan(c+dx)}{24d} + \frac{11ab^2(a+b\sec(c+dx))^2\tan(c+dx)}{12d} + \frac{b^2
$$

$$= a^5x + \frac{b\left(40a^4+40a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b^3\left(58a^2+9b^2\right)\sec(c+dx)\tan(c+
$$

$$= a^5x + \frac{b\left(40a^4+40a^2b^2+3b^4\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{ab^2\left(53a^2+20b^2\right)\tan(c+dx)}{6d} + \frac{b^3(
$$

Mathematica [A] time = 0.565435, size = 114, normalized size = 0.72

$$\frac{3b\left(40a^2b^2+40a^4+3b^4\right)\tanh^{-1}(\sin(c+dx))+3b^2\tan(c+dx)\left(b\left(40a^2+3b^2\right)\sec(c+dx)+40a\left(2a^2+b^2\right)+2b^3\sec^3(\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^5,x]
```

```
[Out] (24*a^5*d*x + 3*b*(40*a^4 + 40*a^2*b^2 + 3*b^4)*ArcTanh[Sin[c + d*x]] + 3*b
^2*(40*a*(2*a^2 + b^2) + b*(40*a^2 + 3*b^2)*Sec[c + d*x] + 2*b^3*Sec[c + d*
x]^3)*Tan[c + d*x] + 40*a*b^4*Tan[c + d*x]^3)/(24*d)
```

Maple [A] time = 0.042, size = 205, normalized size = 1.3

$$a^5x+\frac{a^5c}{d}+5\,\frac{a^4b\ln(\sec(dx+c)+\tan(dx+c))}{d}+10\,\frac{a^3b^2\tan(dx+c)}{d}+5\,\frac{a^2b^3\sec(dx+c)\tan(dx+c)}{d}+5\,\frac{a^2b^3\ln(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^5,x)
```

```
[Out] a^5*x+1/d*a^5*c+5/d*a^4*b*ln(sec(d*x+c)+tan(d*x+c))+10/d*a^3*b^2*tan(d*x+c)
+5/d*a^2*b^3*sec(d*x+c)*tan(d*x+c)+5/d*a^2*b^3*ln(sec(d*x+c)+tan(d*x+c))+10
/3/d*a*b^4*tan(d*x+c)+5/3/d*a*b^4*tan(d*x+c)*sec(d*x+c)^2+1/4/d*b^5*tan(d*x
+c)*sec(d*x+c)^3+3/8/d*b^5*sec(d*x+c)*tan(d*x+c)+3/8/d*b^5*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 1.21271, size = 267, normalized size = 1.69

$$a^5x+\frac{5\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)ab^4}{3\,d}-\frac{b^5\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 5/3*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^4/d - 1/16*b^5*(2*(3*sin(
d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*lo
```

```
g(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))/d - 5/2*a^2*b^3*(2*sin(d*x +
 c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
 + 5*a^4*b*log(sec(d*x + c) + tan(d*x + c))/d + 10*a^3*b^2*tan(d*x + c)/d
```

Fricas [A] time = 1.80867, size = 444, normalized size = 2.81

$$\frac{48\,a^5dx\cos\left(dx+c\right)^4+3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/48*(48*a^5*d*x*cos(d*x + c)^4 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*x
 + c)^4*log(sin(d*x + c) + 1) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(40*a*b^4*cos(d*x + c) + 6*b^5 + 80*(3*a^3
*b^2 + a*b^4)*cos(d*x + c)^3 + 3*(40*a^2*b^3 + 3*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(c+dx\right)\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**5, x)
```

Giac [B] time = 1.18383, size = 513, normalized size = 3.25

$$\frac{24\left(dx+c\right)a^5+3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(40\,a^4b+40\,a^2b^3+3\,b^5\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a^5 + 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - 3*(40*a^4*b + 40*a^2*b^3 + 3*b^5)*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) - 2*(240*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*a^2*b^3*tan(1/
2*d*x + 1/2*c)^7 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^7 - 15*b^5*tan(1/2*d*x +
1/2*c)^7 - 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 120*a^2*b^3*tan(1/2*d*x + 1
/2*c)^5 - 200*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 9*b^5*tan(1/2*d*x + 1/2*c)^5 +
 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 +
200*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*b^5*tan(1/2*d*x + 1/2*c)^3 - 240*a^3*b
^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a*b^4*tan(
1/2*d*x + 1/2*c) - 15*b^5*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

3.487 $\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d}-\frac{a(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan(c+dx)\sec(c+dx)}{2b^2d}$$

```
[Out] -(a*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^4*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3
*a^2 + 2*b^2)*Tan[c + d*x])/(3*b^3*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^
2*d) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.484825, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3851, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(3a^2+2b^2)\tan(c+dx)}{3b^3d}-\frac{a(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan(c+dx)\sec(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```


```
[Out] -(a*(2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) + (2*a^4*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3
*a^2 + 2*b^2)*Tan[c + d*x])/(3*b^3*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^
2*d) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d)
```


Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)\left(2a+2b\sec(c+dx)-3a\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{3b}$$

$$= -\frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec(c+dx)\left(-3a^2+ab\sec(c+dx)+2\left(3a^2+2b^2\right)\right.}{a+b\sec(c+dx)}}{6b^2}$$

$$= \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \int \frac{\sec(c+dx)}{}$$

$$= \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{a^4\int \frac{\sec(}{a+b\,\mathrm{se}}}{b}$$

$$= -\frac{a\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} +$$

$$= -\frac{a\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{\left(3a^2+2b^2\right)\tan(c+dx)}{3b^3d} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} +$$

$$= -\frac{a\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{2a^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d} + \frac{\left(3a^2+2b^2\right)\tan(c+dx}{3b^3d}$$

Mathematica [A] time = 2.35536, size = 258, normalized size = 1.64

$$\frac{\frac{1}{2}\sec^3(c+dx)\left(4b\sin(c+dx)\left(\left(3a^2+2b^2\right)\cos(2(c+dx))+3a^2-3ab\cos(c+dx)+4b^2\right)+9a\left(2a^2+b^2\right)\cos(c+dx)\left(\mathrm{l}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-24*a^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (Sec[c + d*x]^3*(9*a*(2*a^2 + b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*a*(2*
a^2 + b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
```

```
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*b*(3*a^2 + 4*b^2 - 3*a*b*Cos[c +
 d*x] + (3*a^2 + 2*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*b^4*d)
```

Maple [B] time = 0.056, size = 400, normalized size = 2.6

$$2\,\frac{a^4}{db^4\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-\frac{1}{3\,db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{a}{2\,db^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))-1/3/d/b/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1
)^2*a+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2-1
/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a-1/d/b/(tan(1/2*d*x+1/2*c)+1)-1/d*a^3/b^4*
ln(tan(1/2*d*x+1/2*c)+1)-1/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)-1/3/d/b/(tan(
1/2*d*x+1/2*c)-1)^3-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a-1/2/d/b/(tan(1/2*d
*x+1/2*c)-1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2/(tan(1/2*d*x+1/
2*c)-1)*a-1/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a^3/b^4*ln(tan(1/2*d*x+1/2*c)-1)
+1/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.92305, size = 1243, normalized size = 7.92

$$\left[\frac{6\sqrt{a^2-b^2}a^4\cos(dx+c)^3\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-3\left(2\,a^5-a^3b^2-ab^4\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(a^2 - b^2)*a^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(2*a
^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3*(2*a^5 - a^3
*b^2 - a*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*a^2*b^3 - 2*b^5
+ 2*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*
x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*sqrt(-a^
2 + b^2)*a^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c)))*cos(d*x + c)^3 - 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log
(sin(d*x + c) + 1) + 3*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3*log(-sin(d*
x + c) + 1) + 2*(2*a^2*b^3 - 2*b^5 + 2*(3*a^4*b - a^2*b^3 - 2*b^5)*cos(d*x
+ c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d
*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.3424, size = 386, normalized size = 2.46

$$\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^4}{\sqrt{-a^2+b^2}\,b^4}-\frac{3\left(2a^3+ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}+\frac{3\left(2a^3+ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^4/(sqrt(-a^
2 + b^2)*b^4) - 3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 +
3*(2*a^3 + a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(6*a^2*tan(1/2
*d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)
^5 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*t
an(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

3.488 $\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*a^3*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*Tan
[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.27524, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3851, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(2a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^3d}-\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^3*d) - (2*a^3*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*Tan
[c + d*x])/(b^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
```

```
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(a+b\sec(c+dx)-2a\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2b}$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)\left(ab+\left(2a^2+b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2b^2}$$

$$= -\frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{a^3\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^3} + \frac{\left(2a^2+b^2\right)\int \sec(c+dx)\,d}{2b^3}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{a^3\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}}{b^4}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2bd} - \frac{\left(2a^3\right)\mathrm{Subst}\Big(\int}{}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^3d} - \frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\sec(c+d}{}$$

Mathematica [A] time = 1.02696, size = 238, normalized size = 2.

$$\frac{\frac{8a^3\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right) - 4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((8*a^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^
2] - 4*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*b^2*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]] + 4*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ 2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + b^2/(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^2 - b^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a*b*T
an[c + d*x])/(4*b^3*d)
```

Maple [B] time = 0.052, size = 262, normalized size = 2.2

$$-2\,\frac{a^3}{db^3\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)\,(a-b)}}\right)-\frac{1}{2\,db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{1}{2\,db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*a^3/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tan(1/2*d*x+1/2*c)+1)
+1/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a+1/d/b^3*ln(tan(1/2*d*x+1/2*c)+1)*a^2+1/2/
d/b*ln(tan(1/2*d*x+1/2*c)+1)+1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tan(
1/2*d*x+1/2*c)-1)+1/d/b^2/(tan(1/2*d*x+1/2*c)-1)*a-1/d/b^3*ln(tan(1/2*d*x+1
/2*c)-1)*a^2-1/2/d/b*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.85141, size = 1076, normalized size = 9.04

$$\left[\frac{2\,\sqrt{a^2-b^2}a^3\cos(dx+c)^2\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+\left(2\,a^4-a^2b^2-b^4\right)\cos}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(a^2 - b^2)*a^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) - (a^2 -
 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c
```

```
) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*a^4
- a^2*b^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^4 - a^2*b^2 -
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^2*b^2 - b^4 - 2*(a^3*b -
a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2), -1/4
*(4*sqrt(-a^2 + b^2)*a^3*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c)))*cos(d*x + c)^2 - (2*a^4 - a^2*b^2 - b^4)*cos(d*x +
c)^2*log(sin(d*x + c) + 1) + (2*a^4 - a^2*b^2 - b^4)*cos(d*x + c)^2*log(-si
n(d*x + c) + 1) - 2*(a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*
x + c))/((a^2*b^3 - b^5)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.26155, size = 285, normalized size = 2.39

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3}{\sqrt{-a^2+b^2}\,b^3}-\frac{(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3}+\frac{(2a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3}-}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3/(sqrt(-a^
2 + b^2)*b^3) - (2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + (2*a
^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 2*(2*a*tan(1/2*d*x + 1/2
*c)^3 + b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^2))/d
```

3.489 $\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\tan(c+dx)}{bd}$$

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) + (2*a^2*ArcTanh[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + Tan[c + d*x]/(b*d
)
```

Rubi [A] time = 0.162578, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3790, 3789, 3770, 3831, 2659, 208}

$$\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```


```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) + (2*a^2*ArcTanh[(Sqrt[a - b]*Tan[(c +
 d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) + Tan[c + d*x]/(b*d
)
```


Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\tan(c+dx)}{bd} - \frac{a\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}\\
&= \frac{\tan(c+dx)}{bd} - \frac{a\int \sec(c+dx)\,dx}{b^2} + \frac{a^2\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^2}\\
&= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{\tan(c+dx)}{bd} + \frac{a^2\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^3}\\
&= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{\tan(c+dx)}{bd} + \frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^3d}\\
&= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}d} + \frac{\tan(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.37087, size = 115, normalized size = 1.35

$$\frac{-\frac{2a^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\tan}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-2*a^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b
^2] + a*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + b*Tan[c + d*x])/(b^2*d)
```

Maple [A] time = 0.046, size = 134, normalized size = 1.6

$$2\,\frac{a^2}{db^2\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-\frac{1}{db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{a}{db^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))-1/d/b/(tan(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)-
1/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.18313, size = 902, normalized size = 10.61

$$\left[\frac{\sqrt{a^2-b^2}a^2\cos(dx+c)\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-\left(a^3-ab^2\right)\cos(dx+c)}{2\left(a^2b^2-b^4\right)d\cos(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*a^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b
^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^3 - a*b^
2)*cos(d*x + c)*log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(-sin
(d*x + c) + 1) + 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x +
 c)), 1/2*(2*sqrt(-a^2 + b^2)*a^2*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (a^3 - a*b^2)*cos(d*x + c)*
log(sin(d*x + c) + 1) + (a^3 - a*b^2)*cos(d*x + c)*log(-sin(d*x + c) + 1) +
 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.21458, size = 205, normalized size = 2.41

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^2}{\sqrt{-a^2+b^2}b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}+\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}-\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2/(sqrt(-a^2 + b
^2)*b^2) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 + a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*
b))/d
```

3.490 $\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b*d) - (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.108916, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3789, 3770, 3831, 2659, 208}

$$\frac{\tanh^{-1}(\sin(c+dx))}{bd}-\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b*d) - (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\int \sec(c+dx)\,dx}{b} - \frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{a\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^2} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{bd} - \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}
\end{aligned}
$$

Mathematica [A] time = 0.074775, size = 102, normalized size = 1.5

$$
\frac{\frac{2a\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
 - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]])/(b*d)
```

Maple [A] time = 0.043, size = 88, normalized size = 1.3

$$-2\,{\frac {a}{db\sqrt {(a+b)(a-b)}}{\rm Artanh} \left({\frac {(a-b)\tan(1/2\,dx+c/2)}{\sqrt {(a+b)(a-b)}}}\right)}+{\frac {1}{db}\ln \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}}\right)+1\right)}-{\frac {1}{db}\ln \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*a/b/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))+1/d/b*ln(tan(1/2*d*x+1/2*c)+1)-1/d/b*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08207, size = 657, normalized size = 9.66

$$\left[\frac{\sqrt{a^2-b^2}a\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2\right.}{2\left(a^2b-b^3\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*a*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^2 - b^2)*log(sin(d*x +
c) + 1) - (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d), -1/2*(2*sq
rt(-a^2 + b^2)*a*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)
*sin(d*x + c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log(-sin(
d*x + c) + 1))/((a^2*b - b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.29399, size = 162, normalized size = 2.38

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{\sqrt{-a^2+b^2}\,b}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/(sqrt(-a^2 + b^
2)*b) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/b + log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b)/d
```

3.491 $\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
 + b]*d)
```

Rubi [A] time = 0.0589132, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3831, 2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
 + b]*d)
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}\end{aligned}$$

Mathematica [A] time = 0.0388272, size = 48, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2]*
d)
```

Maple [A] time = 0.04, size = 44, normalized size = 0.9

$$2\,\frac{1}{d\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69718, size = 419, normalized size = 8.55

$$\left[\frac{\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,\sqrt{a^2-b^2}d}, \frac{\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{\left(a^2-b^2\right)\sin(dx+c)}\right)}{\left(a^2-b^2\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 -
b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + b^2))/(sqrt(a^2 - b^2)*d), sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))/((a^2 - b^
2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.21241, size = 104, normalized size = 2.12

$$-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(2a-2b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*d)
```

3.492 $\int \frac{1}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```

Rubi [A] time = 0.0505581, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3783, 2659, 208}

$$\frac{x}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-1),x]
```


```
[Out] x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```


Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\sec(c+dx)}\,dx &= \frac{x}{a}-\frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a}\\
&= \frac{x}{a}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\\
&= \frac{x}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] time = 0.084559, size = 60, normalized size = 1.02

$$\frac{\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{d\sqrt{a^2-b^2}}+\frac{c}{d}+x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-1),x]
```

```
[Out] (c/d + x + (2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt
[a^2 - b^2]*d))/a
```

Maple [A] time = 0.05, size = 67, normalized size = 1.1

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}-2\,\frac{b}{da\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan\left(1/2\,dx+c/2\right)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a*arctan(tan(1/2*d*x+1/2*c))-2/d*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75805, size = 502, normalized size = 8.51

$$\left[\frac{2\left(a^2-b^2\right)dx+\sqrt{a^2-b^2}b\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\left(a^3-ab^2\right)d},\frac{\left(a^2-b^2\right)dx-\sqrt{-a^2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/((a^3 -
 a*b^2)*d), ((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*
(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.24557, size = 128, normalized size = 2.17

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}a}-\frac{dx+c}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^
2)*a) - (d*x + c)/a)/d
```

3.493 $\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sin(c+dx)}{ad}$$

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + Sin[c + d*x]/(a*d)
```

Rubi [A] time = 0.101524, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3853, 12, 3783, 2659, 208}

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + Sin[c + d*x]/(a*d)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{\sin(c+dx)}{ad} - \frac{\int \frac{b}{a+b\sec(c+dx)}\,dx}{a} \\
&= \frac{\sin(c+dx)}{ad} - \frac{b\int \frac{1}{a+b\sec(c+dx)}\,dx}{a} \\
&= -\frac{bx}{a^2} + \frac{\sin(c+dx)}{ad} + \frac{b\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2} \\
&= -\frac{bx}{a^2} + \frac{\sin(c+dx)}{ad} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d} \\
&= -\frac{bx}{a^2} + \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} + \frac{\sin(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.138573, size = 72, normalized size = 0.95

$$\frac{-\frac{2b^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\sin(c+dx) - b(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-(b*(c + d*x)) - (2*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + a*Sin[c + d*x])/(a^2*d)
```

Maple [A] time = 0.076, size = 102, normalized size = 1.3

$$2\,\frac{\tan(1/2\,dx+c/2)}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{b\arctan(\tan(1/2\,dx+c/2))}{da^2}+2\,\frac{b^2}{da^2\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+}{\sqrt{(a+b)\,(a-b)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d/a^2*b*arctan(tan(1/2*
d*x+1/2*c))+2/d*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79363, size = 599, normalized size = 7.88

$$\left[\frac{\sqrt{a^2-b^2}b^2\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\left(a^2b-b^3\right)dx+2\left(a^3-ab^2\right)\sin(}{2\left(a^4-a^2b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*b^2*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
 c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^2*b - b^3)*d*x + 2*
(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d), (sqrt(-a^2 + b^2)*b^2*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
2*b - b^3)*d*x + (a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.26341, size = 170, normalized size = 2.24

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{\sqrt{-a^2+b^2}\,a^2}-\frac{(dx+c)b}{a^2}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^2/(sqrt(-a^2 + b
^2)*a^2) - (d*x + c)*b/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 + 1)*a))/d
```

3.494 $\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2+2b^2\right)}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}$$

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - (b*Sin[c + d*x])/(a^2*d) + (
Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.277757, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3853, 4104, 3919, 3831, 2659, 208}

$$-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2+2b^2\right)}{2a^3}-\frac{b\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/S
qrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - (b*Sin[c + d*x])/(a^2*d) + (
Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int \frac{\cos(c+dx)\left(-2b+a\sec(c+dx)+b\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{2a}$$

$$= -\frac{b\sin(c+dx)}{a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int \frac{-a^2-2b^2-ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{2a^2}$$

$$= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{b\sin(c+dx)}{a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{b^3\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^3}$$

$$= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{b\sin(c+dx)}{a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{b^2\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^3}$$

$$= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{b\sin(c+dx)}{a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \text{ta}\right.}{a^3d}$$

$$= \frac{\left(a^2+2b^2\right)x}{2a^3} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} - \frac{b\sin(c+dx)}{a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

Mathematica [A] time = 0.229584, size = 97, normalized size = 0.88

$$\frac{2\left(a^2+2b^2\right)(c+dx) + \frac{8b^3\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a^2\sin(2(c+dx)) - 4ab\sin(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*(c + d*x) + (8*b^3*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] - 4*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)])
/(4*a^3*d)
```

Maple [B] time = 0.072, size = 222, normalized size = 2.

$$-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2} - 2\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^3 b}{da^2\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^2} + \frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/a^2/(1+tan(1/2*d
*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(
1/2*d*x+1/2*c)-2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*b+1/d/
a*arctan(tan(1/2*d*x+1/2*c))+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))*b^2-2/d*b^3
/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83928, size = 730, normalized size = 6.64

$$\left[\frac{\sqrt{a^2-b^2}b^3\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+\left(a^4+a^2b^2-2\,b^4\right)dx-\left(2\,a^3b-2\,ab^3\right.}{2\left(a^5-a^3b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a^2 - b^2)*b^3*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
 c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/
(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (a^4 + a^2*b^2 - 2*b^4)*
d*x - (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^
5 - a^3*b^2)*d), -1/2*(2*sqrt(-a^2 + b^2)*b^3*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4 + a^2*b^2 - 2*b^4)*d*x
+ (2*a^3*b - 2*a*b^3 - (a^4 - a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 -
```

```
a^3*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.24599, size = 240, normalized size = 2.18

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^3}{\sqrt{-a^2+b^2}\,a^3}-\frac{\left(a^2+2b^2\right)(dx+c)}{a^3}+\frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^3/(sqrt(-a^
2 + b^2)*a^3) - (a^2 + 2*b^2)*(d*x + c)/a^3 + 2*(a*tan(1/2*d*x + 1/2*c)^3 +
 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.495 $\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{bx\left(a^2+2b^2\right)}{2a^4}-\frac{b\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}$$

```
[Out] -(b*(a^2 + 2*b^2)*x)/(2*a^4) + (2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2*a^2 + 3*b^2)*Sin[c +
d*x])/(3*a^3*d) - (b*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2
*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.458556, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3853, 4104, 3919, 3831, 2659, 208}

$$\frac{(2a^2+3b^2)\sin(c+dx)}{3a^3d}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{bx\left(a^2+2b^2\right)}{2a^4}-\frac{b\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{\sin(c+dx)\cos^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(b*(a^2 + 2*b^2)*x)/(2*a^4) + (2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2*a^2 + 3*b^2)*Sin[c +
d*x])/(3*a^3*d) - (b*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2
*Sin[c + d*x])/(3*a*d)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} + \frac{\int \frac{\cos^2(c+dx)\left(-3b+2a\sec(c+dx)+2b\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{3a}$$

$$= -\frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{\int \frac{\cos(c+dx)\left(-2\left(2a^2+3b^2\right)-ab\sec(c+dx)+3b^2\sec\right.}{a+b\sec(c+dx)}}{6a^2}$$

$$= \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3ad} + \frac{\int \frac{-3b\left(a^2+2b^2\right)-}{a+b\,\mathrm{se}}}{6}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+d}{3ad}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+d}{3ad}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+d}{3ad}$$

$$= -\frac{b\left(a^2+2b^2\right)x}{2a^4} + \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{3a^3d} - \frac{b\cos(c+dx)\sin}{2a^2d}$$

Mathematica [A] time = 0.31313, size = 122, normalized size = 0.82

$$\frac{-6b\left(a^2+2b^2\right)(c+dx)+3a\left(3a^2+4b^2\right)\sin(c+dx)-\frac{24b^4\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-3a^2b\sin(2(c+dx))+a^3\sin(3(c+dx))}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-6*b*(a^2 + 2*b^2)*(c + d*x) - (24*b^4*ArcTanh[((-a + b)*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sin[c + d*x] - 3*a
^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*a^4*d)
```

Maple [B] time = 0.07, size = 367, normalized size = 2.5

$$2\,\frac{(\tan(1/2\,dx+c/2))^5}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^3} + \frac{b}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3} + 2\,\frac{(\tan(1/2\,dx+c/2))^5\,b^2}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+1/d/a^2/(1+tan(1/2*d*
x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*b+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan
(1/2*d*x+1/2*c)^5*b^2+4/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)
^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*b^2+2/d/a/(1+tan
(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)+2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^3*
tan(1/2*d*x+1/2*c)*b^2-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c
)*b-1/d/a^2*b*arctan(tan(1/2*d*x+1/2*c))-2/d/a^4*arctan(tan(1/2*d*x+1/2*c))
*b^3+2/d*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87652, size = 875, normalized size = 5.91

$$\left[\frac{3\sqrt{a^2-b^2}b^4\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-3\left(a^4b+a^2b^3-2\,b^5\right)dx+\left(4\,a^5+\right.}{6\left(a^6-a^4b^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(a^2 - b^2)*b^4*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 3*(a^4*b + a^2*b^3 - 2
*b^5)*d*x + (4*a^5 + 2*a^3*b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x + c)^2
```

```
 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d), 1/6
*(6*sqrt(-a^2 + b^2)*b^4*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c))) - 3*(a^4*b + a^2*b^3 - 2*b^5)*d*x + (4*a^5 + 2*a^3*
b^2 - 6*a*b^4 + 2*(a^5 - a^3*b^2)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34216, size = 336, normalized size = 2.27

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^4}{\sqrt{-a^2+b^2}a^4}-\frac{3\left(a^2b+2b^3\right)(dx+c)}{a^4}+\frac{2\left(6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6b^2\tan\left(\right.\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^4/(sqrt(-a^
2 + b^2)*a^4) - 3*(a^2*b + 2*b^3)*(d*x + c)/a^4 + 2*(6*a^2*tan(1/2*d*x + 1/
2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*a^
2*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*
x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

3.496 $\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d}+\frac{\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(4a^2b^2+3a^4+8\right.}{8a^5}$$

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) - (b*(2*a^2 + 3*
b^2)*Sin[c + d*x])/(3*a^4*d) + ((3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) - (b*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (Cos[c + d*x]^3*Sin
[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.686522, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3853, 4104, 3919, 3831, 2659, 208}

$$-\frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d}+\frac{\left(3a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(4a^2b^2+3a^4+8\right.}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]*d) - (b*(2*a^2 + 3*
b^2)*Sin[c + d*x])/(3*a^4*d) + ((3*a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) - (b*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (Cos[c + d*x]^3*Sin
[c + d*x])/(4*a*d)
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int \frac{\cos^3(c+dx)\left(-4b+3a\sec(c+dx)+3b\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{4a}$$

$$= -\frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \frac{\cos^2(c+dx)\left(-3\left(3a^2+4b^2\right)-ab\sec(c+dx)+8b}{a+b\sec(c+dx)}}{12a^2}$$

$$= \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

$$= -\frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b\cos^2(c+dx)\sin(c+dx)}{3a^2d}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b}{}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b}{}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{\left(3a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{b}{}$$

$$= \frac{\left(3a^4+4a^2b^2+8b^4\right)x}{8a^5} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}d} - \frac{b\left(2a^2+3b^2\right)\sin(c+dx)}{3a^4d} + \frac{(3a^2+}{}$$

Mathematica [A] time = 0.563935, size = 153, normalized size = 0.79

$$\frac{12\left(4a^2b^2+3a^4+8b^4\right)(c+dx) - 24ab\left(3a^2+4b^2\right)\sin(c+dx) + 24a^2\left(a^2+b^2\right)\sin(2(c+dx)) + \frac{192b^5\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}}}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(c + d*x) + (192*b^5*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*Si
n[c + d*x] + 24*a^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a^3*b*Sin[3*(c + d*x)]
 + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

Maple [B] time = 0.083, size = 672, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] -5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-2/d/a^2/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*
tan(1/2*d*x+1/2*c)^7*b^2-2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2
*c)^7*b^3+3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-10/3/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b-6/d/a^4/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^3-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan
(1/2*d*x+1/2*c)^5*b^2-3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)
^3+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^2-10/3/d/a^2/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b-6/d/a^4/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^3+5/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^2-2/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b-2/d/a^4/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^3+3/4/d/a*arctan(tan(1/2*d*x+1/2*c))+1/d/a
^3*arctan(tan(1/2*d*x+1/2*c))*b^2+2/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^4-2/
d*b^5/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b)
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98027, size = 1057, normalized size = 5.48

$$\left[\frac{12\sqrt{a^2-b^2}b^5\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+3\left(3\,a^6+a^4b^2+4\,a^2b^4-8\,b^6\right)dx\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(12*sqrt(a^2 - b^2)*b^5*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d
*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b
^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 3*(3*a^6 + a^4*b^2 +
 4*a^2*b^4 - 8*b^6)*d*x - (16*a^5*b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a^4*b
^2)*cos(d*x + c)^3 + 8*(a^5*b - a^3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a^4*b^
2 - 4*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d), -1/24*(24*s
qrt(-a^2 + b^2)*b^5*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - 3*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*d*x + (16*a^5*
b + 8*a^3*b^3 - 24*a*b^5 - 6*(a^6 - a^4*b^2)*cos(d*x + c)^3 + 8*(a^5*b - a^
3*b^3)*cos(d*x + c)^2 - 3*(3*a^6 + a^4*b^2 - 4*a^2*b^4)*cos(d*x + c))*sin(d
*x + c))/((a^7 - a^5*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.28458, size = 531, normalized size = 2.75

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)b^5}{\sqrt{-a^2+b^2}\,a^5}-\frac{3\left(3a^4+4a^2b^2+8b^4\right)(dx+c)}{a^5}+\frac{2\left(15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24a^2b\tan\left(\frac{1}{2}dx+\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*ta
n(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b^5/(sqrt(-
a^2 + b^2)*a^5) - 3*(3*a^4 + 4*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 + 2*(15*a^3*t
an(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*
d*x + 1/2*c)^7 + 24*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*tan(1/2*d*x + 1/2*c)
^5 + 40*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72
*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b*tan(1
/2*d*x + 1/2*c)^3 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*b^3*tan(1/2*d*x +
1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) - 12
*a*b^2*tan(1/2*d*x + 1/2*c) + 24*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x +
1/2*c)^2 + 1)^4*a^4))/d
```

3.497 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=222

$$-\frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3\left(3a^2-4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c}{bd\left(a^2-b^2\right.}$$

```
[Out] ((6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(3*a^2 - 4*b^2)*Ar
cTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a +
b)^(3/2)*d) - (a*(3*a^2 - 2*b^2)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*a^
2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*
x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.612422, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3845, 4092, 4082, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3d\left(a^2-b^2\right)}+\frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3\left(3a^2-4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c}{bd\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ((6*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(3*a^2 - 4*b^2)*Ar
cTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a +
b)^(3/2)*d) - (a*(3*a^2 - 2*b^2)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*a^
2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*
x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
```

```
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-ab\sec(c+dx)-\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-a\left(3a^2-b^2\right)\right.}{}}{}$$

$$= -\frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c}{b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= -\frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c}{b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= \frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\sec(c+dx)\tan(}{2b^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(3a^2-2b^2\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\sec(c+dx)\tan(}{2b^2\left(a^2-b^2\right)d}$$

$$= \frac{\left(6a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a^3\left(3a^2-4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d} - \frac{a\left(3a^2-2b\right.}{b^3\left(a^2\right.}$$

Mathematica [A] time = 6.12183, size = 357, normalized size = 1.61

$$\frac{a^4\sin(c+dx)}{b^3d(b-a)(a+b)(a\cos(c+dx)+b)} + \frac{2a^3\left(4b^2-3a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}\left(b^2-a^2\right)} + \frac{\left(-6a^2-b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\mathrm{s}\right.}{2b^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^3*(-3*a^2 + 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(b^4*Sqrt[a^2 - b^2]*(-a^2 + b^2)*d) + ((-6*a^2 - b^2)*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/(2*b^4*d) + ((6*a^2 + b^2)*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]])/(2*b^4*d) + 1/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^2) - (2*a*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])) - 1/(4*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (2*a*Sin[(c + d
*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (a^4*Sin[c + d*x])/
(b^3*(-a + b)*(a + b)*d*(b + a*Cos[c + d*x]))
```

Maple [A] time = 0.072, size = 405, normalized size = 1.8

$$2\,{\frac {{a}^{4}\tan \left(1/2\,dx+c/2 \right) }{d{b}^{3} \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}a- \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}b-a-b \right) }}-6\,{\frac {{a}^{5}}{d{b}^{4} \left(a+b \right) \left(a-b \right) \sqrt { \left(a+b \right) \left(a-b \right) }}{\rm Artanh} \left(

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.95439, size = 2024, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((3*a^6 - 4*a^4*b^2)*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*x
 + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + ((6*a^7 - 11*a^5*b^2 + 4*
a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*
cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 +
a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*cos(d*x +
c)^2)*log(-sin(d*x + c) + 1) + 2*(a^4*b^3 - 2*a^2*b^5 + b^7 - 2*(3*a^6*b -
5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos
(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 +
(a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2), -1/4*(4*((3*a^6 - 4*a^4*b^2)
*cos(d*x + c)^3 + (3*a^5*b - 4*a^3*b^3)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (
(6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4
*b^3 + 4*a^2*b^5 + b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*a^7 - 1
1*a^5*b^2 + 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (6*a^6*b - 11*a^4*b^3 + 4*a
^2*b^5 + b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^4*b^3 - 2*a^2*b
^5 + b^7 - 2*(3*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 - 3*(a^5*b^2
- 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^4 - 2*a^3*b^6 + a*
b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.35202, size = 404, normalized size = 1.82

$$\frac{\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{4\left(3\,a^5-4\,a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{\left(6\,a^2+b^2\right)\cdots}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2))
+ (6*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - (6*a^2 + b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(
1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) + b*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

3.498 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{(2a^2-b^2)\tan(c+dx)}{b^2d(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{2a\tanh^{-1}(\sin(}{b^3d}$$

```
[Out] (-2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*a^2*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d)
 + ((2*a^2 - b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]*Tan
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.346423, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3845, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(2a^2-b^2)\tan(c+dx)}{b^2d(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c+dx)\sec(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{2a\tanh^{-1}(\sin(}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (-2*a*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*a^2*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d)
 + ((2*a^2 - b^2)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]*Tan
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
```

```
]))
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(a^2-ab\sec(c+dx)-\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-b^2\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(a^2b+2a\left(a^2-b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-b^2\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{(2a)\int\sec(c+dx)\,dx}{b^3} + \frac{\left(a^2\left(2a\right.\right.}{}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{\left(2a^2-b^2\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(a^2\right.}{}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{\left(2a^2-b^2\right)\tan(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2a\right.}{}$$

$$= -\frac{2a\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{2a^2\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{\left(2a^2-b^2\right)\tan(c\ldots}{b^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 1.41855, size = 162, normalized size = 0.99

$$\frac{-\frac{2a^2\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a^3b\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} + 2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2a\log\left(\sin\left(\frac{1}{2}(\ldots\right.\right.}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(3/2) + 2*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*a*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*Sin[c + d*x])/((a - b)*(a
 + b)*(b + a*Cos[c + d*x])) + b*Tan[c + d*x])/(b^3*d)
```

Maple [A] time = 0.058, size = 275, normalized size = 1.7

$$-2\frac{a^{3}\tan\left(1/2\,dx+c/2\right)}{db^{2}\left(a^{2}-b^{2}\right)\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)}+4\frac{a^{4}}{db^{3}\left(a+b\right)\left(a-b\right)\sqrt{\left(a+b\right)\left(a-b\right)}}\mathrm{Artanh}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b-a-b)+4/d*a^4/b^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-6/d*a^2/b/(a+b)/(a-b)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/d/b^2/(tan
(1/2*d*x+1/2*c)+1)-2/d*a/b^3*ln(tan(1/2*d*x+1/2*c)+1)+2/d*a/b^3*ln(tan(1/2*
d*x+1/2*c)-1)-1/d/b^2/(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.21063, size = 1704, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x +
 c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*
cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*((a^6 - 2*a^4*b^2 + a^2*b^4
```

```
)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x +
c) + 1) + 2*((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^
3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b^2 - 2*a^2*b^4 +
b^6 + (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 -
 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*
x + c)), (((2*a^5 - 3*a^3*b^2)*cos(d*x + c)^2 + (2*a^4*b - 3*a^2*b^3)*cos(d
*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a
^2 - b^2)*sin(d*x + c))) - ((a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a
^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((a^6 - 2*a
^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)
)*log(-sin(d*x + c) + 1) + (a^4*b^2 - 2*a^2*b^4 + b^6 + (2*a^5*b - 3*a^3*b^
3 + a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos
(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.32944, size = 447, normalized size = 2.73

$$2\left(\frac{\left(2a^4-3a^2b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^2b^3-b^5\right)\sqrt{-a^2+b^2}}-\frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*
```

```
b*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x +
 1/2*c)^3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2
*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^
4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2
 - b^4)) - a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^3)/d
```

3.499 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=117

$$-\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) - (2*a*(a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - (a^2*Tan
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.218582, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3839, 3998, 3770, 3831, 2659, 208}

$$-\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) - (2*a*(a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - (a^2*Tan
[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-ab-\left(a^2-b^2\right)\sec(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \sec(c+dx)\,dx}{b^2} - \frac{\left(a\left(a^2-2b^2\right)\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2 d} - \frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(a\left(a^2-2b^2\right)\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^3\left(a^2-b^2\right)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2 d} - \frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a\left(a^2-2b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1-\cdots}\right.}{b^3\left(a^2-b\cdots\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2 d} - \frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d} - \frac{a^2\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c\cdots\right.}$$

Mathematica [A] time = 0.356342, size = 146, normalized size = 1.25

$$\frac{\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{a^2 b\sin(c+dx)}{(b-a)(a+b)(a\cos(c+dx)+b)} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\cdots\right.}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a*(a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + (a^2*b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a
*Cos[c + d*x])))/(b^2*d)
```

Maple [B] time = 0.059, size = 225, normalized size = 1.9

$$2\,\frac{a^2\tan(1/2\,dx+c/2)}{db\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2 a - (\tan(1/2\,dx+c/2))^2 b - a - b\right)} - 2\,\frac{a^3}{db^2(a+b)(a-b)\sqrt{(a+b)(a-b)}}\text{Artanh}\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)-2/d*a^3/b^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+4/d*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2
)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+1/d/b^2*ln(tan(1/2*
d*x+1/2*c)+1)-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.10252, size = 1346, normalized size = 11.5

$$\left[\frac{\left(a^3b-2\,ab^3+\left(a^4-2\,a^2b^2\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*lo
g((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b
*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + b^2)) + (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)
*cos(d*x + c))*log(sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*
a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^4*b - a^2*b^3)
*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2
*a^2*b^5 + b^7)*d), -1/2*(2*(a^3*b - 2*a*b^3 + (a^4 - 2*a^2*b^2)*cos(d*x +
```

```
c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 -
b^2)*sin(d*x + c))) - (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*
cos(d*x + c))*log(sin(d*x + c) + 1) + (a^4*b - 2*a^2*b^3 + b^5 + (a^5 - 2*a
^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(a^4*b - a^2*b^3)*
sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 - 2*
a^2*b^5 + b^7)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.35248, size = 274, normalized size = 2.34

$$\frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^2b-b^3)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{2\left(a^3-2\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^2-b^4)\sqrt{-a^2+b^2}}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*ta
n(1/2*d*x + 1/2*c)^2 - a - b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) + log(abs(
tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2)/d
```

3.500 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] (-2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(
a + b)^(3/2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.126998, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3836, 12, 3831, 2659, 208}

$$\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(
a + b)^(3/2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{\int \frac{b\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{-a^2+b^2}\\
&= \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{b\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2}\\
&= \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2-b^2}\\
&= \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\
&= -\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}+\frac{a\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.196392, size = 83, normalized size = 0.98

$$\frac{\frac{2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/
2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

Maple [A] time = 0.051, size = 118, normalized size = 1.4

$$\frac{1}{d}\left(-2\,\frac{a\tan(1/2\,dx+c/2)}{\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)}-2\,\frac{b}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)-2*b/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75816, size = 745, normalized size = 8.76

$$\left[-\frac{\left(ab\cos\left(dx+c\right)+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-2\left(a^3-ab^2\right)\mathrm{si}}{2\left(\left(a^5-2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b-2\,a^2b^3+b^5\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*b*cos(d*x + c) + b^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*
x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*
(a^3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^
4*b - 2*a^2*b^3 + b^5)*d), -((a*b*cos(d*x + c) + b^2)*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
3 - a*b^2)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b
 - 2*a^2*b^3 + b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.29392, size = 203, normalized size = 2.39

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*b/((a^2 - b^2)*sq
rt(-a^2 + b^2)) + a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan
(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

3.501 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=86

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

```
[Out] (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
 + b)^(3/2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.103098, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3833, 12, 3831, 2659, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
 + b)^(3/2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= -\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\int \frac{a\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{a\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{a\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b\left(a^2-b^2\right)}\\
&= -\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b\left(a^2-b^2\right)d}\\
&= \frac{2a\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.232228, size = 83, normalized size = 0.97

$$\frac{\frac{b\sin(c+dx)}{(b-a)(a+b)(a\cos(c+dx)+b)}-\frac{2a\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*a*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3
/2) + (b*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])))/d
```

Maple [A] time = 0.051, size = 118, normalized size = 1.4

$$\frac{1}{d}\left(2\,\frac{b\tan(1/2\,dx+c/2)}{\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)}+2\,\frac{a}{(a+b)\,(a-b)\,\sqrt{(a+b)\,(a-b)}}\mathrm{Artanh}\left(\frac{(a-}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(2*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)+2*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.80353, size = 744, normalized size = 8.65

$$\left[-\frac{\left(a^2\cos\left(dx+c\right)+ab\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+2\left(a^2b-b^3\right)\sin}{2\left(\left(a^5-2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b-2\,a^2b^3+b^5\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (
a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*
x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*
(a^2*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^
4*b - 2*a^2*b^3 + b^5)*d), ((a^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^2
*b - b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b
- 2*a^2*b^3 + b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.29836, size = 203, normalized size = 2.36

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)a}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a/((a^2 - b^2)*sqrt
(-a^2 + b^2)) - b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c)^2 - a - b)*(a^2 - b^2)))/d
```

3.502 $\int \frac{1}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2b\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{x}{a^2}$$

```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tan[c + d*x])/(a*(a^2 - b^2
)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.168731, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 3919, 3831, 2659, 208}

$$-\frac{2b\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tan[c + d*x])/(a*(a^2 - b^2
)*d*(a + b*Sec[c + d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+b\sec(c+dx))^2}\,dx &= \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+b^2+ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)} \\
&= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2\left(a^2-b^2\right)} \\
&= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2a^2-b^2\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2\left(a^2-b^2\right)} \\
&= \frac{x}{a^2} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(c\right.\right.}{a^2\left(a^2-b^2\right)d} \\
&= \frac{x}{a^2} - \frac{2b\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.441932, size = 138, normalized size = 1.27

$$\frac{\frac{b\left(\left(a^2-b^2\right)(c+dx)+ab\sin(c+dx)\right)+a\left(a^2-b^2\right)(c+dx)\cos(c+dx)}{a\cos(c+dx)+b}-\frac{2b\left(b^2-2a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{a^2d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-2),x]
```

```
[Out] ((-2*b*(-2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] + (a*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + b*((a^2 - b^2)*(
c + d*x) + a*b*Sin[c + d*x]))/(b + a*Cos[c + d*x]))/(a^2*(a - b)*(a + b)*d)
```

Maple [B] time = 0.061, size = 204, normalized size = 1.9

$$2\,{\frac{\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) }{d{a}^{2}}}-2\,{\frac{{b}^{2}\tan \left(1/2\,dx+c/2 \right) }{da \left({a}^{2}-{b}^{2} \right) \left(\left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}a- \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2}b-a-b \right) }}-4\,{\frac{}{d \left(a+b \right) \left(a-b \right. }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*arctan(tan(1/2*d*x+1/2*c))-2/d*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-4/d*b/(a+b)/(a-b)/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d*b^3
/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91523, size = 1053, normalized size = 9.66

$$\left[\frac{2\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)dx\cos\left(dx+c\right)+2\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)dx+\left(2\,a^{2}b^{2}-b^{4}+\left(2\,a^{3}b-ab^{3}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\log}{2\left(\left(a^{7}-2\,a^{5}b^{2}+a^{3}b^{4}\right)d\cos\left(dx+c\right)+\left(a^{6}b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + 2*(a^4*b - 2*a^2*b^3 +
 b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(a^2 - b
^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b
^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + b^2)) + 2*(a^3*b^2 - a*b^4)*sin(d*x + c))/((a^7 - 2*a^
5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((a^5 -
 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x - (2*a
^2*b^2 - b^4 + (2*a^3*b - a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^3*b^2 -
 a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b
- 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-2), x)
```

Giac [A] time = 1.16389, size = 242, normalized size = 2.22

$$-\frac{\frac{2\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{3}-ab^{2}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a-b\right)}+\frac{2\left(2\,a^{2}b-b^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{4}-a^{2}b^{2}\right)\sqrt{-a^{2}+b^{2}}}-\frac{dx+c}{a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*b^2*tan(1/2*d*x + 1/2*c)/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 - a - b)) + 2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) - (d*x +
c)/a^2)/d
```

3.503 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=146

$$\frac{(a^2-2b^2)\sin(c+dx)}{a^2d(a^2-b^2)}+\frac{2b^2(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{2bx}{a^3}$$

```
[Out] (-2*b*x)/a^3 + (2*b^2*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2 - 2*b^2)*Sin[c
+ d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Se
c[c + d*x]))
```

Rubi [A] time = 0.32812, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3847, 4104, 3919, 3831, 2659, 208}

$$\frac{(a^2-2b^2)\sin(c+dx)}{a^2d(a^2-b^2)}+\frac{2b^2(3a^2-2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin(c+dx)}{ad(a^2-b^2)(a+b\sec(c+dx))}-\frac{2bx}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (-2*b*x)/a^3 + (2*b^2*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2 - 2*b^2)*Sin[c
+ d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Se
c[c + d*x]))
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-a^2+2b^2+ab\sec(c+dx)-b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-2b\left(a^2-b^2\right)+ab^2\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(b^2\left(3a^2-2b^2\right)\right)\int \frac{\mathrm{sec}}{a+b\,\mathrm{s}}}{a^3\left(a^2-b^2\right)}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(b\left(3a^2-2b^2\right)\right)\int \frac{}{1+\frac{a\,\mathrm{co}}{}}}{a^3\left(a^2-b^2\right)}$$

$$= -\frac{2bx}{a^3} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2b\left(3a^2-2b^2\right)\right)\mathrm{Subst}}{}$$

$$= -\frac{2bx}{a^3} + \frac{2b^2\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(a^2-2b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\,\mathrm{s}}{a\left(a^2-b^2\right)\,\mathrm{a}}$$

Mathematica [A] time = 0.727932, size = 172, normalized size = 1.18

$$\frac{\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)+\left(a^2-b^2\right)\left(a^2\sin(2(c+dx))-4b^2(c+dx)\right)-4ab\left(a^2-b^2\right)(c+dx)\cos(c+dx)}{a\cos(c+dx)+b} + \frac{4b^2\left(2b^2-3a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2a^3d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + (-4*a*b*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] + 2*a*b*(a
^2 - 2*b^2)*Sin[c + d*x] + (a^2 - b^2)*(-4*b^2*(c + d*x) + a^2*Sin[2*(c + d
*x)]))/(b + a*Cos[c + d*x]))/(2*a^3*(a - b)*(a + b)*d)
```

Maple [A] time = 0.083, size = 242, normalized size = 1.7

$$2\frac{\tan(1/2\,dx+c/2)}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)}-4\frac{b\arctan(\tan(1/2\,dx+c/2))}{da^3}+2\frac{b^3\tan(1/2\,dx+c/2)}{da^2\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^3*b*arctan(tan(1/
2*d*x+1/2*c))+2/d*b^3/a^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)+6/d*b^2/a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-4/d*b^4/a^3/(a+b)/(a-b
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93909, size = 1237, normalized size = 8.47

$$\left[-\frac{4\left(a^5b-2\,a^3b^3+ab^5\right)dx\cos(dx+c)+4\left(a^4b^2-2\,a^2b^4+b^6\right)dx-\left(3\,a^2b^3-2\,b^5+\left(3\,a^3b^2-2\,ab^4\right)\cos(dx+c)\right)\sqrt{a^2-}}{2\left(\left(a^8-2\,a^6b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^4*b^2 - 2*a^2*
b^4 + b^6)*d*x - (3*a^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))*s
qrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*s
```

```
qrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*
x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + (a
^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6*b^2 + a
^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d), -(2*(a^5*b - 2*a
^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x - (3*a
^2*b^3 - 2*b^5 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arcta
n(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^5
*b - 3*a^3*b^3 + 2*a*b^5 + (a^6 - 2*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 +
a^3*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.29059, size = 414, normalized size = 2.84

$$\frac{2\left(\frac{(3a^2b^2-2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^5-a^3b^2)\sqrt{-a^2+b^2}}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((3*a^2*b^2 - 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^5 - a^3*b^2)*sqrt(-a^2 + b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*
tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*x +
 1/2*c)^3 - a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2*t
an(1/2*d*x + 1/2*c) + 2*b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^
```

```
4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^4 - a
^2*b^2)) - (d*x + c)*b/a^3)/d
```

3.504 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$-\frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)}-\frac{2b^3\left(4a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin}{ad\left(a^2-\right.}$$

```
[Out] ((a^2 + 6*b^2)*x)/(2*a^4) - (2*b^3*(4*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(2*a^
2 - 3*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 3*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(a*(a
^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.58549, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3847, 4104, 3919, 3831, 2659, 208}

$$-\frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2-3b^2\right)\sin(c+dx)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)}-\frac{2b^3\left(4a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin}{ad\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ((a^2 + 6*b^2)*x)/(2*a^4) - (2*b^3*(4*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(2*a^
2 - 3*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 3*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(a*(a
^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
```

```
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(-a^2+3b^2+ab\sec(c+dx)-2b^2\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\cos(c+dx)\left(-2b\left(2a^2-3\right.\right.}{}}{}$$

$$= -\frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\cos(c+dx)\sin(c+}{a\left(a^2-b^2\right)d(a+b\sec(c}$$

$$= \frac{\left(a^2+6b^2\right)x}{2a^4} - \frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\cos(c}{a\left(a^2-b^2\right.}$$

$$= \frac{\left(a^2+6b^2\right)x}{2a^4} - \frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\cos(c}{a\left(a^2-b^2\right.}$$

$$= \frac{\left(a^2+6b^2\right)x}{2a^4} - \frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\cos(c}{a\left(a^2-b^2\right.}$$

$$= \frac{\left(a^2+6b^2\right)x}{2a^4} - \frac{2b^3\left(4a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\left(2a^2-3b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a\right.}{}$$

<u>Mathematica</u> [A] time = 0.724508, size = 144, normalized size = 0.69

$$\frac{2\left(a^2+6b^2\right)(c+dx) - \frac{8b^3\left(3b^2-4a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + a^2\sin(2(c+dx)) + \frac{4ab^4\sin(c+dx)}{(a-b)(a+b)(a\cos(c+dx)+b)} - 8ab\sin(c+dx)}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + 6*b^2)*(c + d*x) - (8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - 8*a*b*Sin[c + d*x] + (4
*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*Sin[2*(c
+ d*x)])/(4*a^4*d)
```

Maple [A] time = 0.083, size = 362, normalized size = 1.7

$$-\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-4\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^3 b}{da^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^2}+\frac{1}{da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-4/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*b+
1/d/a^2*arctan(tan(1/2*d*x+1/2*c))+6/d/a^4*arctan(tan(1/2*d*x+1/2*c))*b^2-2
/d*b^4/a^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)-8/d*b^3/a^2/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+6/d*b^5/a^4/(a+b)/(a-b)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12801, size = 1445, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b +
4*a^4*b^3 - 11*a^2*b^5 + 6*b^7)*d*x + (4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3*a
*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a
^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (4*a^5*b^2 - 1
0*a^3*b^4 + 6*a*b^6 - (a^7 - 2*a^5*b^2 + a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b
 - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9 - 2*a^7*b^2 + a^5
*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*d), 1/2*((a^7 + 4*a^5*
b^2 - 11*a^3*b^4 + 6*a*b^6)*d*x*cos(d*x + c) + (a^6*b + 4*a^4*b^3 - 11*a^2*
b^5 + 6*b^7)*d*x - 2*(4*a^2*b^4 - 3*b^6 + (4*a^3*b^3 - 3*a*b^5)*cos(d*x + c
))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) - (4*a^5*b^2 - 10*a^3*b^4 + 6*a*b^6 - (a^7 - 2*a^5*b^2 +
 a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))*si
n(d*x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^
3 + a^4*b^5)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.28963, size = 356, normalized size = 1.71

$$-\frac{\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{4\left(4\,a^2b^3-3\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{-a^2+b^2}}-\frac{(a^2+6}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b^2))
 - (a^2 + 6*b^2)*(d*x + c)/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*
d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/((tan(1
/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d
```

3.505 $\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$\frac{\left(7a^2b^2+2a^4-12b^4\right)\sin(c+dx)}{3a^4d\left(a^2-b^2\right)}+\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{2b^4\,(5a\cdots}{\cdots}$$

```
[Out] -((b*(a^2 + 4*b^2)*x)/a^5) + (2*b^4*(5*a^2 - 4*b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((2*a^4
 + 7*a^2*b^2 - 12*b^4)*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) - (b*(a^2 - 2*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 4*b^2)*Cos[c +
d*x]^2*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Cos[c + d*x]^2*Sin[c + d*
x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.835403, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3847, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(7a^2b^2+2a^4-12b^4\right)\sin(c+dx)}{3a^4d\left(a^2-b^2\right)}+\frac{\left(a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{b\left(a^2-2b^2\right)\sin(c+dx)\cos(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{2b^4\,(5a\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b*(a^2 + 4*b^2)*x)/a^5) + (2*b^4*(5*a^2 - 4*b^2)*ArcTanh[(Sqrt[a - b]*Ta
n[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((2*a^4
 + 7*a^2*b^2 - 12*b^4)*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) - (b*(a^2 - 2*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2 - 4*b^2)*Cos[c +
d*x]^2*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Cos[c + d*x]^2*Sin[c + d*
x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
```

```
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{b^2\cos^2(c+dx)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^3(c+dx)(-a^2+4b^2+ab\sec(c+dx)-3b^2\sec^2(c+dx))}{a+b\sec(c+dx)}\,dx}{a(a^2-b^2)}$$

$$= \frac{(a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)d} + \frac{b^2\cos^2(c+dx)\sin(c+dx)}{a(a^2-b^2)d(a+b\sec(c+dx))} + \frac{\int \frac{\cos^2(c+dx)(-6b(a^2-}{}}{}$$

$$= -\frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3(a^2-b^2)d} + \frac{(a^2-4b^2)\cos^2(c+dx)\sin(c+dx)}{3a^2(a^2-b^2)d} + \frac{b^2\cos^2(c}{a(a^2-b^2)d}$$

$$= \frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4(a^2-b^2)d} - \frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3(a^2-b^2)d} + \frac{(a^2-4b^2)\cos^2(}{3a^2(a}$$

$$= -\frac{b(a^2+4b^2)x}{a^5} + \frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4(a^2-b^2)d} - \frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3(a^2-b^2)d}$$

$$= -\frac{b(a^2+4b^2)x}{a^5} + \frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4(a^2-b^2)d} - \frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3(a^2-b^2)d}$$

$$= -\frac{b(a^2+4b^2)x}{a^5} + \frac{(2a^4+7a^2b^2-12b^4)\sin(c+dx)}{3a^4(a^2-b^2)d} - \frac{b(a^2-2b^2)\cos(c+dx)\sin(c+dx)}{a^3(a^2-b^2)d}$$

$$= -\frac{b(a^2+4b^2)x}{a^5} + \frac{2b^4(5a^2-4b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} + \frac{(2a^4+7a^2b^2-12b^4)\sin(c}{3a^4(a^2-b^2)d}$$

Mathematica [C] time = 1.05719, size = 176, normalized size = 0.67

$$\frac{9a(a^2+4b^2)\sin(c+dx) + \frac{24b^4(4b^2-5a^2)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - 6a^2b\sin(2(c+dx)) + a^3\sin(3(c+dx)) + \frac{12ab^5\sin(c+d}{(b-a)(a+b)(a\cos(c-}}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-12*b*((-I)*a + 2*b)*(I*a + 2*b)*(c + d*x) + (24*b^4*(-5*a^2 + 4*b^2)*ArcT
anh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 9*a*(
```

```
a^2 + 4*b^2)*Sin[c + d*x] + (12*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b +
a*Cos[c + d*x])) - 6*a^2*b*Sin[2*(c + d*x)] + a^3*Sin[3*(c + d*x)])/(12*a^5
*d)
```

Maple [B] time = 0.087, size = 508, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+2/d/a^3/(1+tan(1/2*
d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*t
an(1/2*d*x+1/2*c)^5*b^2+4/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/
2*c)^3+12/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3*b^2+2/d/a^2
/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-2/d/a^3/(1+tan(1/2*d*x+1/2*c
)^2)^3*tan(1/2*d*x+1/2*c)*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+
1/2*c)*b^2-2/d/a^3*b*arctan(tan(1/2*d*x+1/2*c))-8/d/a^5*arctan(tan(1/2*d*x+
1/2*c))*b^3+2/d*b^5/a^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)+10/d*b^4/a^3/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-8/d*b^6/a^5/(a+b)/(a-
b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23767, size = 1661, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 6*(a^
6*b^2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^3*
b^4 - 4*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
 c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*
a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^3 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + 2*(a^8 + a^6
*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^
2 + a^6*b^4)*d*cos(d*x + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d), -1/3*(3*(a^
7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*b^2 + 2*a^
4*b^4 - 7*a^2*b^6 + 4*b^8)*d*x - 3*(5*a^2*b^5 - 4*b^7 + (5*a^3*b^4 - 4*a*b^
6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a
*b^7 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^3 - 2*(a^7*b - 2*a^5*b^3 +
a^3*b^5)*cos(d*x + c)^2 + 2*(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x
 + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) + (a^9*b
- 2*a^7*b^3 + a^5*b^5)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32735, size = 452, normalized size = 1.73

$$\frac{6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^6-a^4b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{6\left(5\,a^2b^4-4\,b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-a^5b^2\right)\sqrt{-a^2+b^2}}-\frac{3\left(a^2b+4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^2))
- 3*(a^2*b + 4*b^3)*(d*x + c)/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a*b
*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 + 2*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) - 3
*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2
*c)^2 + 1)^3*a^4))/d
```

3.506 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=230

$$\frac{(3a^2-2b^2)\tan(c+dx)}{2b^3d(a^2-b^2)}+\frac{3a^2(-5a^2b^2+2a^4+4b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}+-\frac{}{2}$$

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a +
 b)^(5/2)*d) + ((3*a^2 - 2*b^2)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (a^2*
Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (
3*a^3*(a^2 - 2*b^2)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x
]))
```

Rubi [A] time = 0.707457, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3845, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{(3a^2-2b^2)\tan(c+dx)}{2b^3d(a^2-b^2)}+\frac{3a^2(-5a^2b^2+2a^4+4b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}+-\frac{}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a +
 b)^(5/2)*d) + ((3*a^2 - 2*b^2)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (a^2*
Sec[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (
3*a^3*(a^2 - 2*b^2)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x
]))
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
```

```
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-2ab\sec(c+dx)-\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(3a^2b\left(\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\tan(c+\cdots}{2b^3\left(a^2-b^2\right)^2 d(a+b\sec(c\cdots}$$

$$= \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\tan(c+\cdots}{2b^3\left(a^2-b^2\right)^2 d(a+b\sec(c\cdots}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \cdots$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(3a^2-2b^2\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \cdots$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d} + \frac{\left(3a^2-2b\cdots\right.}{2b^3\left(\cdots\right.}$$

Mathematica [A] time = 4.80484, size = 205, normalized size = 0.89

$$\frac{\frac{a^3 b\sin(c+dx)\left(a\left(4a^2-7b^2\right)\cos(c+dx)+5a^2b-8b^3\right)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2}-\frac{6a^2\left(-5a^2b^2+2a^4+4b^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a^3*b*(5*a^2*b - 8*
b^3 + a*(4*a^2 - 7*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b
 + a*Cos[c + d*x])^2) + 2*b*Tan[c + d*x])/(2*b^4*d)
```

Maple [B] time = 0.064, size = 735, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+8/d*a^
3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3+4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d*a^4/b^2/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(
1/2*d*x+1/2*c)-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+6/d*a^6/b^4/(a^4-2*a^2*b^2+b^4
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+12/d*a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/d/b^3/(tan(1
/2*d*x+1/2*c)+1)-3/d*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-1/d/b^3/(tan(1/2*d*x+1/
2*c)-1)+3/d*a/b^4*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.63501, size = 2921, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^3 + 2*(2*a^7*b - 5*a^
5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos
(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*((a^9 - 3*a^7*b^2 + 3
*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2
*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x +
c))*log(sin(d*x + c) + 1) + 6*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(
d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (
a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(-sin(d*x + c) +
1) + 2*(2*a^6*b^3 - 6*a^4*b^5 + 6*a^2*b^7 - 2*b^9 + (6*a^8*b - 17*a^6*b^3 +
 13*a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*
b^6 - 4*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^
8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^1
1)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x +
 c)), 1/2*(3*((2*a^8 - 5*a^6*b^2 + 4*a^4*b^4)*cos(d*x + c)^3 + 2*(2*a^7*b -
 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^2 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6
)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
 a)/((a^2 - b^2)*sin(d*x + c))) - 3*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6
)*cos(d*x + c)^3 + 2*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)
^2 + (a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(sin(d*x +
c) + 1) + 3*((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^3 + 2*(a^
```

```
8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + (a^7*b^2 - 3*a^5*b^
4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*a^6*b^3 -
6*a^4*b^5 + 6*a^2*b^7 - 2*b^9 + (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*
b^7)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*cos(d
*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(
d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2
+ (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

Giac [A] time = 1.33193, size = 517, normalized size = 2.25

$$-\frac{\frac{3\left(2a^6-5a^4b^2+4a^2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4b^4-2a^2b^6+b^8\right)\sqrt{-a^2+b^2}}+\frac{4a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7a^4b^2\tan\left(\frac{1}{2}dx+\cdots\right.}{\left(a^4\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2
*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-
a^2 + b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(-a^2 + b^2)) + (4*a^6*tan(1/
2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*tan(1/2*d*x + 1/2*c) -
 5*a^5*b*tan(1/2*d*x + 1/2*c) + 7*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*a^3*b^3*
tan(1/2*d*x + 1/2*c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*a*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/b^4 - 3*a*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 2*tan(1/2*d*x + 1
```

```
/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

3.507 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=188

$$-\frac{a\left(-5a^2b^2+2a^4+6b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b^3*d) - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d
) - (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^2) - (a^2*(2*a^2 - 5*b^2)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[
c + d*x]))
```

Rubi [A] time = 0.409982, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3845, 4080, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(-5a^2b^2+2a^4+6b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{a^2\tan(c+dx)\sec(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(b^3*d) - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d
) - (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^2) - (a^2*(2*a^2 - 5*b^2)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[
c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
```

```
1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(a^2-2ab\sec(c+dx)-2\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\sec(c+dx)\left(ab\left(a^2-4\right.\right.}{a+}}{2b^2}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \sec(c+dx)\,dx}{b^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\tan(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a\left(2a^4-5a^2b^2+6b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} - \frac{a^2\sec(c+dx)}{2b\left(a^2-b^2\right)d(a+}$$

Mathematica [A] time = 1.26872, size = 194, normalized size = 1.03

$$\frac{-\frac{a^2b\sin(c+dx)\left(a\left(2a^2-5b^2\right)\cos(c+dx)+3b\left(a^2-2b^2\right)\right)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{2a\left(-5a^2b^2+2a^4+6b^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]])/(a^2 - b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(3*b*(a^2 - 2*b^2) +
 a*(2*a^2 - 5*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*
Cos[c + d*x])^2))/(2*b^3*d)
```

Maple [B] time = 0.066, size = 685, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*a^2/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*
tan(1/2*d*x+1/2*c)^3-2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*a^3/b/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c)+6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)
/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+5/d*a^3/
b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/
((a+b)*(a-b))^(1/2))-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+1/d/b^3*ln(tan(1/2*d*x+1/2*
c)+1)-1/d/b^3*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.56752, size = 2466, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)*co
s(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2 -
 b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 -
 b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
+ 2*a*b*cos(d*x + c) + b^2)) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (
a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^
3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*(a^6*b^2 - 3
*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*
x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(-sin
(d*x + c) + 1) - 2*(3*a^6*b^2 - 9*a^4*b^4 + 6*a^2*b^6 + (2*a^7*b - 7*a^5*b^
3 + 5*a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^
7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10
)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*a^
5*b^2 - 5*a^3*b^4 + 6*a*b^6 + (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4)*cos(d*x + c)^
2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arct
an(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))
*log(sin(d*x + c) + 1) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8 + (a^8 - 3*
a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^
3*b^5 - a*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (3*a^6*b^2 - 9*a^4*b^
4 + 6*a^2*b^6 + (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5)*cos(d*x + c))*sin(d*x + c
))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b
^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7
+ 3*a^2*b^9 - b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Giac [A] time = 1.4412, size = 468, normalized size = 2.49

$$\frac{\frac{\left(2\,a^{5}-5\,a^{3}b^{2}+6\,ab^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)}{\left(a^{4}b^{3}-2\,a^{2}b^{5}+b^{7}\right)\sqrt{-a^{2}+b^{2}}}+\frac{2\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.}{\left(a^{4}b^{2}\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a^2 + b^2)) + (2*a^5*tan(1/2*d*x
 + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x + 1/2*
c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*a^5*tan(1/2*d*x + 1/2*c) - 3*a^
4*b*tan(1/2*d*x + 1/2*c) + 5*a^3*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1
/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^
3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3)/d
```

3.508 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$\frac{(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{2bd(a^2-b^2)^2(a+b\sec(c+dx))}$$

```
[Out] ((a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(5/2)*(a + b)^(5/2)*d) - (a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])^2) + (a*(a^2 - 4*b^2)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*
Sec[c + d*x]))
```

Rubi [A] time = 0.231627, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3839, 4003, 12, 3831, 2659, 208}

$$\frac{(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\tan(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{a(a^2-4b^2)\tan(c+dx)}{2bd(a^2-b^2)^2(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(5/2)*(a + b)^(5/2)*d) - (a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])^2) + (a*(a^2 - 4*b^2)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*
Sec[c + d*x]))
```

Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(-2ab-\left(a^2-2b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{b\left(a^2+2b^2\right)\sec(c+dx}{a+b\sec(c+dx)}}{2b\left(a^2-b^2\right)^2}$$

$$= -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(a^2+2b^2\right)\int \frac{\sec}{a+b\,\mathrm{se}}}{2\left(a^2-b^2\right.}$$

$$= -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(a^2+2b^2\right)\int \frac{}{1+\frac{a\,\mathrm{c}}{}}}{2b\left(a^2-b^2\right.}$$

$$= -\frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right)\tan(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(a^2+2b^2\right)\mathrm{Subst}}{}$$

$$= \frac{\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-4b^2\right.}{2b\left(a^2-b^2\right)^2 d(a}$$

Mathematica [A] time = 0.418227, size = 113, normalized size = 0.76

$$\frac{\frac{a\sin(c+dx)\left(a^2-3ab\cos(c+dx)-4b^2\right)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} - \frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(5/2) + (a*(a^2 - 4*b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/((a -
 b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2))/(2*d)
```

Maple [A] time = 0.056, size = 184, normalized size = 1.2

$$\frac{1}{d}\left(-2\,\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)^{2}}\left(-1/2\,\frac{(a+4\,b)\,a\left(\tan\left(1/2\,dx+c/2\right)\right)^{3}}{(a-b)\left(a^{2}+2\,ab+b^{2}\right)}-1/2\,\frac{(a-4\,b)\,a\,t}{(a+b)\,(a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(a+4*b)*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(a-4
*b)*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^2+(a^2+2*b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96033, size = 1291, normalized size = 8.66

$$\left[\frac{\left(a^{2}b^{2}+2\,b^{4}+\left(a^{4}+2\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{2}+2\left(a^{3}b+2\,ab^{3}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^{2}-2\,b^{2}\right)\cos(dx+c)^{2}+2\,\sqrt{}}{a^{2}\cos(dx+c)^{2}+2}\right.}{4\left(\left(a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b-3\,a^{5}b^{3}+3\,a^{3}b^{5}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3*b + 2*a*
b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
```

```
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(a^5 - 5*a^3*
b^2 + 4*a*b^4 - 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)
, 1/2*((a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2)*cos(d*x + c)^2 + 2*(a^3*b + 2*a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x +
c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^5 - 5*a^3*b^2 + 4*a*b^4 - 3*(a^4*b
 - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Giac [A] time = 1.40527, size = 342, normalized size = 2.3

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)(a^2+2b^2)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}-\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^3\tan\left(\frac{1}{2}\right.}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(a^2 + 2*b^2)/((a^4 -
 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
```

```
- a - b)^2))/d
```

3.509 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{\left(a^2+2b^2\right)\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (-3*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)
*(a + b)^(5/2)*d) + (a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + ((a^2 + 2*b^2)*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.190906, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3836, 4003, 12, 3831, 2659, 208}

$$\frac{\left(a^2+2b^2\right)\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)
*(a + b)^(5/2)*d) + (a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + ((a^2 + 2*b^2)*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
```

```
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)(-2b+a\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{3ab\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(3ab)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(3a)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1-}\right.}{\left(a^2\right.}$$

$$= -\frac{3ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\left(a^2+2b^2\right)\tan(c\ -}{2\left(a^2-b^2\right)^2 d(a+b\sec}$$

Mathematica [A] time = 0.364625, size = 115, normalized size = 0.86

$$\frac{\frac{\sin(c+dx)\left(a\left(2a^2+b^2\right)\cos(c+dx)+b\left(a^2+2b^2\right)\right)}{(a\cos(c+dx)+b)^2} + \frac{6ab\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^
2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(b + a
*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

Maple [A] time = 0.051, size = 195, normalized size = 1.5

$$\frac{1}{d}\left(2\frac{1}{\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)^2}\left(-1/2\,\frac{\left(2\,a^2+ab+2\,b^2\right)(\tan(1/2\,dx+c/2))^3}{(a-b)\left(a^2+2\,ab+b^2\right)}+1/2\,\frac{\left(2\,a^2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(2*(-1/2*(2*a^2+a*b+2*b^2)/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1
/2*(2*a^2-a*b+2*b^2)/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-3*a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89356, size = 1222, normalized size = 9.12

$$\left[\frac{3\left(a^3b\cos(dx+c)^2+2\,a^2b^2\cos(dx+c)+ab^3\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{4\left(\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\cos(dx+c)^2+2\left(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7\right)d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^3*b*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(a^2 -
b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 -
b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
```

```
 2*a*b*cos(d*x + c) + b^2)) + 2*(a^4*b + a^2*b^3 - 2*b^5 + (2*a^5 - a^3*b^2
 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b
^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x
+ c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), -1/2*(3*(a^3*b*cos(d*x +
 c)^2 + 2*a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (a^4*b + a^2*b^3
- 2*b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.35931, size = 374, normalized size = 2.79

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}-\frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}dx+\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
 + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*b^
2 + b^4)*sqrt(-a^2 + b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*
d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^
3 - 2*a^3*tan(1/2*d*x + 1/2*c) - a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*tan(1/2
*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan
```

```
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

3.510 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\tan(c+dx)}{2d(a^2-b^2)^2(a+b\sec(c+dx))}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

```
[Out] ((2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(5/2)*(a + b)^(5/2)*d) - (b*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) - (3*a*b*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.175449, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3833, 4003, 12, 3831, 2659, 208}

$$\frac{(2a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\tan(c+dx)}{2d(a^2-b^2)^2(a+b\sec(c+dx))}-\frac{b\tan(c+dx)}{2d(a^2-b^2)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((2*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(5/2)*(a + b)^(5/2)*d) - (b*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) - (3*a*b*Tan[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
```

```
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2a+b\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\left(2a^2+b^2\right)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(2a^2+b^2\right)\int \frac{\sec(c+}{a+b\sec(c}}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(2a^2+b^2\right)\int \frac{1}{1+\frac{a\cos(c}{b}}}{2b\left(a^2-b^2\right)^2}$$

$$= -\frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(2a^2+b^2\right)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{3ab\tan(c}{2\left(a^2-b^2\right)^2 d(a+}$$

Mathematica [A] time = 0.45444, size = 115, normalized size = 0.86

$$\frac{\frac{b\sin(c+dx)\left(\left(b^2-4a^2\right)\cos(c+dx)-3ab\right)}{(a\cos(c+dx)+b)^2} - \frac{2\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + (b*(-3*a*b + (-4*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(b
+ a*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

Maple [A] time = 0.061, size = 186, normalized size = 1.4

$$\frac{1}{d}\left(-2\,\frac{1}{\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)^2}\left(-1/2\,\frac{(4\,a+b)\,b\,(\tan(1/2\,dx+c/2))^3}{(a-b)\left(a^2+2\,ab+b^2\right)}+1/2\,\frac{(4\,a-b)\,b\,\mathrm{t}}{(a+b)\,(\mathrm{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*(4*a
-b)*b/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.88728, size = 1296, normalized size = 9.74

$$\left[\frac{\left(2\,a^2b^2+b^4+\left(2\,a^4+a^2b^2\right)\cos(dx+c)^2+2\left(2\,a^3b+ab^3\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{}}{a^2\cos(dx+c)^2+2}\right.}{4\left(\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)d\cos(dx+c)^2+2\left(a^7b-3\,a^5b^3+3\,a^3b^5\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2*a^3*b + a*
b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
```

```
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(3*a^3*b^2 -
3*a*b^4 + (4*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3
*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
d), 1/2*((2*a^2*b^2 + b^4 + (2*a^4 + a^2*b^2)*cos(d*x + c)^2 + 2*(2*a^3*b +
 a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c))) - (3*a^3*b^2 - 3*a*b^4 + (4*a^4*b - 5
*a^2*b^3 + b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 -
 a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*co
s(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.37075, size = 343, normalized size = 2.58

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)(2a^2+b^2)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}-\frac{4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4a^2b\tan\cdots}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\cdots\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x
+ 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(2*a^2 + b^2)/((a^4 -
 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*b*tan(1/2*
d*x + 1/2*c) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
```

```
- a - b)^2))/d
```

3.511 $\int \frac{1}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+d$$

```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tan[c + d*x])/(
2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tan[c + d*
x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.309837, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3785, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3),x]
```


```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tan[c + d*x])/(
2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tan[c + d*
x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2\left(a^2-b^2\right)+2ab\sec(c+dx)-b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b\right.}{a+b\sec(c+}}{2a^2\left(a^2-\right.}$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\left(b\left(6a^4-5a^2\right.\right.}{}$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\left(6a^4-5a^2b^2\right.}{2a}$$

$$= \frac{x}{a^3} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\left(6a^4-5a^2b^2\right.}{}$$

$$= \frac{x}{a^3} - \frac{b\left(6a^4-5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} +$$

Mathematica [A] time = 0.760963, size = 205, normalized size = 1.18

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(\frac{3ab^2\left(2a^2-b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} + \frac{2b\left(-5a^2b^2+6a^4+2b^4\right)(a\cos(c+dx)+b)^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{ab}{}\right.}{2a^3d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(2*(c + d*x)*(b + a*Cos[c + d*x])^2 +
(2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sin[c + d*x])
/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cos[c + d*x])*Sin[c + d
*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.07, size = 664, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*arctan(tan(1/2*d*x+1/2*c))-6/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d/a*b^3/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)
*tan(1/2*d*x+1/2*c)^3+2/d/a^2*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+6/d*b^2/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*
x+1/2*c)-1/d/a*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a
+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d/a^2*b^4/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-6/d*
a*b/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2))+5/d/a*b^3/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-2/d/a^3*b^5/(a^4-2*a^2*
b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13457, size = 1960, normalized size = 11.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 + 8*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*d*x + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7 + (6*a^6*b - 5*a
^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*co
s(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(5*a^5*b^3 - 7*a^3*b
^5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x + c))*sin(d*x +
c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b
- 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 +
3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x
*cos(d*x + c)^2 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c
) + 2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x - (6*a^4*b^3 - 5*a^2*b^5
+ 2*b^7 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 -
 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (5*a^5*b^3 - 7*a^3*b^
5 + 2*a*b^7 + 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)*cos(d*x + c))*sin(d*x + c
))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b -
 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3
*a^5*b^6 - a^3*b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\sec{(c + dx)})^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-3), x)
```

Giac [B] time = 1.29495, size = 435, normalized size = 2.51

$$\frac{\frac{\left(6\,a^4b-5\,a^2b^3+2\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}} - \frac{6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(a^6-2\,a^4\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b^2)) - (6*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*x +
1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) -
5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x + 1/2*c) + 2*b^5*tan(1
/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (d*x + c)/a^3)/d
```

3.512 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=223

$$\frac{\left(-11a^2b^2+2a^4+6b^4\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+}$$

```
[Out] (-3*b*x)/a^4 + (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4 -
 11*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Sin[c + d
*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (3*b^2*(2*a^2 - b^2)*Sin[
c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.623448, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3847, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(-11a^2b^2+2a^4+6b^4\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{3b^2\left(2a^2-b^2\right)\sin(c+dx}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*b*x)/a^4 + (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4 -
 11*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Sin[c + d
*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (3*b^2*(2*a^2 - b^2)*Sin[
c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
```

```
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a^2+3b^2+2ab\sec(c+dx)-2b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2\left(2a^2-b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\cos(c+dx)\left(2a^4-11a^2\right.}{}}{}$$

$$= \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2\left(2a^2-b^2\right)s}{2a^2\left(a^2-b^2\right)^2 d(a+}$$

$$= -\frac{3bx}{a^4} + \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2\left(2a\right.}{2a^2\left(a^2-b^2\right.}$$

$$= -\frac{3bx}{a^4} + \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2\left(2a\right.}{2a^2\left(a^2-b^2\right.}$$

$$= -\frac{3bx}{a^4} + \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3b^2\left(2a\right.}{2a^2\left(a^2-b^2\right.}$$

$$= -\frac{3bx}{a^4} + \frac{3b^2\left(4a^4-5a^2b^2+2b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\left(2a^4-11a^2b^2+6b^4\right)\sin(c+d}{2a^3\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 0.881031, size = 229, normalized size = 1.03

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(-\frac{ab^3\left(8a^2-5b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} - \frac{6b^2\left(-5a^2b^2+4a^4+2b^4\right)(a\cos(c+dx)+b)^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \right.}{2a^4d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(-6*b*(c + d*x)*(b + a*Cos[c + d*x])^2
 - (6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^4*Sin[c +
d*x])/((a - b)*(a + b)) - (a*b^3*(8*a^2 - 5*b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^2*(a + b)^2) + 2*a*(b + a*Cos[c + d*x])^2*Sin[c + d*x]))/
(2*a^4*d*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 0.086, size = 702, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a^4*b*arctan(tan(1/
2*d*x+1/2*c))+8/d/a*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d/a^2*b^4/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3-4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b
)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-8/d/a*b^3/(tan(1/2*d*x+1/2*c)^2*a-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d/a^
2*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*
b+b^2)*tan(1/2*d*x+1/2*c)+4/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+12/d*b^2/(a^4-2*a
^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-
b))^(1/2))-15/d*b^4/a^2/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34654, size = 2225, normalized size = 9.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 24
*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c) + 12*(a^6*b^3 -
 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (4*a
^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b^5 +
 2*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*
b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^7*b
^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8 + 2*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a
^3*b^6)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*co
s(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(
d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) +
(a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d), -1/2*(6*(a^8*b - 3*a^6*b^3
 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c)^2 + 12*(a^7*b^2 - 3*a^5*b^4 + 3*a^
3*b^6 - a*b^8)*d*x*cos(d*x + c) + 6*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)
*d*x - 3*(4*a^4*b^4 - 5*a^2*b^6 + 2*b^8 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^
6)*cos(d*x + c)^2 + 2*(4*a^5*b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(d*x + c))*sqrt(
-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(
d*x + c))) - (2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8 + 2*(a^9 - 3*a^
7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^
4*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*
b^4 - a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b
^7)*d*cos(d*x + c) + (a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [A] time = 1.35167, size = 482, normalized size = 2.16

$$-\frac{\frac{3\left(4a^4b^2-5a^2b^4+2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^8-2a^6b^2+a^4b^4\right)\sqrt{-a^2+b^2}}-\frac{8a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5ab^5\tan\left(\frac{1}{2}\right.}{\left(a\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2
*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-
a^2 + b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(-a^2 + b^2)) - (8*a^3*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^5*tan(1/2*d
*x + 1/2*c)^3 + 4*b^6*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*b^3*tan(1/2*d*x + 1/2*
c) - 7*a^2*b^4*tan(1/2*d*x + 1/2*c) + 5*a*b^5*tan(1/2*d*x + 1/2*c) + 4*b^6*
tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + 3*(d*x + c)*b/a^4 - 2*tan(1/2*d*
x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

3.513 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=296

$$-\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(-10a^2b^2+a^4+6b^4\right)\sin(c+dx)\cos(c+dx)}{2a^3d\left(a^2-b^2\right)^2}-\frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan}{a^5d(a-b)^{5/2}(a+}$$

```
[Out] ((a^2 + 12*b^2)*x)/(2*a^5) - (b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh[(S
qrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)
*d) - (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^2*d
) + ((a^4 - 10*a^2*b^2 + 6*b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^
2)^2*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) + (b^2*(7*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.998154, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3847, 4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(-10a^2b^2+a^4+6b^4\right)\sin(c+dx)\cos(c+dx)}{2a^3d\left(a^2-b^2\right)^2}-\frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan}{a^5d(a-b)^{5/2}(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((a^2 + 12*b^2)*x)/(2*a^5) - (b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh[(S
qrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)
*d) - (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^2*d
) + ((a^4 - 10*a^2*b^2 + 6*b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^
2)^2*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
 d*x])^2) + (b^2*(7*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-2a^2+4b^2+2ab\sec(c+dx)-3b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(7a^2-4b^2\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\cos^2(c+dx)\left(2\left(a^4\right.\right.}{}}{}$$

$$= \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(7a^2-\right.}{2a^2\left(a^2\right.}$$

$$= -\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2 d} + \frac{b^2}{2a\left(a^2\right.}$$

$$= \frac{\left(a^2+12b^2\right)x}{2a^5} - \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+dx)\sin(c}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2+12b^2\right)x}{2a^5} - \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+dx)\sin(c}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2+12b^2\right)x}{2a^5} - \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\cos(c+dx)\sin(c}{2a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(a^2+12b^2\right)x}{2a^5} - \frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} - \frac{3b\left(2a^4-7a^2b^2+4b\right.}{2a^4\left(a^2-b\right.}$$

Mathematica [A] time = 2.04199, size = 199, normalized size = 0.67

$$\frac{2\left(a^2+12b^2\right)(c+dx)+\frac{2ab^4\left(10a^2-7b^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a\cos(c+dx)+b)}+\frac{4b^3\left(-29a^2b^2+20a^4+12b^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}+a^2\sin(2(c+dx))+\frac{2\cdots}{(b-a)(a\cdots}}{4a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(a^2 + 12*b^2)*(c + d*x) + (4*b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh
[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 12*a*b*S
in[c + d*x] + (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])
^2) + (2*a*b^4*(10*a^2 - 7*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*C
os[c + d*x])) + a^2*Sin[2*(c + d*x)])/(4*a^5*d)
```

Maple [B] time = 0.092, size = 827, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/a^4/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)*b+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+
1/d/a^3*arctan(tan(1/2*d*x+1/2*c))+12/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^2-
10/d/a^2*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+6/d*b^
6/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)/(a^2+2*a*
b+b^2)*tan(1/2*d*x+1/2*c)^3+10/d/a^2*b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*
x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*b^5/a^3/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*t
an(1/2*d*x+1/2*c)-6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-20/d/a*b^3/(a^4-2*a^2*b^2
+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))+29/d/a^3*b^5/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-12/d*b^7/a^5/(a^4-2*a^2*b^2+b^4)/((a+b
```

```
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54114, size = 2541, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*d*x*cos(d
*x + c)^2 + 4*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*d*x*
cos(d*x + c) + 2*(a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*
d*x + (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^2
*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(6*a^7*b^3 - 27*a^5*b^5 + 33*a^
3*b^7 - 12*a*b^9 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*cos(d*x + c)^3
+ 4*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c)^2 + (11*a^8*b^2
- 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^13
- 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*b - 3*a^10*b
^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^
6 - a^5*b^8)*d), 1/2*((a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*
b^8)*d*x*cos(d*x + c)^2 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 -
12*a*b^9)*d*x*cos(d*x + c) + (a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8
 - 12*b^10)*d*x - (20*a^4*b^5 - 29*a^2*b^7 + 12*b^9 + (20*a^6*b^3 - 29*a^4*
b^5 + 12*a^2*b^7)*cos(d*x + c)^2 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*c
os(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)
/((a^2 - b^2)*sin(d*x + c))) - (6*a^7*b^3 - 27*a^5*b^5 + 33*a^3*b^7 - 12*a*
```

```
b^9 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*cos(d*x + c)^3 + 4*(a^9*b -
3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c)^2 + (11*a^8*b^2 - 43*a^6*b^4
+ 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2
+ 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^
5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*
d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.4158, size = 1037, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(20*a^4*b^3 - 29*a^2*b^5 + 12*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(-a^2 + b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(-a^2 + b^2)) + 2*(a
^7*tan(1/2*d*x + 1/2*c)^7 + 4*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*a^5*b^2*tan
(1/2*d*x + 1/2*c)^7 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*b^4*tan(1/2
*d*x + 1/2*c)^7 - 17*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*a*b^6*tan(1/2*d*x
+ 1/2*c)^7 + 12*b^7*tan(1/2*d*x + 1/2*c)^7 - 3*a^7*tan(1/2*d*x + 1/2*c)^5 -
 4*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 26*a^4
*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 67*a^2*b^
5*tan(1/2*d*x + 1/2*c)^5 - 18*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*b^7*tan(1/2
*d*x + 1/2*c)^5 + 3*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*b*tan(1/2*d*x + 1/2*
c)^3 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*a^4*b^3*tan(1/2*d*x + 1/2*c)^3
 - 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 +
```

```
18*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*b^7*tan(1/2*d*x + 1/2*c)^3 - a^7*tan(1
/2*d*x + 1/2*c) + 4*a^6*b*tan(1/2*d*x + 1/2*c) + 13*a^5*b^2*tan(1/2*d*x + 1
/2*c) - 2*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*a^3*b^4*tan(1/2*d*x + 1/2*c) -
17*a^2*b^5*tan(1/2*d*x + 1/2*c) + 18*a*b^6*tan(1/2*d*x + 1/2*c) + 12*b^7*ta
n(1/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4
- b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (a^2
+ 12*b^2)*(d*x + c)/a^5)/d
```

3.514 $\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=316

$$\frac{\left(-23a^2b^2+12a^4+6b^4\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-28a^4b^2+35a^2b^4+8a^6-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(4a^2-9b^2\right)}{6b^2d\left(a^2-b^2\right.}$$

```
[Out] (-4*a*ArcTanh[Sin[c + d*x]])/(b^5*d) + (a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^
4 - 20*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*b^5*(a + b)^(7/2)*d) + ((12*a^4 - 23*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(6
*b^4*(a^2 - b^2)^2*d) - (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Sec[c + d*x])^3) - (a^2*(4*a^2 - 9*b^2)*Sec[c + d*x]^2*Tan[c + d*x
])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a^3*(4*a^4 - 11*a^2*b^
2 + 12*b^4)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.10616, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3845, 4098, 4090, 4082, 3998, 3770, 3831, 2659, 208}

$$\frac{\left(-23a^2b^2+12a^4+6b^4\right)\tan(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-28a^4b^2+35a^2b^4+8a^6-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(4a^2-9b^2\right)}{6b^2d\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (-4*a*ArcTanh[Sin[c + d*x]])/(b^5*d) + (a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^
4 - 20*b^6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*b^5*(a + b)^(7/2)*d) + ((12*a^4 - 23*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(6
*b^4*(a^2 - b^2)^2*d) - (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(3*b*(a^2 - b^2)*
d*(a + b*Sec[c + d*x])^3) - (a^2*(4*a^2 - 9*b^2)*Sec[c + d*x]^2*Tan[c + d*x
])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a^3*(4*a^4 - 11*a^2*b^
2 + 12*b^4)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
```

```
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
```

```
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)\left(3a^2-3ab\sec(c+dx)-\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\sec^2(c+dx)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \int \frac{\sec^2(c+dx)(\cdots}{\cdots}$$

$$= -\frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\sec^2(c+dx)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a^3\left(4a^4-11\cdots\right.}{2b^4\left(a^2-b\cdots\right.}$$

$$= \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\sec\cdots}{6b^2\left(a^2-b^2\right)^2 d(\cdots}$$

$$= \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\sec\cdots}{6b^2\left(a^2-b^2\right)^2 d(\cdots}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^3(c+dx)\tan(c+\cdots}{3b\left(a^2-b^2\right)d(a+b\sec(c\cdots}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^3(c+dx)\tan(c+\cdots}{3b\left(a^2-b^2\right)d(a+b\sec(c\cdots}$$

$$= -\frac{4a\tanh^{-1}(\sin(c+dx))}{b^5d} + \frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d} + \cdots$$

Mathematica [A] time = 6.23323, size = 416, normalized size = 1.32

$$-\frac{a^3\sin(c+dx)}{3b^2d(b-a)(a+b)(a\cos(c+dx)+b)^3} + \frac{6a^5\sin(c+dx)-11a^3b^2\sin(c+dx)}{6b^3d(b-a)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{50a^5b^2\sin(c+dx)-47a^3b^4\sin(c+\cdots}{6b^4d(b-a)^3(a+b)^3(a\cos\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((a^2*(-8*a^6 + 28*a^4*b^2 - 35*a^2*b^4 + 20*b^6)*ArcTanh[((-a + b)*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)^3*d)) + (4*
a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(b^5*d) - (4*a*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]])/(b^5*d) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(b^4*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) - (a^3*Sin[c + d*x])/(3*b^2*(-a + b)*(a + b)*d*(b + a*Cos[c +
 d*x])^3) + (6*a^5*Sin[c + d*x] - 11*a^3*b^2*Sin[c + d*x])/(6*b^3*(-a + b)^
2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (-18*a^7*Sin[c + d*x] + 50*a^5*b^2*
Sin[c + d*x] - 47*a^3*b^4*Sin[c + d*x])/(6*b^4*(-a + b)^3*(a + b)^3*d*(b +
a*Cos[c + d*x]))
```

Maple [B] time = 0.07, size = 1481, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -6/d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/d*a^6/b^3/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+18/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-5/d*a^4/b/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+
b^3)*tan(1/2*d*x+1/2*c)^5-20/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+12/d*a^7
/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/
(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-116/3/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3+40/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*a^7/b^4/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*t
an(1/2*d*x+1/2*c)-2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+18/d*a^5/b^2/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)+5/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-20/d*a^
3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+
3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+8/d*a^8/b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-28
/d*a^6/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
```

```
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+35/d*a^4/b/(a^6-3*a^4*b^2+3*a^2*b^4
-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))-20/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/d/b^4/(tan(1/2*d*x+1/2*c)+1)
-4/d*a/b^5*ln(tan(1/2*d*x+1/2*c)+1)-1/d/b^4/(tan(1/2*d*x+1/2*c)-1)+4/d*a/b^
5*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.2749, size = 4581, normalized size = 14.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*((8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6)*cos(d*x + c)^4 +
3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c)^3 + 3*(8*a
^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + (8*a^8*b^3
- 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((
2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*co
s(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos
(d*x + c) + b^2)) - 24*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^
8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9
)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^
10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)
*cos(d*x + c))*log(sin(d*x + c) + 1) + 24*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 -
 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 -
4*a^5*b^7 + a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 -
 4*a^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 -
4*a^3*b^9 + a*b^11)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(6*a^8*b^4 - 2
```

```
4*a^6*b^6 + 36*a^4*b^8 - 24*a^2*b^10 + 6*b^12 + (24*a^11*b - 92*a^9*b^3 + 1
33*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9)*cos(d*x + c)^3 + 3*(20*a^10*b^2 - 77*a
^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c)^2 + (44*a^9*b^
3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c))*sin(
d*x + c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d*cos
(d*x + c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)
*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^
15)*d*cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^
16)*d*cos(d*x + c)), 1/6*(3*((8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6
)*cos(d*x + c)^4 + 3*(8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(
d*x + c)^3 + 3*(8*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x +
 c)^2 + (8*a^8*b^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c))*sq
rt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*s
in(d*x + c))) - 12*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*c
os(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*co
s(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*
cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cos
(d*x + c))*log(sin(d*x + c) + 1) + 12*((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a
^6*b^6 + a^4*b^8)*cos(d*x + c)^4 + 3*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^
5*b^7 + a^3*b^9)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a
^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^
3*b^9 + a*b^11)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (6*a^8*b^4 - 24*a^6*
b^6 + 36*a^4*b^8 - 24*a^2*b^10 + 6*b^12 + (24*a^11*b - 92*a^9*b^3 + 133*a^7
*b^5 - 71*a^5*b^7 + 6*a^3*b^9)*cos(d*x + c)^3 + 3*(20*a^10*b^2 - 77*a^8*b^4
 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c)^2 + (44*a^9*b^3 - 16
9*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c))*sin(d*x +
c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^13)*d*cos(d*x +
 c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14)*d*cos
(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*
cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*
cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*sec(c + d*x))**4, x)
```

Giac [A] time = 1.43354, size = 799, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*s
qrt(-a^2 + b^2)) + (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*x +
1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x + 1/
2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x + 1/2*
c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*a^9*tan(1/2*d*x + 1/2*c)^3 +
152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 236*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 1
20*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) + 42*a^8*b*
tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*b^3*tan(1/
2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*tan(1/2*d*x
+ 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b
^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)
 + 12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 - 12*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/b^5 + 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^
4))/d
```

3.515 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=259

$$-\frac{a\left(-7a^4b^2+8a^2b^4+2a^6-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a^3\left(3a^2-8b^2\right)\mathrm{ta}}{6b^3d\left(a^2-b^2\right)^2(a+}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b^4*d) - (a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*
ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a
+ b)^(7/2)*d) - (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b
*Sec[c + d*x])^3) + (a^3*(3*a^2 - 8*b^2)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2
*d*(a + b*Sec[c + d*x])^2) - (a^2*(9*a^4 - 28*a^2*b^2 + 34*b^4)*Tan[c + d*x
])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.751795, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3845, 4090, 4080, 3998, 3770, 3831, 2659, 208}

$$-\frac{a\left(-7a^4b^2+8a^2b^4+2a^6-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a^3\left(3a^2-8b^2\right)\mathrm{ta}}{6b^3d\left(a^2-b^2\right)^2(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(b^4*d) - (a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*
ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a
+ b)^(7/2)*d) - (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b
*Sec[c + d*x])^3) + (a^3*(3*a^2 - 8*b^2)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2
*d*(a + b*Sec[c + d*x])^2) - (a^2*(9*a^4 - 28*a^2*b^2 + 34*b^4)*Tan[c + d*x
])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
```

```
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 3998

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec^2(c+dx)\left(2a^2-3ab\sec(c+dx)-3\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(2a^2b\right.}{}}{}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{a^2\left(9a^4-28a^2\right.}{6b^3\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{a^2\left(9a^4-28a^2\right.}{6b^3\left(a^2-b^2\right)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4 d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4 d} - \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\tan(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\sec(c+\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4 d} - \frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d} - \frac{a^2\,\mathrm{se}}{3b\left(a^2-\right.}$$

Mathematica [A] time = 4.18369, size = 250, normalized size = 0.97

$$\frac{-\frac{a^2 b \sin(c+dx)\left(a^2\left(-17a^2b^2+6a^4+26b^4\right)\cos^2(c+dx)+15ab\left(-3a^2b^2+a^4+4b^4\right)\cos(c+dx)-32a^2b^4+11a^4b^2+36b^6\right)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)^3}+\frac{6a\left(-7a^4b^2+8a^2b^4+2a^6-8b^6\right)\tanh^{-1}\left(\frac{(b-a)\,\mathrm{ta}}{\surd}\right.}{\left(a^2-b^2\right)^{7/2}}}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a^2*b*(11*a^4*
b^2 - 32*a^2*b^4 + 36*b^6 + 15*a*b*(a^4 - 3*a^2*b^2 + 4*b^4)*Cos[c + d*x] +
 a^2*(6*a^4 - 17*a^2*b^2 + 26*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^3
*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*b^4*d)
```

Maple [B] time = 0.069, size = 1429, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^
3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-1/d*a^5/b^2/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5-6/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+4/d*a^3/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5+12/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-4/d*a^6/b^3/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-
2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+44/3/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^
3-24/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a
*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+2/d*a^6/b^3/(tan(1/2*d*x+1/2*c
)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c)+1/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
```

```
^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-6/d*a^4/b/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*
tan(1/2*d*x+1/2*c)-4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+12/d*a^2*b/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)-2/d*a^7/b^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+7/d*a^5/b^2
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*
x+1/2*c)/((a+b)*(a-b))^(1/2))-8/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*
(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+8/d*a*b^
2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))+1/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)-1/d/b^4*ln(
tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 20.7488, size = 3977, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9 + (2*a^10 - 7*a^8*b^2
 + 8*a^6*b^4 - 8*a^4*b^6)*cos(d*x + c)^3 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5*b
^5 - 8*a^3*b^7)*cos(d*x + c)^2 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^4*b^6 - 8*a
^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^
2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2
*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 6*(a^8*b^3 -
 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
 4*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
```

```
4*a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
4*a^3*b^8 + a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) - 6*(a^8*b^3 - 4*a^
6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^
5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4
*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3
*b^8 + a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(11*a^8*b^3 - 43*a^
6*b^5 + 68*a^4*b^7 - 36*a^2*b^9 + (6*a^10*b - 23*a^8*b^3 + 43*a^6*b^5 - 26*
a^4*b^7)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*
cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10
 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4
*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 -
4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4
*a^2*b^13 + b^15)*d), -1/6*(3*(2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9
+ (2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6)*cos(d*x + c)^3 + 3*(2*a^9*b
- 7*a^7*b^3 + 8*a^5*b^5 - 8*a^3*b^7)*cos(d*x + c)^2 + 3*(2*a^8*b^2 - 7*a^6*
b^4 + 8*a^4*b^6 - 8*a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*(a^8*b^3 - 4
*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4
*a^5*b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*
a^4*b^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*
a^3*b^8 + a*b^10)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*(a^8*b^3 - 4*a^6*
b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11 + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*
b^6 + a^3*b^8)*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b
^7 + a^2*b^9)*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b
^8 + a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (11*a^8*b^3 - 43*a^6*b^
5 + 68*a^4*b^7 - 36*a^2*b^9 + (6*a^10*b - 23*a^8*b^3 + 43*a^6*b^5 - 26*a^4*
b^7)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*cos(
d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a
^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^1
1 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^
3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2
*b^13 + b^15)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.53588, size = 755, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(-a
^2 + b^2)) + (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^
5 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 -
6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a
^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 12*a^8*tan(1/2*d*x + 1/2*c)^3 + 56*a^6*b^2*
tan(1/2*d*x + 1/2*c)^3 - 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 72*a^2*b^6*ta
n(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1/2*d*x +
1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x + 1/2*c) -
 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c) + 36*a^2*
b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*log(abs(tan(1/
2*d*x + 1/2*c) + 1))/b^4 - 3*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4)/d
```

3.516 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=222

$$-\frac{b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a\left(\right.}{6}$$

```
[Out] -((b*(3*a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
(a - b)^(7/2)*(a + b)^(7/2)*d)) - (a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^3) - (a^2*(2*a^2 - 7*b^2)*Tan[c + d*x])/(6*b
^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4
)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.421282, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3845, 4080, 4003, 12, 3831, 2659, 208}

$$-\frac{b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a\left(\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] -((b*(3*a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
(a - b)^(7/2)*(a + b)^(7/2)*d)) - (a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^3) - (a^2*(2*a^2 - 7*b^2)*Tan[c + d*x])/(6*b
^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4
)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
```

```
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(a^2-3ab\sec(c+dx)-\left(2a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(2ab\left(a^2\right.\right.}{}}{}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^4-5a^2b^2+\right.}{6b^2\left(a^2-b^2\right)^3 d}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^4-5a^2b^2+\right.}{6b^2\left(a^2-b^2\right)^3 d}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^4-5a^2b^2+\right.}{6b^2\left(a^2-b^2\right)^3 d}$$

$$= -\frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\tan(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^4-5a^2b^2+\right.}{6b^2\left(a^2-b^2\right)^3 d}$$

$$= -\frac{b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{a^2\left(2a^2\right.}{6b^2\left(a^2-b^2\right.}$$

Mathematica [A] time = 1.01167, size = 158, normalized size = 0.71

$$\frac{\frac{a\sin(c+dx)\left(a^2\left(4a^2+11b^2\right)\cos^2(c+dx)+3ab\left(a^2+9b^2\right)\cos(c+dx)-5a^2b^2+2a^4+18b^4\right)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)^3}+\frac{6b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*b*(3*a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^2 - b^2)^(7/2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4 + 3*a*b*(a^2 + 9*b^2)*C
os[c + d*x] + a^2*(4*a^2 + 11*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^3
*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Maple [A] time = 0.064, size = 285, normalized size = 1.3

$$\frac{1}{d}\left(2\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)^{3}}\left(-1/2\,\frac{\left(2\,a^{2}+3\,ab+6\,b^{2}\right)a\left(\tan\left(1/2\,dx+c/2\right)\right)^{5}}{\left(a-b\right)\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}+2/3\,\frac{(a}{(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(2*(-1/2*(2*a^2+3*a*b+6*b^2)*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*
d*x+1/2*c)^5+2/3*(a^2+9*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+
1/2*c)^3-1/2*(2*a^2-3*a*b+6*b^2)*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*
d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3-b*(3*a^2+
2*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19083, size = 1975, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(3*a^2*b^4 + 2*b^6 + (3*a^5*b + 2*a^3*b^3)*cos(d*x + c)^3 + 3*(3*
a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^7 - 7*a^5*b^2 + 23*a^3*b^4
 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*b +
8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a
^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*
a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a
^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(3*a^2*b^4 + 2*b^6 + (3*a^5*b + 2*a^
3*b^3)*cos(d*x + c)^3 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 3*(3*a^3
*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*
cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^7 - 7*a^5*b^2 + 23*a^3
*b^4 - 18*a*b^6 + (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4)*cos(d*x + c)^2 + 3*(a^6*
b + 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 +
 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3
+ 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^
4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 +
 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x)
```

Giac [A] time = 1.4136, size = 544, normalized size = 2.45

$$-\frac{\frac{3\left(3a^2b+2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{6a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(3*a^2*b + 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan(1/
2*d*x + 1/2*c)^5 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b^2*tan(1/2*d*x +
 1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*
c)^5 - 4*a^5*tan(1/2*d*x + 1/2*c)^3 - 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 3
6*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*b*tan(1
/2*d*x + 1/2*c) + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*a^2*b^3*tan(1/2*d*x +
 1/2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.517 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=206

$$\frac{a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{(-10a}{6bd\,(a}$$

```
[Out] (a*(a^2 + 4*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
 b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*S
ec[c + d*x])^3) + (a*(a^2 - 6*b^2)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a +
b*Sec[c + d*x])^2) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Tan[c + d*x])/(6*b*(a^2 -
b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.353413, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3839, 4003, 12, 3831, 2659, 208}

$$\frac{a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{(-10a}{6bd\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (a*(a^2 + 4*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
 b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*S
ec[c + d*x])^3) + (a*(a^2 - 6*b^2)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a +
b*Sec[c + d*x])^2) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Tan[c + d*x])/(6*b*(a^2 -
b^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
```

]

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)\left(-3ab-\left(a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(2b\left(2a^2+ \right.\right.}{(a+}}{6b}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\left(a^4-10a^2b^2-6\right.}{6b\left(a^2-b^2\right)^3 d(a}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\left(a^4-10a^2b^2-6\right.}{6b\left(a^2-b^2\right)^3 d(a}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\left(a^4-10a^2b^2-6\right.}{6b\left(a^2-b^2\right)^3 d(a}$$

$$= -\frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\tan(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\left(a^4-10a^2b^2-6\right.}{6b\left(a^2-b^2\right)^3 d(a}$$

$$= \frac{a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a\left(a^2-6b\right.}{6b\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 1.0865, size = 165, normalized size = 0.8

$$\frac{\frac{\sin(c+dx)\left(-a^2b\left(13a^2+2b^2\right)\cos^2(c+dx)+3a\left(-9a^2b^2+a^4-2b^4\right)\cos(c+dx)+b\left(-10a^2b^2+a^4-6b^4\right)\right)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)^3} - \frac{6a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-6*a*(a^2 + 4*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(7/2) + ((b*(a^4 - 10*a^2*b^2 - 6*b^4) + 3*a*(a^4 - 9*a^2*b^2 -
 2*b^4)*Cos[c + d*x] - a^2*b*(13*a^2 + 2*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])
/((a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Maple [A] time = 0.064, size = 294, normalized size = 1.4

$$\frac{1}{d}\left(-2\,\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}b-a-b\right)^{3}}\left(-1/2\,\frac{\left(a^{3}+6\,a^{2}b+2\,ab^{2}+2\,b^{3}\right)\left(\tan\left(1/2\,dx+c/2\right)\right)^{5}}{\left(a-b\right)\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^3+6*a^2*b+2*a*b^2+2*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5+2/3*(7*a^2+3*b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3+1/2*(a^3-6*a^2*b+2*a*b^2-2*b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a
-b)^3+a*(a^2+4*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21951, size = 1976, normalized size = 9.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3*b^3 + 4*a*b^5 + (a^6 + 4*a^4*b^2)*cos(d*x + c)^3 + 3*(a^5*b
+ 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c))*sqrt(a^
2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^
```

```
2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7
 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 + 3*(a^7 - 10*a^5*b^2
 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*
a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6
*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*
a^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(a^3*b^3 + 4*a*b^5 + (a^6 + 4*a^4*b^
2)*cos(d*x + c)^3 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c)^2 + 3*(a^4*b^2 + 4*a
^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c))) + (a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6
*b^7 - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 + 3*(a^7 - 10*a^5
*b^2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2
+ 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3
 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b
^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5
+ 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.44319, size = 582, normalized size = 2.83

$$-\frac{\frac{3\left(a^3+4\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^5*tan(1/2*d*
x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1
/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^
5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 - 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 16*a^2
*b^3*tan(1/2*d*x + 1/2*c)^3 + 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*tan(1/2
*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2*c) + 6*b
^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.518 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=192

$$-\frac{b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{}{3d\left(a^2\right.}$$

```
[Out] -((b*(4*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(7/2)*(a + b)^(7/2)*d)) + (a*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Se
c[c + d*x])^3) + ((2*a^2 + 3*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*S
ec[c + d*x])^2) + (a*(2*a^2 + 13*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
 b*Sec[c + d*x]))
```

Rubi [A] time = 0.307318, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3836, 4003, 12, 3831, 2659, 208}

$$-\frac{b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(2a^2+13b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{}{3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((b*(4*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(7/2)*(a + b)^(7/2)*d)) + (a*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Se
c[c + d*x])^3) + ((2*a^2 + 3*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*S
ec[c + d*x])^2) + (a*(2*a^2 + 13*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
 b*Sec[c + d*x]))
```

Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)(-3b+2a\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)\left(10ab-\left(2a^2+3\right.\right.}{(a+b\sec(c+d}}{6\left(a^2-b\right.}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2+13b^2\right)\tan}{6\left(a^2-b^2\right)^3 d(a+b\,\mathrm{s}}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2+13b^2\right)\tan}{6\left(a^2-b^2\right)^3 d(a+b\,\mathrm{s}}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2+13b^2\right)\tan}{6\left(a^2-b^2\right)^3 d(a+b\,\mathrm{s}}$$

$$= \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{a\left(2a^2+13b^2\right)\tan}{6\left(a^2-b^2\right)^3 d(a+b\,\mathrm{s}}$$

$$= -\frac{b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\left(2a^2+3b\right.}{6\left(a^2-b^2\right)^2 d(}$$

Mathematica [A] time = 1.20827, size = 164, normalized size = 0.85

$$\frac{\frac{\sin(c+dx)\left(a\left(10a^2b^2+6a^4-b^4\right)\cos^2(c+dx)-3b\left(-9a^2b^2-2a^4+b^4\right)\cos(c+dx)+2a^3b^2+13ab^4\right)}{(a-b)^3(a+b)^3(a\cos(c+dx)+b)^3} + \frac{6b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((6*b*(4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + ((2*a^3*b^2 + 13*a*b^4 - 3*b*(-2*a^4 - 9*a^2*b^2 + b^4)*
Cos[c + d*x] + a*(6*a^4 + 10*a^2*b^2 - b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/(
(a - b)^3*(a + b)^3*(b + a*Cos[c + d*x])^3))/(6*d)
```

Maple [A] time = 0.06, size = 297, normalized size = 1.6

$$\frac{1}{d}\left(2\,\frac{1}{\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)^3}\left(-1/2\,\frac{\left(2\,a^3+2\,a^2b+6\,ab^2+b^3\right)(\tan(1/2\,dx+c/2))^5}{(a-b)\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(2*(-1/2*(2*a^3+2*a^2*b+6*a*b^2+b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*ta
n(1/2*d*x+1/2*c)^5+2/3*(3*a^2+7*b^2)*a/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(
1/2*d*x+1/2*c)^3-1/2*(2*a^3-2*a^2*b+6*a*b^2-b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^3-b*(4*a^2+b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.19169, size = 1978, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(4*a^2*b^4 + b^6 + (4*a^5*b + a^3*b^3)*cos(d*x + c)^3 + 3*(4*a^4*
b^2 + a^2*b^4)*cos(d*x + c)^2 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c))*sqrt(a^
2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^
```

```
2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6 + (6
*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4*
b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*
a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6
*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*
a^4*b^7 - 4*a^2*b^9 + b^11)*d), -1/6*(3*(4*a^2*b^4 + b^6 + (4*a^5*b + a^3*b
^3)*cos(d*x + c)^3 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + 3*(4*a^3*b^3
+ a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
 + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6
 + (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7
*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2
 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^
3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*
b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5
 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.35889, size = 582, normalized size = 3.03

$$-\frac{\frac{3\left(4a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{6a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(4*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^5*tan(1/2*
d*x + 1/2*c)^5 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x +
1/2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^4*tan(1/2*d*x + 1/2*c
)^5 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*a^5*tan(1/2*d*x + 1/2*c)^3 - 16*a^3
*b^2*tan(1/2*d*x + 1/2*c)^3 + 28*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1
/2*d*x + 1/2*c) + 6*a^4*b*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1
/2*c) + 27*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a*b^4*tan(1/2*d*x + 1/2*c) - 3
*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*
d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.519 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{5ab\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{}{3d\left(a^2-\right.}$$

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(7/2)*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])^3) - (5*a*b*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]
)^2) - (b*(11*a^2 + 4*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sec[c +
d*x]))
```

Rubi [A] time = 0.307289, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3833, 4003, 12, 3831, 2659, 208}

$$\frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{5ab\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{}{3d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(7/2)*(a + b)^(7/2)*d) - (b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec
[c + d*x])^3) - (5*a*b*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]
)^2) - (b*(11*a^2 + 4*b^2)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Sec[c +
d*x]))
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3a+2b\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)\left(2\left(3a^2+2b^2\right)\right.}{(a+b\sec(c+}}{6\left(a^2-\right.}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b\left(11a^2+4b^2\right)\mathrm{ta}}{6\left(a^2-b^2\right)^3 d(a+b}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b\left(11a^2+4b^2\right)\mathrm{ta}}{6\left(a^2-b^2\right)^3 d(a+b}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b\left(11a^2+4b^2\right)\mathrm{ta}}{6\left(a^2-b^2\right)^3 d(a+b}$$

$$= -\frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\tan(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} - \frac{b\left(11a^2+4b^2\right)\mathrm{ta}}{6\left(a^2-b^2\right)^3 d(a+b}$$

$$= \frac{a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{5ab\,\mathrm{ta}}{6\left(a^2-b^2\right)^2 d(}$$

Mathematica [A] time = 1.10894, size = 163, normalized size = 0.89

$$-\frac{\frac{b\sin(c+dx)\left(6ab\left(9a^2+b^2\right)\cos(c+dx)+\left(-5a^2b^2+18a^4+2b^4\right)\cos(2(c+dx))+17a^2b^2+18a^4+10b^4\right)}{(a\cos(c+dx)+b)^3}+\frac{12a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{12d(a-b)^3(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -((12*a*(2*a^2 + 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] + (b*(18*a^4 + 17*a^2*b^2 + 10*b^4 + 6*a*b*(9*a^2 + b^2)
*Cos[c + d*x] + (18*a^4 - 5*a^2*b^2 + 2*b^4)*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(b + a*Cos[c + d*x])^3)/(12*(a - b)^3*(a + b)^3*d)
```

Maple [A] time = 0.061, size = 284, normalized size = 1.5

$$\frac{1}{d}\left(-2\,\frac{1}{\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)^3}\left(-1/2\,\frac{\left(6\,a^2+3\,ab+2\,b^2\right)b\,(\tan(1/2\,dx+c/2))^5}{(a-b)\left(a^3+3\,a^2b+3\,ab^2+b^3\right)}+2/3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2
*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2
*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3+a*(2*a^2
+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25008, size = 1974, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^3*b^3 + 3*a*b^5 + (2*a^6 + 3*a^4*b^2)*cos(d*x + c)^3 + 3*(2*
a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))
*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2
```

```
*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(11*a^4*b^3 - 7*a^2*b^5 - 4*b^7
 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b^
2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a
^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*
a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a
^4*b^7 - 4*a^2*b^9 + b^11)*d), 1/6*(3*(2*a^3*b^3 + 3*a*b^5 + (2*a^6 + 3*a^4
*b^2)*cos(d*x + c)^3 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 3*(2*a^4*b^
2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (11*a^4*b^3 - 7*a^2*b^5 - 4*
b^7 + (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*(9*a^5
*b^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 +
6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 +
 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4
 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 +
6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Giac [B] time = 1.34987, size = 544, normalized size = 2.96

$$-\frac{\frac{3\left(2a^3+3ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{18a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-27a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
+ arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (18*a^4*b*tan(1
/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^3*tan(1/2*d
*x + 1/2*c)^5 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*b^5*tan(1/2*d*x + 1/2*c)
^5 - 36*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 32*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 +
4*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*t
an(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x
+ 1/2*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

3.520 $\int \frac{1}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=242

$$-\frac{b\left(-8a^4b^2+7a^2b^4+8a^6-2b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-17a^2b^2+26a^4+6b^4\right)\tan(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{b^2\left(8a^2-3b\right.}{6a^2d\left(a^2-b^2\right)^2(}$$

```
[Out] x/a^4 - (b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d) + (b^2*Tan
[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b^2*(8*a^2 - 3*b^2
)*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (b^2*(26*a
^4 - 17*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c
+ d*x]))
```

Rubi [A] time = 0.535264, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3785, 4060, 3919, 3831, 2659, 208}

$$-\frac{b\left(-8a^4b^2+7a^2b^4+8a^6-2b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-17a^2b^2+26a^4+6b^4\right)\tan(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{b^2\left(8a^2-3b\right.}{6a^2d\left(a^2-b^2\right)^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-4),x]
```


```
[Out] x/a^4 - (b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTanh[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d) + (b^2*Tan
[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b^2*(8*a^2 - 3*b^2
)*Tan[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (b^2*(26*a
^4 - 17*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c
+ d*x]))
```


Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
```

```
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{-3\left(a^2-b^2\right)+3ab\sec(c+dx)-2b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{6\left(a^2-b^2\right)^2-2ab\left(6a^2 \cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{b^2\left(26a^4-17a^2b^2 \cdots\right.}{6a^3\left(a^2-b^2\right)^3 d \cdots}$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{b^2\left(26a^4-1 \cdots\right.}{6a^3\left(a^2- \cdots\right.}$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{b^2\left(26a^4-1 \cdots\right.}{6a^3\left(a^2- \cdots\right.}$$

$$= \frac{x}{a^4} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{b^2\left(26a^4-1 \cdots\right.}{6a^3\left(a^2- \cdots\right.}$$

$$= \frac{x}{a^4} - \frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+ \cdots\right.}$$

Mathematica [A] time = 1.51756, size = 268, normalized size = 1.11

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b)\left(-\frac{ab^3\left(12a^2-7b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2} + \frac{ab^2\left(-32a^2b^2+36a^4+11b^4\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{(a-b)^3(a+b)^3} - \frac{6b\left(8a^4b^2-7 \cdots\right.}{\cdots}\right.}{6a^4d(a+b\sec(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-4),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(6*(c + d*x)*(b + a*Cos[c + d*x])^3 -
(6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (2*a*b^
```

```
4*Sin[c + d*x])/((a - b)*(a + b)) - (a*b^3*(12*a^2 - 7*b^2)*(b + a*Cos[c +
d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) + (a*b^2*(36*a^4 - 32*a^2*b^2 + 1
1*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a + b)^3)))/(6*a^4*
d*(a + b*Sec[c + d*x])^4)
```

Maple [B] time = 0.073, size = 1408, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 2/d/a^4*arctan(tan(1/2*d*x+1/2*c))-12/d*b^2*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)
^5-4/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+6/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5+1/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3
/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/d*b^6/a^3/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5+24/d*b^2*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-44/3/d*b^
4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(
a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+4/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3-12/d*b^2*a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)
/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+4/d*b^3/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*
x+1/2*c)+6/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-1/d*b^5/a^2/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan
(1/2*d*x+1/2*c)-2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-8/d*a^2*b/(a^6-3*
a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2))+8/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(
1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-7/d*b^5/a^2/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))+2/d*b^7/a^4/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(
a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6181, size = 3174, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x
+ c)^3 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(
d*x + c)^2 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*
cos(d*x + c) + 12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*x
+ 3*(8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10 + (8*a^9*b - 8*a^7*b^3 + 7*
a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^3 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6
- 2*a^2*b^8)*cos(d*x + c)^2 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^3*b^7 - 2*a*b^
9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
- b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(26*a^7*b^4 - 4
3*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10 + (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 -
 11*a^3*b^8)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b
^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^
6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8
*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4
*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 -
4*a^6*b^9 + a^4*b^11)*d), 1/6*(6*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6
+ a^3*b^8)*d*x*cos(d*x + c)^3 + 18*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*
b^7 + a^2*b^9)*d*x*cos(d*x + c)^2 + 18*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4
*a^3*b^8 + a*b^10)*d*x*cos(d*x + c) + 6*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 -
4*a^2*b^9 + b^11)*d*x - 3*(8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10 + (8*
a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^3 + 3*(8*a^8*b^2 -
8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c)^2 + 3*(8*a^7*b^3 - 8*a^5*b^
5 + 7*a^3*b^7 - 2*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
```

```
b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (26*a^7*b^4 - 43*a
^5*b^6 + 23*a^3*b^8 - 6*a*b^10 + (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11
*a^3*b^8)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)
*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 +
 a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^
7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^
7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a
^6*b^9 + a^4*b^11)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\sec{(c + dx)})^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-4), x)
```

Giac [B] time = 1.24683, size = 718, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a
^2 + b^2)) - (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*x +
1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x + 1/2*
c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*c)^5 +
 6*b^8*tan(1/2*d*x + 1/2*c)^5 - 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 116*a^4
*b^4*tan(1/2*d*x + 1/2*c)^3 - 56*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 12*b^8*ta
n(1/2*d*x + 1/2*c)^3 + 36*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*a^5*b^3*tan(1/2
*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*a^3*b^5*tan(1/2*d*x + 1
/2*c) - 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*a*b^7*tan(1/2*d*x + 1/2*c) + 6*
b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1
```

```
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(d*x + c)/a^4)
/d
```

3.521 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=299

$$\frac{\left(-65a^4b^2+68a^2b^4+6a^6-24b^6\right)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b^2\left(-35a^4b^2+28a^2b^4+20a^6-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-\right.}{2a^3}$$

```
[Out] (-4*b*x)/a^5 + (b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2]])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d
) + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Sin[c + d*x])/(6*a^4*(a^2 -
 b^2)^3*d) + (b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3)
+ (b^2*(9*a^2 - 4*b^2)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c +
d*x])^2) + (b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(2*a^3*(a^2 - b
^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.03704, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3847, 4100, 4104, 3919, 3831, 2659, 208}

$$\frac{\left(-65a^4b^2+68a^2b^4+6a^6-24b^6\right)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b^2\left(-35a^4b^2+28a^2b^4+20a^6-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-\right.}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (-4*b*x)/a^5 + (b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2]])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d
) + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Sin[c + d*x])/(6*a^4*(a^2 -
 b^2)^3*d) + (b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3)
+ (b^2*(9*a^2 - 4*b^2)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c +
d*x])^2) + (b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(2*a^3*(a^2 - b
^2)^3*d*(a + b*Sec[c + d*x]))
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-3a^2+4b^2+3ab\sec(c+dx)-3b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{\int \frac{\cos(c+dx)\left(6a^4-2\right.}{}}{}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2\left(9a^2-4b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^2} + \frac{b^2\left(12a^4-11a^2\right.}{2a^3\left(a^2-b^2\right)^3}$$

$$= \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b^2}{6a^2\left(a^2\right.}$$

$$= -\frac{4bx}{a^5} + \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} +$$

$$= -\frac{4bx}{a^5} + \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} +$$

$$= -\frac{4bx}{a^5} + \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\sin(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} +$$

$$= -\frac{4bx}{a^5} + \frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} + \frac{\left(6a^6-65a^4b^2+6\right.}{6a^4}$$

Mathematica [A] time = 1.67384, size = 293, normalized size = 0.98

$$\frac{\sec^4(c+dx)(a\cos(c+dx)+b)\left(\frac{5ab^4\left(3a^2-2b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^2(a+b)^2}-\frac{ab^3\left(-71a^2b^2+60a^4+26b^4\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{(a-b)^3(a+b)^3}+\frac{6b^2\left(35a^4b^2-2\ldots\right.}{\ldots}\right.}{6a^5d(\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^4*(-24*b*(c + d*x)*(b + a*Cos[c + d*x])^
3 + (6*b^2*(-20*a^6 + 35*a^4*b^2 - 28*a^2*b^4 + 8*b^6)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2)
+ (2*a*b^5*Sin[c + d*x])/((-a + b)*(a + b)) + (5*a*b^4*(3*a^2 - 2*b^2)*(b +
 a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) - (a*b^3*(60*a^4 - 71*
a^2*b^2 + 26*b^4)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/((a - b)^3*(a + b)^3
) + 6*a*(b + a*Cos[c + d*x])^3*Sin[c + d*x]))/(6*a^5*d*(a + b*Sec[c + d*x])
^4)
```

Maple [B] time = 0.097, size = 1448, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^4,x)
```

```
[Out] 2/d/a^4*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-8/d/a^5*b*arctan(tan(1/
2*d*x+1/2*c))+20/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+5/d*b^4/a/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)
*tan(1/2*d*x+1/2*c)^5-18/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/d*b^6/
a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+6/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*
c)^5-40/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*
a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+116/3/d*b^5/a^2/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*ta
```

```
n(1/2*d*x+1/2*c)^3-12/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+20/d*b^3/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)-5/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-18/d*b^
5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^
2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*
c)+6/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^
2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))-35/d*b^4/a/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+28/d*b^6/a^3/(a^6-3*a^4*b
^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))-8/d*b^8/a^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.91176, size = 3573, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(48*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d
*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d
*x*cos(d*x + c)^2 + 144*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^
11)*d*x*cos(d*x + c) + 48*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b
^12)*d*x - 3*(20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11 + (20*a^9*b^2 -
```

```
 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c)^3 + 3*(20*a^8*b^3 - 35*a
^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + 3*(20*a^7*b^4 - 35*a^5*b^
6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x
 + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b
^2)) - 2*(6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11 + 6
*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^3 + (18
*a^11*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c
)^2 + 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*
cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 +
 a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^
7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a
^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*
a^7*b^9 + a^5*b^11)*d), -1/6*(24*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^
7 + a^3*b^9)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*
a^4*b^8 + a^2*b^10)*d*x*cos(d*x + c)^2 + 72*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^
7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d*x + c) + 24*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*
b^8 - 4*a^2*b^10 + b^12)*d*x - 3*(20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*
b^11 + (20*a^9*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c)^3 +
3*(20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + 3*(20
*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
) - (6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11 + 6*(a^1
2 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*cos(d*x + c)^3 + (18*a^11
*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c)^2 +
 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*cos(d
*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*
b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a
^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^
8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b
^9 + a^5*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47294, size = 761, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*s
qrt(-a^2 + b^2)) - (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan(1/2
*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(1/2*d
*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*d*x +
1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 - 120*a^6*b^3*tan(1/2*d*x + 1/2*c)
^3 + 236*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 - 152*a^2*b^7*tan(1/2*d*x + 1/2*c)^
3 + 36*b^9*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*tan(1/2*d*x + 1/2*c) + 105*a
^5*b^4*tan(1/2*d*x + 1/2*c) - 24*a^4*b^5*tan(1/2*d*x + 1/2*c) - 117*a^3*b^6
*tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*a*b^8*tan(1/2*
d*x + 1/2*c) + 18*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4
- a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3)
 + 12*(d*x + c)*b/a^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1
)*a^4))/d
```

3.522 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=387

$$-\frac{b\left(-146a^4b^2+167a^2b^4+24a^6-60b^6\right)\sin(c+dx)}{6a^5d\left(a^2-b^2\right)^3}+\frac{\left(-23a^4b^2+27a^2b^4+a^6-10b^6\right)\sin(c+dx)\cos(c+dx)}{2a^4d\left(a^2-b^2\right)^3}-\frac{b^3\left(-84\right.}{}$$

```
[Out] ((a^2 + 20*b^2)*x)/(2*a^6) - (b^3*(40*a^6 - 84*a^4*b^2 + 69*a^2*b^4 - 20*b^
6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*(a - b)^(7/2)*
(a + b)^(7/2)*d) - (b*(24*a^6 - 146*a^4*b^2 + 167*a^2*b^4 - 60*b^6)*Sin[c +
 d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6 - 23*a^4*b^2 + 27*a^2*b^4 - 10*b^6)*
Cos[c + d*x]*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b^2*Cos[c + d*x]*Sin[
c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (5*b^2*(2*a^2 - b^2)
*Cos[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2)
+ (b^2*(48*a^4 - 53*a^2*b^2 + 20*b^4)*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^
2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.45597, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3847, 4100, 4104, 3919, 3831, 2659, 208}

$$-\frac{b\left(-146a^4b^2+167a^2b^4+24a^6-60b^6\right)\sin(c+dx)}{6a^5d\left(a^2-b^2\right)^3}+\frac{\left(-23a^4b^2+27a^2b^4+a^6-10b^6\right)\sin(c+dx)\cos(c+dx)}{2a^4d\left(a^2-b^2\right)^3}-\frac{b^3\left(-84\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```


```
[Out] ((a^2 + 20*b^2)*x)/(2*a^6) - (b^3*(40*a^6 - 84*a^4*b^2 + 69*a^2*b^4 - 20*b^
6)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*(a - b)^(7/2)*
(a + b)^(7/2)*d) - (b*(24*a^6 - 146*a^4*b^2 + 167*a^2*b^4 - 60*b^6)*Sin[c +
 d*x])/(6*a^5*(a^2 - b^2)^3*d) + ((a^6 - 23*a^4*b^2 + 27*a^2*b^4 - 10*b^6)*
Cos[c + d*x]*Sin[c + d*x])/(2*a^4*(a^2 - b^2)^3*d) + (b^2*Cos[c + d*x]*Sin[
c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (5*b^2*(2*a^2 - b^2)
*Cos[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2)
+ (b^2*(48*a^4 - 53*a^2*b^2 + 20*b^4)*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^
2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(-3a^2+5b^2+3ab\sec(c+dx)-4b^2\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^3}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{5b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(2(}{}}{}$$

$$= \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{5b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^2} + \frac{b^2\left(48a^4-53\right.}{6a^3\left(\right.}$$

$$= \frac{\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\cos(c+dx)\sin(c+dx)}{2a^4\left(a^2-b^2\right)^3d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}$$

$$= -\frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3d} + \frac{\left(a^6-23a^4b^2+27a^2b^4-10b^6\right)\cos(c}{2a^4\left(a^2-b^2\right)^3d}$$

$$= \frac{\left(a^2+20b^2\right)x}{2a^6} - \frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3d} + \frac{\left(a^6-23a^4b^2+27a^2b^4\right.}{2a^4}$$

$$= \frac{\left(a^2+20b^2\right)x}{2a^6} - \frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3d} + \frac{\left(a^6-23a^4b^2+27a^2b^4\right.}{2a^4}$$

$$= \frac{\left(a^2+20b^2\right)x}{2a^6} - \frac{b\left(24a^6-146a^4b^2+167a^2b^4-60b^6\right)\sin(c+dx)}{6a^5\left(a^2-b^2\right)^3d} + \frac{\left(a^6-23a^4b^2+27a^2b^4\right.}{2a^4}$$

$$= \frac{\left(a^2+20b^2\right)x}{2a^6} - \frac{b^3\left(40a^6-84a^4b^2+69a^2b^4-20b^6\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\left(24a^6-\right.}{}$$

Mathematica [A] time = 6.35884, size = 326, normalized size = 0.84

$$\frac{\left(a^2+20b^2\right)(c+dx)}{2a^6d} - \frac{b^6\sin(c+dx)}{3a^5d(b-a)(a+b)(a\cos(c+dx)+b)^3} + \frac{13b^7\sin(c+dx)-18a^2b^5\sin(c+dx)}{6a^5d(b-a)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{122a^2b^6\sin}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((a^2 + 20*b^2)*(c + d*x))/(2*a^6*d) + (b^3*(-40*a^6 + 84*a^4*b^2 - 69*a^2*
b^4 + 20*b^6)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*Sq
rt[a^2 - b^2]*(-a^2 + b^2)^3*d) - (4*b*Sin[c + d*x])/(a^5*d) - (b^6*Sin[c +
 d*x])/(3*a^5*(-a + b)*(a + b)*d*(b + a*Cos[c + d*x])^3) + (-18*a^2*b^5*Sin
[c + d*x] + 13*b^7*Sin[c + d*x])/(6*a^5*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c
 + d*x])^2) + (-90*a^4*b^4*Sin[c + d*x] + 122*a^2*b^6*Sin[c + d*x] - 47*b^8
*Sin[c + d*x])/(6*a^5*(-a + b)^3*(a + b)^3*d*(b + a*Cos[c + d*x])) + Sin[2*
(c + d*x)]/(4*a^4*d)
```

Maple [B] time = 0.101, size = 1576, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x)
```

```
[Out] -1/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-8/d/a^5/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+1/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-8/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*b+
1/d/a^4*arctan(tan(1/2*d*x+1/2*c))+20/d/a^6*arctan(tan(1/2*d*x+1/2*c))*b^2-
30/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-6/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d
*x+1/2*c)^5+34/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+3/d*b^7/a^4/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+
b^3)*tan(1/2*d*x+1/2*c)^5-12/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+60/d
*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2
)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-212/3/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d
*x+1/2*c)^3+24/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)^3/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-30/d*b^4/a/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)+6/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+34/d*b^6/a
^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b
+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-3/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/
2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-
```

```
12/d*b^8/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^3/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-40/d*b^3/(a^6-3*a^4*b^2+3*a^2*b^
4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))+84/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctan
h((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-69/d*b^7/a^4/(a^6-3*a^4*b^2
+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))+20/d*b^9/a^6/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/
2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.31759, size = 4038, normalized size = 10.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(6*(a^13 + 16*a^11*b^2 - 74*a^9*b^4 + 116*a^7*b^6 - 79*a^5*b^8 + 20*a
^3*b^10)*d*x*cos(d*x + c)^3 + 18*(a^12*b + 16*a^10*b^3 - 74*a^8*b^5 + 116*a
^6*b^7 - 79*a^4*b^9 + 20*a^2*b^11)*d*x*cos(d*x + c)^2 + 18*(a^11*b^2 + 16*a
^9*b^4 - 74*a^7*b^6 + 116*a^5*b^8 - 79*a^3*b^10 + 20*a*b^12)*d*x*cos(d*x +
c) + 6*(a^10*b^3 + 16*a^8*b^5 - 74*a^6*b^7 + 116*a^4*b^9 - 79*a^2*b^11 + 20
*b^13)*d*x + 3*(40*a^6*b^6 - 84*a^4*b^8 + 69*a^2*b^10 - 20*b^12 + (40*a^9*b
^3 - 84*a^7*b^5 + 69*a^5*b^7 - 20*a^3*b^9)*cos(d*x + c)^3 + 3*(40*a^8*b^4 -
 84*a^6*b^6 + 69*a^4*b^8 - 20*a^2*b^10)*cos(d*x + c)^2 + 3*(40*a^7*b^5 - 84
*a^5*b^7 + 69*a^3*b^9 - 20*a*b^11)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) - 2*(24*a^9*b^4 - 170*a^7*b^6 + 313*a^5*b^8 - 227*a^3*b^10 + 6
0*a*b^12 - 3*(a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^7*b^6 + a^5*b^8)*cos(d*x
```

```
+ c)^4 + 15*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7 + a^4*b^9)*cos(d*x
 + c)^3 + (63*a^11*b^2 - 342*a^9*b^4 + 590*a^7*b^6 - 421*a^5*b^8 + 110*a^3*
b^10)*cos(d*x + c)^2 + 3*(23*a^10*b^3 - 146*a^8*b^5 + 263*a^6*b^7 - 190*a^4
*b^9 + 50*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2 + 6*a^1
3*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*b^3 + 6
*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4*a^13*b
^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 - 4*a^12
*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d), 1/6*(3*(a^13 + 16*a^11*b^2 -
74*a^9*b^4 + 116*a^7*b^6 - 79*a^5*b^8 + 20*a^3*b^10)*d*x*cos(d*x + c)^3 + 9
*(a^12*b + 16*a^10*b^3 - 74*a^8*b^5 + 116*a^6*b^7 - 79*a^4*b^9 + 20*a^2*b^1
1)*d*x*cos(d*x + c)^2 + 9*(a^11*b^2 + 16*a^9*b^4 - 74*a^7*b^6 + 116*a^5*b^8
 - 79*a^3*b^10 + 20*a*b^12)*d*x*cos(d*x + c) + 3*(a^10*b^3 + 16*a^8*b^5 - 7
4*a^6*b^7 + 116*a^4*b^9 - 79*a^2*b^11 + 20*b^13)*d*x - 3*(40*a^6*b^6 - 84*a
^4*b^8 + 69*a^2*b^10 - 20*b^12 + (40*a^9*b^3 - 84*a^7*b^5 + 69*a^5*b^7 - 20
*a^3*b^9)*cos(d*x + c)^3 + 3*(40*a^8*b^4 - 84*a^6*b^6 + 69*a^4*b^8 - 20*a^2
*b^10)*cos(d*x + c)^2 + 3*(40*a^7*b^5 - 84*a^5*b^7 + 69*a^3*b^9 - 20*a*b^11
)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
 a)/((a^2 - b^2)*sin(d*x + c))) - (24*a^9*b^4 - 170*a^7*b^6 + 313*a^5*b^8 -
 227*a^3*b^10 + 60*a*b^12 - 3*(a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^7*b^6 +
a^5*b^8)*cos(d*x + c)^4 + 15*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7 +
 a^4*b^9)*cos(d*x + c)^3 + (63*a^11*b^2 - 342*a^9*b^4 + 590*a^7*b^6 - 421*a
^5*b^8 + 110*a^3*b^10)*cos(d*x + c)^2 + 3*(23*a^10*b^3 - 146*a^8*b^5 + 263*
a^6*b^7 - 190*a^4*b^9 + 50*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4
*a^15*b^2 + 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b
 - 4*a^14*b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^
15*b^2 - 4*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (
a^14*b^3 - 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**4,x)

[Out] Timed out
```

Giac [A] time = 1.38853, size = 830, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(6*(40*a^6*b^3 - 84*a^4*b^5 + 69*a^2*b^7 - 20*b^9)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d
*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*
sqrt(-a^2 + b^2)) - 2*(90*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 162*a^5*b^5*tan(
1/2*d*x + 1/2*c)^5 - 48*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 213*a^3*b^7*tan(1/
2*d*x + 1/2*c)^5 - 48*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 81*a*b^9*tan(1/2*d*x
 + 1/2*c)^5 + 36*b^10*tan(1/2*d*x + 1/2*c)^5 - 180*a^6*b^4*tan(1/2*d*x + 1/
2*c)^3 + 392*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 284*a^2*b^8*tan(1/2*d*x + 1/2
*c)^3 + 72*b^10*tan(1/2*d*x + 1/2*c)^3 + 90*a^6*b^4*tan(1/2*d*x + 1/2*c) +
162*a^5*b^5*tan(1/2*d*x + 1/2*c) - 48*a^4*b^6*tan(1/2*d*x + 1/2*c) - 213*a^
3*b^7*tan(1/2*d*x + 1/2*c) - 48*a^2*b^8*tan(1/2*d*x + 1/2*c) + 81*a*b^9*tan
(1/2*d*x + 1/2*c) + 36*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a^9*b^2 + 3*a^
7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a -
 b)^3) + 3*(a^2 + 20*b^2)*(d*x + c)/a^6 - 6*(a*tan(1/2*d*x + 1/2*c)^3 + 8*b
*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 8*b*tan(1/2*d*x + 1/2*c)
)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d
```

3.523 $\int \frac{1}{3+5\sec(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{5\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{6d}-\frac{x}{12}$$

```
[Out] -x/12 + (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(6*d)
```

Rubi [A] time = 0.0308961, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3783, 2657}

$$\frac{5\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{6d}-\frac{x}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-1),x]
```

```
[Out] -x/12 + (5*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(6*d)
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{3+5\sec(c+dx)}\,dx = \frac{x}{3} - \frac{1}{3}\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}\,dx$$
$$= -\frac{x}{12} + \frac{5\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{6d}$$

Mathematica [A] time = 0.0555576, size = 30, normalized size = 0.97

$$\frac{2(c+dx)+5\tan^{-1}\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-1),x]
```

```
[Out] (2*(c + d*x) + 5*ArcTan[2*Cot[(c + d*x)/2]])/(6*d)
```

Maple [A] time = 0.039, size = 34, normalized size = 1.1

$$\frac{2}{3d}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{5}{6d}\arctan\left(\frac{1}{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c)),x)
```

```
[Out] 2/3/d*arctan(tan(1/2*d*x+1/2*c))-5/6/d*arctan(1/2*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.77395, size = 63, normalized size = 2.03

$$\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) - 5\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(4*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 5*arctan(1/2*sin(d*x + c)/
(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.6259, size = 89, normalized size = 2.87

$$\frac{4\,dx + 5\,\arctan\left(\frac{5\,\cos(dx+c)+3}{4\,\sin(dx+c)}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*d*x + 5*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{5\sec\left(c + dx\right) + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x)
```

```
[Out] Integral(1/(5*sec(c + d*x) + 3), x)
```

Giac [A] time = 1.24765, size = 41, normalized size = 1.32

$$-\frac{dx + c - 10\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(d*x + c - 10*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

3.524 $\int \frac{1}{(3+5\sec(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}+\frac{35\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{288d}+\frac{29x}{576}$$

```
[Out] (29*x)/576 + (35*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(288*d) - (25*Tan
[c + d*x])/(48*d*(3 + 5*Sec[c + d*x]))
```

Rubi [A] time = 0.0796837, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3785, 3919, 3831, 2657}

$$-\frac{25\tan(c+dx)}{48d(5\sec(c+dx)+3)}+\frac{35\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{288d}+\frac{29x}{576}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-2),x]
```

```
[Out] (29*x)/576 + (35*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(288*d) - (25*Tan
[c + d*x])/(48*d*(3 + 5*Sec[c + d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5\sec(c+dx))^2}\,dx &= -\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}+\frac{1}{48}\int \frac{16+15\sec(c+dx)}{3+5\sec(c+dx)}\,dx\\
&= \frac{x}{9}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}-\frac{35}{144}\int \frac{\sec(c+dx)}{3+5\sec(c+dx)}\,dx\\
&= \frac{x}{9}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}-\frac{7}{144}\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}\,dx\\
&= \frac{29x}{576}+\frac{35\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{288d}-\frac{25\tan(c+dx)}{48d(3+5\sec(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.171145, size = 73, normalized size = 1.3

$$\frac{160(c+dx)-150\sin(c+dx)+96(c+dx)\cos(c+dx)+35(3\cos(c+dx)+5)\tan^{-1}\left(2\cot\left(\frac{1}{2}(c+dx)\right)\right)}{288d(3\cos(c+dx)+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-2),x]
```

```
[Out] (160*(c + d*x) + 96*(c + d*x)*Cos[c + d*x] + 35*ArcTan[2*Cot[(c + d*x)/2]]*
(5 + 3*Cos[c + d*x]) - 150*Sin[c + d*x])/(288*d*(5 + 3*Cos[c + d*x]))
```

Maple [A] time = 0.044, size = 63, normalized size = 1.1

$$\frac{2}{9d}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{25}{48d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+4\right)^{-1}-\frac{35}{288d}\arctan\left(\frac{1}{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c))^2,x)
```

```
[Out] 2/9/d*arctan(tan(1/2*d*x+1/2*c))-25/48/d*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/
2*c)^2+4)-35/288/d*arctan(1/2*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 2.24934, size = 119, normalized size = 2.12

$$-\frac{\frac{150\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+4\right)(\cos(dx+c)+1)}-64\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+35\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{288d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/288*(150*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)*(cos(d*
x + c) + 1)) - 64*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 35*arctan(1/2*s
in(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.63778, size = 211, normalized size = 3.77

$$\frac{192\,dx\cos(dx+c)+320\,dx+35\,(3\cos(dx+c)+5)\arctan\left(\frac{5\cos(dx+c)+3}{4\sin(dx+c)}\right)-300\sin(dx+c)}{576\,(3\,d\cos(dx+c)+5\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/576*(192*d*x*cos(d*x + c) + 320*d*x + 35*(3*cos(d*x + c) + 5)*arctan(1/4*
(5*cos(d*x + c) + 3)/sin(d*x + c)) - 300*sin(d*x + c))/(3*d*cos(d*x + c) +
```

```
5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5\sec(c+dx)+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**2,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-2), x)
```

Giac [A] time = 1.12016, size = 80, normalized size = 1.43

$$\frac{29\,dx+29\,c-\frac{300\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4}+70\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{576\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/576*(29*d*x + 29*c - 300*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 4
) + 70*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

3.525 $\int \frac{1}{(3+5\sec(c+dx))^3}\,dx$

Optimal. Leaf size=81

$$-\frac{125\tan(c+dx)}{4608d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}+\frac{3055\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{27648d}-\frac{1007x}{55296}$$

```
[Out] (-1007*x)/55296 + (3055*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(27648*d)
- (25*Tan[c + d*x])/(96*d*(3 + 5*Sec[c + d*x])^2) - (125*Tan[c + d*x])/(460
8*d*(3 + 5*Sec[c + d*x]))
```

Rubi [A] time = 0.116301, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 4060, 3919, 3831, 2657}

$$-\frac{125\tan(c+dx)}{4608d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{96d(5\sec(c+dx)+3)^2}+\frac{3055\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{27648d}-\frac{1007x}{55296}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-3),x]
```

```
[Out] (-1007*x)/55296 + (3055*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(27648*d)
- (25*Tan[c + d*x])/(96*d*(3 + 5*Sec[c + d*x])^2) - (125*Tan[c + d*x])/(460
8*d*(3 + 5*Sec[c + d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
```

```
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3+5\sec(c+dx))^3}\,dx &= -\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}+\frac{1}{96}\int \frac{32+30\sec(c+dx)-25\sec^2(c+dx)}{(3+5\sec(c+dx))^2}\,dx\\
&= -\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}+\frac{\int \frac{512-165\sec(c+dx)}{3+5\sec(c+dx)}\,dx}{4608}\\
&= \frac{x}{27}-\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}-\frac{3055\int \frac{\sec(c+dx)}{3+5\sec(c+dx)}\,dx}{13824}\\
&= \frac{x}{27}-\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}-\frac{611\int \frac{1}{1+\frac{3}{5}\cos(c+dx)}\,dx}{13824}\\
&= -\frac{1007x}{55296}+\frac{3055\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{27648d}-\frac{25\tan(c+dx)}{96d(3+5\sec(c+dx))^2}-\frac{125\tan(c+dx)}{4608d(3+5\sec(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.340921, size = 108, normalized size = 1.33

$$\frac{-3750\sin(c+dx)-4725\sin(2(c+dx))+30720(c+dx)\cos(c+dx)+4608c\cos(2(c+dx))+4608dx\cos(2(c+dx))+}{27648d(3\cos(c+dx)+5)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-3),x]
```

```
[Out] (30208*c + 30208*d*x + 30720*(c + d*x)*Cos[c + d*x] + 3055*ArcTan[2*Cot[(c
+ d*x)/2]]*(5 + 3*Cos[c + d*x])^2 + 4608*c*Cos[2*(c + d*x)] + 4608*d*x*Cos[
2*(c + d*x)] - 3750*Sin[c + d*x] - 4725*Sin[2*(c + d*x)])/(27648*d*(5 + 3*C
os[c + d*x])^2)
```

Maple [A] time = 0.051, size = 94, normalized size = 1.2

$$\frac{2}{27\,d}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{475}{4608\,d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+4\right)^{-2}-\frac{275}{1152\,d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c))^3,x)
```

```
[Out] 2/27/d*arctan(tan(1/2*d*x+1/2*c))+475/4608/d/(tan(1/2*d*x+1/2*c)^2+4)^2*tan
(1/2*d*x+1/2*c)^3-275/1152/d/(tan(1/2*d*x+1/2*c)^2+4)^2*tan(1/2*d*x+1/2*c)-
3055/27648/d*arctan(1/2*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 2.30583, size = 177, normalized size = 2.19

$$-\frac{\frac{150\left(\frac{44\sin(dx+c)}{\cos(dx+c)+1}-\frac{19\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+16}-2048\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+3055\arctan\left(\frac{\sin(dx+c)}{2(\cos(dx+c)+1)}\right)}{27648\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/27648*(150*(44*sin(d*x + c)/(cos(d*x + c) + 1) - 19*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 16) - 2048*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 3
055*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.70344, size = 348, normalized size = 4.3

$$\frac{18432\,dx\cos(dx+c)^2 + 61440\,dx\cos(dx+c) + 51200\,dx + 3055\left(9\,\cos(dx+c)^2 + 30\,\cos(dx+c) + 25\right)\arctan\left(\frac{5\cos(\cdots}{4\sin\cdots}\right.}{55296\left(9\,d\cos(dx+c)^2 + 30\,d\cos(dx+c) + 25\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(18432*d*x*cos(d*x + c)^2 + 61440*d*x*cos(d*x + c) + 51200*d*x + 30
55*(9*cos(d*x + c)^2 + 30*cos(d*x + c) + 25)*arctan(1/4*(5*cos(d*x + c) + 3
)/sin(d*x + c)) - 300*(63*cos(d*x + c) + 25)*sin(d*x + c))/(9*d*cos(d*x + c
)^2 + 30*d*cos(d*x + c) + 25*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5\sec(c+dx)+3)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**3,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-3), x)
```

Giac [A] time = 1.25186, size = 101, normalized size = 1.25

$$-\frac{1007\,dx + 1007\,c - \frac{300\left(19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 44\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^2} - 6110\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{55296\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/55296*(1007*d*x + 1007*c - 300*(19*tan(1/2*d*x + 1/2*c)^3 - 44*tan(1/2*d
*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 4)^2 - 6110*arctan(sin(d*x + c)/(cos
(d*x + c) + 3)))/d
```

3.526 $\int \frac{1}{(3+5\sec(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$-\frac{16925\tan(c+dx)}{221184d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{4608d(5\sec(c+dx)+3)^2}-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}+\frac{11215\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1327104d}+\frac{215}{265}$$

```
[Out] (21553*x)/2654208 + (11215*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(132710
4*d) - (25*Tan[c + d*x])/(144*d*(3 + 5*Sec[c + d*x])^3) - (25*Tan[c + d*x])
/(4608*d*(3 + 5*Sec[c + d*x])^2) - (16925*Tan[c + d*x])/(221184*d*(3 + 5*Se
c[c + d*x]))
```

Rubi [A] time = 0.157904, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 4060, 3919, 3831, 2657}

$$-\frac{16925\tan(c+dx)}{221184d(5\sec(c+dx)+3)}-\frac{25\tan(c+dx)}{4608d(5\sec(c+dx)+3)^2}-\frac{25\tan(c+dx)}{144d(5\sec(c+dx)+3)^3}+\frac{11215\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+3}\right)}{1327104d}+\frac{215}{265}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*Sec[c + d*x])^(-4),x]
```

```
[Out] (21553*x)/2654208 + (11215*ArcTan[Sin[c + d*x]/(3 + Cos[c + d*x])])/(132710
4*d) - (25*Tan[c + d*x])/(144*d*(3 + 5*Sec[c + d*x])^3) - (25*Tan[c + d*x])
/(4608*d*(3 + 5*Sec[c + d*x])^2) - (16925*Tan[c + d*x])/(221184*d*(3 + 5*Se
c[c + d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
```

```
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{(3+5\sec(c+dx))^4}\,dx = -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}+\frac{1}{144}\int \frac{48+45\sec(c+dx)-50\sec^2(c+dx)}{(3+5\sec(c+dx))^3}\,dx$$

$$= -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}+\frac{\int \frac{1536-870\sec(c+dx)-75\sec^2(c+dx)}{(3+5\sec(c+dx))^2}dx}{13824}$$

$$= -\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+dx)}{221184d(3+5\sec(c+dx))}+\frac{\int 2}{}$$

$$= \frac{x}{81}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+dx)}{221184d(3+5\sec(c+dx))}-$$

$$= \frac{x}{81}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx))^2}-\frac{16925\tan(c+dx)}{221184d(3+5\sec(c+dx))}-$$

$$= \frac{21553x}{2654208}+\frac{11215\tan^{-1}\left(\frac{\sin(c+dx)}{3+\cos(c+dx)}\right)}{1327104d}-\frac{25\tan(c+dx)}{144d(3+5\sec(c+dx))^3}-\frac{25\tan(c+dx)}{4608d(3+5\sec(c+dx)}$$

Mathematica [A] time = 0.529908, size = 141, normalized size = 1.33

$$\frac{-5660475\sin(c+dx)-3082500\sin(2(c+dx))-582975\sin(3(c+dx))+8036352(c+dx)\cos(c+dx)+2211840c\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*Sec[c + d*x])^(-4),x]
```

```
[Out] (6307840*c + 6307840*d*x + 8036352*(c + d*x)*Cos[c + d*x] + 22430*ArcTan[2*
Cot[(c + d*x)/2]]*(5 + 3*Cos[c + d*x])^3 + 2211840*c*Cos[2*(c + d*x)] + 221
1840*d*x*Cos[2*(c + d*x)] + 221184*c*Cos[3*(c + d*x)] + 221184*d*x*Cos[3*(c
 + d*x)] - 5660475*Sin[c + d*x] - 3082500*Sin[2*(c + d*x)] - 582975*Sin[3*(
c + d*x)])/(2654208*d*(5 + 3*Cos[c + d*x])^3)
```

Maple [A] time = 0.05, size = 125, normalized size = 1.2

$$\frac{2}{81\,d}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{25925}{221184\,d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+4\right)^{-3}-\frac{3575}{6912\,d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{d}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+5*sec(d*x+c))^4,x)
```

```
[Out] 2/81/d*arctan(tan(1/2*d*x+1/2*c))-25925/221184/d/(tan(1/2*d*x+1/2*c)^2+4)^3
*tan(1/2*d*x+1/2*c)^5-3575/6912/d/(tan(1/2*d*x+1/2*c)^2+4)^3*tan(1/2*d*x+1/
2*c)^3-17675/13824/d/(tan(1/2*d*x+1/2*c)^2+4)^3*tan(1/2*d*x+1/2*c)-11215/13
27104/d*arctan(1/2*tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.95531, size = 231, normalized size = 2.18

$$-\frac{\frac{150\left(\frac{11312\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{4576\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1037\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{12\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+64}-32768\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+11215\,\arctan\left(\frac{\sin(dx+c)}{2\,(\cos(dx+c)+1)}\right)}{1327104\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1327104*(150*(11312*sin(d*x + c)/(cos(d*x + c) + 1) + 4576*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 1037*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 64) - 32768*arctan(sin(d*x + c)/(cos(
d*x + c) + 1)) + 11215*arctan(1/2*sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.73109, size = 504, normalized size = 4.75

$$\frac{884736\,dx\cos\left(dx+c\right)^3+4423680\,dx\cos\left(dx+c\right)^2+7372800\,dx\cos\left(dx+c\right)+4096000\,dx+11215\left(27\,\cos\left(dx+c\right)^3 +\right.}{2654208\left(27\,d\cos\left(dx+c\right)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2654208*(884736*d*x*cos(d*x + c)^3 + 4423680*d*x*cos(d*x + c)^2 + 7372800
*d*x*cos(d*x + c) + 4096000*d*x + 11215*(27*cos(d*x + c)^3 + 135*cos(d*x +
c)^2 + 225*cos(d*x + c) + 125)*arctan(1/4*(5*cos(d*x + c) + 3)/sin(d*x + c)
) - 300*(7773*cos(d*x + c)^2 + 20550*cos(d*x + c) + 16925)*sin(d*x + c))/(2
7*d*cos(d*x + c)^3 + 135*d*cos(d*x + c)^2 + 225*d*cos(d*x + c) + 125*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\sec\left(c+dx\right)+3\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))**4,x)
```

```
[Out] Integral((5*sec(c + d*x) + 3)**(-4), x)
```

Giac [A] time = 1.21062, size = 119, normalized size = 1.12

$$\frac{21553\,dx+21553\,c-\frac{300\left(1037\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4576\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+11312\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\right)^3}+22430\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+3}\right)}{2654208\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+5*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2654208*(21553*d*x + 21553*c - 300*(1037*tan(1/2*d*x + 1/2*c)^5 + 4576*ta
n(1/2*d*x + 1/2*c)^3 + 11312*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
+ 4)^3 + 22430*arctan(sin(d*x + c)/(cos(d*x + c) + 3)))/d
```

3.527 $\int \frac{1}{5+3\sec(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}+\frac{x}{5}$$

```
[Out] x/5 + (3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(20*d) - (3*Log[2*Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

Rubi [A] time = 0.0349103, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3783, 2659, 206}

$$\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}+\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-1),x]
```

```
[Out] x/5 + (3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(20*d) - (3*Log[2*Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{5+3\sec(c+dx)}\,dx = \frac{x}{5}-\frac{1}{5}\int \frac{1}{1+\frac{5}{3}\cos(c+dx)}\,dx$$
$$= \frac{x}{5}-\frac{2\,\text{Subst}\left(\int \frac{1}{\frac{8}{3}-\frac{2x^2}{3}}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{5d}$$
$$= \frac{x}{5}+\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}-\frac{3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

Mathematica [A] time = 0.0588145, size = 69, normalized size = 0.99

$$\frac{4(c+dx)+3\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-1),x]
```

```
[Out] (4*(c + d*x) + 3*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[2*Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d)
```

Maple [A] time = 0.039, size = 51, normalized size = 0.7

$$\frac{2}{5\,d}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3}{20\,d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)+\frac{3}{20\,d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c)),x)
```

```
[Out] 2/5/d*arctan(tan(1/2*d*x+1/2*c))-3/20/d*ln(tan(1/2*d*x+1/2*c)+2)+3/20/d*ln(
tan(1/2*d*x+1/2*c)-2)
```

Maxima [A] time = 1.70804, size = 95, normalized size = 1.36

$$\frac{8 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) - 3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 2\right) + 3 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - 2\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/20*(8*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 3*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 2) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Fricas [A] time = 1.68041, size = 154, normalized size = 2.2

$$\frac{8\,dx - 3 \log\left(\frac{3}{2} \cos\left(dx+c\right) + 2 \sin\left(dx+c\right) + \frac{5}{2}\right) + 3 \log\left(\frac{3}{2} \cos\left(dx+c\right) - 2 \sin\left(dx+c\right) + \frac{5}{2}\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*d*x - 3*log(3/2*cos(d*x + c) + 2*sin(d*x + c) + 5/2) + 3*log(3/2*co
s(d*x + c) - 2*sin(d*x + c) + 5/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3\sec\left(c+dx\right)+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x)
```

```
[Out] Integral(1/(3*sec(c + d*x) + 5), x)
```

Giac [A] time = 1.27425, size = 58, normalized size = 0.83

$$\frac{4\,dx + 4\,c - 3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 2\right|\right) + 3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 2\right|\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/20*(4*d*x + 4*c - 3*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 3*log(abs(tan(1/
2*d*x + 1/2*c) - 2)))/d
```

3.528 $\int \frac{1}{(5+3\sec(c+dx))^2} dx$

Optimal. Leaf size=95

$$\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}-\frac{123\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}$$

```
[Out] x/25 + (123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(1600*d) - (123*Log
[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(1600*d) + (9*Tan[c + d*x])/(80*d*
(5 + 3*Sec[c + d*x]))
```

Rubi [A] time = 0.0926357, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3785, 3919, 3831, 2659, 206}

$$\frac{9\tan(c+dx)}{80d(3\sec(c+dx)+5)}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}-\frac{123\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-2),x]
```

```
[Out] x/25 + (123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(1600*d) - (123*Log
[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(1600*d) + (9*Tan[c + d*x])/(80*d*
(5 + 3*Sec[c + d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(5+3\sec(c+dx))^2}\,dx &= \frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}-\frac{1}{80}\int \frac{-16+15\sec(c+dx)}{5+3\sec(c+dx)}\,dx\\
&= \frac{x}{25}+\frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}-\frac{123}{400}\int \frac{\sec(c+dx)}{5+3\sec(c+dx)}\,dx\\
&= \frac{x}{25}+\frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}-\frac{41}{400}\int \frac{1}{1+\frac{5}{3}\cos(c+dx)}\,dx\\
&= \frac{x}{25}+\frac{9\tan(c+dx)}{80d(5+3\sec(c+dx))}-\frac{41\,\mathrm{Subst}\left(\int \frac{1}{\frac{8}{3}-\frac{2x^2}{3}}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{200d}\\
&= \frac{x}{25}+\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{1600d}-\frac{123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c\right.\right.}{1600d}
\end{aligned}$$

Mathematica [A] time = 0.156953, size = 162, normalized size = 1.71

$$\frac{5\cos(c+dx)\left(64(c+dx)+123\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-123\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)\right)\right.}{1600d(5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-2),x]
```

```
[Out] (5*Cos[c + d*x]*(64*(c + d*x) + 123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]] - 123*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*(64*c + 64*d*x +
123*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 123*Log[2*Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + 60*Sin[c + d*x]))/(1600*d*(3 + 5*Cos[c + d*x]))
```

Maple [A] time = 0.048, size = 87, normalized size = 0.9

$$\frac{2}{25\,d}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{9}{160\,d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-1}-\frac{123}{1600\,d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)-\frac{9}{160\,d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^2,x)
```

```
[Out] 2/25/d*arctan(tan(1/2*d*x+1/2*c))-9/160/d/(tan(1/2*d*x+1/2*c)+2)-123/1600/d
*ln(tan(1/2*d*x+1/2*c)+2)-9/160/d/(tan(1/2*d*x+1/2*c)-2)+123/1600/d*ln(tan(
1/2*d*x+1/2*c)-2)
```

Maxima [A] time = 1.76822, size = 150, normalized size = 1.58

$$-\frac{\frac{180\,\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-4\right)(\cos(dx+c)+1)}-128\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+123\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-123\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{1600\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1600*(180*sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4)*(cos(d
*x + c) + 1)) - 128*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 123*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 2) - 123*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 2))/d
```

Fricas [A] time = 1.64609, size = 309, normalized size = 3.25

$$\frac{640\,dx\cos\left(dx+c\right)+384\,dx-123\left(5\,\cos\left(dx+c\right)+3\right)\log\left(\frac{3}{2}\,\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)+\frac{5}{2}\right)+123\left(5\,\cos\left(dx+c\right)+3\right.}{3200\left(5\,d\cos\left(dx+c\right)+3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3200*(640*d*x*cos(d*x + c) + 384*d*x - 123*(5*cos(d*x + c) + 3)*log(3/2*c
os(d*x + c) + 2*sin(d*x + c) + 5/2) + 123*(5*cos(d*x + c) + 3)*log(3/2*cos(
d*x + c) - 2*sin(d*x + c) + 5/2) + 360*sin(d*x + c))/(5*d*cos(d*x + c) + 3*
d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3\sec\left(c+dx\right)+5\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**2,x)
```

```
[Out] Integral((3*sec(c + d*x) + 5)**(-2), x)
```

Giac [A] time = 1.27848, size = 93, normalized size = 0.98

$$\frac{64\,dx+64\,c-\frac{180\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4}-123\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+123\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{1600\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1600*(64*d*x + 64*c - 180*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 -
4) - 123*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 123*log(abs(tan(1/2*d*x + 1/2
*c) - 2)))/d
```

3.529 $\int \frac{1}{(5+3\sec(c+dx))^3}\,dx$

Optimal. Leaf size=120

$$\frac{963\tan(c+dx)}{12800d(3\sec(c+dx)+5)}+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}+\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d}-\frac{8361\log\left(\sin\right.}{}$$

```
[Out] x/125 + (8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(256000*d) - (836
1*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(256000*d) + (9*Tan[c + d*x])
/(160*d*(5 + 3*Sec[c + d*x])^2) + (963*Tan[c + d*x])/(12800*d*(5 + 3*Sec[c
+ d*x]))
```

Rubi [A] time = 0.133031, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3785, 4060, 3919, 3831, 2659, 206}

$$\frac{963\tan(c+dx)}{12800d(3\sec(c+dx)+5)}+\frac{9\tan(c+dx)}{160d(3\sec(c+dx)+5)^2}+\frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d}-\frac{8361\log\left(\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-3),x]
```

```
[Out] x/125 + (8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(256000*d) - (836
1*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(256000*d) + (9*Tan[c + d*x])
/(160*d*(5 + 3*Sec[c + d*x])^2) + (963*Tan[c + d*x])/(12800*d*(5 + 3*Sec[c
+ d*x]))
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
```

```
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(5+3\sec(c+dx))^3}\,dx = \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} - \frac{1}{160}\int \frac{-32+30\sec(c+dx)-9\sec^2(c+dx)}{(5+3\sec(c+dx))^2}\,dx$$
$$= \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} + \frac{\int \frac{512-1365\sec(c+dx)}{5+3\sec(c+dx)}\,dx}{12800}$$
$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{8361\int \frac{\sec(c+dx)}{5+3\sec(c+dx)}\,dx}{64000}$$
$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{2787\int \frac{1}{1+\frac{5}{3}\cos(c+dx)}\,dx}{64000}$$
$$= \frac{x}{125} + \frac{9\tan(c+dx)}{160d(5+3\sec(c+dx))^2} + \frac{963\tan(c+dx)}{12800d(5+3\sec(c+dx))} - \frac{2787\,\text{Subst}\left(\int \frac{1}{\frac{8}{3}-\frac{2x^2}{3}}\,dx, x\right.}{32000d}$$
$$= \frac{x}{125} + \frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256000d} - \frac{8361\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.}{256000d}$$

Mathematica [B] time = 0.317709, size = 241, normalized size = 2.01

$$\frac{115560\sin(c+dx)+110700\sin(2(c+dx))+359523\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(c+dx)\left(2048(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-3),x]
```

```
[Out] (88064*c + 88064*d*x + 359523*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
60*Cos[c + d*x]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - 8361*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 25*Cos[2*(c + d
*x)]*(2048*(c + d*x) + 8361*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 83
61*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 359523*Log[2*Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]] + 115560*Sin[c + d*x] + 110700*Sin[2*(c + d*x)])/(5
12000*d*(3 + 5*Cos[c + d*x])^2)
```

Maple [A] time = 0.049, size = 123, normalized size = 1.

$$\frac{2}{125d}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{27}{2560d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-2} - \frac{1323}{25600d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-1} - \frac{8361}{256000d}\ln\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^3,x)
```

```
[Out] 2/125/d*arctan(tan(1/2*d*x+1/2*c))+27/2560/d/(tan(1/2*d*x+1/2*c)+2)^2-1323/
25600/d/(tan(1/2*d*x+1/2*c)+2)-8361/256000/d*ln(tan(1/2*d*x+1/2*c)+2)-27/25
60/d/(tan(1/2*d*x+1/2*c)-2)^2-1323/25600/d/(tan(1/2*d*x+1/2*c)-2)+8361/2560
00/d*ln(tan(1/2*d*x+1/2*c)-2)
```

Maxima [A] time = 1.64843, size = 209, normalized size = 1.74

$$-\frac{\frac{540\left(\frac{156\sin(dx+c)}{\cos(dx+c)+1}-\frac{49\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-16}-4096\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-8361\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-2\right)}{256000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/256000*(540*(156*sin(d*x + c)/(cos(d*x + c) + 1) - 49*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3)/(8*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 16) - 4096*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) +
 8361*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 8361*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 2))/d
```

Fricas [A] time = 1.69625, size = 482, normalized size = 4.02

$$\frac{102400\,dx\cos(dx+c)^2+122880\,dx\cos(dx+c)+36864\,dx-8361\left(25\cos(dx+c)^2+30\cos(dx+c)+9\right)\log\left(\frac{3}{2}\cos(\right.}{5120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/512000*(102400*d*x*cos(d*x + c)^2 + 122880*d*x*cos(d*x + c) + 36864*d*x -
 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*cos(d*x + c) + 2*si
n(d*x + c) + 5/2) + 8361*(25*cos(d*x + c)^2 + 30*cos(d*x + c) + 9)*log(3/2*
```

```
cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(205*cos(d*x + c) + 107)*sin(d*
x + c))/(25*d*cos(d*x + c)^2 + 30*d*cos(d*x + c) + 9*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\sec(c+dx)+5)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**3,x)
```

```
[Out] Integral((3*sec(c + d*x) + 5)**(-3), x)
```

Giac [A] time = 1.25572, size = 115, normalized size = 0.96

$$\frac{2048\,dx + 2048\,c - \frac{540\left(49\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 156\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 4\right)^2} - 8361\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right) + 8361\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{256000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/256000*(2048*d*x + 2048*c - 540*(49*tan(1/2*d*x + 1/2*c)^3 - 156*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 4)^2 - 8361*log(abs(tan(1/2*d*x + 1
/2*c) + 2)) + 8361*log(abs(tan(1/2*d*x + 1/2*c) - 2)))/d
```

3.530 $\int \frac{1}{(5+3\sec(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$\frac{38733\tan(c+dx)}{1024000d(3\sec(c+dx)+5)}+\frac{519\tan(c+dx)}{12800d(3\sec(c+dx)+5)^2}+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}+\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{20480000d}$$

```
[Out] x/625 + (278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(20480000*d) -
(278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(20480000*d) + (3*Tan[c
 + d*x])/(80*d*(5 + 3*Sec[c + d*x])^3) + (519*Tan[c + d*x])/(12800*d*(5 + 3
*Sec[c + d*x])^2) + (38733*Tan[c + d*x])/(1024000*d*(5 + 3*Sec[c + d*x]))
```

Rubi [A] time = 0.180382, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3785, 4060, 3919, 3831, 2659, 206}

$$\frac{38733\tan(c+dx)}{1024000d(3\sec(c+dx)+5)}+\frac{519\tan(c+dx)}{12800d(3\sec(c+dx)+5)^2}+\frac{3\tan(c+dx)}{80d(3\sec(c+dx)+5)^3}+\frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\right.}{20480000d}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*Sec[c + d*x])^(-4),x]
```


```
[Out] x/625 + (278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(20480000*d) -
(278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(20480000*d) + (3*Tan[c
 + d*x])/(80*d*(5 + 3*Sec[c + d*x])^3) + (519*Tan[c + d*x])/(12800*d*(5 + 3
*Sec[c + d*x])^2) + (38733*Tan[c + d*x])/(1024000*d*(5 + 3*Sec[c + d*x]))
```


Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
```

```
a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(5+3\sec(c+dx))^4}\,dx = \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} - \frac{1}{240}\int \frac{-48+45\sec(c+dx)-18\sec^2(c+dx)}{(5+3\sec(c+dx))^3}\,dx$$

$$= \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{\int \frac{1536-4230\sec(c+dx)+1557\sec^2(c+dx)}{(5+3\sec(c+dx))^2}\,dx}{38400}$$

$$= \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(c+dx)}{1024000d(5+3\sec(c+dx))} - \int$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(c+dx)}{1024000d(5+3\sec(c+dx)}$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(c+dx)}{1024000d(5+3\sec(c+dx)}$$

$$= \frac{x}{625} + \frac{3\tan(c+dx)}{80d(5+3\sec(c+dx))^3} + \frac{519\tan(c+dx)}{12800d(5+3\sec(c+dx))^2} + \frac{38733\tan(c+dx)}{1024000d(5+3\sec(c+dx)}$$

$$= \frac{x}{625} + \frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{20480000d} - \frac{278151\log\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\text{s}\right.}{20480000d}$$

Mathematica [B] time = 0.520187, size = 344, normalized size = 2.37

$$\frac{52174260\sin(c+dx)+51462000\sin(2(c+dx))+24286500\sin(3(c+dx))+4096000c\cos(3(c+dx))+4096000dx\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Sec[c + d*x])^(-4),x]
```

```
[Out] (18284544*c + 18284544*d*x + 4096000*c*Cos[3*(c + d*x)] + 4096000*d*x*Cos[3
*(c + d*x)] + 155208258*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 347688
75*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 915*Cos[c
+ d*x]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - 278151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 450*Cos[2*(c + d*x)
]*(32768*(c + d*x) + 278151*Log[2*Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 27
8151*Log[2*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 155208258*Log[2*Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] - 34768875*Cos[3*(c + d*x)]*Log[2*Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + 52174260*Sin[c + d*x] + 51462000*Sin[2*(c + d*x)
] + 24286500*Sin[3*(c + d*x)])/(81920000*d*(3 + 5*Cos[c + d*x])^3)
```

Maple [A] time = 0.053, size = 159, normalized size = 1.1

$$\frac{2}{625\,d}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{27}{10240\,d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-3}+\frac{1431}{102400\,d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{-2}-\frac{69093}{2048000\,d}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5+3*sec(d*x+c))^4,x)
```

```
[Out] 2/625/d*arctan(tan(1/2*d*x+1/2*c))-27/10240/d/(tan(1/2*d*x+1/2*c)+2)^3+1431
/102400/d/(tan(1/2*d*x+1/2*c)+2)^2-69093/2048000/d/(tan(1/2*d*x+1/2*c)+2)-2
78151/20480000/d*ln(tan(1/2*d*x+1/2*c)+2)-27/10240/d/(tan(1/2*d*x+1/2*c)-2)
^3-1431/102400/d/(tan(1/2*d*x+1/2*c)-2)^2-69093/2048000/d/(tan(1/2*d*x+1/2*
c)-2)+278151/20480000/d*ln(tan(1/2*d*x+1/2*c)-2)
```

Maxima [A] time = 1.6035, size = 262, normalized size = 1.81

$$-\frac{\frac{540\left(\frac{26384\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{16032\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2559\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{\frac{48\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{12\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-64}-65536\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+278151\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+2\right)-278151\,\log\left(\right.}{20480000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/20480000*(540*(26384*sin(d*x + c)/(cos(d*x + c) + 1) - 16032*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 2559*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(48*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 - 12*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 64) - 65536*arctan(sin(d*x + c)/(co
s(d*x + c) + 1)) + 278151*log(sin(d*x + c)/(cos(d*x + c) + 1) + 2) - 278151
*log(sin(d*x + c)/(cos(d*x + c) + 1) - 2))/d
```

Fricas [A] time = 1.75805, size = 672, normalized size = 4.63

$$8192000\,dx\cos(dx+c)^3+14745600\,dx\cos(dx+c)^2+8847360\,dx\cos(dx+c)+1769472\,dx-278151\left(125\,\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/40960000*(8192000*d*x*cos(d*x + c)^3 + 14745600*d*x*cos(d*x + c)^2 + 8847
360*d*x*cos(d*x + c) + 1769472*d*x - 278151*(125*cos(d*x + c)^3 + 225*cos(d
*x + c)^2 + 135*cos(d*x + c) + 27)*log(3/2*cos(d*x + c) + 2*sin(d*x + c) +
5/2) + 278151*(125*cos(d*x + c)^3 + 225*cos(d*x + c)^2 + 135*cos(d*x + c) +
 27)*log(3/2*cos(d*x + c) - 2*sin(d*x + c) + 5/2) + 1080*(44975*cos(d*x + c
)^2 + 47650*cos(d*x + c) + 12911)*sin(d*x + c))/(125*d*cos(d*x + c)^3 + 225
*d*cos(d*x + c)^2 + 135*d*cos(d*x + c) + 27*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\sec(c+dx)+5)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))**4,x)
```

```
[Out] Integral((3*sec(c + d*x) + 5)**(-4), x)
```

Giac [A] time = 1.2832, size = 132, normalized size = 0.91

$$\frac{32768\,dx+32768\,c-\frac{540\left(2559\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-16032\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+26384\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\right)^{3}}-278151\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)+}{20480000\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/20480000*(32768*d*x + 32768*c - 540*(2559*tan(1/2*d*x + 1/2*c)^5 - 16032*
tan(1/2*d*x + 1/2*c)^3 + 26384*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^
2 - 4)^3 - 278151*log(abs(tan(1/2*d*x + 1/2*c) + 2)) + 278151*log(abs(tan(1
/2*d*x + 1/2*c) - 2)))/d
```

3.531 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=292

$$\frac{2(a-b)\sqrt{a+b}(2a+9b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^2d}+\frac{2(a-b)\sqrt{a}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a^2 - 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) + (2*(a - b
)*Sqrt[a + b]*(2*a + 9*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*d) - (4*a*Sqrt[a + b*Sec[c
+ d*x]]*Tan[c + d*x])/(15*b*d) + (2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(5*b*d)
```

Rubi [A] time = 0.441508, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3840, 4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}\left(2a^2-9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{2(a-b)\sqrt{a+b}(2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(2*a^2 - 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) + (2*(a - b
)*Sqrt[a + b]*(2*a + 9*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*d) - (4*a*Sqrt[a + b*Sec[c
+ d*x]]*Tan[c + d*x])/(15*b*d) + (2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(5*b*d)
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
```

```
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{2\int \sec(c+dx)\left(\frac{3b}{2} - a\sec(c+dx)\right)\sqrt{a+}}{5b}$$

$$= -\frac{4a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{4\int}{}$$

$$= -\frac{4a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15bd} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{((a}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(2a^2-9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{15b^3d}$$

Mathematica [A] time = 13.595, size = 401, normalized size = 1.37

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\left(2b\left(-2a^2+7ab+9b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\text{si}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(2*a^3
 + 2*a^2*b - 9*a*b^2 - 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + 2*b*(-2*a^2 + 7*a*b + 9*b^2)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (2*a^2 - 9*b^2)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b
^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]) + (S
qrt[a + b*Sec[c + d*x]]*((2*(-2*a^2 + 9*b^2)*Sin[c + d*x])/(15*b^2) + (2*a*
Tan[c + d*x])/(15*b) + (2*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.614, size = 1582, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-9*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*b^3+2*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-2*cos(d*x+c)^3*a^2*b+9*co
s(d*x+c)^3*b^3-2*cos(d*x+c)^4*a^3+2*cos(d*x+c)^3*a^3-6*cos(d*x+c)^2*b^3-3*b
^3+9*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*b^3+2*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-9*sin(d*x+c)*cos(d*x+c)^2*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3+9*sin(d*x+c)
*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*b^3+cos(d*x+c)^4*a^2*b+9*cos(d*x+c)^4*a*b^2-5*cos(d*x+c)^3*a*b^2+cos(d*
x+c)^2*a^2*b-4*cos(d*x+c)*a*b^2+2*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-9*sin(d*x+c)*cos(d*x+
c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2
-2*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^2*b+7*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+2*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-9*sin(d*
x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a*b^2-2*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+7*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2)/(b+a*cos(d*x+c))
/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)+a}\sec(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(c+dx)}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.532 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=241

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd}-\frac{2a(a-b)\sqrt{a+b}\cot}{}$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*(a - b)*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(3*b*d) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.277633, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3835, 4005, 3832, 4004}

$$-\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2\tan(c+dx)\sqrt{a+b\sec(c}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) - (2*(a - b)*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(3*b*d) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx &= \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}\int\frac{\sec(c+dx)(b+a\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx\\ &= \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}a\int\frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx+\frac{1}{3}(-a\\ &= -\frac{2a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{3b^2d}\end{aligned}$$

Mathematica [A] time = 10.5518, size = 293, normalized size = 1.22

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2a\sin(c+dx)}{3b}+\frac{2}{3}\tan(c+dx)\right)}{d}-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(-2b(a+b)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos}{(a+b)(}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(2*a*(a + b)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*
d*(b + a*Cos[c + d*x])) + (Sqrt[a + b*Sec[c + d*x]]*((2*a*Sin[c + d*x])/(3*
b) + (2*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.373, size = 913, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/b*(-1+cos(d*x+c))^2*(cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+cos(d*x+c)^2*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-cos(d*x+c)^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a^2-cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a*b+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-cos(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-co
s(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*a*b+cos(d*x+c)^3*a^2+cos(d*x+c)^3*a*b-cos(d*x+c)^2*a^2+cos(d*x+c)
^2*a*b+cos(d*x+c)^2*b^2-2*cos(d*x+c)*a*b-b^2)*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(c+dx)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.533 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=209

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(b*d)
```

Rubi [A] time = 0.163459, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3829, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*(a - b)*Sqrt[a + b]*Cot[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(b*d)
```


Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = (a-b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + b\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{bd}$$

Mathematica [A] time = 10.5016, size = 232, normalized size = 1.11

$$\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d} - \frac{2\sqrt{a+b\sec(c+dx)}\left(\frac{(a+b)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left(E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c\right.\right.\right.}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}}\right.}{d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d - (2*Sqrt[a + b*Sec[c + d*x]]*(
((a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b + a*Co
s[c + d*x])*Tan[(c + d*x)/2]))/(d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2
]^2]*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]])
```

Maple [B] time = 0.308, size = 814, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2
*(cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)*a+cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)*b-cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-cos(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+a*cos
(d*x+c)^2-a*cos(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec\left(c+dx\right)}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

3.534 $\int \sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.0288398, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3780}

$$-\frac{2\cot(c+dx)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(\sec(c+dx)+1)}{a+b\sec(c+dx)}}(a+b\sec(c+dx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[c
+ d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[c + d*x]))/(a + b*Sec[c + d*x
]))]*Sqrt[(b*(1 + Sec[c + d*x]))/(a + b*Sec[c + d*x])]*(a + b*Sec[c + d*x])
)/(Sqrt[a + b]*d)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,dx = -\frac{2\cot(c+dx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(c+dx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(c+dx))}{a+b\sec(c+dx)}}\sqrt{\frac{b(1+\sec(c+dx))}{a+b\sec(c+dx)}}(a+b\sec(c}{\sqrt{a+b}d}$$

Mathematica [A] time = 1.56896, size = 153, normalized size = 1.22

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\sqrt{a+b\sec(c+dx)}\left((a-b)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)\right.}{d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((a - b)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/(d*(b + a*Cos[c + d*x]))
```

Maple [A] time = 0.265, size = 215, normalized size = 1.7

$$-2\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{d(b+a\cos(dx+c))(\sin(dx+c))^2}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\left(\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*
x+c))*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec{(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a), x)
```

3.535 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=330

$$\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx}}{d}$$

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (
b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/d
```

Rubi [A] time = 0.323031, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3857, 4059, 3921, 3784, 3832, 4004}

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*Cot[c + d*x]*Ellipt
icF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (
b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/d
```


Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\int \frac{b-b\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\int \frac{b+b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx - \frac{1}{2}b\int \frac{\sec(c+dx}{\sqrt{a+}}$$

$$= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(}{a-}}}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(}{a-}}}{bd}$$

Mathematica [C] time = 18.8184, size = 2713, normalized size = 8.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Cos[c + d*x]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
(2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))
/(Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(
c + d*x)/2]^4]*((Sec[(c + d*x)/2]^2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*T
an[(c + d*x)/2]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + (2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b
)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2]))/(Sqrt[(-a +
 b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]
) + (a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(I*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sq
```

```
rt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*I)*b*EllipticPi[
-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
+ b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - Sqr
t[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Co
s[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(2*Sqrt[(-a + b)/(
a + b)]*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]) -
 (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(I*(a - b)*EllipticE[I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*I)*b*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)
/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c
 + d*x)/2])*(-(Sec[(c + d*x)/2]^4*Sin[c + d*x]) + 2*Cos[c + d*x]*Sec[(c + d
*x)/2]^4*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]^2))/(2*Sqrt[(-a + b)/(a +
 b)]*Sqrt[b + a*Cos[c + d*x]]*(Cos[c + d*x]*Sec[(c + d*x)/2]^4)^(3/2)) + (S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-((Sqrt[(-a
 + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/Sqrt[2]) + Sqrt[2]*a*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sin[c + d*x]*Tan[(c + d*x)/2] - (Sqrt[(-a + b)/(a
 + b)]*(b + a*Cos[c + d*x])*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])
^2 - Sin[c + d*x]/(1 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(Sqrt[2]*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]) + ((I/2)*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(c + d*x)/2]^2*
Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(a + b)))/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ (I*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a +
 b)) + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)))
/Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (b*Sqrt[(-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)])/((1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b))*Sqrt[1 + ((-a + b)*
Tan[(c + d*x)/2]^2)/(a - b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)
]) - ((a - b)*Sqrt[(-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)
/(a - b)])/(2*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])))/(Sqrt[(-a
+ b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4
]) + ((I*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)
] + (2*I)*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2)/(a + b)] - Sqrt[2]*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2])*(-1 + Tan[(c + d*x)/2]
^2)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2
*Sec[c + d*x]*Tan[c + d*x]))/(2*Sqrt[(-a + b)/(a + b)]*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^4]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
```

```
d*x]])))
```

Maple [B] time = 0.287, size = 829, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(2*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-cos(d*x+c)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a-cos(d*x+c)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*b+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b*sin(d*x+c)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*b*sin(d*x+c)-cos(d*x+c)^3*a+a*cos(d*x+c)^2-b*cos(d*x+c)^
2+b*cos(d*x+c))*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*c
os(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec\left(c+dx\right)}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

3.536 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=396

$$\frac{\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4ad}-\frac{\sqrt{a+b}\left(4a^2-b^2\right)\cot}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(2*a + b)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^
2*d) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (Cos[c + d*x]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.598746, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3857, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\frac{b\sin(c+dx)\sqrt{a+b}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```



```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + (Sqrt[a + b]*(2*a + b)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2 - b^2)*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^
2*d) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (Cos[c + d*x]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
```

```
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} + \frac{1}{4}\int \frac{\cos(c+dx)\left(b+2a\sec(c+d\right.}{\sqrt{a+b\sec(c}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} -$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4ad} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} -$$

$$= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{4ad}$$

$$= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a}}}{4ad}$$

Mathematica [C] time = 18.5592, size = 1173, normalized size = 2.96

$$\frac{\sqrt{a+b\sec(c+dx)}\sin(2(c+dx))}{4d} + \frac{\sqrt{a+b\sec(c+dx)}\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(-b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d*x
]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c +
 d*x)/2]^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSi
nh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)
*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2
*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
 I*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*
a^2 - a*b - b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a*
Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-1 +
Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]
*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.286, size = 1257, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a*(-1+cos(d*x+c))^2*(4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2*cos(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(
```

```
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a*b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^2-8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2+2*cos(d*x+c)*b^2*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))+4*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+
c)-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin
(d*x+c)-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^
2*sin(d*x+c)-8*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*sin(d*x+c)+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*cos(d*x+c)^4*a^2-3*cos(d*x+c)^3*a*b+2*
cos(d*x+c)^2*a^2+cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2+2*cos(d*x+c)*a*b+cos(d*x
+c)*b^2)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+
c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec{(c + dx)}} \cos^2{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(dx + c\right) + a} \cos\left(dx + c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.537 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=405

$$-\frac{2(a-b)\sqrt{a+b}\left(6a^2b+8a^3+39ab^2-147b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{315b^3d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b
^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^3 + 6*a^2*b + 39*a*b^2 - 147*b^3)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(315*b^3*d) + (2*a*(8*a^2 + 39*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c
 + d*x])/(315*b^2*d) + (2*(8*a^2 + 49*b^2)*(a + b*Sec[c + d*x])^(3/2)*Tan[c
 + d*x])/(315*b^2*d) - (8*a*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^
2*d) + (2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.841505, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3865, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\left(8a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315b^2d}+\frac{2a\left(8a^2+39b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}-\frac{2(a-b)\sqrt{a+b}\left(6a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b
^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^3 + 6*a^2*b + 39*a*b^2 - 147*b^3)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(315*b^3*d) + (2*a*(8*a^2 + 39*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c
 + d*x])/(315*b^2*d) + (2*(8*a^2 + 49*b^2)*(a + b*Sec[c + d*x])^(3/2)*Tan[c
 + d*x])/(315*b^2*d) - (8*a*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^
2*d) + (2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)
```


Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
```

```
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(c+dx)(a+b\sec(c}{}$$

$$= -\frac{8a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63b^2d} + \frac{2\sec(c+dx)(a+b\sec(c+dx))^{5/2}\text{ta}}{9bd}$$

$$= \frac{2\left(8a^2+49b^2\right)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315b^2d} - \frac{8a(a+b\sec(c+dx))^{5/2}\text{ta}}{63b^2d}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a+b\sec(c}{315b^2}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a+b\sec(c}{315b^2}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(8a^4+33a^2b^2+147b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right|}{315b^4d}$$

Mathematica [A] time = 18.5738, size = 550, normalized size = 1.36

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2\left(33a^2b^2+8a^4+147b^4\right)\sin(c+dx)}{315b^3}+\frac{2\sec^2(c+dx)\left(3a^2\sin(c+dx)+49b^2\sin(c+dx)\right)}{315b}+\frac{8\sec(c+dx)\left(22ab^2\sin(c+}{315b^2}\right.}{d(a\cos(c+dx)+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(8*
a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^4 +
```

```
2*a^3*b + 33*a^2*b^2 + 186*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 + 33*a^2*b^2 + 147*b^4)*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315
*b^3*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2))
+ (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(8*a^4 + 33*a^2*b^2 + 147*b^
4)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x]^2*(3*a^2*Sin[c + d*x] + 49*b^2
*Sin[c + d*x]))/(315*b) + (8*Sec[c + d*x]*(-(a^3*Sin[c + d*x]) + 22*a*b^2*S
in[c + d*x]))/(315*b^2) + (20*a*Sec[c + d*x]^2*Tan[c + d*x])/63 + (2*b*Sec[
c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x]))
```

Maple [B] time = 1.041, size = 2522, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/315/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(33*cos(d*x+c)^6*a^3*b^2+147*cos(d*x+c)^6*a*b^4+8*cos(d*x+c)^5*a^4
*b-34*cos(d*x+c)^5*a^3*b^2+33*cos(d*x+c)^5*a^2*b^3-10*cos(d*x+c)^5*a*b^4-4*
cos(d*x+c)^4*a^4*b-68*cos(d*x+c)^4*a^2*b^3+cos(d*x+c)^3*a^3*b^2-52*cos(d*x+
c)^3*a*b^4-53*cos(d*x+c)^2*a^2*b^3-85*cos(d*x+c)*a*b^4-4*cos(d*x+c)^6*a^4*b
-8*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^5+147*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+88*cos(d*x+c)^6*a^2*b^3-35*b^5+8
*cos(d*x+c)^6*a^5-8*cos(d*x+c)^5*a^5+147*cos(d*x+c)^5*b^5-98*cos(d*x+c)^4*b
^5-14*cos(d*x+c)^2*b^5-147*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+147*sin(d*x+c)*cos(d*x+c)^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-8*sin(d*
x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^5-147*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*b^5+8*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+2*sin(d*x+c)*cos(d*
```

```
x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3
*b^2+33*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b^3+186*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-8*sin(d*x+c)*cos(d*x+
c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b
-33*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b^2-33*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*sin(d*x+c)*cos(d*x+c
)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+
8*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^4*b+2*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+33*sin(d*x+c)*cos(d*x+c)^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+186*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^4-8*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-33*sin(d*x+c)*cos(d*x+c)^4*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-33*sin(d
*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^3-147*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^4/sin
(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(dx+c\right)^5+a\sec\left(dx+c\right)^4\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

3.538 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=342

$$\frac{2(a-b)\sqrt{a+b}\left(6a^2+57ab-25b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{105b^2d}-$$

```
[Out] (4*a*(a - b)*Sqrt[a + b]*(3*a^2 - 41*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(a
 - b)*Sqrt[a + b]*(6*a^2 + 57*a*b - 25*b^2)*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) - (2*
(6*a^2 - 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) - (4*a*(a
 + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*(a + b*Sec[c + d*x])^(
5/2)*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.598772, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3840, 4002, 4005, 3832, 4004}

$$-\frac{2\left(6a^2-25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105bd}+\frac{2(a-b)\sqrt{a+b}\left(6a^2+57ab-25b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec}{}}}{105b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (4*a*(a - b)*Sqrt[a + b]*(3*a^2 - 41*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(a
 - b)*Sqrt[a + b]*(6*a^2 + 57*a*b - 25*b^2)*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) - (2*
(6*a^2 - 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) - (4*a*(a
 + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*(a + b*Sec[c + d*x])^(
5/2)*Tan[c + d*x])/(7*b*d)
```


Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd} + \frac{2\int \sec(c+dx)\left(\frac{5b}{2}-a\sec(c+dx)\right)(a\ \ldots}{7b}$$

$$= -\frac{4a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{35bd} + \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd} + \ldots$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105bd} - \frac{4a(a+b\sec(c+dx))^{3/2}\tan\ldots}{35bd}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105bd} - \frac{4a(a+b\sec(c+dx))^{3/2}\tan\ldots}{35bd}$$

$$= \frac{4a(a-b)\sqrt{a+b}\left(3a^2-41b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\ldots}{\ldots}}\ldots}{105b^3d}$$

Mathematica [A] time = 14.1161, size = 471, normalized size = 1.38

$$\frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\left(b\left(51a^2b-6a^3+82ab^2+25b^3\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\mathrm{E}\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*a*(3
*a^3 + 3*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] + b*(-6*a^3 + 51*a^2*b + 82*a*b^2 + 25*b^3
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 a*(3*a^2 - 41*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]
*Sec[c + d*x]^(3/2)) + (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-4*a*(3*a
^2 - 41*b^2)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]*(3*a^2*Sin[c + d*x]
+ 25*b^2*Sin[c + d*x]))/(105*b) + (16*a*Sec[c + d*x]*Tan[c + d*x])/35 + (2*
b*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.658, size = 1852, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/105/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-6*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^4-25*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-6*cos(d*x+c)^4*sin(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-25
*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^4-6*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-3*cos(d*x+c)^3*a^3*b+15*b^4+82*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2+82*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-51*cos(d*x+c)
^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b^2-82*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+82*cos(d*x+c)^3*si
n(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b^2+82*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^3+6*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-51*cos(d*x+c)^3*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^
```

```
2-82*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^3+6*cos(d*x+c)^5*a^4-6*cos(d*x+c)^4*a^4-25*cos(d*x+c
)^4*b^4+10*cos(d*x+c)^2*b^4+68*cos(d*x+c)^3*a*b^3+27*cos(d*x+c)^2*a^2*b^2+3
9*cos(d*x+c)*a*b^3-3*cos(d*x+c)^5*a^3*b-82*cos(d*x+c)^5*a^2*b^2-25*cos(d*x+
c)^5*a*b^3+6*cos(d*x+c)^4*a^3*b+55*cos(d*x+c)^4*a^2*b^2-82*cos(d*x+c)^4*a*b
^3)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec(dx+c)^4+a\sec(dx+c)^3\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

3.539 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=282

$$-\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{5bd}-\frac{2(a-b)\sqrt{a+}}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(5*b^2*d) - (2*(a - 3*b
)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))]])/(5*b*d) + (2*a*Sqrt[a + b*Sec[c + d*x]]
*Tan[c + d*x])/(5*d) + (2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.409832, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3835, 4002, 4005, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{5b^2d}+\frac{2\tan(c+dx)(a+}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(5*b^2*d) - (2*(a - 3*b
)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))]])/(5*b*d) + (2*a*Sqrt[a + b*Sec[c + d*x]]
*Tan[c + d*x])/(5*d) + (2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```


Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{3}{5}\int \sec(c+dx)(b+a\sec(c+dx))\sqrt{a+}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5d} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5d} + \frac{2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} - \frac{1}{5}($$

$$= -\frac{2(a-b)\sqrt{a+b}\left(a^2+3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a+}}}{5b^2d}$$

Mathematica [A] time = 13.4665, size = 408, normalized size = 1.45

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{5b}+\frac{4}{5}a\tan(c+dx)+\frac{2}{5}b\tan(c+dx)\sec(c+dx)\right)}{d(a\cos(c+dx)+b)} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(2*(a^
3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(5*b*d*(b +
 a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(3/2)) + (Cos[c +
d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(a^2 + 3*b^2)*Sin[c + d*x])/(5*b) + (4*
a*Tan[c + d*x])/5 + (2*b*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d
*x]))
```

Maple [B] time = 0.497, size = 1566, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*a^2*b+4*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+3*sin(d*x+c)*cos(d*x+c)^3*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3-sin(d*x
+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*a^3-sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a^2*b-3*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-3*sin(d*x+c)*cos(d*x+c
)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3+si
n(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*a^2*b+4*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+3*sin(d*x+c)*cos(d*x+c)^2*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3-sin(d*x+c)*cos(
d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
^3-sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*a^2*b-3*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-3*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3+cos(d*x+c)
^4*a^3+2*cos(d*x+c)^4*a^2*b+3*cos(d*x+c)^4*a*b^2-cos(d*x+c)^3*a^3+cos(d*x+c
)^3*a^2*b+3*cos(d*x+c)^3*b^3-3*cos(d*x+c)^2*a^2*b-2*cos(d*x+c)^2*b^3-3*cos(
d*x+c)*a*b^2-b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(dx+c\right)^{3}+a\sec\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

3.540 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=249

$$\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd}+\frac{2b\tan(c+dx)}{}$$

```
[Out] (-8*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*(a - b)*(3*a - b)*Sqrt
[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(3*b*d) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(3*d)
```

Rubi [A] time = 0.291653, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3830, 4005, 3832, 4004}

$$\frac{2b\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2(a-b)(3a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-8*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*(a - b)*(3*a - b)*Sqrt
[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(3*b*d) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])
/(3*d)
```


Rule 3830

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*
m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^
```

```
2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{2b\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{\sec(c+dx)\left(\frac{3a^2}{2}+\frac{b^2}{2}+2ab\sec(c+a\right.}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{1}{3}((a-b)(3a-b))\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,d$$

$$= -\frac{8a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{3bd}$$

Mathematica [A] time = 9.93595, size = 304, normalized size = 1.22

$$-\frac{2\sqrt{a+b\sec(c+dx)}\left(-2\left(3a^2+4ab+b^2\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*Sec[c + d*x]]*(8*a*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*(3*a^2 + 4*a*
b + b^2)*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] - 5*a*b*Sin[c + d*x] - 2*a^2*Sin[2*(c + d*x)] + 4*
a*b*Cos[c + d*x]*Tan[(c + d*x)/2] + 4*a^2*Cos[c + d*x]^2*Tan[(c + d*x)/2] -
 b^2*Tan[c + d*x]))/(3*d*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.331, size = 1106, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))^2*(3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+4*cos(d*x+c)^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+cos(d*x+c
)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*b^2-4*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a^2-4*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2+4*cos(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a*b+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*b^2-4*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-4*cos(d*x+c)*(cos(d*x+c)/(cos(d
```

```
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*cos(d*x+c)^
3*a^2+cos(d*x+c)^3*a*b-4*cos(d*x+c)^2*a^2+4*cos(d*x+c)^2*a*b+cos(d*x+c)^2*b
^2-5*cos(d*x+c)*a*b-b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)
^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{3}{2}} \sec (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec (dx + c)^2 + a \sec (dx + c)\right)\sqrt{b \sec (dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sec (c + dx))^{\frac{3}{2}} \sec (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.541 $\int (a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=309

$$\frac{2(2a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{a+b}\cot(\ldots}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(2*a - b)*Sqrt[a + b]*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/d - (2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rubi [A] time = 0.219808, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3781, 3921, 3784, 3832, 4004}

$$\frac{2(2a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*(2*a - b)*Sqrt[a + b]*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/d - (2*a*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```


Rule 3781

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Dist[b^2, Int[(Csc[
c + d*x]*(1 + Csc[c + d*x]))/Sqrt[a + b*Csc[c + d*x]], x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}\,dx = b^2\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{a^2+(2a-b)b\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{d}$$

Mathematica [C] time = 17.9729, size = 882, normalized size = 2.85

$$\frac{2b\cos(c+dx)\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{d(b+a\cos(c+dx))} + \frac{2\left(-b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)+ab\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)-2ab\sqrt{\frac{b-a}{a+b}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c
+ d*x])) + (2*(a + b*Sec[c + d*x])^(3/2)*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (2*I)*a^2*EllipticPi
[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*a^2*EllipticPi[-((a + b)/(a -
b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*T
an[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)^2*EllipticF[I*ArcSinh[Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/
2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
```

```
*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.306, size = 1199, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*
(sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2-2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+cos(d*x+c)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+sin(d*x+
c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b^2-2*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,((a-b)/(a+b))^(1/2))*a^2+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*a^2*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)-cos(d*x
+c)^2*a*b+cos(d*x+c)*a*b-cos(d*x+c)*b^2+b^2)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^(3/2), x)
```

3.542 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=334

$$\frac{\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

```
[Out] (a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(a + 2*b)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (3*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (a*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.334163, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3864, 4058, 3921, 3784, 3832, 4004}

$$\frac{a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}+\frac{\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(a + 2*b)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (3*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (a*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/d
```


Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}\int \frac{-3ab-2b^2\sec(c+dx)+ab\sec^2(c+$$

$$= \frac{a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}\int \frac{-3ab+\left(-ab-2b^2\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx -$$

$$= \frac{a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{bd}$$

$$= \frac{a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{bd}$$

Mathematica [C] time = 11.5793, size = 439, normalized size = 1.31

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)(a+b\sec(c+dx))^{3/2}\left(4ib(a-b)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{b-a}{a+b}}\text{t}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-2*I)*a*(a -
b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)] + (4*I)*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*
ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - (12*I)
*a*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + a*Sqrt[(-a + b)/(a + b)]
*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(S
qrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.289, size = 1029, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(-1+cos(d*x+c))^2*(4*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-
cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*a*b-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*b^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-6*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a*b-co
s(d*x+c)^3*a^2+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*b+cos(d*x+c)*a*b)*(cos(d*x+c
)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c)\sec(dx+c)+a\cos(dx+c))\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.543 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=390

$$\frac{\sqrt{a+b}(2a+5b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\left(4a^2+3b^2\right)\text{co}}{}$$

```
[Out] (5*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a + 5*b)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(4*d) - (Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*EllipticPi[(a +
b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*
a*d) + (5*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*Cos[c + d*x]*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.543815, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3864, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{5b\sin(c+dx)\sqrt{a+b}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (5*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a + 5*b)*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(4*d) - (Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*EllipticPi[(a +
b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*
a*d) + (5*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*Cos[c + d*x]*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
```

```
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} - \frac{1}{4}\int \frac{\cos(c+dx)\left(-5ab-2\left(a^2 \right.\right.}{\sqrt{a+}}$$

$$= \frac{5b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{2d}$$

$$= \frac{5b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{a\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx}{2d}$$

$$= \frac{5(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{4d}$$

$$= \frac{5(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{4d}$$

Mathematica [C] time = 18.341, size = 1159, normalized size = 2.97

$$\frac{a\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(2(c+dx))}{4d(b+a\cos(c+dx))} - \frac{(a+b\sec(c+dx))^{3/2}\left(-5b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)+5ab\sqrt{\frac{b-a}{a+b}}\tan^5\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[2*(c + d*x)])/(4*d*(b + a*Co
s[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a*b*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2] + 5*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 10*a*b*Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 5*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2]^5 - 5*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^2
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*b^2*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^2*EllipticPi[-((a + b)/(a - b)),
I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c
 + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*b^2*EllipticPi[-((a + b)/(a - b)
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (5*I)*(a - b)*b*EllipticE[I*ArcSinh
[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*a^2 - a*b - b^2)*EllipticF[I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Co
s[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-
1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.257, size = 1440, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(5*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+5*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-4*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
```

```
*a^2+2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*a*b-8*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2+6*cos(d*x+c)
*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))+5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*a*b*sin(d*x+c)+5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^2*sin(d*x+c)-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-8*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+8*a^2*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)+6*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+2*cos(d*
x+c)^4*a^2+7*cos(d*x+c)^3*a*b-2*cos(d*x+c)^2*a^2-5*cos(d*x+c)^2*a*b+5*cos(d
*x+c)^2*b^2-2*cos(d*x+c)*a*b-5*cos(d*x+c)*b^2)*(cos(d*x+c)+1)^2*((b+a*cos(d
*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{2}\sec\left(dx+c\right)+a\cos\left(dx+c\right)^{2}\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

3.544 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=463

$$-\frac{2(a-b)\sqrt{a+b}\left(57a^2b^2+6a^3b+8a^4-606ab^3+135b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a-}}{}\right.\right.}{693b^3d}$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(693
*b^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^4 + 6*a^3*b + 57*a^2*b^2 - 606*a*b^3
+ 135*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(693*b^3*d) + (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(693*b^2*d) + (2*a*(8*a^2 + 67*b^2)*(a
 + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(693*b^2*d) + (2*(8*a^2 + 81*b^2)*(a
 + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(693*b^2*d) - (8*a*(a + b*Sec[c + d*
x])^(7/2)*Tan[c + d*x])/(99*b^2*d) + (2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(
7/2)*Tan[c + d*x])/(11*b*d)
```

Rubi [A] time = 1.04078, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3865, 4082, 4002, 4005, 3832, 4004}

$$\frac{2\left(8a^2+81b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2+67b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{693b^2d}+\frac{2\left(57a^2b^2+8a^4+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Cot[c + d*x]*Ellip
ticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(693
*b^4*d) - (2*(a - b)*Sqrt[a + b]*(8*a^4 + 6*a^3*b + 57*a^2*b^2 - 606*a*b^3
+ 135*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(693*b^3*d) + (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sq
rt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(693*b^2*d) + (2*a*(8*a^2 + 67*b^2)*(a
 + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(693*b^2*d) + (2*(8*a^2 + 81*b^2)*(a
 + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(693*b^2*d) - (8*a*(a + b*Sec[c + d*
```

```
x])^(7/2)*Tan[c + d*x])/(99*b^2*d) + (2*Sec[c + d*x]*(a + b*Sec[c + d*x])^(
7/2)*Tan[c + d*x])/(11*b*d)
```

Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{11bd} + \frac{2\int \sec(c+dx)(a+b\sec(c+ \cdots}{\cdots}$$

$$= -\frac{8a(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{99b^2d} + \frac{2\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan \cdots}{11bd}$$

$$= \frac{2\left(8a^2+81b^2\right)(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{693b^2d} - \frac{8a(a+b\sec(c+dx))^{7/2}\tan \cdots}{99b^2d}$$

$$= \frac{2a\left(8a^2+67b^2\right)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{693b^2d} + \frac{2\left(8a^2+81b^2\right)(a+b\sec(\cdots}{693b^2 \cdots}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{693b^2d} + \frac{2a\left(8a^2+67b^2\right)(a \cdots}{\cdots}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{693b^2d} + \frac{2a\left(8a^2+67b^2\right)(a \cdots}{\cdots}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\left(8a^4+51a^2b^2+741b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle| \cdots\right.}{693b^4d}$$

Mathematica [A] time = 16.8228, size = 615, normalized size = 1.33

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2a\left(51a^2b^2+8a^4+741b^4\right)\sin(c+dx)}{693b^3} + \frac{2}{693}\sec^3(c+dx)\left(113a^2\sin(c+dx)+81b^2\sin(c+dx)\right) + \cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*(
8*a^5 + 8*a^4*b + 51*a^3*b^2 + 51*a^2*b^3 + 741*a*b^4 + 741*b^5)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^5
+ 2*a^4*b + 51*a^3*b^2 + 663*a^2*b^3 + 741*a*b^4 + 135*b^5)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(8*a^4 + 51*a^
2*b^2 + 741*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(693*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Se
c[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*a*(8*a^4
 + 51*a^2*b^2 + 741*b^4)*Sin[c + d*x])/(693*b^3) + (2*Sec[c + d*x]^3*(113*a
^2*Sin[c + d*x] + 81*b^2*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(3*a^3*Sin[
c + d*x] + 229*a*b^2*Sin[c + d*x]))/(693*b) + (2*Sec[c + d*x]*(-4*a^4*Sin[c
 + d*x] + 205*a^2*b^2*Sin[c + d*x] + 135*b^4*Sin[c + d*x]))/(693*b^2) + (46
*a*b*Sec[c + d*x]^3*Tan[c + d*x])/99 + (2*b^2*Sec[c + d*x]^4*Tan[c + d*x])/
11))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 1.365, size = 2806, normalized size = 6.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/693/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d
*x+c))^2*(-8*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^6+135*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^6-63*b^6-4*cos(d*x+c)^7*
a^5*b+51*cos(d*x+c)^7*a^4*b^2+205*cos(d*x+c)^7*a^3*b^3+741*cos(d*x+c)^7*a^2
*b^4+135*cos(d*x+c)^7*a*b^5+8*cos(d*x+c)^6*a^5*b-52*cos(d*x+c)^6*a^4*b^2+51
*cos(d*x+c)^6*a^3*b^3-307*cos(d*x+c)^6*a^2*b^4+741*cos(d*x+c)^6*a*b^5-4*cos
(d*x+c)^5*a^5*b-140*cos(d*x+c)^5*a^3*b^3-566*cos(d*x+c)^5*a*b^5+cos(d*x+c)^
4*a^4*b^2-160*cos(d*x+c)^4*a^2*b^4-116*cos(d*x+c)^3*a^3*b^3-86*cos(d*x+c)^3
*a*b^5-274*cos(d*x+c)^2*a^2*b^4-224*cos(d*x+c)*a*b^5+8*cos(d*x+c)^7*a^6-8*c
os(d*x+c)^6*a^6+135*cos(d*x+c)^6*b^6-54*cos(d*x+c)^4*b^6-18*cos(d*x+c)^2*b^
6+135*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
```

(a-b)/(a+b))^(1/2))*b^6-8*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6-51*sin(d*x+c)*cos(d*x+c)^5*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2-51*sin
(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3*b^3-741*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4-741*sin(d*x+c)*cos(d*x+c)^5*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5+8*sin(
d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a^5*b+2*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2+51*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^3+663*sin(d*
x+c)*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b^4+741*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5-8*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^5*b-51*sin(d*x+c)
*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^4*b^2-51*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^3-741*sin(d*x+c)*cos(d*x+c)^6*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^4-741*sin(d*x+
c)*cos(d*x+c)^6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^5+8*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^5*b+2*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b^2+51*sin(d*x+c)*cos
(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^3*b^3+663*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2*b^4+741*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(

```
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^5-8*sin(d*x+c)*cos(
d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^5*b)/(b+a*cos(d*x+c))/cos(d*x+c)^5/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \sec\left(dx+c\right)^6 + 2\,ab \sec\left(dx+c\right)^5 + a^2 \sec\left(dx+c\right)^4\right)\sqrt{b \sec\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^6 + 2*a*b*sec(d*x + c)^5 + a^2*sec(d*x + c)^4)*s
qrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

3.545 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=399

$$\frac{2(a-b)\sqrt{a+b}\left(165a^2b+10a^3-114ab^2+147b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{315b^2d}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a^4 - 279*a^2*b^2 - 147*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*
b^3*d) + (2*(a - b)*Sqrt[a + b]*(10*a^3 + 165*a^2*b - 114*a*b^2 + 147*b^3)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(315*b^2*d) - (4*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(315*b*d) - (2*(10*a^2 - 49*b^2)*(a + b*Sec[c + d*x])^(3/2)*T
an[c + d*x])/(315*b*d) - (4*a*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*
b*d) + (2*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.779678, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3840, 4002, 4005, 3832, 4004}

$$-\frac{2\left(10a^2-49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315bd}-\frac{4a\left(5a^2-57b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315bd}+\frac{2(a-b)\sqrt{a+b}\,\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a^4 - 279*a^2*b^2 - 147*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*
b^3*d) + (2*(a - b)*Sqrt[a + b]*(10*a^3 + 165*a^2*b - 114*a*b^2 + 147*b^3)*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(315*b^2*d) - (4*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(315*b*d) - (2*(10*a^2 - 49*b^2)*(a + b*Sec[c + d*x])^(3/2)*T
an[c + d*x])/(315*b*d) - (4*a*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*
b*d) + (2*(a + b*Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```


Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} + \frac{2\int \sec(c+dx)\left(\frac{7b}{2}-a\sec(c+dx)\right)(a-}{9b}$$

$$= -\frac{4a(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{63bd} + \frac{2(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} +$$

$$= -\frac{2\left(10a^2-49b^2\right)(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{315bd} - \frac{4a(a+b\sec(c+dx))^{5/2}}{63bd}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)(a+b\sec}{315}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)(a+b\sec}{315}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(10a^4-279a^2b^2-147b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right.}{315b^3d}$$

Mathematica [A] time = 16.1654, size = 552, normalized size = 1.38

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\left(2b\left(279a^2b^2+155a^3b-10a^4+261ab^3+147b^4\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(10*
a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(-10*a
^4 + 155*a^3*b + 279*a^2*b^2 + 261*a*b^3 + 147*b^4)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (10*a^4 - 279*a^2*b^2 -
147*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*
x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-10*a^4 + 279*a
^2*b^2 + 147*b^4)*Sin[c + d*x])/(315*b^2) + (2*Sec[c + d*x]^2*(75*a^2*Sin[c
 + d*x] + 49*b^2*Sin[c + d*x]))/315 + (2*Sec[c + d*x]*(5*a^3*Sin[c + d*x] +
```

```
163*a*b^2*Sin[c + d*x]))/(315*b) + (38*a*b*Sec[c + d*x]^2*Tan[c + d*x])/63
+ (2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 1.017, size = 2523, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/315/d/b^2*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-279*cos(d*x+c)^6*a^3*b^2-147*cos(d*x+c)^6*a*b^4+10*cos(d*x+c)^5*a
^4*b+199*cos(d*x+c)^5*a^3*b^2-279*cos(d*x+c)^5*a^2*b^3-65*cos(d*x+c)^5*a*b^
4-5*cos(d*x+c)^4*a^4*b+272*cos(d*x+c)^4*a^2*b^3+80*cos(d*x+c)^3*a^3*b^2+82*
cos(d*x+c)^3*a*b^4+170*cos(d*x+c)^2*a^2*b^3+130*cos(d*x+c)*a*b^4-5*cos(d*x+
c)^6*a^4*b-10*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^5-147*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-163*cos(d*x+c)^6*a^2*
b^3+35*b^5+10*cos(d*x+c)^6*a^5-10*cos(d*x+c)^5*a^5-147*cos(d*x+c)^5*b^5+98*
cos(d*x+c)^4*b^5+14*cos(d*x+c)^2*b^5+147*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5-147*sin(d*x+c)*c
os(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b^5-10*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^5+147*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^5+10*sin(d*x+c)*cos(d*x+c)^5
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-155
*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3*b^2-279*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-261*sin(d*x+c)*cos(d*x+c)^
5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10
*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
```

```
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^4*b+279*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+279*sin(d*x+c)*cos(d*x+c)^5*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+14
7*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^4+10*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-155*sin(d*x+c)*cos(d*x+c)^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-279*
sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^2*b^3-261*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-10*sin(d*x+c)*cos(d*x+c)^4*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+279*si
n(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a^3*b^2+279*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+147*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4)/(b+a
*cos(d*x+c))/cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec\left(dx+c\right)^5+2\,ab\sec\left(dx+c\right)^4+a^2\sec\left(dx+c\right)^3\right)\sqrt{b\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^5 + 2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3)*s
qrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

3.546 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=333

$$-\frac{2(a-b)\sqrt{a+b}\left(3a^2-24ab+5b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{21bd}+$$

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(3*a^2 + 29*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b^2*d) - (2*(a
 - b)*Sqrt[a + b]*(3*a^2 - 24*a*b + 5*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b*d) + (2*(3*a
^2 + 5*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*(a + b*Sec
[c + d*x])^(3/2)*Tan[c + d*x])/(7*d) + (2*(a + b*Sec[c + d*x])^(5/2)*Tan[c
+ d*x])/(7*d)
```

Rubi [A] time = 0.565556, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3835, 4002, 4005, 3832, 4004}

$$\frac{2\left(3a^2+5b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{21d}-\frac{2(a-b)\sqrt{a+b}\left(3a^2-24ab+5b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{21bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*a*(a - b)*Sqrt[a + b]*(3*a^2 + 29*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b^2*d) - (2*(a
 - b)*Sqrt[a + b]*(3*a^2 - 24*a*b + 5*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(21*b*d) + (2*(3*a
^2 + 5*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(21*d) + (2*a*(a + b*Sec
[c + d*x])^(3/2)*Tan[c + d*x])/(7*d) + (2*(a + b*Sec[c + d*x])^(5/2)*Tan[c
+ d*x])/(7*d)
```


Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{5}{7}\int \sec(c+dx)(b+a\sec(c+dx))(a+$$

$$= \frac{2a(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{7d} + \frac{2(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{2}{7}$$

$$= \frac{2\left(3a^2+5b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{21d} + \frac{2a(a+b\sec(c+dx))^{3/2}\tan(c}{7d}$$

$$= \frac{2\left(3a^2+5b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{21d} + \frac{2a(a+b\sec(c+dx))^{3/2}\tan(c}{7d}$$

$$= -\frac{2a(a-b)\sqrt{a+b}\left(3a^2+29b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1}{}}}{21b^2d}$$

Mathematica [A] time = 13.735, size = 474, normalized size = 1.42

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2a\left(3a^2+29b^2\right)\sin(c+dx)}{21b} + \frac{2}{21}\sec(c+dx)\left(9a^2\sin(c+dx)+5b^2\sin(c+dx)\right) + \frac{6}{7}ab\tan(c\right.}{d(a\cos(c+dx)+b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*a*(
3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(3*a^3 + 27*a^2*b + 29*a*b^2 + 5*b^
3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ a*(3*a^2 + 29*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(21*b*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*S
ec[c + d*x]^(5/2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*a*(3*a^
2 + 29*b^2)*Sin[c + d*x])/(21*b) + (2*Sec[c + d*x]*(9*a^2*Sin[c + d*x] + 5*
b^2*Sin[c + d*x]))/21 + (6*a*b*Sec[c + d*x]*Tan[c + d*x])/7 + (2*b^2*Sec[c
+ d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.648, size = 1852, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x)

[Out] 2/21/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^4-5*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+3*cos(d*x+c)^4*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-5*cos(d
*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^4+3*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^4+12*cos(d*x+c)^3*a^3*b+3*b^4+29*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^2*b^2+29*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-3*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-27*cos(d*x+c)^4*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^
2*b^2-29*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a*b^3+3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+29*cos(d*x+c)^3*sin(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2
+29*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^3-3*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-27*cos(d*x+c)^3*sin(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-29*c
os(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a*b^3-3*cos(d*x+c)^5*a^4+3*cos(d*x+c)^4*a^4-5*cos(d*x+c)^4*b^4
+2*cos(d*x+c)^2*b^4+22*cos(d*x+c)^3*a*b^3+18*cos(d*x+c)^2*a^2*b^2+12*cos(d*

```
x+c)*a*b^3-9*cos(d*x+c)^5*a^3*b-29*cos(d*x+c)^5*a^2*b^2-5*cos(d*x+c)^5*a*b^
3-3*cos(d*x+c)^4*a^3*b+11*cos(d*x+c)^4*a^2*b^2-29*cos(d*x+c)^4*a*b^3)/(b+a*
cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec(dx+c)^4+2\,ab\sec(dx+c)^3+a^2\sec(dx+c)^2\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

3.547 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=296

$$\frac{2(a-b)\sqrt{a+b}\left(15a^2-8ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15bd}-\frac{2(}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(23*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) + (2*(a - b
)*Sqrt[a + b]*(15*a^2 - 8*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) + (16*a*b*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*b*(a + b*Sec[c + d*x])^(3/2
)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.450473, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3830, 4002, 4005, 3832, 4004}

$$\frac{2(a-b)\sqrt{a+b}\left(15a^2-8ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}-\frac{2(a-b)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(23*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) + (2*(a - b
)*Sqrt[a + b]*(15*a^2 - 8*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) + (16*a*b*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*b*(a + b*Sec[c + d*x])^(3/2
)*Tan[c + d*x])/(5*d)
```

Rule 3830

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1))/(f*m), x] +
Dist[1/m, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*
```

```
m + a*b*(2*m - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^
2 - b^2, 0] && GtQ[m, 1] && IntegerQ[2*m]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\left(\frac{5a}{2}\right.$$

$$= \frac{16ab\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} +$$

$$= \frac{16ab\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2b(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} +$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(23a^2+9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{}}}{15bd}$$

Mathematica [A] time = 16.2857, size = 440, normalized size = 1.49

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{2}{15}\left(23a^2+9b^2\right)\sin(c+dx)+\frac{22}{15}ab\tan(c+dx)+\frac{2}{5}b^2\tan(c+dx)\sec(c+dx)\right)}{d(a\cos(c+dx)+b)^2} - \frac{2(a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a + b*Sec[c + d*x])^(5/2)*(-2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*(
15*a^3 + 23*a^2*b + 17*a*b^2 + 9*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] - (23*a^2 + 9*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*d*(b + a*Cos[c + d*x
])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x
])^(5/2)*((2*(23*a^2 + 9*b^2)*Sin[c + d*x])/15 + (22*a*b*Tan[c + d*x])/15 +
 (2*b^2*Sec[c + d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.515, size = 1775, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-9*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*b^3-23*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+23*cos(d*x+c)^3*a^2*b+9*cos(
d*x+c)^3*b^3+23*cos(d*x+c)^4*a^3-23*cos(d*x+c)^3*a^3-6*cos(d*x+c)^2*b^3-3*b
^3+9*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*b^3-23*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-9*sin(d*x+c)*cos(d*x+c)^2*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3+9*sin(d*x+c
)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*b^3+11*cos(d*x+c)^4*a^2*b+9*cos(d*x+c)^4*a*b^2+5*cos(d*x+c)^3*a*b^2-34
*cos(d*x+c)^2*a^2*b-14*cos(d*x+c)*a*b^2-23*sin(d*x+c)*cos(d*x+c)^3*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-9*sin(d*x+c)
*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a*b^2+23*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+17*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2-23*sin(d*x+c)*cos(d
*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
2*b-9*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*a*b^2+23*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+17*sin(d*x+c)*cos(d*x+c)^2
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+15*
cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a^3+15*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d
*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec(dx+c)^3+2\,ab\sec(dx+c)^2+a^2\sec(dx+c)\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.548 $\int (a + b\sec(c + dx))^{5/2} \, dx$

Optimal. Leaf size=352

$$\frac{2\sqrt{a+b}\left(9a^2-7ab+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3d}-\frac{2a^2\sqrt{a+b}}{}$$

```
[Out] (-14*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) + (2*Sqrt[a + b]*(9*a^2 - 7*
a*b + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(3*d) - (2*a^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/d + (2*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.334114, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3782, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(9a^2-7ab+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3d}-\frac{2a^2\sqrt{a+b}\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-14*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
 d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*d) + (2*Sqrt[a + b]*(9*a^2 - 7*
a*b + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a +
b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + S
ec[c + d*x]))/(a - b))])/(3*d) - (2*a^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/d + (2*b^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/2}\,dx = \frac{2b^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int\frac{\frac{3a^3}{2}+\frac{1}{2}b\left(9a^2+b^2\right)\sec(c+dx)+\frac{7}{2}ab^2\sec^2(c+$$

$$=\frac{2b^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d}+\frac{2}{3}\int\frac{\frac{3a^3}{2}+\left(-\frac{7ab^2}{2}+\frac{1}{2}b\left(9a^2+b^2\right)\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx+$$

$$=-\frac{14a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx)}{a-b}}}{3d}$$

$$=-\frac{14a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx)}{a-b}}}{3d}$$

Mathematica [C] time = 17.6498, size = 713, normalized size = 2.03

$$\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{14}{3}ab\sin(c+dx)+\frac{2}{3}b^2\tan(c+dx)\right)}{d(a\cos(c+dx)+b)^2}+\frac{2(a+b\sec(c+dx))^{5/2}\left(-i\left(-9a^2b+3a^3+7a\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(5/2)*((-7*I)*a*(a - b)*b*EllipticE[I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] - I*(3*a^3 - 9*a^2*b + 7*a*b^2 - b^3)*EllipticF[I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*a^3*EllipticPi[-((a + b)/(a
 - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 7*a*b*Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]*(b - b*Tan[(c + d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^
```

```
2)^2)))/(3*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]
^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + T
an[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]) + (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^
(5/2)*((14*a*b*Sin[c + d*x])/3 + (2*b^2*Tan[c + d*x])/3))/(d*(b + a*Cos[c +
 d*x])^2)
```

Maple [B] time = 0.347, size = 1514, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))^2*(3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*sin(d*x+c)*cos(d*x+c)^2*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-7*sin(d*
x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a*b^2-sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*b^3-6*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+7*sin(d*x+c)*cos(d*
x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2
*b+7*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a*b^2+3*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-9*cos(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-7*cos(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a*b^2-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*b^3-6*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(
```

```
d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+7*cos(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+7*cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a*b^2-7*cos(d*x+c)^3*a^2*b-cos(d*x+c)^3*a*b^2+7*cos(d*x+c)^2*a^2*b-7*c
os(d*x+c)^2*a*b^2-cos(d*x+c)^2*b^3+8*cos(d*x+c)*a*b^2+b^3)*((b+a*cos(d*x+c)
)/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)
^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2), x)
```

3.549 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=353

$$\frac{\sqrt{a+b}\left(a^2+6ab-2b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+}}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(a^2
+ 6*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/d - (5*a*b*Sqrt[a + b]*Cot[c + d*x]*Elliptic
Pi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/d + (a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.344536, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3841, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(a^2+6ab-2b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+b}\left(a^2-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(a^2
+ 6*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/d - (5*a*b*Sqrt[a + b]*Cot[c + d*x]*Elliptic
Pi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/d + (a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
```

```
2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{\frac{5a^2b}{2} + 3ab^2\sec(c+dx) - \frac{1}{2}b\left(a^2-2b^2\right)}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}\left(b\left(a^2-2b^2\right)\right)\int \frac{\sec(c+dx)(1+\sec(c+}{\sqrt{a+b\sec(c+dx}}$$

$$= \frac{(a-b)\sqrt{a+b}\left(a^2-2b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{bd}$$

$$= \frac{(a-b)\sqrt{a+b}\left(a^2-2b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{bd}$$

Mathematica [B] time = 16.5455, size = 784, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}(a+b\sec(c+dx))^{5/2}\left(2b\left(-3a^2+3ab+b^2\right)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c\right.}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*b^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*(b + a*Co
s[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^
(-1)]*(a^3*Tan[(c + d*x)/2] + a^2*b*Tan[(c + d*x)/2] - 2*a*b^2*Tan[(c + d*x
)/2] - 2*b^3*Tan[(c + d*x)/2] - 2*a^3*Tan[(c + d*x)/2]^3 + 4*a*b^2*Tan[(c +
 d*x)/2]^3 + a^3*Tan[(c + d*x)/2]^5 - a^2*b*Tan[(c + d*x)/2]^5 - 2*a*b^2*Ta
n[(c + d*x)/2]^5 + 2*b^3*Tan[(c + d*x)/2]^5 - 10*a^2*b*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 10*a^2*b*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + (a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[ArcSin[Ta
```

```
n[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
+ d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] + 2*b*(-3*a^2 + 3*a*b + b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*(b + a*Cos
[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)
])
```

Maple [B] time = 0.366, size = 1640, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(10*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+cos(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+cos(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^2*b-2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*a*b^2-2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-6*cos(d*x+c)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+6*cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a*b^2+2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*b^3+10*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+a^3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+a^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*b^2*(cos(d*x+c)/(c
```

```
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-2*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*a^
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b+6*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b^3*sin(d*x+c)+cos(d*x+c)^3*a^3-cos(d*x+c)^2*a^3+cos(d*x+c)^2*a^2*b+2
*cos(d*x+c)^2*a*b^2-cos(d*x+c)*a^2*b-2*cos(d*x+c)*a*b^2+2*cos(d*x+c)*b^3-2*
b^3)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/
sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}}\cos (dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos (dx+c)\sec (dx+c)^2+2\,ab\cos (dx+c)\sec (dx+c)+a^2\cos (dx+c)\right)\sqrt{b\sec (dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c)
 + a^2*cos(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.550 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=399

$$\frac{\sqrt{a+b}\left(2a^2+9ab+8b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\left(4a\right.}{}$$

```
[Out] (9*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a^2 + 9*a*b
+ 8*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2 + 15*b^2)*Cot[c + d*x]*E
llipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(4*d) + (9*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (
a^2*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.63201, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\sqrt{a+b}\left(2a^2+9ab+8b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\left(4a^2+15b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (9*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) + (Sqrt[a + b]*(2*a^2 + 9*a*b
+ 8*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2 + 15*b^2)*Cot[c + d*x]*E
llipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(4*d) + (9*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (
a^2*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
```

```
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\cos(c+dx)\left(\frac{9a^2b}{2}+a\left(a^2\right.\right.}{}$$

$$= \frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c -}{2d}$$

$$= \frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{a^2\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c -}{2d}$$

$$= \frac{9a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{4d}$$

$$= \frac{9a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{4d}$$

Mathematica [C] time = 23.3032, size = 4588, normalized size = 11.5

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*Sin[2*(c + d*x)])/(4*d*(b +
a*Cos[c + d*x])^2) + ((a^3/(2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

```
+ (3*a*b^2)/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (11*a^2*b*Sqrt[
Sec[c + d*x]])/(8*Sqrt[b + a*Cos[c + d*x]]) + (b^3*Sqrt[Sec[c + d*x]])/Sqrt
[b + a*Cos[c + d*x]] + (9*a^2*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(8*Sqr
t[b + a*Cos[c + d*x]]))*(a + b*Sec[c + d*x])^(5/2)*((18*I)*a*(a - b)*b*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]]
, (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)
/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a -
 b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c +
 d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*(-(Sqrt[Sec[(c + d*x)/2]^2]*Tan[(
c + d*x)/2]*((18*I)*a*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2
*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos
[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(4*Sqr
t[(-a + b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(-1 + Tan[(c + d*x)/2]^2)^2) + (a*Sin[c + d*x]*((18*I)*a*(a - b)*b*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
 b)/(a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a
+ b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(8*Sqrt[(-a + b)/(a + b)]*(b + a*Cos[c
 + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x
]]*(-1 + Tan[(c + d*x)/2]^2)) - (Tan[(c + d*x)/2]*((18*I)*a*(a - b)*b*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(
```

a - b)] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(8*Sqrt[(-a + b)/(a + b)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) + ((-9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((9*I)*a*(a - b)*b*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - ((2*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((2*I)*a*(4*a^2 + 15*b^2)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((9*I)*a*(a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - ((2*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + ((2*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + 9*a^2*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + 9*a*b*Sqrt[(-a + b)/(a + b)]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - 9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 + (2*Sqrt[(-a + b)/(a + b)]*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - (2*a*Sqrt[(-a + b)/(a + b)]*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/((1 - ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b))*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b)]*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)]) - (9*a*(a - b)*b*Sqrt[(-a + b)/(a + b)]*Sqrt[Cos[c

```
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a - b)])
/Sqrt[1 + ((-a + b)*Tan[(c + d*x)/2]^2)/(a + b)])/(4*Sqrt[(-a + b)/(a + b)]
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)) - (((18*I)*a*(a - b)*b*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
b)/(a - b)] - (4*I)*(2*a^3 - a^2*b + 3*a*b^2 - 4*b^3)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)
] + (4*I)*a*(4*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-((a + b)/(a - b
)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)] -
9*a*b*Sqrt[(-a + b)/(a + b)]*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2])*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2])
 + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(8*Sqrt[(-a + b)/(a + b)]
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2))))
```

Maple [B] time = 0.283, size = 1646, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))^2*(4*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-2*cos(d*x+c)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+24*cos(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b^2-8*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*b^3-9*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-9*cos(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-8*cos(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
```

```
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*s
in(d*x+c)*a^3-30*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)
/(a+b))^(1/2))*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+4*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*a^2*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+24*b^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-8*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-9*a
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b-9*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a-8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+
b))^(1/2))*a^3*sin(d*x+c)-30*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*cos(d*x+c)^4*a^3-11*cos(d*x+c)^3*
a^2*b+2*cos(d*x+c)^2*a^3+9*cos(d*x+c)^2*a^2*b-9*cos(d*x+c)^2*a*b^2+2*cos(d*
x+c)*a^2*b+9*cos(d*x+c)*a*b^2)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c
))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^2\sec(dx+c)^2+2\,ab\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.551 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=460

$$\frac{\sqrt{a+b}\left(16a^2+26ab+33b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{24d}+\frac{(16a^2 -$$

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2 + 33*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d) + (Sqrt[a + b
]*(16*a^2 + 26*a*b + 33*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (5*b*Sqrt[a + b]*(4*a^2
 + b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2 + 33*b^2)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(24*d) + (13*a*b*Cos[c + d*x]*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(12*d) + (a^2*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.933337, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{\left(16a^2+33b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(16a^2+26ab+33b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2 + 33*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d) + (Sqrt[a + b
]*(16*a^2 + 26*a*b + 33*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (5*b*Sqrt[a + b]*(4*a^2
 + b^2)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/
Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-(
(b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2 + 33*b^2)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(24*d) + (13*a*b*Cos[c + d*x]*Sqrt[a + b*Sec[c +
```

```
 d*x]]*Sin[c + d*x])/(12*d) + (a^2*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(3*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\cos^2(c+dx)\left(\frac{13a^2b}{2}+a\right.}{}$$

$$= \frac{13ab\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d} + \frac{a^2\cos^2(c+dx)\sqrt{a+b\sec(}{3d}$$

$$= \frac{\left(16a^2+33b^2\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d} + \frac{13ab\cos(c+dx)\sqrt{a+b\sec(c}{12d}$$

$$= \frac{\left(16a^2+33b^2\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d} + \frac{13ab\cos(c+dx)\sqrt{a+b\sec(c}{12d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2+33b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{se}}{a}}}{24bd}$$

$$= \frac{(a-b)\sqrt{a+b}\left(16a^2+33b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{se}}{a}}}{24bd}$$

Mathematica [B] time = 17.241, size = 1026, normalized size = 2.23

$$\frac{\cos^2(c+dx)\left(\frac{1}{12}\sin(c+dx)a^2+\frac{1}{12}\sin(3(c+dx))a^2+\frac{13}{24}b\sin(2(c+dx))a\right)(a+b\sec(c+dx))^{5/2}}{d(b+a\cos(c+dx))^2}+\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(16a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((a^2*Sin[c + d*x])/12 + (13*a*b
*Sin[2*(c + d*x)])/24 + (a^2*Sin[3*(c + d*x)])/12))/(d*(b + a*Cos[c + d*x])
^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a
^3*Tan[(c + d*x)/2] + 16*a^2*b*Tan[(c + d*x)/2] + 33*a*b^2*Tan[(c + d*x)/2]
 + 33*b^3*Tan[(c + d*x)/2] - 32*a^3*Tan[(c + d*x)/2]^3 - 66*a*b^2*Tan[(c +
d*x)/2]^3 + 16*a^3*Tan[(c + d*x)/2]^5 - 16*a^2*b*Tan[(c + d*x)/2]^5 + 33*a*
b^2*Tan[(c + d*x)/2]^5 - 33*b^3*Tan[(c + d*x)/2]^5 - 120*a^2*b*EllipticPi[-
1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b
^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] - 120*a^2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*b^3*EllipticPi[-1, -ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + (16*a^3 + 16*a^2*b + 33*a*b^2 + 33*b^3)*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]
^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2
*b*(38*a^2 - 13*a*b + 24*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*d*(b + a*Cos[c
 + d*x])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.324, size = 1881, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x)

[Out] -1/24/d*(-1+cos(d*x+c))^2*(16*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-33*cos(d*x+c)^2*a*b^2+33*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+8*cos(d*x+c)^3*a^3+33*cos(d*x+c)^2*b^3+33*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-76*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+26*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-16*cos(d*x+c)*a^2*b+16*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))+33*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-48*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-16*cos(d*x+c)^2*a^3-33*cos(d*x+c)*b^3-48*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+34*cos(d*x+c)^4*a^2*b+59*cos(d*x+c)^3*a*b^2-18*cos(d*x+c)^2*a^2*b-26*cos(d*x+c)*a*b^2+8*cos(d*x+c)^5*a^3+120*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+30*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+30*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+120*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+16*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-76*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+26*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+16*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+33*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))

```
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*a*b^2)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c
))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{5}{2}} \cos (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2 \cos (dx + c)^3 \sec (dx + c)^2 + 2\,ab \cos (dx + c)^3 \sec (dx + c) + a^2 \cos (dx + c)^3\right)\sqrt{b \sec (dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^3*sec(d*x
+ c) + a^2*cos(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.552 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=530

$$\frac{\sqrt{a+b}\left(284a^2b+72a^3+118ab^2+15b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{192ad}$$

```
[Out] ((a - b)*Sqrt[a + b]*(284*a^2 + 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) + (Sqrt[a +
 b]*(72*a^3 + 284*a^2*b + 118*a*b^2 + 15*b^3)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sq
rt[a + b]*(48*a^4 + 120*a^2*b^2 - 5*b^4)*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^2*
d) + (b*(284*a^2 + 15*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d)
 + ((36*a^2 + 59*b^2)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
96*d) + (17*a*b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d
) + (a^2*Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 1.30013, antiderivative size = 530, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4104, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(284a^2+15b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192ad}+\frac{\left(36a^2+59b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{96d}+\frac{\sqrt{a+b}\left(28\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(284*a^2 + 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) + (Sqrt[a +
 b]*(72*a^3 + 284*a^2*b + 118*a*b^2 + 15*b^3)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sq
rt[a + b]*(48*a^4 + 120*a^2*b^2 - 5*b^4)*Cot[c + d*x]*EllipticPi[(a + b)/a,
 ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^2*
```

```
d) + (b*(284*a^2 + 15*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d)
 + ((36*a^2 + 59*b^2)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
96*d) + (17*a*b*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d
) + (a^2*Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
```

```
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{a^2\cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}\int \frac{\cos^3(c+dx)\left(\frac{17a^2b}{2}+3\right.}{}$$

$$= \frac{17ab\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d} + \frac{a^2\cos^3(c+dx)\sqrt{a+b\sec}}{4d}$$

$$= \frac{\left(36a^2+59b^2\right)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{96d} + \frac{17ab\cos^2(c+dx)}{}$$

$$= \frac{b\left(284a^2+15b^2\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{192ad} + \frac{\left(36a^2+59b^2\right)\cos(c+dx)}{}$$

$$= \frac{b\left(284a^2+15b^2\right)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{192ad} + \frac{\left(36a^2+59b^2\right)\cos(c+dx)}{}$$

$$= \frac{(a-b)\sqrt{a+b}\left(284a^2+15b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{192ad}$$

$$= \frac{(a-b)\sqrt{a+b}\left(284a^2+15b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{192ad}$$

<u>Mathematica</u> [C] time = 17.0028, size = 1688, normalized size = 3.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((17*a*b*Sin[c + d*x])/96 + ((48
*a^2 + 59*b^2)*Sin[2*(c + d*x)])/192 + (17*a*b*Sin[3*(c + d*x)])/96 + (a^2*
Sin[4*(c + d*x)])/32))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(
5/2)*(-284*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 284*a^2*b^2*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 15*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2] - 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 568*a^3*b*Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 30*a*b^3*Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]^3 - 284*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 2
84*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*a*b^3*Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]^5 + 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]^5 + (288*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
```

```
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (720*
I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*b^4*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (288*I)*a^4*EllipticPi[-((a +
b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (720*I)*a^2*b^2*EllipticP
i[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (
a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*b^4*Ellip
ticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(284*a^3
 - 284*a^2*b + 15*a*b^2 - 15*b^3)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] - (2*I)*(72*a^4 - 36*a^3*b + 38*a^2*b^2 - 59*a*b^3 - 15*b^4)*Elli
pticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(192*a*Sqrt[(-a + b)/(a +
b)]*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)
/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)])
```

Maple [B] time = 0.398, size = 2330, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/192/d/a*(-1+cos(d*x+c))^2*(48*cos(d*x+c)^6*a^4-72*cos(d*x+c)^2*a^4-15*co
s(d*x+c)*b^4+15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)*b^4+172*cos(d*x+c)^3*a^3*b+288*cos(d*x+c)*sin(d
*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*(co
```

s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)+24*cos(d*x+c)^4*a^4+15*cos(d*x+c)^2*b^4+133*cos(d*x+c)^3*a*b^3+30*cos(d*x+c)^2*a^2*b^2-118*cos(d*x+c)*a*b^3+184*cos(d*x+c)^5*a^3*b+254*cos(d*x+c)^4*a^2*b^2-30*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)-144*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^4+284*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+284*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)+720*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+720*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+72*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b-644*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+118*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+284*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+284*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-284*cos(d*x+c)^2*a^3*b-15*cos(d*x+c)^2*a*b^3-72*cos(d*x+c)*a^3*b-284*cos(d*x+c)*a^2*b^2+15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+288*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)-30*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)-144*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)+72*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-644*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+

```
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+118*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*a*sin(d*x+c))*(cos(d*x+c)+1)^2*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}}\cos (dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x)
```

3.553 $\int (a+b\sec(c+dx))^{7/2}\,dx$

Optimal. Leaf size=403

$$\frac{2\sqrt{a+b}\left(-58a^2b+60a^3+22ab^2-9b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15d}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(58*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d) + (2*Sqrt[a +
 b]*(60*a^3 - 58*a^2*b + 22*a*b^2 - 9*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d) - (2*a^3*Sq
rt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (26*a*b^2*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(15*d) + (2*b^2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d
)
```

Rubi [A] time = 0.499469, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3782, 4056, 4058, 3921, 3784, 3832, 4004}

$$\frac{2\sqrt{a+b}\left(-58a^2b+60a^3+22ab^2-9b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15d}-\frac{2(a-b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(58*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d) + (2*Sqrt[a +
 b]*(60*a^3 - 58*a^2*b + 22*a*b^2 - 9*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*d) - (2*a^3*Sq
rt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]
]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[
-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (26*a*b^2*Sqrt[a + b*Sec[c + d*x]]*
Tan[c + d*x])/(15*d) + (2*b^2*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d
```

)

Rule 3782

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2
*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4056

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> -Simp[(C*Cot[
e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a +
 b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)*C
sc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{7/2}\,dx = \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\sec(c+dx)}\left(\frac{5a^3}{2}+\frac{3}{2}b\left(5a^2+b^2\right)\sec(c$$

$$= \frac{26ab^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{4}{15}\int \frac{\frac{15a^4}{4}}{}$$

$$= \frac{26ab^2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15d} + \frac{2b^2(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{4}{15}\int \frac{\frac{15a^4}{4}}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(58a^2+9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{15d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(58a^2+9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{15d}$$

Mathematica [C] time = 15.7774, size = 1150, normalized size = 2.85

$$\frac{2\left(-9b^4\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)+9ab^3\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)-58a^2b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)+58a^3b\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(7/2)*(58*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2] + 58*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 9*a*b^3*Sqrt[(-
a + b)/(a + b)]*Tan[(c + d*x)/2] + 9*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2] - 116*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 18*a*b^3*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 58*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan
[(c + d*x)/2]^5 - 58*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 9*
a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 9*b^4*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]^5 + (30*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSin
h[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] + (30*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + I*b*(-58*a^3 + 58*a^2*b - 9*a*b^2 + 9*b^3)*EllipticE[I*ArcSin
h[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(15*a^4 - 60*a^3*b + 58*a^2*b^2 - 22*a
*b^3 + 9*b^4)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(15*Sqrt
[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(1
- Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]
^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + T
an[(c + d*x)/2]^2)]) + (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(7/2)*((2*b*(58
*a^2 + 9*b^2)*Sin[c + d*x])/15 + (32*a*b^2*Tan[c + d*x])/15 + (2*b^3*Sec[c
+ d*x]*Tan[c + d*x])/5))/(d*(b + a*Cos[c + d*x])^3)
```

Maple [B] time = 0.534, size = 2185, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(7/2),x)
```

```
[Out] 2/15/d*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(15*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a^4-9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
```

(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4+58*cos(d*x+c)^3*a^3*b-58*cos(d*x+c)^3*a^2*b^2+3*b^4+58*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+58*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+9*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+58*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+58*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+9*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-60*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-58*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-22*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+6*cos(d*x+c)^2*b^4-10*cos(d*x+c)^3*a*b^3+74*cos(d*x+c)^2*a^2*b^2+19*cos(d*x+c)*a*b^3-58*cos(d*x+c)^4*a^3*b-16*cos(d*x+c)^4*a^2*b^2-9*cos(d*x+c)^4*a*b^3-9*cos(d*x+c)^3*b^4-60*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-58*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-22*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-30*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+9*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+15*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4-9*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-30*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4+9*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c

```
)*b^4)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3\right)\sqrt{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(7/2), x)
```

3.554 $\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=359

$$\frac{2\sqrt{a+b}\left(-12a^2b+48a^3+44ab^2+25b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+}{a-}\right.}{105b^4d}$$

```
[Out] (8*a*(a - b)*Sqrt[a + b]*(12*a^2 + 11*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^5*d) + (2*S
qrt[a + b]*(48*a^3 - 12*a^2*b + 44*a*b^2 + 25*b^3)*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^4*d
) + (2*(24*a^2 + 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^3*d)
 - (12*a*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b^2*d) + (
2*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.672518, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3860, 4092, 4082, 4005, 3832, 4004}

$$\frac{2\left(24a^2+25b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^3d}+\frac{2\sqrt{a+b}\left(-12a^2b+48a^3+44ab^2+25b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{\right.}{105b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (8*a*(a - b)*Sqrt[a + b]*(12*a^2 + 11*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^5*d) + (2*S
qrt[a + b]*(48*a^3 - 12*a^2*b + 44*a*b^2 + 25*b^3)*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^4*d
) + (2*(24*a^2 + 25*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^3*d)
 - (12*a*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b^2*d) + (
2*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*b*d)
```


Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
```

```
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{7bd} + \frac{\int \frac{\sec^2(c+dx)\left(4a+5b\sec(c+dx)-6a\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{7b}$$

$$= -\frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{35b^2d} + \frac{2\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+}{7bd}$$

$$= \frac{2\left(24a^2+25b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^3d} - \frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c}{35b^2d}$$

$$= \frac{2\left(24a^2+25b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{105b^3d} - \frac{12a\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c}{35b^2d}$$

$$= \frac{8a(a-b)\sqrt{a+b}\left(12a^2+11b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{105b^5d}$$

Mathematica [A] time = 14.4955, size = 463, normalized size = 1.29

$$\frac{4\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(b\left(-12a^2b-48a^3-44ab^2+25b^3\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{Ellipti}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*a*(12*a^3 +
12*a^2*b + 11*a*b^2 + 11*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] + b*(-48*a^3 - 12*a^2*b - 44*a*b^2 + 25*b^3)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*
```

```
(12*a^2 + 11*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(105*b^4*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]
) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((-8*a*(12*a^2 + 11*b^2)*Sin[c + d*x
])/(105*b^4) + (2*Sec[c + d*x]*(24*a^2*Sin[c + d*x] + 25*b^2*Sin[c + d*x]))
/(105*b^3) - (12*a*Sec[c + d*x]*Tan[c + d*x])/(35*b^2) + (2*Sec[c + d*x]^2*
Tan[c + d*x])/(7*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.683, size = 1852, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/105/d/b^4*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*
x+c))^2*(-48*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^4-25*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4-48*cos(d*x+c)^4*sin(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-
25*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^4-48*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4-24*cos(d*x+c)^3*a^3*b+15*b^4-44*c
os(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*b^2-44*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+48*cos(d*x+c)^4*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+12*cos(d
*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2+44*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-48*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-44*cos(d*x+c
```

```
)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b^2-44*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+48*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+12*cos(d*x+c)^3*
sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a^2*b^2+44*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b^3+48*cos(d*x+c)^5*a^4-48*cos(d*x+c)^4*a^4-25*
cos(d*x+c)^4*b^4+10*cos(d*x+c)^2*b^4-16*cos(d*x+c)^3*a*b^3+6*cos(d*x+c)^2*a
^2*b^2-3*cos(d*x+c)*a*b^3-24*cos(d*x+c)^5*a^3*b+44*cos(d*x+c)^5*a^2*b^2-25*
cos(d*x+c)^5*a*b^3+48*cos(d*x+c)^4*a^3*b-50*cos(d*x+c)^4*a^2*b^2+44*cos(d*x
+c)^4*a*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^5}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*sec(d*x + c) + a), x)
```

3.555 $\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=301

$$-\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^3d}-\frac{2(a-b)}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4*d) - (2*Sqrt[
a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) - (8*a*Sqrt[a + b*S
ec[c + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.423095, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3860, 4082, 4005, 3832, 4004}

$$-\frac{2\sqrt{a+b}\left(8a^2-2ab+9b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-\frac{2(a-b)\sqrt{a+b}\,(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*(8*a^2 + 9*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4*d) - (2*Sqrt[
a + b]*(8*a^2 - 2*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3*d) - (8*a*Sqrt[a + b*S
ec[c + d*x]]*Tan[c + d*x])/(15*b^2*d) + (2*Sec[c + d*x]*Sqrt[a + b*Sec[c +
d*x]]*Tan[c + d*x])/(5*b*d)
```


Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
```

```
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd} + \frac{\int \frac{\sec(c+dx)\left(2a+3b\sec(c+dx)-4a\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{5b}$$

$$= -\frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd} + \frac{2\int -$$

$$= -\frac{8a\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{15b^2d} + \frac{2\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5bd} + \frac{1}{15}\Bigg(9$$

$$= -\frac{2(a-b)\sqrt{a+b}\left(8a^2+9b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1-}{}}}{15b^4d}$$

Mathematica [A] time = 14.2012, size = 365, normalized size = 1.21

$$\frac{2\sqrt{\sec(c+dx)}\left(\sqrt{\sec(c+dx)}(a\cos(c+dx)+b)\left(\left(8a^2+9b^2\right)\sin(c+dx)+b\tan(c+dx)(3b\sec(c+dx)-4a)\right)-\frac{\sqrt{\cos^2(}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(8*a^3 +
 8*a^2*b + 9*a*b^2 + 9*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 +
 Sec[c + d*x]))] - 2*b*(8*a^2 + 2*a*b + 9*b^2)*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c +
d*x])/((a + b)*(1 + Sec[c + d*x]))] + (8*a^2 + 9*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2]
) + (b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((8*a^2 + 9*b^2)*Sin[c + d*x] +
 b*(-4*a + 3*b*Sec[c + d*x])*Tan[c + d*x])))/(15*b^3*d*Sqrt[a + b*Sec[c + d
*x]])
```

Maple [B] time = 0.534, size = 1584, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d/b^3*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x
+c))^2*(9*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*b^3+8*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3-8*cos(d*x+c)^3*a^2*b-9*cos(
d*x+c)^3*b^3-8*cos(d*x+c)^4*a^3+8*cos(d*x+c)^3*a^3+6*cos(d*x+c)^2*b^3+3*b^3
-9*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*b^3+8*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+9*sin(d*x+c)*cos(d*x+c)^2*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^3-9*sin(d*x+c)*c
os(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*b^3+4*cos(d*x+c)^4*a^2*b-9*cos(d*x+c)^4*a*b^2+10*cos(d*x+c)^3*a*b^2+4*cos
(d*x+c)^2*a^2*b-cos(d*x+c)*a*b^2+8*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+9*sin(d*x+c)*cos(d*x
+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^
2-8*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^2*b-2*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2+8*sin(d*x+c)*cos(d*x+c)^2*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+9*sin(d
*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a*b^2-8*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b-2*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^2)/(b+a*cos(d*x+c)
)/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*sec(d*x + c) + a), x)
```

3.556 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d}+\frac{4a(a-b)\sqrt{a+b}\cot}{}$$

```
[Out] (4*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) + (2*Sqrt[a + b]*(2*a + b)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(3*b^2*d) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d
)
```

Rubi [A] time = 0.274916, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3840, 4005, 3832, 4004}

$$\frac{2\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{4a(a-b)\sqrt{a+b}\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*a*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) + (2*Sqrt[a + b]*(2*a + b)
*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x
]))/(a - b))])/(3*b^2*d) + (2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b*d
)
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
```

```
, 0] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd} + \frac{2\int \frac{\sec(c+dx)\left(\frac{b}{2}-a\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3bd} - \frac{(2a)\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b} + \frac{(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{3b}$$

$$= \frac{4a(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^3d} + \cdots$$

Mathematica [A] time = 13.3255, size = 341, normalized size = 1.4

$$\frac{4\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-b(2a-b)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+d\right.\right.\right.\right.}{3b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (4*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*a*(a + b)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (2*
a - b)*b*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] + a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]))/(3*b^2*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) + ((b +
a*Cos[c + d*x])*Sec[c + d*x]*((-4*a*Sin[c + d*x])/(3*b^2) + (2*Tan[c + d*x]
)/(3*b)))/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.358, size = 919, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d/b^2*(-1+cos(d*x+c))^2*(2*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)^2*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-2*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^2-2*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(c
```

```
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-2*cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^2-2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a*b+2*cos(d*x+c)^3*a^2-cos(d*x+c)^3*a*b-2*cos(d*x+c)
^2*a^2+2*cos(d*x+c)^2*a*b-cos(d*x+c)^2*b^2-cos(d*x+c)*a*b+b^2)*((b+a*cos(d*
x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*
x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

3.557 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=204

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{}}{}$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*d)
```

Rubi [A] time = 0.156083, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3837, 3832, 4004}

$$-\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c}{a+b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*d)
```


Rule 3837

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[(Csc[e + f
*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = -\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$$
$$= -\frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2d} -$$

Mathematica [B] time = 18.6494, size = 2189, normalized size = 10.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Tan[c + d*x])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + ((-(
1/(Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) - (a*Sqrt[Sec[c + d*x]])/(
b*Sqrt[b + a*Cos[c + d*x]]) - (a*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*Sq
rt[b + a*Cos[c + d*x]]))*Sqrt[Sec[c + d*x]]*(-2*(a + b)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x] + 2*b*Ell
ipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Se
c[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] -
 (b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b*d*((1 + Cos[
c + d*x])^(-1))^(3/2)*Sqrt[1 + Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*((a*S
```

in[c + d*x]*(-2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*Sec[c + d*x] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Se
c[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]))/(2*b*((1 + Cos[c + d*x])^(-1))^(3/2)*(b + a*Cos
[c + d*x])^(3/2)*Sqrt[1 + Sec[c + d*x]]) - (3*Sin[c + d*x]*(-2*(a + b)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c +
 d*x] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*
x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Se
c[c + d*x]))] - (b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/
(2*b*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[b + a*Cos[c + d*x]]*Sqrt[1 + Sec[c
+ d*x]]) - (Sec[c + d*x]*(-2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x] + 2*b*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*S
qrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] - (b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Tan[c + d*x])/(2*b*((1 + Cos[c + d*
x])^(-1))^(3/2)*Sqrt[b + a*Cos[c + d*x]]*(1 + Sec[c + d*x])^(3/2)) + (-((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 - ((a + b)*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Sec[c + d*x]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (
(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sec[c + d*x]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x
])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + a*Sec[(c
+ d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*Sec[(c + d*x)/2]^2*Sec[c + d*x]*Sqrt[(1 +
Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))]
)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)]) - ((a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
x])/((a + b)(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sec[c + d*x]*Sqrt[1 -
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2] - 2*(a
 + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sec[c + d*x]*Tan[c + d*x] + 2*b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*Sec[c + d*x]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(a + b*Sec[c +
 d*x])/((a + b)*(1 + Sec[c + d*x]))]*Tan[c + d*x] - b*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]^2*((1 + Sec[c + d*x])^(-1))^(3
/2)*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))]*Tan[c + d*x] +
(b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sec[c + d*x]*Sqrt[(
1 + Sec[c + d*x])^(-1)]*((b*Sec[c + d*x]*Tan[c + d*x])/((a + b)*(1 + Sec[c
+ d*x])) - (Sec[c + d*x]*(a + b*Sec[c + d*x])*Tan[c + d*x])/((a + b)*(1 + S

```
ec[c + d*x])^2)))/Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))])/
(b*((1 + Cos[c + d*x])^(-1))^(3/2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[1 + Sec[c
+ d*x]]))
```

Maple [B] time = 0.311, size = 639, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d/b*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)*b-cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)*a-cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+a*cos(d*x+c)^2-a*cos
(d*x+c)+b*cos(d*x+c)-b)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.558 $\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$

```
[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(b*d)
```

Rubi [A] time = 0.0397408, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3832}

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(b*d)
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\frac{2\sqrt{a+b}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

Mathematica [A] time = 1.247, size = 93, normalized size = 0.94

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.245, size = 143, normalized size = 1.4

$$2\,\frac{(-1+\cos(dx+c))(\cos(dx+c)+1)^2}{d\,(b+a\cos(dx+c))(\sin(dx+c))^2}\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-1+co
s(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2/(b+a*cos(d*x
+c))/sin(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(c+dx\right)}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.559 $\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rubi [A] time = 0.0216419, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx=-\frac{2\sqrt{a+b}\cot(c+dx)\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{ad}$$

Mathematica [A] time = 1.24805, size = 140, normalized size = 1.32

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec(c+dx)\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\left(\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{a-b}{a+b}\right)+2\Pi\left(-1;-\text{s}\right.\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] + 2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.245, size = 178, normalized size = 1.7

$$-2\,\frac{(\cos(dx+c)+1)^2(-1+\cos(dx+c))}{d(b+a\cos(dx+c))(\sin(dx+c))^2}\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\left(\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*
x+c))*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)))/(b+a*cos(d*x+c))/sin
(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(d*x + c) + a), x)
```

3.560 $\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=338

$$\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}+\frac{b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(}{a+}}}{}$$

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*
d) + (b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.265468, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3861, 4059, 3921, 3784, 3832, 4004}

$$\frac{b\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (Sqrt[a + b]*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*
d) + (b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(a*d)
```

Rule 3861

```
Int[1/(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]),
 x_Symbol] :> -Simp[(Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]])/(a*f), x] - Dis
t[b/(2*a), Int[(1 + Csc[e + f*x]^2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4059

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f
*x]], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{b\int \frac{1+\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{ad} - \frac{b\int \frac{1-\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{2a} - \frac{b\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{abd} + \frac{\sqrt{}}{}$$

$$= \frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{abd} + \frac{\sqrt{}}{}$$

Mathematica [C] time = 23.5991, size = 5060, normalized size = 14.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.285, size = 649, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(-1+cos(d*x+c))^2*(cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*a+cos(d*x+c)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b-2*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b+(
```

```
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c
)+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*
x+c)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*b*sin(d*x+c)+cos(d*x+c)^3*a-a*cos(d*x+c)^2+b*cos(d*x+c)^2-b*cos(d*x+c))*(c
os(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x
+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

3.561 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=401

$$\frac{(2a-3b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4a^2d}-\frac{\sqrt{a+b}\left(4a^2+3b^2\right)\text{co}}{}$$

```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((2*a - 3*b)*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*Ellipti
cPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*a^3*d) - (3*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (
Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.509854, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3863, 4104, 4058, 3921, 3784, 3832, 4004}

$$-\frac{\sqrt{a+b}\left(4a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d}-\frac{3b\sin(c+dx)\sqrt{a+b}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((2*a - 3*b)*Sqrt[a + b]*
Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b
)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]
))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(4*a^2 + 3*b^2)*Cot[c + d*x]*Ellipti
cPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*a^3*d) - (3*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (
Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
```

```
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(3b-2a\sec(c+dx)-b\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{4a}$$

$$= -\frac{3b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2ad} + \frac{\int \frac{\frac{1}{2}\left(4a^2+3b\right.}{}}{}$$

$$= -\frac{3b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4a^2d} + \frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2ad} + \frac{\int \frac{\frac{1}{2}\left(4a^2+3b\right.}{\sqrt{}}}{}$$

$$= -\frac{3(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^2d} - \frac{3}{}$$

$$= -\frac{3(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^2d} + \frac{(2}{}$$

Mathematica [C] time = 18.306, size = 1195, normalized size = 2.98

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\sin(2(c+dx))}{4ad\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{\frac{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(-3b^{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Sec[
c + d*x]]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(3*a*b*
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 3*b^2*Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2] - 6*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 3*a*b*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]^5 + (8*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (
6*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a^2*Ellipti
cPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
 (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*b^2*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (3*I)*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*(2*a^2 - a*
b + 3*b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^2*Sqrt
[(-a + b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*Sqr
t[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)
/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.286, size = 1258, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/a^2*(-1+cos(d*x+c))^2*(4*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2-2*cos(d*x+c)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
```

```
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*cos(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a*b+3*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b^2-8*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2-6*cos(d*x+c)*b^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))+4*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c
)-2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*si
n(d*x+c)+3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b*sin(d*x+c)+3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^2*sin(d*x+c)-8*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,((a-b)/(a+b))^(1/2))*sin(d*x+c)-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*cos(d*x+c)^4*a^2+cos(d*x+c)^
3*a*b+2*cos(d*x+c)^2*a^2-3*cos(d*x+c)^2*a*b+3*cos(d*x+c)^2*b^2+2*cos(d*x+c)
*a*b-3*cos(d*x+c)*b^2)*(cos(d*x+c)+1)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)
/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

3.562 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=399

$$-\frac{2(4a+3b)\left(4a^2+b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{5b^4d\sqrt{a+b}}-\frac{2a^2\tan(c+d}{bd\left(a^2-b^2\right)\sqrt{a}}$$

```
[Out] (-2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^5*Sqrt[a + b]*d) - (2*(
4*a + 3*b)*(4*a^2 + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^4*Sqrt[a + b]*d) - (2*a^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*a*(
8*a^2 - 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^3*(a^2 - b^2)*d)
 + (2*(6*a^2 - b^2)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.774744, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3845, 4092, 4082, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2a\left(8a^2-3b^2\right)\tan(c+dx)}{5b^3d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^5*Sqrt[a + b]*d) - (2*(
4*a + 3*b)*(4*a^2 + b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(5*b^4*Sqrt[a + b]*d) - (2*a^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*a*(
8*a^2 - 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*b^3*(a^2 - b^2)*d)
 + (2*(6*a^2 - b^2)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b^2*(a^2 - b^2)*d)
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4092

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
)/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m
+ 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec^2(c+dx)\left(2a^2-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(6a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(6a^2-b^2\right)\sec(c+dx)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5b^3\left(a^2-b^2\right)d} + \frac{2\left(6a^2\right.}{}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{5b^3\left(a^2-b^2\right)d} + \frac{2\left(6a^2\right.}{}$$

$$= -\frac{2\left(16a^4-8a^2b^2-3b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(}{a-}}}{5b^5\sqrt{a+b}d}$$

Mathematica [A] time = 15.0895, size = 498, normalized size = 1.25

$$\frac{2\left(4a^2+b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a\cos(c+dx)+b)\left(2b\left(-4a^2-ab+3b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c-}{(a+b)(\cos(}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(4*a^2 + b^2)*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*(2*(4*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(-4*a^2 - a*b
 + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + (4*a^2 - 3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/(5*b^4*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b
*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-16*a^4
 + 8*a^2*b^2 + 3*b^4)*Sin[c + d*x])/(5*b^4*(-a^2 + b^2)) + (2*a^4*Sin[c + d
*x])/(b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x])) - (6*a*Tan[c + d*x])/(5*b^3) +
 (2*Sec[c + d*x]*Tan[c + d*x])/(5*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.851, size = 2477, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/5/d/(a-b)/(a+b)/b^4*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-a^2*b^
3-8*cos(d*x+c)^4*a^4*b+3*cos(d*x+c)^4*a^2*b^3+6*cos(d*x+c)^3*a^3*b^2+5*cos(
d*x+c)^3*a*b^4+6*cos(d*x+c)^2*a^2*b^3-2*cos(d*x+c)*a*b^4-3*sin(d*x+c)*cos(d
*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^
5-16*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^5+b^5+16*cos(d*x+c)^4*a^5-16*cos(d*x+c)^3*a^5-3*cos(d*
x+c)^3*b^5+2*cos(d*x+c)^2*b^5-8*cos(d*x+c)^4*a^3*b^2-3*cos(d*x+c)^4*a*b^4+1
6*cos(d*x+c)^3*a^4*b-8*cos(d*x+c)^3*a^2*b^3-8*cos(d*x+c)^2*a^4*b+2*cos(d*x+
c)*a^3*b^2+3*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*b^5-3*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5-16*sin(d*x+c)*cos(d*x+c)
^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5+3*s
in(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*b^5+16*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
```

```
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+4*sin(d*x+c)*cos(d*x+c)^3*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-8*sin(d*x+c
)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^2*b^3+sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a*b^4-16*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+8*sin(d*x+c)*cos(d*
x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3
*b^2+8*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a^2*b^3+3*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4+16*sin(d*x+c)*cos(d*x+c)
^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+4
*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*a^3*b^2-8*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+sin(d*x+c)*cos(d*x+c)^2*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4-16*sin(d
*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a^4*b+8*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+8*sin(d*x+c)*cos(d*x+c)^2*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+3*sin(d*x+c)
*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a*b^4)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{5}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{5}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{5}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(3/2), x)
```

3.563 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=325

$$\frac{2(2a+b)(4a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^3d\sqrt{a+b}}-\frac{2a^2\tan(c+dx)\sec}{bd\left(a^2-b^2\right)\sqrt{a+b\,\mathrm{s}}}$$

```
[Out] (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) + (2*(2*a + b)*(4*
a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) - (2*a^2*Sec[c + d*x]*Tan[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a +
b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.503577, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3845, 4082, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)\sec(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(4a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)}+\frac{2a\left(8a^2-5b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx}{a+b}}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*a*(8*a^2 - 5*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) + (2*(2*a + b)*(4*
a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) - (2*a^2*Sec[c + d*x]*Tan[c + d*
x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a +
b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
```

```
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(a^2-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(4a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{4\int \frac{\sec(c}{}}{}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{((2a+b}{}$$

$$= \frac{2a\left(8a^2-5b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3b^4\sqrt{a+b}d} + \frac{2}{}$$

Mathematica [A] time = 14.0506, size = 470, normalized size = 1.45

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(-\frac{2a\left(5b^2-8a^2\right)\sin(c+dx)}{3b^3\left(b^2-a^2\right)} - \frac{2a^3\sin(c+dx)}{b^2\left(b^2-a^2\right)(a\cos(c+dx)+b)} + \frac{2\tan(c+dx)}{3b^2}\right)}{d(a+b\sec(c+dx))^{3/2}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*(2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^3 + 2*a^2*b - 5*a*b
^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + a*(8*a^2 - 5*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a +
 b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*a*(-8
*a^2 + 5*b^2)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) - (2*a^3*Sin[c + d*x])/(b^
2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*b^2)))/(d*(a + b
*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.474, size = 1789, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/3/d/(a-b)/(a+b)/b^3*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(a^2*b^2-
8*cos(d*x+c)^2*a^4-8*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-4*cos(d*x+c)^3*a^3*b
-5*cos(d*x+c)^3*a^2*b^2-b^4-8*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+5*cos(d*x+c)^2*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+5
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a*b^3-8*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4+cos(d*x+c)^2*b^4+8*cos(d*x+c)^3*a^4+
cos(d*x+c)^3*a*b^3+4*cos(d*x+c)^2*a^2*b^2+4*cos(d*x+c)*a*b^3+8*cos(d*x+c)^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^3*b+2*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*a^2*b^2-5*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+cos(d*x+c)^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+5*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)*a*b^3+8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-5*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*
b^3-8*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
```

```
c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*cos(d*x+c)*a^3*b+5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+8*cos(d*x+c)^2*a^3*b-5*cos(d*
x+c)^2*a*b^3-4*cos(d*x+c)*a^3*b)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{4}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{4}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{4}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(3/2), x)
```

3.564 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{2(2a+b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+}}$$

```
[Out] (-2*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) - (2*(2*a + b)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(b^2*Sqrt[a + b]*d) - (2*a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rubi [A] time = 0.319486, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3839, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(2a^2-b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^3d\sqrt{a+b}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b
*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) - (2*(2*a + b)*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(b^2*Sqrt[a + b]*d) - (2*a^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```


Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
```

```
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{ab}{2}-\frac{1}{2}\left(2a^2-b^2\right)\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{(2a+b)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b(a+b)} + \frac{\left(2a^2-b^2\right)\int \frac{\sec(c+dx)(1+}{\sqrt{a+b\sec}}}{b\left(a^2-b^2\right)}$$

$$= -\frac{2\left(2a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^3\sqrt{a+b}d} - $$

Mathematica [A] time = 13.2423, size = 440, normalized size = 1.71

$$\frac{2\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a\cos(c+dx)+b)\left(2b\left(-2a^2-ab+b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{Ellip}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-2*a^2 + b^2)*Sin[c + d*x])/(b^
2*(-a^2 + b^2)) + (2*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(b + a*Cos[c + d*x])
)))/(d*(a + b*Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(
3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(2*a^3 + 2*a^2*b - a*b^2 - b^
3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
+ 2*b*(-2*a^2 - a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)] + (2*a^2 - b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b^2*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)
/2]^2]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.366, size = 1451, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/b^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^2*b+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a*b^2-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-2*cos(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-2*cos(d*x+c)
```

```
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a^2*b+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b^2+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+2*a^2*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+b^2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-2*a^3*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*a^2
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b+b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^3*sin(d*x+c)+2*cos(d*x+c)^2*a^3-cos(d*x+c)^2*a^2*b-cos(d*x+c)^2*a*b^2-2*
cos(d*x+c)*a^3+2*cos(d*x+c)*a^2*b+cos(d*x+c)*a*b^2-cos(d*x+c)*b^3-a^2*b+b^3
)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{3}}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x)
```

3.565 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}+\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a}{}$$

```
[Out] (2*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d)
 + (2*a*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.292973, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3836, 4005, 3832, 4004}

$$\frac{2a\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+\frac{2\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
 d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d)
 + (2*a*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
```

```
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx &= \frac{2a\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2\int \frac{\sec(c+dx)\left(-\frac{b}{2}-\frac{1}{2}a\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}\\
&= \frac{2a\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a+b}-\frac{a\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}\\
&= \frac{2a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b^2\sqrt{a+b}d}+\frac{2\cot(c+dx)}{}
\end{aligned}$$

Mathematica [A] time = 8.55713, size = 249, normalized size = 1.05

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-4b(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sec[c + d*x]*(4*a*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*b*(a + b)*Cos[(c +
d*x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] +
 a*(a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/(b*(a^2 - b^2)*d*Sqr
t[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.272, size = 837, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/b/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a*b+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*b^2-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+b^2*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
```

```
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+cos
(d*x+c)^2*a^2-cos(d*x+c)^2*a*b-cos(d*x+c)*a^2+cos(d*x+c)*a*b)/(b+a*cos(d*x+
c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^2}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.566 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\text{co}}{}$$

```
[Out] (-2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) -
(2*b*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.228511, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3833, 21, 3829, 3832, 4004}

$$-\frac{2b\tan(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}-\frac{2\cot(c+a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(b*Sqrt[a + b]*d) + (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*Sqrt[a + b]*d) -
(2*b*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2b\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2b\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{2b\tan(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a+b} + \frac{b\int \frac{\sec(c+dx)(1+\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d} + \frac{2\cot(c+dx)}{}$$

Mathematica [A] time = 9.36891, size = 244, normalized size = 1.03

$$-\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-4(a+b)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((Sec[(c + d*x)/2]*Sec[c + d*x]*(4*(a + b)*Cos[(c + d*x)/2]^3*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] - 4*(a + b)*Cos[(c + d*
x)/2]^3*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + (
a - b)*(Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2])))/((a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]]))
```

Maple [B] time = 0.251, size = 817, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
+cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)*b-cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*a-cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b+sin(d*x+c)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a+(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+a*cos(d*x+c)^2-b*cos
(d*x+c)^2-a*cos(d*x+c)+b*cos(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b*
sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.567 $\int \frac{1}{(a+b\sec(c+dx))^{3/2}} dx$

Optimal. Leaf size=347

$$-\frac{2\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}+\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-$$

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.316301, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3785, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) - (
2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(c+dx)+\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int \frac{\sec(c+dx)(1+\sec(c+}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2b^2\,\text{t}}{a\left(a^2-b^2\right)d}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{2\cot(c+dx)}{}$$

Mathematica [C] time = 18.2849, size = 1249, normalized size = 3.6

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b*Sin[c + d*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Se
c[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
```

```
2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
- a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)
*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.268, size = 1209, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/d/a/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(sin(d*x+c)*c
os(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2-2*sin(d*x+c)*cos(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2+2
*cos(d*x+c)*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*sin(d*x+c)+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
```

```
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-2*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)+2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+cos(d*x+c)^2
*a*b-cos(d*x+c)^2*b^2-cos(d*x+c)*a*b+cos(d*x+c)*b^2)/(b+a*cos(d*x+c))/sin(d
*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c)
+ a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-3/2), x)
```

3.568 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=396

$$\frac{(a+3b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}}+\frac{b\left(a^2-3b^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+}$$

```
[Out] ((a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((a + 3*b)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a^2*Sqrt[a + b]*d) + (3*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a
, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*d)
 + Sin[c + d*x]/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(a^2 - 3*b^2)*Tan[c + d
*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.497515, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3846, 4061, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(a^2-3b^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(a^2-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2bd\sqrt{a+b}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] ((a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((a + 3*b)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(a^2*Sqrt[a + b]*d) + (3*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a
, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*d)
 + Sin[c + d*x]/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(a^2 - 3*b^2)*Tan[c + d
*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3846

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*
Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
```

```
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{-\frac{3b}{2}+\frac{1}{2}b\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b\left(a^2-3b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{4}b\left(a^2-b^2\right)-\frac{1}{2}ab^2\sec(c+dx)+\frac{1}{4}b(}{\sqrt{a+b\sec(c+dx}}}{a^2\left(a^2-b^2\right)}$$

$$= \frac{\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b\left(a^2-3b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{4}b\left(a^2-b^2\right)+\left(-\frac{ab^2}{2}-\frac{1}{4}b\left(a^2-3b^2\right)\right)}{\sqrt{a+b\sec(c+dx)}}}{a^2\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2b\sqrt{a+b}d} + \frac{}{ad\sqrt{}}$$

$$= \frac{\left(a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{a^2b\sqrt{a+b}d} + \frac{(a+}{}$$

Mathematica [B] time = 15.4708, size = 1077, normalized size = 2.72

$$\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(-\frac{2\sin(c+dx)b^3}{a^2\left(a^2-b^2\right)(b+a\cos(c+dx))}-\frac{2\sin(c+dx)b^2}{a^2\left(b^2-a^2\right)}\right)}{d(a+b\sec(c+dx))^{3/2}} - \frac{(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c\right.}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b^2*Sin[c + d*x])/(a^2*(-a^2 +
b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a
 + b*Sec[c + d*x])^(3/2)) - ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a^3*Tan[(c + d*x)/2] + a^2*b
*Tan[(c + d*x)/2] - 3*a*b^2*Tan[(c + d*x)/2] - 3*b^3*Tan[(c + d*x)/2] - 2*a
^3*Tan[(c + d*x)/2]^3 + 6*a*b^2*Tan[(c + d*x)/2]^3 + a^3*Tan[(c + d*x)/2]^5
 - a^2*b*Tan[(c + d*x)/2]^5 - 3*a*b^2*Tan[(c + d*x)/2]^5 + 3*b^3*Tan[(c + d
*x)/2]^5 + 6*a^2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] - 6*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)
/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a^2*b*EllipticPi[-1, -ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
 6*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] + (a^3 + a^2*b - 3*a*b^2 - 3*b^3)*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] + 2*a*b*(a + b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a^2*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^(3/2)*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c
+ d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.27, size = 1662, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d/a^2/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x
+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^3+3*cos(d*x+c)^2*a*b^2-3*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
```

```
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+cos(d*x+c)^3*a^3-3*cos(d*x+
c)^2*b^3-3*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a+2*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*b+2*b^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-cos(d*x+c)*a^2*b+a^3*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-3*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-cos(d*x+c)
^2*a^3+3*cos(d*x+c)*b^3-cos(d*x+c)^3*a*b^2+cos(d*x+c)^2*a^2*b-2*cos(d*x+c)*
a*b^2-6*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*sin(d*x+c)*a^2*b+6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*b^3-6*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticPi
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+a^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*cos(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^2*b+2*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b^2+cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b-3*cos(d*x+c)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+6*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(
d*x+c)*b^3)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)}{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*b*
sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.569 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=470

$$\frac{\left(2a^2-5ab-15b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{4a^3d\sqrt{a+b}}-\frac{b^2\left(7a^2-15b^2\right)}{4a^3d\left(a^2-b^2\right)\sqrt{a}}$$

```
[Out] -((7*a^2 - 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) + ((2*a^2 - 5*a*b -
15*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(4*a^2 + 15*b^2)
*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(4*a^4*d) - (5*b*Sin[c + d*x])/(4*a^2*d*Sqrt[a +
 b*Sec[c + d*x]]) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b*Sec[c + d
*x]]) - (b^2*(7*a^2 - 15*b^2)*Tan[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b
*Sec[c + d*x]])
```

Rubi [A] time = 0.77522, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3846, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b^2\left(7a^2-15b^2\right)\tan(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-5ab-15b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{4a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -((7*a^2 - 15*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) + ((2*a^2 - 5*a*b -
15*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[
c + d*x]))/(a - b))])/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(4*a^2 + 15*b^2)
*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
```

```
 Sec[c + d*x]))/(a - b))])/(4*a^4*d) - (5*b*Sin[c + d*x])/(4*a^2*d*Sqrt[a +
 b*Sec[c + d*x]]) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b*Sec[c + d
*x]]) - (b^2*(7*a^2 - 15*b^2)*Tan[c + d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b
*Sec[c + d*x]])
```

Rule 3846

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*
Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
```

```
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{5b}{2}+a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{2a}$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\frac{1}{4}\left(-4a^2-15b^2\right)-\frac{3}{2}ab\sec(c+dx)+\frac{5}{4}b^2\sec^2(c+}{(a+b\sec(c+dx))^{3/2}}}{2a^2}$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b^2\left(7a^2-15b^2\right)\tan(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{5b\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b^2\left(7a^2-15b^2\right)\tan(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} +$$

$$= -\frac{\left(7a^2-15b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^3\sqrt{a+b}d} - \frac{}{4}$$

$$= -\frac{\left(7a^2-15b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{4a^3\sqrt{a+b}d} + \frac{(}{}$$

Mathematica [C] time = 14.3102, size = 1745, normalized size = 3.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b^3*Sin[c + d*x])/(a^3*(-a^2 + b
^2)) + (2*b^4*Sin[c + d*x])/(a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])) + Sin[2*
(c + d*x)]/(4*a^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-7*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2] - 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*b^3*S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*b^4*Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2] + 14*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 30*a*b^
3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 7*a^3*b*Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2]^5 + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5
+ 15*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]^5 - (8*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*A
```

```
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - (22*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + (30*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^4*E
llipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (22*I)*a
^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
(30*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + I*b*(7*a^3 - 7*a^2*b - 15*a*b^2 + 15*b^3)*EllipticE[I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] + (2*I)*(2*a^4 - a^3*b + 9*a^2*b^2 + 5*a*b^3 - 15*b^4
)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
- b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^3*Sqrt[(-a + b)
/(a + b)]*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2
)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c +
 d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.353, size = 2298, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d/a^3/(a+b)/(a-b)*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*cos(d*
x+c)^2*a^4+15*cos(d*x+c)*b^4-15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^4+5*cos(d*x+c)^3*a^3*b-8*cos(
d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1
/2))*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
```

```
*x+c)+1))^(1/2)-2*cos(d*x+c)^4*a^4-15*cos(d*x+c)^2*b^4-5*cos(d*x+c)^3*a*b^3
+5*cos(d*x+c)^2*a^2*b^2-10*cos(d*x+c)*a*b^3+2*cos(d*x+c)^4*a^2*b^2+30*cos(d
*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*b^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)+4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a^4+7*a^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+7*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-15*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-22*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*El
lipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x
+c)-15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*a*b^3-22*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2-2*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+4*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)*a^2*b^2+10*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3+7*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+7*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a
^2*b^2-7*cos(d*x+c)^2*a^3*b+15*cos(d*x+c)^2*a*b^3+2*cos(d*x+c)*a^3*b-7*cos(
d*x+c)*a^2*b^2-15*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^4*sin(d*x+c)-8*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,((a-b)/(a+b))^(1/2))*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)+30*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)+4*sin(d*x+c)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)-2*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+4*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+10
```

```
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*a*sin
(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^2}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x)
```

3.570 $\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=427

$$\frac{2\left(-16a^2b^2+12a^3b+16a^4-9ab^3-b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right.}{3b^4d(a-b)(a+b)^{3/2}}$$

```
[Out] (8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)
*d) + (2*(16*a^4 + 12*a^3*b - 16*a^2*b^2 - 9*a*b^3 - b^4)*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*
(a - b)*b^4*(a + b)^(3/2)*d) - (2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^
2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a^3*(3*a^2 - 5*b^2)*Tan[c + d*x
])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(2*a^2 - b^2)*Sqrt
[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.932791, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3845, 4090, 4082, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2-b^2\right)\tan(c+dx)\sqrt{a+b\sec(c+dx}}{3b^3d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)
*d) + (2*(16*a^4 + 12*a^3*b - 16*a^2*b^2 - 9*a*b^3 - b^4)*Cot[c + d*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*
(a - b)*b^4*(a + b)^(3/2)*d) - (2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*(a^
2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a^3*(3*a^2 - 5*b^2)*Tan[c + d*x
])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(2*a^2 - b^2)*Sqrt
```

```
[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4090

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*(-(a*(b*B
 - a*C)) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
```

```
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^2(c+dx)\left(2a^2-\frac{3}{2}ab\sec(c+dx)-\frac{3}{2}\left(2a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4\int \frac{\sec(c+dx)(}{}}{}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(2a^2-b^2\right)}{}$$

$$= -\frac{2a^2\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\tan(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(2a^2-b^2\right)}{}$$

$$= \frac{8a\left(4a^4-7a^2b^2+2b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{3(a-b)b^5(a+b)^{3/2}d}$$

Mathematica [A] time = 17.3404, size = 578, normalized size = 1.35

$$\frac{4\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a\cos(c+dx)+b)^2\left(b\left(28a^3b^2+7a^2b^3-4a^4b-16a^5-8ab^4+b^5\right)\sqrt{\frac{\cos(}{\cos(c}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (4*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
+ d*x]]*(4*a*(4*a^5 + 4*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4 + 2*b^5)*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + b*(
-16*a^5 - 4*a^4*b + 28*a^3*b^2 + 7*a^2*b^3 - 8*a*b^4 + b^5)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*a*(4*a^4 - 7*a
^2*b^2 + 2*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c +
 d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-8*a*(4*a^4 - 7*a^
2*b^2 + 2*b^4)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2)^2) - (2*a^3*Sin[c + d*x])/
(3*b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) - (2*(-7*a^5*Sin[c + d*x] + 11*
a^3*b^2*Sin[c + d*x]))/(3*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + (2*Tan
[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.926, size = 4176, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d/(a-b)^2/(a+b)^2/b^4*4^(1/2)*(-2*a^2*b^5-16*sin(d*x+c)*cos(d*x+c)^3*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^7-sin(d*x+
c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*b^7-8*cos(d*x+c)^4*a^6*b+16*cos(d*x+c)^3*a^2*b^5-2*cos(d*x+c)^3*a*b^6
-24*cos(d*x+c)^2*a^6*b+16*cos(d*x+c)^2*a^5*b^2+42*cos(d*x+c)^2*a^4*b^3-28*c
os(d*x+c)^2*a^3*b^4-13*cos(d*x+c)^2*a^2*b^5+8*cos(d*x+c)^2*a*b^6-6*cos(d*x+
c)*a^5*b^2+12*cos(d*x+c)*a^3*b^4-6*cos(d*x+c)*a*b^6+16*cos(d*x+c)^4*a^7-16*
cos(d*x+c)^3*a^7-cos(d*x+c)^2*b^7+a^4*b^3-28*cos(d*x+c)^4*a^5*b^2+13*cos(d*
x+c)^4*a^4*b^3+8*cos(d*x+c)^4*a^3*b^4-cos(d*x+c)^4*a^2*b^5+32*cos(d*x+c)^3*
a^6*b+18*cos(d*x+c)^3*a^5*b^2-56*cos(d*x+c)^3*a^4*b^3+8*cos(d*x+c)^3*a^3*b^
4-16*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a^7-sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^7+28*sin(d*x+c)*cos(d*x+c)*EllipticE((
```

-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^4-8*sin(d*x+c)*
cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*a^2*b^5-8*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a*b^6+16*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6*b+4*sin(d*x+c)*cos(d*x+c)
^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b^2
-28*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^4*b^3-7*sin(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^4+8*sin(d*x+c)*cos(d*x+c)^3
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^5-s
in(d*x+c)*cos(d*x+c)^3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a*b^6-16*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^6*b+28*sin(d*x+c)*cos(d*x+c)^3*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b^2+28*sin(d
*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^4*b^3-8*sin(d*x+c)*cos(d*x+c)^3*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^4-8*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^5+16*sin(d*x
+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*a^6*b+20*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b^2-24*sin(d*x+c)*cos(d*x+c)^2*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^3-35*sin(d*x+c
)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^3*b^4+sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^5+7*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/

```
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^6-32*sin(d*x+c)*cos(
d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a
^6*b+12*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a^5*b^2+56*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^3+20*sin(d*x+c)*cos(d*
x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3
*b^4-16*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*a^2*b^5-8*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^6+16*sin(d*x+c)*cos(d*x+c
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b^2+
4*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*a^4*b^3-28*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^4-7*sin(d*x+c)*cos(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^5+8*sin(d*
x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a*b^6-16*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^6*b-16*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b^2+28*sin(d*x+c)*cos(d
*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*
b^3+b^7)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))^2/sin(d*x+c)/
cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{5}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^5/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{5}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{5}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(b*sec(d*x + c) + a)^(5/2), x)
```

3.571 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=362

$$-\frac{2\left(6a^2b+8a^3-9ab^2-3b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^3d(a-b)(a+b)^{3/2}}-\frac{8a^2\left(\cdots\right.}{3b^2d\left(a^2\cdots\right.}$$

```
[Out] (-2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^4*(a + b)^(3/2)
*d) - (2*(8*a^3 + 6*a^2*b - 9*a*b^2 - 3*b^3)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) - (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a +
 b*Sec[c + d*x])^(3/2)) - (8*a^2*(a^2 - 2*b^2)*Tan[c + d*x])/(3*b^2*(a^2 -
b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.591205, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3845, 4080, 4005, 3832, 4004}

$$-\frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\tan(c+dx)\sec(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(6a^2b+8a^3-9ab^2-3b^3\right)\cot(c+dx)\sqrt{\frac{b(1\cdots}{}}}{3b^3d(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^4*(a + b)^(3/2)
*d) - (2*(8*a^3 + 6*a^2*b - 9*a*b^2 - 3*b^3)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) - (2*a^2*Sec[c + d*x]*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a +
 b*Sec[c + d*x])^(3/2)) - (8*a^2*(a^2 - 2*b^2)*Tan[c + d*x])/(3*b^2*(a^2 -
b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4080

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^
(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), In
t[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1
) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x],
 x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2,
0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
```

```
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(a^2-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{\sec(c+dx)\left(\frac{1}{2}\right.}{}}{}$$

$$= -\frac{2a^2\sec(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\tan(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{\left(8a^3+6a^2b-\right.}{3}$$

$$= -\frac{2\left(8a^4-15a^2b^2+3b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(}{a-}}}{3(a-b)b^4(a+b)^{3/2}d}$$

Mathematica [A] time = 18.4109, size = 556, normalized size = 1.54

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)^3\left(\frac{2\left(-15a^2b^2+8a^4+3b^4\right)\sin(c+dx)}{3b^3\left(b^2-a^2\right)^2}+\frac{2a^2\sin(c+dx)}{3b\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}+\frac{8\left(2a^2b^2\sin(c+dx)-a^4\sin(c+dx)\right)}{3b^2\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^{5/2}} - \frac{2\sec^{\frac{5}{2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Sin
[c + d*x])/(3*b^3*(-a^2 + b^2)^2) + (2*a^2*Sin[c + d*x])/(3*b*(-a^2 + b^2)*
(b + a*Cos[c + d*x])^2) + (8*(-(a^4*Sin[c + d*x]) + 2*a^2*b^2*Sin[c + d*x])
)/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/
2)) - (2*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*
Sec[c + d*x]]*(2*(8*a^5 + 8*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 3*a*b^4 + 3*b
^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - 2*b*(8*a^4 + 2*a^3*b - 15*a^2*b^2 - 6*a*b^3 + 3*b^4)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (8*a^4 - 15*a^2*b^2
```

```
+ 3*b^4)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^
(5/2))
```

Maple [B] time = 0.582, size = 3674, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d/(a-b)^2/(a+b)^2/b^3*4^(1/2)*(4*cos(d*x+c)^3*a^5*b+3*a^4*b^2-6*a^2*b^4
-8*cos(d*x+c)^3*a^6+8*cos(d*x+c)^2*a^6-3*cos(d*x+c)*b^6+8*sin(d*x+c)*cos(d*
x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6
+8*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*a^6+3*b^6-8*cos(d*x+c)^3*a^3*b^3-6*cos(d*x+c)^2*a^2*b^4+6*co
s(d*x+c)*a*b^5+3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*b^6-3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*b^6+15*cos(d*x+c)^3*a^4*b^2-3*cos(d*x+c)^3*a^2*b^4
-16*cos(d*x+c)^2*a^5*b-10*cos(d*x+c)^2*a^4*b^2+30*cos(d*x+c)^2*a^3*b^3-6*co
s(d*x+c)^2*a*b^5+12*cos(d*x+c)*a^5*b-8*cos(d*x+c)*a^4*b^2-22*cos(d*x+c)*a^3
*b^3+15*cos(d*x+c)*a^2*b^4-8*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-2*sin(d*x+c)*cos(d*x+c)^2*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2+15
*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*a^3*b^3+6*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4-3*sin(d*x+c)*cos(d*x+c)^2*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5-7*sin(
d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*a^4*b^2-12*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos
```

(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4-8*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-10*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2+13*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+21*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4+3*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5+16*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-30*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+6*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5+8*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-15*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2-15*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+3*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4+3*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5+3*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^6-3*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^6+8*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b+8*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2-15*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3-15*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4+3*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(

```
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5-8*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2-2*sin(d*x+c)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+15*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b
^4+6*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a*b^5)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c)
)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^4}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/2), x)
```

3.572 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=337

$$\frac{2\left(2a^2+3ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3b^2d(a-b)(a+b)^{3/2}}-\frac{2a^2\tan}{3bd\left(a^2-b^2\right)(a}$$

```
[Out] (4*a*(a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(a + b)^(3/2)*d) + (2*(2*a^
2 + 3*a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*d) - (2*a^2*T
an[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*(a^2 - 3
*b^2)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.501503, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3839, 4003, 4005, 3832, 4004}

$$-\frac{2a^2\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(2a^2+3ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+d}{a+b}}}{3b^2d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (4*a*(a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(a + b)^(3/2)*d) + (2*(2*a^
2 + 3*a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*d) - (2*a^2*T
an[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*a*(a^2 - 3
*b^2)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3ab}{2}-\frac{1}{2}\left(2a^2-3b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{\sec(c+dx)\left(\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{2a^2\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\tan(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(2a^2+3ab-\cdots\right.}{3(a\cdots}$$

$$= \frac{4a\left(a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b^3(a+b)^{3/2}d} + \cdots$$

Mathematica [A] time = 14.0179, size = 503, normalized size = 1.49

$$\frac{4\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a\cos(c+dx)+b)^2\left(b\left(a^2b-2a^3+6ab^2+3b^3\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx\cdots}{(a+b)(\cos(c+\cdots}}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((4*a*(-a^2 + 3*b^2)*Sin[c + d*x])/(
3*b^2*(-a^2 + b^2)^2) - (2*a*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c + d
*x])^2) - (2*(-(a^3*Sin[c + d*x]) + 5*a*b^2*Sin[c + d*x]))/(3*b*(-a^2 + b^2
)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (4*(b + a*Cos[
c + d*x])^2*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*a*(
a^3 + a^2*b - 3*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)] + b*(-2*a^3 + a^2*b + 6*a*b^2 + 3*b^3)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + a*(a^2 -
3*b^2)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(3*(-(a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2*(a + b*Sec[c + d*x])^(
5/2))
```

Maple [B] time = 0.329, size = 2731, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d/(a-b)^2/(a+b)^2/b^2*4^(1/2)*(6*cos(d*x+c)^3*a^3*b^2+12*cos(d*x+c)^2*
a^2*b^3+6*cos(d*x+c)*a*b^4+3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5+2*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-6*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-6*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4
-2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a^3*b^2+EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*a^2*b^3+6*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a*b^4+2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5-4*cos(d*x+c)^2*a^3*b^2-6*co
s(d*x+c)^2*a*b^4+3*cos(d*x+c)*a^4*b-7*cos(d*x+c)*a^2*b^3+2*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-2*cos(d*x
+c)^3*a^5+cos(d*x+c)^3*a^4*b-5*cos(d*x+c)^3*a^2*b^3-4*cos(d*x+c)^2*a^4*b-2*
cos(d*x+c)*a^3*b^2+2*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5+2*cos(d*x+c)^2*a^5-4*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3
*b^2-6*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*a*b^4-2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+7*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(co
```

```
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a^2*b^3+9*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a*b^4+4*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-12*sin(d*x+c)*cos(d*x+c
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+
3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*b^5-2*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^4*b+sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+6*sin(d*x+c)*cos(d*x+c)^
2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+
3*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a*b^4+2*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-6*sin(d*x+c)*cos(d*x+c)^2*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-6*sin(d
*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a^2*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d
*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^3}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2), x)
```

3.573 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$\frac{2(a-3b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd(a-b)(a+b)^{3/2}}+\frac{2\left(a^2+3b^2\right)\tan(c+dx}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c}}$$

```
[Out] (2*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a - 3*
b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2)*d) + (2*a*Tan[c + d*x])/(3*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2 + 3*b^2)*Tan[c + d*x])/(3*(
a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.436419, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3836, 4003, 4005, 3832, 4004}

$$\frac{2\left(a^2+3b^2\right)\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(a^2+3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c}{a}}}{3b^2d(a-b)(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*d) + (2*(a - 3*
b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d
*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2)*d) + (2*a*Tan[c + d*x])/(3*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2 + 3*b^2)*Tan[c + d*x])/(3*(
a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```


Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4\int \frac{\sec(c+dx)\left(ab+\frac{1}{4}(\cdots\right.}{\sqrt{a+b\,\cdots}}}{3\left(a^2\cdots\right.}$$

$$= \frac{2a\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{(a-3b)\int \frac{\sec(\cdots}{\sqrt{a+b\,\mathrm{se}\cdots}}}{3(a-b)(a+\cdots}$$

$$= \frac{2\left(a^2+3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d} + \frac{2(\cdots}{\cdots}$$

Mathematica [A] time = 13.3361, size = 486, normalized size = 1.53

$$\frac{2\sec^{\frac{5}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}(a\cos(c+dx)+b)^2\left(-2b\left(a^2+4ab+3b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2 + 3*b^2)*Sin[c + d*x])/(3*
b*(-a^2 + b^2)^2) + (2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x])
^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b + a*Co
s[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*S
ec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a^3 + a^2*b + 3
*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] - 2*b*(a^2 + 4*a*b + 3*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2 + 3*b^2)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b^2)^2*d*Sqr
t[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.276, size = 2411, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/3/d/(a-b)^2/(a+b)^2/b*4^(1/2)*(-cos(d*x+c)^2*a^4+3*cos(d*x+c)*b^4-3*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*b^4-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*a^4+3*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4+3*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^4-2*cos(d*x+c)^3*a^3*b+
3*cos(d*x+c)^3*a^2*b^2-cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-3*cos(d*x+c)^2*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-3*cos(d*
x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a*b^3-EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a^4-3*cos(d*x+c)^2*b^4+cos(d*x+c)^3*a^4-2*cos(d*x
+c)^3*a*b^3-4*cos(d*x+c)^2*a^2*b^2-4*cos(d*x+c)*a*b^3+cos(d*x+c)^2*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b+4
*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a^2*b^2+3*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-a^3*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)-3*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c)-6*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*co
```

```
s(d*x+c)*a*b^3+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*cos(d*x+c)*a^3*b+5*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+7*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^3-
2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*cos(d*x+c)*a^3*b-4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b^2+2*cos(d*x+c)^2*a^3*b+6*cos(d*x+c)
^2*a*b^3+cos(d*x+c)*a^2*b^2-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+4*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*a*sin(d*x+c))*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{2}}{b^{3}\sec\left(dx+c\right)^{3}+3\,ab^{2}\sec\left(dx+c\right)^{2}+3\,a^{2}b\sec\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

3.574 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=304

$$\frac{2(3a-b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3bd(a-b)(a+b)^{3/2}}-\frac{8ab\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+}}$$

```
[Out] (-8*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))]])/(3*(a - b)*b*(a + b)^(3/2)*d) + (2*(3*a - b)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)]])/(3*(a - b)*b*(a + b)^(3/2)*d) - (2*b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^(3/2)) - (8*a*b*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a +
 b*Sec[c + d*x]])
```

Rubi [A] time = 0.407995, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3833, 4003, 4005, 3832, 4004}

$$-\frac{8ab\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2(3a-b)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+}{a-}}}{3bd(a-b)(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-8*a*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))]])/(3*(a - b)*b*(a + b)^(3/2)*d) + (2*(3*a - b)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)]])/(3*(a - b)*b*(a + b)^(3/2)*d) - (2*b*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^(3/2)) - (8*a*b*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a +
 b*Sec[c + d*x]])
```


Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8ab\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{\sec(c+dx)\left(\frac{1}{4}(3 \cdots\right.}{\sqrt{a+\cdots}}}{3\left(a \cdots\right.}$$

$$= -\frac{2b\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{8ab\tan(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{(3a-b)\int \frac{\sec \cdots}{\sqrt{a+b \cdots}}}{3(a-b)(a \cdots}$$

$$= -\frac{8a\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3(a-b)b(a+b)^{3/2}d} + \frac{2(3a-b) \cdots}{\cdots}$$

Mathematica [A] time = 7.81277, size = 360, normalized size = 1.18

$$-\frac{2\sec^3(c+dx)(a\cos(c+dx)+b)\left(2a\cos^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)\left(-\left(3a^2+4ab+b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+ \cdots}{(a+b)(\cos(c \cdots}} \cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^3*(b^2*(-a^2 + b^2)*Sin[c + d*x] - b*
(-5*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] - 4*a^2*(b + a*Cos[c + d*x
])^2*Sin[c + d*x] + 2*a*Cos[(c + d*x)/2]^2*(b + a*Cos[c + d*x])*(4*a*(a + b
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] -
 (3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)] + 2*a*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2])))/(3*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.263, size = 1781, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d/(a-b)^2/(a+b)^2*4^(1/2)*(-4*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-4*cos(d*x+c)^2*a*b^2-5*c
os(d*x+c)^3*a^2*b+cos(d*x+c)^3*b^3+4*cos(d*x+c)^3*a^3-4*b^2*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+3*a^2*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+4*b
^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a-3*cos(d*x+c)*a^2*b+(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^3*sin(d*x+c)-4*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3+3*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-4*cos
(d*x+c)^2*a^3-cos(d*x+c)*b^3+3*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+cos(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+8*cos(d*x+c)
^2*a^2*b+4*cos(d*x+c)*a*b^2-4*a^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+7*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+5*cos(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^
2-8*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a^2*b-4*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-4*sin(d*x+c)*cos(d*x+c)^2*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b+4*sin(d*x+c
)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^2*b+sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c
```

```
))/(cos(d*x+c)+1))^(1/2)*a*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x
+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b^
2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.575 $\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=448

$$-\frac{2\left(6a^2-ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3a^2d(a-b)(a+b)^{3/2}}+\frac{2b^2\left(7a^2-3b\ldots\right.}{3a^2d\left(a^2-b^2\right)^2\ldots}$$

```
[Out] (2*(7*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*(a + b)^(3/2)*d) - (2*(6*a^
2 - a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*(a + b)^(3/2)*d) - (2*Sqrt[a
+ b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (2*b^2*Tan[c + d*x])/(3*a*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(7*a^2 - 3*b^2)*Tan[c + d*x])/(3
*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.563737, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3785, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2\left(6a^2-ab-3b^2\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\ldots}{3a^2d(a-\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-5/2),x]
```


```
[Out] (2*(7*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*(a + b)^(3/2)*d) - (2*(6*a^
2 - a*b - 3*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*(a + b)^(3/2)*d) - (2*Sqrt[a
+ b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (2*b^2*Tan[c + d*x])/(3*a*(a^2 - b
```

```
^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(7*a^2 - 3*b^2)*Tan[c + d*x])/(3
*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(a^2-b^2\right)+\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{\frac{3}{4}\left(a^2-b^2\right)^2-\cdots}{\cdots}}{\cdots}$$

$$= \frac{2b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{\frac{3}{4}\left(a^2-b^2\right)^2+\cdots}{\cdots}}{\cdots}$$

$$= \frac{2\left(7a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d} + \cdots$$

$$= \frac{2\left(7a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d} - \cdots$$

Mathematica [C] time = 14.1443, size = 1798, normalized size = 4.01

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b*(-7*a^2 + 3*b^2)*Sin[c + d*x])
/(3*a^2*(-a^2 + b^2)^2) - (2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*Co
s[c + d*x])^2) - (8*(-2*a^2*b^2*Sin[c + d*x] + b^4*Sin[c + d*x]))/(3*a^2*(a
^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) + (2*(b
+ a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*a^3*b*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2] + 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2] - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2] - 14*a^3*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]^3 + 6*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 7*a^3*b*Sqrt[(-a
 + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 7*a^2*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]^5 - 3*a*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*b^4*Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (6*I)*a^4*EllipticPi[-((a + b)/(a
 - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)] + (12*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - (6*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (
6*I)*a^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
+ (12*I)*a^2*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] - (6*I)*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] - I*b*(7*a^3 - 7*a^2*b - 3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + I*(3*a^4 + 6*a^3*b - 13*a^2*b^2 - 2*a*b^3 +
6*b^4)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b
)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*a^2*Sqrt[(-a
 + b)/(a + b)]*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*(-1 + Tan[(c + d*
x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + T
an[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.31, size = 3889, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d/(a-b)^2/(a+b)^2/a^2*4^(1/2)*(8*cos(d*x+c)^3*a^3*b^2-4*cos(d*x+c)^3*a
*b^4+4*cos(d*x+c)^2*a^2*b^3-2*cos(d*x+c)*a*b^4+7*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+7*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-3*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b
^4-6*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^3*b^2-EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*a^2*b^3+2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*a*b^4-14*cos(d*x+c)^2*a^3*b^2+6*cos(d*x+c)^2*a*b^
4-7*cos(d*x+c)*a^2*b^3-12*sin(d*x+c)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2+6*sin(d*x+c)*cos(d*x+c)
^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b
^4+6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*a^4*b-12*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-12*sin(d*x+c)*cos(d*x
+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^
2*b^3+6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*a*b^4-3*cos(d*x+c)^2*b^5+3*cos(d*x+c)*b^5-7*cos(d*x
+c)^3*a^4*b+3*cos(d*x+c)^3*a^2*b^3+7*cos(d*x+c)^2*a^4*b+6*cos(d*x+c)*a^3*b^
2-3*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^5+6*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5+6*sin(d*x+c)*cos(d*x+c)*Ellipti
```

cPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5-3*sin(d*x
+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^5-3*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*b^5+6*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-12*sin(d*x+c)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-3*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+6*sin(d*x+
c)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a^5+14*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^2-6*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4-9*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*
b-7*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*a^3*b^2+EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+2*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4+7*sin(d*x+c
)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*a^4*b+4*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*a^2*b^3+6*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5-3*sin(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^5-6*sin(d*x+c)*cos(d*x+c)^2*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b-sin(
d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*a^3*b^2+2*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d
x+c),((a-b)/(a+b))^(1/2))(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3+7*sin(d*x+c)*cos(d*x+c)^2*Ellipti

```
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b+7*sin(d*x+c
)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*a^3*b^2-3*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^3-3*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^4)*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c
)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/2), x)
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}} dx$

Optimal. Leaf size=510

$$\frac{\left(21a^2b+3a^3-5ab^2-15b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{3a^3d(a-b)(a+b)^{3/2}}+\frac{b\left(-26a\right.}{3a^3d\left(\right.}$$

```
[Out] ((3*a^4 - 26*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*b*(a + b)^(3/2)
*d) + ((3*a^3 + 21*a^2*b - 5*a*b^2 - 15*b^3)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*(
a + b)^(3/2)*d) + (5*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + Sin[
c + d*x]/(a*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^2 - 5*b^2)*Tan[c + d*x]
)/(3*a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^4 - 26*a^2*b^2
 + 15*b^4)*Tan[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.814156, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3846, 4061, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{b\left(-26a^2b^2+3a^4+15b^4\right)\tan(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{b\left(3a^2-5b^2\right)\tan(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{\left(21a^2b+3a^3-5ab^2-15b^3\right)\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] ((3*a^4 - 26*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*b*(a + b)^(3/2)
*d) + ((3*a^3 + 21*a^2*b - 5*a*b^2 - 15*b^3)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*(a - b)*(
a + b)^(3/2)*d) + (5*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
```

```
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + Sin[
c + d*x]/(a*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^2 - 5*b^2)*Tan[c + d*x]
)/(3*a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^4 - 26*a^2*b^2
 + 15*b^4)*Tan[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3846

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*
Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 4061

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 + a^2*C)*Cot[e + f*x]*(a + b*Csc[
e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
```

```
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{-\frac{5b}{2}+\frac{3}{2}b\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx}{a}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2-5b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{15}{4}b\left(a^2-b^2\right)-\frac{3}{2}ab^2\sec(c+dx}{(a+b\sec(}}{3a^2\left(a^2\right.}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2-5b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^4-26a^2b^2+15b^4\right)\mathrm{t}}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^2-5b^2\right)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{b\left(3a^4-26a^2b^2+15b^4\right)\mathrm{t}}{3a^3\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{\left(3a^4-26a^2b^2+15b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{3a^3(a-b)b(a+b)^{3/2}d}$$

$$= \frac{\left(3a^4-26a^2b^2+15b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+}{a-b}}}{3a^3(a-b)b(a+b)^{3/2}d}$$

<u>Mathematica</u> [B] time = 18.5984, size = 1493, normalized size = 2.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-4*b^2*(-5*a^2 + 3*b^2)*Sin[c + d*
x])/(3*a^3*(-a^2 + b^2)^2) + (2*b^4*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*(b + a
*Cos[c + d*x])^2) + (2*(-11*a^2*b^3*Sin[c + d*x] + 7*b^5*Sin[c + d*x]))/(3*
a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) -
((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2
)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan
[(c + d*x)/2]^2)]*(3*a^5*Tan[(c + d*x)/2] + 3*a^4*b*Tan[(c + d*x)/2] - 26*a
^3*b^2*Tan[(c + d*x)/2] - 26*a^2*b^3*Tan[(c + d*x)/2] + 15*a*b^4*Tan[(c + d
*x)/2] + 15*b^5*Tan[(c + d*x)/2] - 6*a^5*Tan[(c + d*x)/2]^3 + 52*a^3*b^2*Ta
n[(c + d*x)/2]^3 - 30*a*b^4*Tan[(c + d*x)/2]^3 + 3*a^5*Tan[(c + d*x)/2]^5 -
 3*a^4*b*Tan[(c + d*x)/2]^5 - 26*a^3*b^2*Tan[(c + d*x)/2]^5 + 26*a^2*b^3*Ta
```

```
n[(c + d*x)/2]^5 + 15*a*b^4*Tan[(c + d*x)/2]^5 - 15*b^5*Tan[(c + d*x)/2]^5
+ 30*a^4*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] - 60*a^2*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^5*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a^4*b*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] - 60*a^2*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Ta
n[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*b^5*EllipticPi[-1, -
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + (3*a^5 + 3*a^4*b - 26*a^3*b^2 - 26*a^2*b^3 + 15*a*b^4 + 15*b^5)*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + 2*a*b*(6*a^3 + 9*a^2*b - 2*a*b^2 - 5*b^3)*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*Sqrt
[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x
)/2]^2)))
```

Maple [B] time = 0.45, size = 4580, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 1/6/d/(a-b)^2/(a+b)^2/a^3*4^(1/2)*(-6*cos(d*x+c)^3*a^5*b+3*cos(d*x+c)^3*a^6
+15*cos(d*x+c)*b^6-3*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6-3*sin(d*x+c)*cos(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^6+30*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*b^6+6*cos(d*x+c)^4*a^4*b^2-3*cos(d*x+c)^4*a^2*b^4+34*cos(d*x+c)^3*a^3*b^3-
```

20*cos(d*x+c)^3*a*b^5+14*cos(d*x+c)^2*a^2*b^4-10*cos(d*x+c)*a*b^5-15*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^6-26*cos(d*x+c)^3*a^4*b^2+15*cos(d*x+c)^3*a^2*b^4+6*cos(d*x+c)^2*a^5*b+17*cos(d*x+c)^2*a^4*b^2-52*cos(d*x+c)^2*a^3*b^3+30*cos(d*x+c)^2*a*b^5+3*cos(d*x+c)*a^4*b^2+18*cos(d*x+c)*a^3*b^3-26*cos(d*x+c)*a^2*b^4-3*cos(d*x+c)^4*a^6+30*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-60*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+30*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5+30*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b+30*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2-60*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3-60*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4+30*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5-15*cos(d*x+c)^2*b^6+30*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^6+30*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2-60*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4-12*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-18*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2+4*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+10*sin(d*x+c)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4+23*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2+11*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a

```
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4-12*sin(d*x+c)*cos(d*x+c)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-30*sin(d*x+
c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*a^4*b^2-14*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+14*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4+10*sin(d*x+c)*cos
(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*
b^5-6*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^5*b+52*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3-30*sin(d*x+c)*cos(d*x+c)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5-3*sin(d
*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*a^5*b+26*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2+26*sin(d*x+c)*cos(d*x+c)^2*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3-15*sin(d*x
+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*a^2*b^4-15*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5-15*sin(d*x+c)*cos(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*b^6-3*sin(d*x+c)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^5*b-3*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^
2+26*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*a^3*b^3+26*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*a^2*b^4-15*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*a*b^5-12*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)
```

```
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^4*b^2-18*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^3*b^3+4*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*a^2*b^4+10*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
a*b^5)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\cos(dx+c)}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)/(b^3*sec(d*x + c)^3 + 3*a*b^
2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.577 $\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=562

$$\frac{(a+3b)\left(-45a^2b+6a^3+35b^3\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{12a^4d(a-b)(a+b)^{3/2}}-\frac{b^2\left(-1\right.}{12a}$$

```
[Out] -((33*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(12*a^4*(a - b)*(a + b)^(3
/2)*d) + ((a + 3*b)*(6*a^3 - 45*a^2*b + 35*b^3)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(12*a^4*(a -
b)*(a + b)^(3/2)*d) - (Sqrt[a + b]*(4*a^2 + 35*b^2)*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
]])/(4*a^5*d) - (7*b*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[c + d*x])^(3/2)) + (C
os[c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - (b^2*(27*a^2
 - 35*b^2)*Tan[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2))
- (b^2*(33*a^4 - 170*a^2*b^2 + 105*b^4)*Tan[c + d*x])/(12*a^4*(a^2 - b^2)^2
*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.16665, antiderivative size = 562, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3846, 4104, 4060, 4058, 3921, 3784, 3832, 4004}

$$-\frac{b^2\left(-170a^2b^2+33a^4+105b^4\right)\tan(c+dx)}{12a^4d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{b^2\left(27a^2-35b^2\right)\tan(c+dx)}{12a^3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{(a+3b)\left(-45a^2b+6a^3+35b^3\right)\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] -((33*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(12*a^4*(a - b)*(a + b)^(3
/2)*d) + ((a + 3*b)*(6*a^3 - 45*a^2*b + 35*b^3)*Cot[c + d*x]*EllipticF[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]])/(12*a^4*(a -
```

```
b)*(a + b)^(3/2)*d) - (Sqrt[a + b]*(4*a^2 + 35*b^2)*Cot[c + d*x]*EllipticPi
[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(4*a^5*d) - (7*b*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[c + d*x])^(3/2)) + (C
os[c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - (b^2*(27*a^2
 - 35*b^2)*Tan[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2))
- (b^2*(33*a^4 - 170*a^2*b^2 + 105*b^4)*Tan[c + d*x])/(12*a^4*(a^2 - b^2)^2
*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3846

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*
Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(
d*Csc[e + f*x])^(n + 1)*Simp[b*(m + n + 1) - a*(n + 1)*Csc[e + f*x] - b*(m
+ n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2
 - b^2, 0] && ILtQ[m + 1/2, 0] && ILtQ[n, 0]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 -
 a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^
2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

B, C}, x] && NeQ[a^2 - b^2, 0]

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{7b}{2}+a\sec(c+dx)+\frac{5}{2}b\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{5/2}}dx}{2a}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{4}\left(-4a^2-35b^2\right)-\frac{5}{2}ab\sec(c+dx)+\frac{21}{4}b^2}{(a+b\sec(c+dx))^{5/2}}}{2a^2}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2\left(27a^2-35b^2\right)\tan(c+d}{12a^3\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2\left(27a^2-35b^2\right)\tan(c+d}{12a^3\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{7b\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b^2\left(27a^2-35b^2\right)\tan(c+d}{12a^3\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= -\frac{\left(33a^4-170a^2b^2+105b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{12a^4(a-b)(a+b)^{3/2}d}$$

$$= -\frac{\left(33a^4-170a^2b^2+105b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(}{}}}{12a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 15.4117, size = 2285, normalized size = 4.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b^3*(-13*a^2 + 9*b^2)*Sin[c + d*
x])/(3*a^4*(-a^2 + b^2)^2) - (2*b^5*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*(b + a
*Cos[c + d*x])^2) - (4*(-7*a^2*b^4*Sin[c + d*x] + 5*b^6*Sin[c + d*x]))/(3*a
^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + Sin[2*(c + d*x)]/(4*a^3)))/(d*(a +
 b*Sec[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)
/2]^2)]*(33*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 33*a^4*b^2*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 170*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Ta
```

n[(c + d*x)/2] - 170*a^2*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 105*
a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 105*b^6*Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2] - 66*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 +
340*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 210*a*b^5*Sqrt[(-a
+ b)/(a + b)]*Tan[(c + d*x)/2]^3 + 33*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]^5 - 33*a^4*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 170*a^
3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 170*a^2*b^4*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]^5 + 105*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2]^5 - 105*b^6*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + (24*I)*a^6*El
lipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)
/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
+ d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (162*I)*a^4*b^2*EllipticPi[-
((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (396*I)*a^2*b^4*EllipticPi[-((a + b)/(
a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (210*I)*b^6*EllipticPi[-((a + b)/(a - b)), I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + (24*I)*a^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a +
b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] + (162*I)*a^4*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[S
qrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^
2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(
c + d*x)/2]^2)/(a + b)] - (396*I)*a^2*b^4*EllipticPi[-((a + b)/(a - b)), I*
ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c +
d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + (210*I)*b^6*EllipticPi[-((a + b)/(a - b)
), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(-33*a^5 + 33*a^4*b + 170*a^3*b
^2 - 170*a^2*b^3 - 105*a*b^4 + 105*b^5)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - (2*I)*(6*a^6 - 3*a^5*b + 57*a^4*b^2 + 54*a^3*b^3 - 184*a^
2*b^4 - 35*a*b^5 + 105*b^6)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c +
d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)]))/(12*a^4*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(
5/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c +
d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))

Maple [B] time = 0.579, size = 5638, normalized size = 10.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^2}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2/(b^3*sec(d*x + c)^3 + 3*a*
b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x)
```

3.578 $\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=535

$$-\frac{2\left(-36a^2b^2-13a^3b+45a^4+5ab^3+15b^4\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right),\right.}{15a^3d(a-b)^2(a+b)^{5/2}}$$

```
[Out] (2*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*a^3*(a - b)^2*(a + b)^(
5/2)*d) - (2*(45*a^4 - 13*a^3*b - 36*a^2*b^2 + 5*a*b^3 + 15*b^4)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(15*a^3*(a - b)^2*(a + b)^(5/2)*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a^4*d) + (2*b^2*Tan[c + d*x])/(5*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x
])^(5/2)) + (2*b^2*(13*a^2 - 5*b^2)*Tan[c + d*x])/(15*a^2*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Tan[c +
d*x])/(15*a^3*(a^2 - b^2)^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.860081, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3785, 4060, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\left(-41a^2b^2+58a^4+15b^4\right)\tan(c+dx)}{15a^3d\left(a^2-b^2\right)^3\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{15a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^{3/2}}+\frac{2b^2\tan(c+dx)}{5ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] (2*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*a^3*(a - b)^2*(a + b)^(
5/2)*d) - (2*(45*a^4 - 13*a^3*b - 36*a^2*b^2 + 5*a*b^3 + 15*b^4)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(15*a^3*(a - b)^2*(a + b)^(5/2)*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ellipt
```

icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + (2*b^2*Tan[c + d*x])/(5*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(5/2)) + (2*b^2*(13*a^2 - 5*b^2)*Tan[c + d*x])/(15*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b^2*(58*a^4 - 41*a^2*b^2 + 15*b^4)*Tan[c + d*x])/(15*a^3*(a^2 - b^2)^3*d*Sqrt[a + b*Sec[c + d*x]])

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dist[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
```

```
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{7/2}}\,dx = \frac{2b^2\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{5}{2}\left(a^2-b^2\right)+\frac{5}{2}ab\sec(c+dx)-\frac{3}{2}b^2\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{5a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{15a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} + \frac{4\int \frac{\frac{15}{4}\left(a^2-b^2\right)}{}}{}$$

$$= \frac{2b^2\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{15a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(58a^4-\right.}{15a^3\left(a^2-\right.}$$

$$= \frac{2b^2\tan(c+dx)}{5a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{5/2}} + \frac{2b^2\left(13a^2-5b^2\right)\tan(c+dx)}{15a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))^{3/2}} + \frac{2b^2\left(58a^4-\right.}{15a^3\left(a^2-\right.}$$

$$= \frac{2\left(58a^4-41a^2b^2+15b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

$$= \frac{2\left(58a^4-41a^2b^2+15b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{15a^3(a-b)^2(a+b)^{5/2}d}$$

Mathematica [C] time = 15.5633, size = 2346, normalized size = 4.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-7/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^4*Sec[c + d*x]^4*((2*b*(58*a^4 - 41*a^2*b^2 + 15*b^4)
*Sin[c + d*x])/(15*a^3*(-a^2 + b^2)^3) + (2*b^4*Sin[c + d*x])/(5*a^3*(a^2 -
 b^2)*(b + a*Cos[c + d*x])^3) + (2*(-19*a^2*b^3*Sin[c + d*x] + 11*b^5*Sin[c
 + d*x]))/(15*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (2*(74*a^4*b^2*Si
n[c + d*x] - 65*a^2*b^4*Sin[c + d*x] + 23*b^6*Sin[c + d*x]))/(15*a^3*(a^2 -
 b^2)^3*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(7/2)) + (2*(b + a*
Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(58*a^5*b*Sqrt[(-a + b)/(a
 + b)]*Tan[(c + d*x)/2] + 58*a^4*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
] - 41*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 41*a^2*b^4*Sqrt[(-
a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c +
```

d*x)/2] + 15*b^6*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 116*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 82*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 30*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 58*a^5*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 58*a^4*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 41*a^3*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 41*a^2*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 15*a*b^5*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*b^6*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (30*I)*a^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (90*I)*a^4*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (90*I)*a^2*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*b^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*a^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (90*I)*a^4*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (90*I)*a^2*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*b^6*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*b*(-58*a^5 + 58*a^4*b + 41*a^3*b^2 - 41*a^2*b^3 - 15*a*b^4 + 15*b^5)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(15*a^6 + 45*a^5*b - 103*a^4*b^2 - 23*a^3*b^3 + 86*a^2*b^4 + 10*a*b^5 - 30*b^6)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(15*a^3*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x])^(7/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))

Maple [B] time = 0.5, size = 7838, normalized size = 14.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{b^4\sec(dx+c)^4+4\,ab^3\sec(dx+c)^3+6\,a^2b^2\sec(dx+c)^2+4\,a^3b\sec(dx+c)+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/(b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c
)^3 + 6*a^2*b^2*sec(d*x + c)^2 + 4*a^3*b*sec(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-7/2), x)
```

3.579 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2b\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6b\,\mathrm{s}}{}$$

```
[Out] (-6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.099222, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3768, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6b\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```



```
[Out] (-6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= a\int \sec^{\frac{5}{2}}(c+dx)\,dx + b\int \sec^{\frac{7}{2}}(c+dx)\,dx\\ &= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \sqrt{\sec(c+dx}\\ &= \frac{6b\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b\sec^{\frac{5}{2}}(c+dx)\sin}{5d}\\ &= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6b\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} +\\ &= -\frac{6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{3d}\end{aligned}$$

Mathematica [A] time = 0.319521, size = 97, normalized size = 0.64

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(20a\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+10a\sin(2(c+dx))+21b\sin(c+dx)+9b\sin(3(c+dx))-36b\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-36*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 2
0*a*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*b*Sin[c + d*x] + 10*a
```

```
*Sin[2*(c + d*x)] + 9*b*Sin[3*(c + d*x)]))/(30*d)
```

Maple [B] time = 3.964, size = 502, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*b/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c
)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx+c) + a) \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec(dx+c)^3 + a \sec(dx+c)^2\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.580 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d}$$

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*b*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rubi [A] time = 0.0911815, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3768, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2b\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*b*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx = a\int \sec^{\frac{3}{2}}(c+dx)\,dx + b\int \sec^{\frac{5}{2}}(c+dx)\,dx$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - a\int \frac{1}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \left(a\sqrt{\cos(c+dx)}\sqrt{s\right.$$

$$= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [A] time = 0.226119, size = 85, normalized size = 0.69

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(2b\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx)(3a\cos(c+dx)+b)-6a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-6*a*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
b*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(b + 3*a*Cos[c + d*x])*S
in[c + d*x]))/(3*d)
```

Maple [B] time = 3.412, size = 397, normalized size = 3.2

$$\frac{2}{3d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\mathrm{EllipticF}\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b*sin(1/2*d*x+1/2*c)^2+6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*sin(1/2*d*x+
1/2*c)^2-12*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2*a+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(dx+c\right)^{2}+a\sec\left(dx+c\right)\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a) \sec (dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.581 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d}$$

```
[Out] (-2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0754024, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3771, 2641, 3768, 2639}

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))\,dx &= a\int \sqrt{\sec(c+dx)}\,dx + b\int \sec^{\frac{3}{2}}(c+dx)\,dx\\
&= \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.\\
&= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{d} -\\
&= -\frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{d}\end{aligned}$$

Mathematica [A] time = 0.107918, size = 71, normalized size = 0.73

$$\frac{2\sqrt{\sec(c+dx)}\left(a\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+b\sin(c+dx)-b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-(b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*x]))/d
```

Maple [A] time = 1.346, size = 148, normalized size = 1.5

$$-2\,\frac{\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}a+\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{Ellipti}}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b-2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec(dx+c)+a)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec (c + dx))\sqrt{\sec (c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx + c) + a)\sqrt{\sec (dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.582 $\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.062732, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3787, 3771, 2639, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx = a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + b\int \sqrt{\sec(c+dx)}\,dx$$
$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx + \left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c}}$$
$$= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{d}$$

Mathematica [A] time = 0.0726717, size = 52, normalized size = 0.69

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+aE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)/
2, 2])*Sqrt[Sec[c + d*x]])/d
```

Maple [A] time = 1.273, size = 152, normalized size = 2.

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\left(b\text{EllipticF}\,(\right.}{\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,(\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1/2
```

```
*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\sec(dx+c)+a}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.583 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0772574, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\\
&= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx\\
&= \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.\\
&= \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{3d}\end{aligned}$$

Mathematica [A] time = 0.131439, size = 76, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(a\left(2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\right)+6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + a*(
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d)
```

Maple [A] time = 1.557, size = 228, normalized size = 2.3

$$-\frac{2}{3d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,a\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^4+\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\,\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a-3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.584 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0892598, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + b\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}b\int \sqrt{\sec(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{5}\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx +\\ &= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{3d}\end{aligned}$$

Mathematica [A] time = 0.334702, size = 88, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(10b\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))(3a\cos(c+dx)+5b)+18a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(18*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10
*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*b + 3*a*Cos[c + d*x])*
Sin[2*(c + d*x)]))/(15*d)
```

Maple [A] time = 1.356, size = 262, normalized size = 2.1

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,a\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(24\,a+20\,b\right)\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*b-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\sec\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\sec(c + dx)}{\sec^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx + c) + a}{\sec(dx + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.585 $\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6b\sqrt{c}}{}$$

```
[Out] (6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0981735, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6b\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```


```
[Out] (6*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*b*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx+b\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{1}{7}(5a)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\frac{1}{5}(3b)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{1}{21}(5a)\int \sqrt{\sec(c+dx)}\,dx+\frac{1}{5}\left(3b\right.\\ &= \frac{6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2b\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{10a\,\mathrm{si}}{21d\sqrt{\mathrm{s}}}\\ &= \frac{6b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{10a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{21d}\end{aligned}$$

Mathematica [A] time = 0.535665, size = 99, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(100a\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))(15a\cos(2(c+dx))+65a+42b\cos(c+dx))\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
00*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a + 42*b*Cos[c + d*
x] + 15*a*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 1.26, size = 290, normalized size = 1.9

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,a\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-360\,a-168\,b\right)\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*a
-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
a-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec\left(dx+c\right)+a}{\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx+c)+a}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.586 $\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=200

$$\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(7a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{4ab\sin(c+$$

```
[Out] (-12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (12*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (
2*(7*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a*b*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*b^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7
*d)
```

Rubi [A] time = 0.148329, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2(7a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(7a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)\sec^{\frac{5}{2}}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (-12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (12*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (
2*(7*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a*b*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*b^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7
*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^{\frac{7}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(6ab)\int \sec^{\frac{3}{2}}$$

$$= \frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2\left(7a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{4ab}{}$$

$$= \frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2\left(7a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{4ab}{}$$

$$= -\frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(7a^2+5b^2\right)\sqrt{\cos(c+d}}{}$$

Mathematica [A] time = 0.863991, size = 139, normalized size = 0.7

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(20\left(7a^2+5b^2\right)\cos^{\frac{7}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx)\left(5\left(7a^2+5b^2\right)\cos(2(c+dx))+35a^2+\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2]
+ 20*(7*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(35*a
^2 + 55*b^2 + 273*a*b*Cos[c + d*x] + 5*(7*a^2 + 5*b^2)*Cos[2*(c + d*x)] + 6
3*a*b*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)
```

Maple [B] time = 4.871, size = 689, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*a*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin
(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2
*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\sec(dx+c)^4+2\,ab\sec(dx+c)^3+a^2\sec(dx+c)^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.587 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=175

$$\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(5a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c\right.}}{}$$

```
[Out] (-2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*(5*a^2 + 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d) + (4*a*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*b^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.129039, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{2\left(5a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)\sec}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (-2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*(5*a^2 + 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d) + (4*a*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*b^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(2ab)\int \sqrt{\sec(}$$

$$= \frac{2\left(5a^2+3b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2b^2\sec}{}$$

$$= \frac{4ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2\left(5a^2+3b^2\right)\sqrt{\sec(c+dx)}\sin}{5d}$$

$$= -\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}F}{}$$

Mathematica [A] time = 1.21577, size = 126, normalized size = 0.72

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(40ab\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-12\left(5a^2+3b^2\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\left(\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(5*a^2 + 3*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a
^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(5*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(30*d)
```

Maple [B] time = 4.376, size = 660, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*
d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*(
-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec(dx+c)^3 + 2\,ab\sec(dx+c)^2 + a^2\sec(dx+c)\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.588 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (-4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (4*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*b^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.10965, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2(3a^2+b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (4*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*b^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx = (2ab)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\sec(c+dx)}\left(a^2+b^2\sec^2(c+dx)\right)\,dx$$

$$= \frac{4ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - (2ab)\int \frac{}{\sqrt{\sec(}}$$

$$= \frac{4ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \left(2ab\sqrt{\cos(c+\right.$$

$$= -\frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}}{}$$

Mathematica [A] time = 0.329288, size = 93, normalized size = 0.69

$$\frac{2\sec^{\frac{3}{2}}(c+dx)\left(\left(3a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - 6ab\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + b\sin(c+dx)(6a\,c\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
+ (3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + b*(b + 6*a*C
os[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 3.414, size = 514, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(6*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*
d*x+1/2*c)^2+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2-24*a*b*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-3*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2-Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*b^2-6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b+12*a*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec\left(dx+c\right)^2+2\,ab\sec\left(dx+c\right)+a^2\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(sec(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)^2\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.589 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin}{}$$

```
[Out] (2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.107081, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3788, 3771, 2641, 4046, 2639}

$$\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx = (2ab)\int \sqrt{\sec(c+dx)}\,dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(a^2-b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{4ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(\left(a^2-b^2\right.\right.$$

$$= \frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [A] time = 0.182906, size = 82, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(b\left(2a\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+b\sin(c+dx)\right)+\left(a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + b*(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*Sin[c + d*x]
)))/d
```

Maple [A] time = 1.535, size = 202, normalized size = 1.9

$$-2\,\frac{2\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)ab-\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^{2}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sqrt(sec(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\sec{(c + dx)})^2}{\sqrt{\sec{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec{(dx + c)} + a)^2}{\sqrt{\sec{(dx + c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.590 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{d}$$

```
[Out] (4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.109298, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3788, 3771, 2639, 4045, 2641}

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = (2ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(-a^2-3b^2\right)\int \sqrt{\sec(c+dx)}\,dx + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(\left(-a^2-3b^2\right)\sqrt{\cos(}\right.$$

$$= \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

Mathematica [A] time = 0.179771, size = 87, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a^2\sin(2(c+dx))+12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a^2*Sin[2*(c
```

```
+ d*x)]))/(3*d)
```

Maple [A] time = 1.434, size = 283, normalized size = 2.5

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,a^{2}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2+3*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*b^2-6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a*b-2*a^2*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{2}\sec\left(dx+c\right)^{2}+2\,ab\sec\left(dx+c\right)+a^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sec(d*x + c)^(3/2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\sec(c + dx))^2}{\sec^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx + c) + a)^2}{\sec(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.591 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2}{5d\,\mathrm{s}}$$

```
[Out] (2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.12237, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx = (2ab)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(2ab)\int \sqrt{\sec(c+dx)}\,dx - \frac{1}{5}\left(-3a^2-5b^2\right)\int \frac{}{\sqrt{\sec}}$$

$$= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [A] time = 0.42432, size = 100, normalized size = 0.71

$$\frac{\sqrt{\sec(c+dx)}\left(20ab\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+6\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(2(c+dx)\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(10*b +
3*a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [A] time = 1.712, size = 321, normalized size = 2.3

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,a^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6+\left(24\,a^2+40\,ab\right)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^2+40*a*b)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-6*a^2-20*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-15*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^2}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \sec\left(dx+c\right)^2 + 2\,ab\sec\left(dx+c\right) + a^2}{\sec\left(dx+c\right)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sec(d*x + c)^(5/2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b\sec\left(c + dx\right)\right)^2}{\sec^{\frac{5}{2}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sec(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(dx+c\right) + a\right)^2}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.592 $\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab}{5d\,\mathrm{se}}$$

```
[Out] (12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(21*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(5*a^2 + 7*b^2)*Sin[c + d*x
])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.137916, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{2(5a^2+7b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(5a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{4ab\sin(c+}{5d\sec^{\frac{3}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```


```
[Out] (12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(21*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(5*a^2 + 7*b^2)*Sin[c + d*x
])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx = (2ab)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(6ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{7}\left(-5a^2-7b^2\right)\int \frac{}{\sec^{\frac{3}{2}}}$$

$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} - \frac{1}{21}\left(-5a^2-7b^2\right)\int \sqrt{\sec}$$

$$= \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{}$$

$$= \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{21d}$$

Mathematica [A] time = 0.711744, size = 120, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(20\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\left(15a^2\cos(2(c+dx))+65a^2+84ab\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 20*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*a^2
+ 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])
)/(210*d)
```

Maple [A] time = 1.43, size = 362, normalized size = 2.1

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,a^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-360\,a^2-336\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+25*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2+35*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2-126*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sec(d*x + c)^(7/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^2}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x)
```

3.593 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=234

$$\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2b(21a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(5a^2$$

```
[Out] (-2*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(5*a^2 + 9*b^2)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (32*a*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*b^2
*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.240796, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3842, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2b(21a^2+5b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2a(5a^2+9b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```



```
[Out] (-2*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*(5*a^2 + 9*b^2)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (32*a*b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*b^2
*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
```

```
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a\left(7a^2+\right.\right.$$

$$= \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a\left(7a^2+\right.\right.$$

$$= \frac{2b\left(21a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{32ab^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + 2b$$

$$= \frac{2a\left(5a^2+9b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b\left(21a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{21d}$$

$$= \frac{2b\left(21a^2+5b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2a\left(5a^2+9b^2\right)\sqrt{s}}{}$$

$$= -\frac{2a\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\left(21a^2+5b^2\right)\sqrt{}}{}$$

Mathematica [A] time = 3.27374, size = 177, normalized size = 0.76

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(40b\left(21a^2+5b^2\right)\cos^{\frac{7}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-168a\left(5a^2+9b^2\right)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-168*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(c
 + d*x)/2, 2] + 40*b*(21*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x
)/2, 2] + 2*(210*a^2*b + 110*b^3 + 63*a*(5*a^2 + 13*b^2)*Cos[c + d*x] + 10*
(21*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 105*a^3*Cos[3*(c + d*x)] + 189*a*b^2*
Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] time = 5.777, size = 847, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6*a^2*b*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/
42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/5*a*b^2/(8*sin(1/2*d*x+1/2*c)^6-12
*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x
+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c
)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \sec(dx+c)^4 + 3\,ab^2 \sec(dx+c)^3 + 3\,a^2 b \sec(dx+c)^2 + a^3 \sec(dx+c)\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^4 + 3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c
)^2 + a^3*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.594 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=189

$$\frac{2a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6b\left(5a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{6b\left(5a^2+b^2\right.}{}$$

```
[Out] (-6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (6*b*(5*a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(5*d) + (8*a*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b^2*Sec[
c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.201691, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3842, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{6b\left(5a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (6*b*(5*a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(5*d) + (8*a*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b^2*Sec[
c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx = \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(5a^2\right.\right.$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(5a^2\right.\right.$$

$$= \frac{6b\left(5a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{8ab^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2b^2}{}$$

$$= \frac{6b\left(5a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{8ab^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2b^2}{}$$

$$= -\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\left(a^2+b^2\right)\sqrt{\cos}}{}$$

Mathematica [A] time = 1.42845, size = 134, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(5a\left(a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-3b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\sin(c+dx)\left(3\left(5a^2+b^2\right)\cos\right.}{}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*b*(5*a^2 + b^2)*EllipticE[(c +
 d*x)/2, 2] + 5*a*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (b*(5*(3*a^2 + b^
2) + 10*a*b*Cos[c + d*x] + 3*(5*a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/
(2*Cos[c + d*x]^(5/2))))/(5*d)
```

Maple [B] time = 4.544, size = 738, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
```

```
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a*b
^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^3/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+6*a^2*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.595 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=158

$$\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right|$$

```
[Out] (2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.188966, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3842, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```


Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}a\left(3a^2-b^2\right)+\frac{1}{2}b\left(9a^2+b^2\right)\sec(c+}{\sqrt{\sec(c+dx}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{1}{2}a\left(3a^2-b^2\right)+4ab^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{16ab^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d} + \left(a\left(a^2-\right.\right.$$

$$= \frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{16ab^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}$$

$$= \frac{2a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{3d}$$

Mathematica [A] time = 0.463404, size = 106, normalized size = 0.67

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(b\left(2\left(9a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2b\sin(c+dx)(9a\cos(c+dx)+b)\right)+6a\left(a^2-3b^2\right)\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(6*a*(a^2 - 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*
x)/2, 2] + b*(2*(9*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]
+ 2*b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])))/(3*d)
```

Maple [B] time = 3.526, size = 631, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
```

```
c),2^(1/2))*a^2*b*sin(1/2*d*x+1/2*c)^2+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3*sin(1
/2*d*x+1/2*c)^2-6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2+18*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a*b^2*sin(1/2*d*x+1/2*c)^2-36*a*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*b^3+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+18
*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*b^3*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.596 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=166

$$\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2b\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2b\left(3a^2-b\right.}{}$$

```
[Out] (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b*(a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(3*d) + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]])
```

Rubi [A] time = 0.194054, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3841, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\left(3a^2-b^2\right)\sqrt{\cos\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) - (2*b*(a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(3*d) + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
d*x]])
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
```

```
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{4a^2b+\frac{1}{2}a\left(a^2+9b^2\right)\sec(c+dx)-\frac{1}{2}b\left(a^2-3b^2\right)}{\sqrt{\sec(c+dx)}}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{4a^2b-\frac{1}{2}b\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}\left(a\left(a^2\right.\right.$$

$$= -\frac{2b\left(a^2-3b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \left(b\left(3a^2-b\right.\right.$$

$$= \frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} - \frac{2b\left(a^2-3b^2\right)\sqrt{\sec(c+dx)}\sin}{3d}$$

$$= \frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}F\left(\right.}{3d}$$

Mathematica [A] time = 0.551851, size = 108, normalized size = 0.65

$$\frac{\sqrt{\sec(c+dx)}\left(2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx)\left(a^3\cos(c+dx)+3b^3\right)-6b\left(b^2-3a^2\right)\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*b*(-3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2] + 2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
 + 2*(3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [A] time = 1.615, size = 303, normalized size = 1.8

$$-\frac{2}{3\,d}\left(4\,a^3\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+a^3\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\,\left(\sin(1/2\,dx+c/2)\right)^2-1}\text{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+9*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*b^3-2*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.597 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{5d}$$

```
[Out] (6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*b*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
 + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.192798, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3841, 4047, 3771, 2639, 4045, 2641}

$$\frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```


```
[Out] (6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d + (8*a^2*b*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
 + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{6a^2b+\frac{3}{2}a\left(a^2+5b^2\right)\sec(c+dx)+\frac{1}{2}b\left(a^2+5b^2\right)\text{s}}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{6a^2b+\frac{1}{2}b\left(a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}\left(3a\left(a^2\right.\right.$$

$$= \frac{8a^2b\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \left(b\left(a^2+b^2\right)\right)\int \sqrt{\sec(c+dx)}\,dx +$$

$$= \frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2b\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\,\text{se}}{5d\,\text{s}}$$

$$= \frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{d}$$

Mathematica [A] time = 0.443125, size = 106, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(10b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2\sin(\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
a^2*(5*b + a*Cos[c + d*x])*Sin[2*(c + d*x)]))/(5*d)
```

Maple [A] time = 1.528, size = 376, normalized size = 2.4

$$-\frac{2}{5\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\,a^3\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6+\left(8\,a^3+20\,a^2b\right)\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*a^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(8*a^3+20*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-2*a^3-10*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-3*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a^3-15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \sec(dx+c)^3 + 3\,ab^2 \sec(dx+c)^2 + 3\,a^2 b \sec(dx+c) + a^3}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sec(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x)
```

3.598 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b\left(9a^2+5b^2\right)\sqrt{\text{co}}}{}$$

```
[Out] (2*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (32*a^2*b*Sin[c + d*x])/(35*d*Sec[c
 + d*x]^(3/2)) + (2*a*(5*a^2 + 21*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.227949, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\left(9a^2+5b^2\right)\sqrt{\cos(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (32*a^2*b*Sin[c + d*x])/(35*d*Sec[c
 + d*x]^(3/2)) + (2*a*(5*a^2 + 21*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) + (2*a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
```

```
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b+\frac{1}{2}a\left(5a^2+21b^2\right)\sec(c+dx)+\frac{1}{2}b\left(3a^2+\right.}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b+\frac{1}{2}b\left(3a^2+7b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{7}\left(a\left(5a\right.\right.$$

$$= \frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{5}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{5}$$

$$= \frac{2b\left(9a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}}{2}$$

Mathematica [A] time = 0.90998, size = 132, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(20a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a\sin(2(c+dx))\left(15a^2\cos(2(c+dx))+65a^2+1\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 20*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2] + a*(65*a^2 + 210*b^2 + 126*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)
])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 1.568, size = 421, normalized size = 2.1

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,a^3\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-360\,a^3-504\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-189*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*b^3+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \sec(dx+c)^3 + 3\,ab^2 \sec(dx+c)^2 + 3\,a^2 b \sec(dx+c) + a^3}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x)
```

3.599 $\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(15a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (2*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (40*a^2*b*Sin[c + d*x])/(63*d*Sec
[c + d*x]^(5/2)) + (2*a*(7*a^2 + 27*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(
3/2)) + (2*b*(15*a^2 + 7*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*
a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.240522, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(15a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (2*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (40*a^2*b*Sin[c + d*x])/(63*d*Sec
[c + d*x]^(5/2)) + (2*a*(7*a^2 + 27*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(
3/2)) + (2*b*(15*a^2 + 7*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*
a^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
```

```
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}dx = \frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2}{9}\int \frac{10a^2b+\frac{1}{2}a\left(7a^2+27b^2\right)\sec(c+dx)+\frac{1}{2}b\left(5a^2+9\right.}{\sec^{\frac{7}{2}}(c+dx)}$$

$$=\frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2}{9}\int \frac{10a^2b+\frac{1}{2}b\left(5a^2+9b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}dx+\frac{1}{9}\left(a\left(7a\right.\right.$$

$$=\frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(a+b\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(b\right.$$

$$=\frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(15a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a^2(a+b}{9}$$

$$=\frac{2a\left(7a^2+27b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{40a^2b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a\left(7a^2\right.}{45a}$$

$$=\frac{2a\left(7a^2+27b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}}{21}$$

Mathematica [A] time = 1.24958, size = 159, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(120b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\left(7a\left(43a^2+108b^2\right)\cos(c+dx)+\right.\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*a*(7*a^2 + 27*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 120*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] + (7*a*(43*a^2 + 108*b^2)*Cos[c + d*x] + 5*(234*a^2*b + 84*b^3 + 5
4*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(126
0*d)
```

Maple [A] time = 1.5, size = 470, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^3+2160*a^2*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+105*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*b^3-147*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \sec(dx+c)^3 + 3\,ab^2 \sec(dx+c)^2 + 3\,a^2 b \sec(dx+c) + a^3}{\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/sec(d*x+c)**(9/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^3}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x)
```

3.600 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=287

$$\frac{8ab\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{14b^2\left(7a^2+b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{8ab\left(7\right.}{}$$

```
[Out] (-2*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(15*a^4 + 54*a^2*
b^2 + 7*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (8*a*b*(7*a^2 + 5*b^
2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (14*b^2*(7*a^2 + b^2)*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(45*d) + (44*a*b^3*Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(63*d) + (2*b^2*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(
9*d)
```

Rubi [A] time = 0.409396, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3842, 4076, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{14b^2\left(7a^2+b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{8ab\left(7a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(54a^2b^2+15a^4+7b^4\right)\sin(c+}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (-2*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(15*a^4 + 54*a^2*
b^2 + 7*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (8*a*b*(7*a^2 + 5*b^
2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (14*b^2*(7*a^2 + b^2)*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(45*d) + (44*a*b^3*Sec[c + d*x]^(7/2)*Sin[c + d*x]
)/(63*d) + (2*b^2*Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(
9*d)
```


Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^4\,dx = \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{9d} + \frac{2}{9}\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec$$

$$= \frac{44ab^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+}{9d}$$

$$= \frac{44ab^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2b^2\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+}{9d}$$

$$= \frac{8ab\left(7a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{14b^2\left(7a^2+b^2\right)\sec^{\frac{5}{2}}(c+dx)\sin(c}{45d}$$

$$= \frac{2\left(15a^4+54a^2b^2+7b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{15d} + \frac{8ab\left(7a^2+5b^2\right)\sec^{\frac{3}{2}}(c+}{21d}$$

$$= \frac{8ab\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d} + \frac{2\left(15a^4+54a^2b\right.}{}$$

$$= -\frac{2\left(15a^4+54a^2b^2+7b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{8ab\left(7\right.}{}$$

Mathematica [A] time = 2.1412, size = 256, normalized size = 0.89

$$-\frac{2(a+b\sec(c+dx))^4\left(-60ab\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-1134a^2b^2\sin(c+dx)+21\left(54a^2b^2+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (-2*(a + b*Sec[c + d*x])^4*(21*(15*a^4 + 54*a^2*b^2 + 7*b^4)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2] - 60*a*b*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2] - 315*a^4*Sin[c + d*x] - 1134*a^2*b^2*Sin[c + d*x
] - 147*b^4*Sin[c + d*x] - 420*a^3*b*Tan[c + d*x] - 300*a*b^3*Tan[c + d*x]
- 378*a^2*b^2*Sec[c + d*x]*Tan[c + d*x] - 49*b^4*Sec[c + d*x]*Tan[c + d*x]
- 180*a*b^3*Sec[c + d*x]^2*Tan[c + d*x] - 35*b^4*Sec[c + d*x]^3*Tan[c + d*x
]))/(315*d*(b + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2))
```

Maple [B] time = 6.618, size = 1174, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*a^3*b*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))-12/5*a^2*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*s
in(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d
*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1
/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+
8*a*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/2
1*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2)))+2*b^4*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2
)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^
2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))))+2*a^4*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*si
n(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4 \sec(dx+c)^5 + 4\,ab^3 \sec(dx+c)^4 + 6\,a^2b^2 \sec(dx+c)^3 + 4\,a^3b \sec(dx+c)^2 + a^4 \sec(dx+c)\right)\sqrt{\sec(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^5 + 4*a*b^3*sec(d*x + c)^4 + 6*a^2*b^2*sec(d*x +
 c)^3 + 4*a^3*b*sec(d*x + c)^2 + a^4*sec(d*x + c))*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^4\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

3.601 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx$

Optimal. Leaf size=247

$$\frac{2\left(42a^2b^2+21a^4+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2b^2\left(39a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}$$

```
[Out] (-8*a*b*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a*b*(5*a^2 + 3*b^2
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b^2*(39*a^2 + 5*b^2)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(21*d) + (36*a*b^3*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(35*d) + (2*b^2*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(
7*d)
```

Rubi [A] time = 0.365986, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3842, 4076, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2b^2\left(39a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{8ab\left(5a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(42a^2b^2+21a^4+5b^4\right)\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```


```
[Out] (-8*a*b*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(5*d) + (2*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (8*a*b*(5*a^2 + 3*b^2
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b^2*(39*a^2 + 5*b^2)*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(21*d) + (36*a*b^3*Sec[c + d*x]^(5/2)*Sin[c + d*x]
)/(35*d) + (2*b^2*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(
7*d)
```


Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
```

```
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
```

```
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^4\,dx = \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{\sec(c+dx)}(a+b\,\mathrm{se}$$

$$= \frac{36ab^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+}{7d}$$

$$= \frac{36ab^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2b^2\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+}{7d}$$

$$= \frac{8ab\left(5a^2+3b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b^2\left(39a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin}{21d}$$

$$= \frac{8ab\left(5a^2+3b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2b^2\left(39a^2+5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin}{21d}$$

$$= -\frac{8ab\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(21a^4+42a\right.}{}$$

Mathematica [A] time = 1.53072, size = 168, normalized size = 0.68

$$\frac{2\sec^{\frac{7}{2}}(c+dx)\left(5\left(42a^2b^2+21a^4+5b^4\right)\cos^{\frac{7}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+b\sin(c+dx)\left(5b\left(42a^2+5b^2\right)\cos^2(c+\right.\right.}{105a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (2*Sec[c + d*x]^(7/2)*(-84*a*b*(5*a^2 + 3*b^2)*Cos[c + d*x]^(7/2)*EllipticE
[(c + d*x)/2, 2] + 5*(21*a^4 + 42*a^2*b^2 + 5*b^4)*Cos[c + d*x]^(7/2)*Ellip
ticF[(c + d*x)/2, 2] + b*(15*b^3 + 5*b*(42*a^2 + 5*b^2)*Cos[c + d*x]^2 + 84
*a*(5*a^2 + 3*b^2)*Cos[c + d*x]^3)*Sin[c + d*x] + 42*a*b^3*Sin[2*(c + d*x)]
))/(105*d)
```

Maple [B] time = 5.392, size = 925, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a^
2*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-8/5*a*b^3/(8*sin(1/2*d*x+1/2
*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^
2*(12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6
*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin
(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)+2*b^4*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/
2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2)))+8*a^3*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*si
n(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4 \sec(dx+c)^4 + 4\,ab^3 \sec(dx+c)^3 + 6\,a^2b^2 \sec(dx+c)^2 + 4\,a^3b \sec(dx+c) + a^4\right)\sqrt{\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^4\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.602 $\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=209

$$\frac{8ab\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b^2\left(29a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(-30a\right.}{}$$

```
[Out] (2*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*(29*a^2 + 3*b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (28*a*b^3*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(15*d) + (2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*
x])/(5*d)
```

Rubi [A] time = 0.349171, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3842, 4076, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b^2\left(29a^2+3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(-30a^2b^2+5a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b^2*(29*a^2 + 3*b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (28*a*b^3*Sec[c + d*x]^(3/2)*Sin[c +
 d*x])/(15*d) + (2*b^2*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*
x])/(5*d)
```


Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
```

```
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(\frac{1}{2}a\left(5a^2-b^2\right)+\right.}{\sqrt{}}$$

$$= \frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{4}{15}\int$$

$$= \frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2b^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{4}{15}\int$$

$$= \frac{2b^2\left(29a^2+3b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{28ab^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2b^2\sqrt{\sec(c+}}{}$$

$$= \frac{8ab\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2b^2\left(29a^2+3b^2\right)\sqrt{\sec(c+dx)}}{5d}$$

$$= \frac{2\left(5a^4-30a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab\left(3a^2+b^2\right)\sqrt{\cos(c}}{}$$

Mathematica [A] time = 2.12493, size = 146, normalized size = 0.7

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(b\left(80a\left(3a^2+b^2\right)\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2b\sin(c+dx)\left(9\left(10a^2+b^2\right)\cos(2(c+dx))+15\right.\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(12*(5*a^4 - 30*a^2*b^2 - 3*b^4)*Cos[c + d*x]^(5/2)*Ell
ipticE[(c + d*x)/2, 2] + b*(80*a*(3*a^2 + b^2)*Cos[c + d*x]^(5/2)*EllipticF
[(c + d*x)/2, 2] + 2*b*(15*(6*a^2 + b^2) + 40*a*b*Cos[c + d*x] + 9*(10*a^2
+ b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])))/(30*d)
```

Maple [B] time = 4.679, size = 907, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^4*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a^3*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*a*b^3*(-1/6*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^4/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d
*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c
)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+12*a^2*b
^2*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4\sec(dx+c)^4+4\,ab^3\sec(dx+c)^3+6\,a^2b^2\sec(dx+c)^2+4\,a^3b\sec(dx+c)+a^4}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.603 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=208

$$\frac{2\left(18a^2b^2+a^4+b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{4ab}{}$$

```
[Out] (8*a*b*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*(a^4 + 18*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a*b*(a^2 - 6*b^2)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(3*d) - (2*b^2*(a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*d) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*
x]])
```

Rubi [A] time = 0.348607, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4076, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b^2\left(a^2-b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}-\frac{4ab\left(a^2-6b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2\left(18a^2b^2+a^4+b^4\right)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```


```
[Out] (8*a*b*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/d + (2*(a^4 + 18*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a*b*(a^2 - 6*b^2)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(3*d) - (2*b^2*(a^2 - b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x
])/(3*d) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*
x]])
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
```

```
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4076

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*C*Csc[e + f*x]*Cot[e + f*x]*(d*Csc[e + f*x])^n)
/(f*(n + 2)), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2)
+ (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n +
 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &&
 !LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+b\sec(c+dx))\left(5a^2b+\frac{1}{2}a\left(a^2+9b^2\right)\sec(c\right.}{\sqrt{\sec(c+dx)}}$$

$$= -\frac{2b^2\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{4}{9}\int \frac{\frac{15a^3b}{2}}{}$$

$$= -\frac{2b^2\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{4}{9}\int \frac{\frac{15a^3b}{2}}{}$$

$$= -\frac{4ab\left(a^2-6b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} - \frac{2b^2\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2(a}{}$$

$$= \frac{2\left(a^4+18a^2b^2+b^4\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} - \frac{4ab\left(a^2-6b^2\right)\sqrt{\sec(c+}}{3d}$$

$$= \frac{8ab\left(a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(a^4+18a^2b^2+b^4\right)\sqrt{\cos(c+}}{}$$

Mathematica [A] time = 1.08274, size = 130, normalized size = 0.62

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(2\left(18a^2b^2+a^4+b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+24ab\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)(a^4}{}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(24*a*b*(a^2 - b^2)*EllipticE[(c + d
*x)/2, 2] + 2*(a^4 + 18*a^2*b^2 + b^4)*EllipticF[(c + d*x)/2, 2] + ((a^4 +
2*b^4 + 24*a*b^3*Cos[c + d*x] + a^4*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c +
 d*x]^(3/2)))/(3*d)
```

Maple [B] time = 2.028, size = 777, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(-8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*a*(a^3+6*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^4+12*a*b^3+b^4)*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+18*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2*b^2+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*a^3*b+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)*
sin(1/2*d*x+1/2*c)^2+a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)+18*a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^4-12*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+12*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4\sec(dx+c)^4+4\,ab^3\sec(dx+c)^3+6\,a^2b^2\sec(dx+c)^2+4\,a^3b\sec(dx+c)+a^4}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(3/2), x)
```

3.604 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=207

$$\frac{8ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2\left(30a^2b^2\right.}{}$$

```
[Out] (2*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (28*a^3*b*Sin[c + d*x])/(1
5*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(5*d) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^
(3/2))
```

Rubi [A] time = 0.36943, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4074, 4047, 3771, 2641, 4046, 2639}

$$-\frac{2b^2\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{8ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2\left(30a^2b^2+3a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```



```
[Out] (2*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(5*d) + (8*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (28*a^3*b*Sin[c + d*x])/(1
5*d*Sqrt[Sec[c + d*x]]) - (2*b^2*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(5*d) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^
(3/2))
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
```

```
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+b\sec(c+dx))\left(7a^2b+\frac{3}{2}a\left(a^2+5b^2\right)\sec(c+\right.}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}a^2\left(3a^2+29b^2\right)-5ab\left(a\right.}{}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}a^2\left(3a^2+29b^2\right)+\frac{3}{4}b^2\left(a\right.}{\sqrt{\sec(c+}}$$

$$= \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2b^2\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{8ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{28a^3b\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2b^2\left(a^2-\right.}{}$$

$$= \frac{2\left(3a^4+30a^2b^2-5b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8ab\left(a^2+3b^2\right)\sqrt{\cos(}}{}$$

Mathematica [A] time = 0.58226, size = 138, normalized size = 0.67

$$\frac{\sqrt{\sec(c+dx)}\left(80ab\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2\sin(c+dx)\left(40a^3b\cos(c+dx)+3a^4\cos(2(c+d\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*(3*a^4 + 30*a^2*b^2 - 5*b^4)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, 2] + 80*a*b*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2] + 2*(3*a^4 + 30*b^4 + 40*a^3*b*Cos[c + d*x] + 3*a^4*Cos[2*(
c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] time = 1.752, size = 619, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15*(-24*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*a^3*(3*a+10*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*a^4+20*a^3*b+15*b^4)*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+20*a^3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+60*a*b^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-9*a^4*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-90*a^
2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))+15*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*b^4)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \sec(dx+c)^4 + 4\,ab^3 \sec(dx+c)^3 + 6\,a^2b^2 \sec(dx+c)^2 + 4\,a^3b \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\sec(c + dx))^4}{\sec^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**4/sec(c + d*x)**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx + c) + a)^4}{\sec(dx + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(5/2), x)
```

3.605 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{2\left(42a^2b^2+5a^4+21b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a^2\left(5a^2+39b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{8ab\left(3a^2\right.}{}$$

```
[Out] (8*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (36*a^3*b*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2)) + (2*a^2*(5*a^2 + 39*b^2)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c
 + d*x]^(5/2))
```

Rubi [A] time = 0.362749, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3841, 4074, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a^2\left(5a^2+39b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(42a^2b^2+5a^4+21b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{8ab\left(3a^2+5b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```


```
[Out] (8*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(5*d) + (2*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (36*a^3*b*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2)) + (2*a^2*(5*a^2 + 39*b^2)*Sin[c + d*x])/(21*d*S
qrt[Sec[c + d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c
 + d*x]^(5/2))
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
```

```
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+b\sec(c+dx))\left(9a^2b+\frac{1}{2}a\left(5a^2+21b^2\right)\sec\right.}{\sec^{\frac{5}{2}}(c+d}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}a^2\left(5a^2+39b^2\right)-7ab}{}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}a^2\left(5a^2+39b^2\right)-\frac{5}{4}b^2}{\sec^{\frac{3}{2}}(c}$$

$$= \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2\left(5a^2+39b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} -$$

$$= \frac{8ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{36a^3b\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2\left(5a\right.}{2}$$

$$= \frac{8ab\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(5a^4+42a^2b^2+21b^4\right)\sqrt{c}}{}$$

Mathematica [A] time = 0.799924, size = 142, normalized size = 0.67

$$\frac{\sqrt{\sec(c+dx)}\left(20\left(42a^2b^2+5a^4+21b^4\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a^2\sin(2(c+dx))\left(15a^2\cos(2(c+dx))-\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(336*a*b*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(
c + d*x)/2, 2] + 20*(5*a^4 + 42*a^2*b^2 + 21*b^4)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2] + a^2*(65*a^2 + 420*b^2 + 168*a*b*Cos[c + d*x] + 15*a^2
*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 1.425, size = 476, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^4*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^4-672*a^3*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^4+672*a^3*b+840*a^2*b^2)*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+(-80*a^4-168*a^3*b-420*a^2*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+25*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+210*a^2*b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+105*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-252*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^
3*b-420*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \sec(dx+c)^4 + 4\,ab^3 \sec(dx+c)^3 + 6\,a^2b^2 \sec(dx+c)^2 + 4\,a^3b \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^4}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(7/2), x)
```

3.606 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{8ab\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{14a^2\left(a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8ab\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (44*a^3*b*Sin[c + d*x
])/(63*d*Sec[c + d*x]^(5/2)) + (14*a^2*(a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Se
c[c + d*x]^(3/2)) + (8*a*b*(5*a^2 + 7*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/
2))
```

Rubi [A] time = 0.402599, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3841, 4074, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{14a^2\left(a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8ab\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{8ab\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```


```
[Out] (2*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(15*d) + (8*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (44*a^3*b*Sin[c + d*x
])/(63*d*Sec[c + d*x]^(5/2)) + (14*a^2*(a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Se
c[c + d*x]^(3/2)) + (8*a*b*(5*a^2 + 7*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/
2))
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
```

```
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
```

```
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+b\sec(c+dx))\left(11a^2b+\frac{1}{2}a\left(7a^2+27b^2\right)\sec(\ldots}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{4}{63}\int \frac{-\frac{49}{4}a^2\left(a^2+7b^2\right)-9ab\left(5a\ldots\right.}{\ldots}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{4}{63}\int \frac{-\frac{49}{4}a^2\left(a^2+7b^2\right)-\frac{21}{4}b^2\left(a\ldots\right.}{\sec^{\frac{5}{2}}(c+\ldots}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{14a^2\left(a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{8ab\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\,\ldots}{9\ldots}$$

$$= \frac{44a^3b\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{14a^2\left(a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{8ab\left(5a^2+7b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2a^2(a+b\,\ldots}{9\ldots}$$

$$= \frac{2\left(7a^4+54a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{8ab\left(5a^2+7b^2\right)\sqrt{\cos\ldots}}{\ldots}$$

Mathematica [A] time = 1.47051, size = 168, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(480ab\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a\sin(2(c+dx))\left(7a\left(43a^2+216b^2\right)\cos(c+dx)\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^4 + 54*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*E
llipticE[(c + d*x)/2, 2] + 480*a*b*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2] + a*(7*a*(43*a^2 + 216*b^2)*Cos[c + d*x] + 5*(312*a^2*
b + 336*b^3 + 72*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c + d*x)]))*Sin[2*(c
```

```
+ d*x)]))/(1260*d)
```

Maple [A] time = 1.666, size = 529, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^4*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^4+2880*a^3*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^4-4320*a^3*b-3024*a^2*b^2)*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^4+3360*a^3*b+3024*a^2*b^2+1680*a*b^3)*si
n(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^4-960*a^3*b-756*a^2*b^2-840*a
*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+300*a^3*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+420*a*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4-1
134*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2-315*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^4)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4\sec\left(dx+c\right)^4+4\,ab^3\sec\left(dx+c\right)^3+6\,a^2b^2\sec\left(dx+c\right)^2+4\,a^3b\sec\left(dx+c\right)+a^4}{\sec\left(dx+c\right)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^4}{\sec\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(9/2), x)
```

3.607 $\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=289

$$\frac{2\left(330a^2b^2+45a^4+77b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{231d}+\frac{2a^2\left(9a^2+59b^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8ab\left(7a\right.}{4}$$

```
[Out] (8*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(15*d) + (2*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (52*a^3*b*Sin[c +
d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*a^2*(9*a^2 + 59*b^2)*Sin[c + d*x])/(77
*d*Sec[c + d*x]^(5/2)) + (8*a*b*(7*a^2 + 9*b^2)*Sin[c + d*x])/(45*d*Sec[c +
 d*x]^(3/2)) + (2*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sin[c + d*x])/(231*d*Sqrt
[Sec[c + d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c +
 d*x]^(9/2))
```

Rubi [A] time = 0.459503, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3841, 4074, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2a^2\left(9a^2+59b^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{8ab\left(7a^2+9b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2\left(330a^2b^2+45a^4+77b^4\right)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2\left(330a^2b^2+45\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```


```
[Out] (8*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(15*d) + (2*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (52*a^3*b*Sin[c +
d*x])/(99*d*Sec[c + d*x]^(7/2)) + (2*a^2*(9*a^2 + 59*b^2)*Sin[c + d*x])/(77
*d*Sec[c + d*x]^(5/2)) + (8*a*b*(7*a^2 + 9*b^2)*Sin[c + d*x])/(45*d*Sec[c +
 d*x]^(3/2)) + (2*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sin[c + d*x])/(231*d*Sqrt
[Sec[c + d*x]]) + (2*a^2*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c +
 d*x]^(9/2))
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4074

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[(A*a*Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*n), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^4}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+b\sec(c+dx))\left(13a^2b+\frac{3}{2}a\left(3a^2+11b^2\right)\mathrm{se}\right.}{\sec^{\frac{9}{2}}(c+}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{4}{99}\int \frac{-\frac{9}{4}a^2\left(9a^2+59b^2\right)-11ab}{}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{4}{99}\int \frac{-\frac{9}{4}a^2\left(9a^2+59b^2\right)-\frac{9}{4}b^2}{\sec^{\frac{7}{2}}(c}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2\left(9a^2+59b^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{8ab\left(7a^2+9b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+}{}$$

$$= \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2\left(9a^2+59b^2\right)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)} + \frac{8ab\left(7a^2+9b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(45a^4\right.}{}$$

$$= \frac{8ab\left(7a^2+9b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{52a^3b\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a^2\left(9a\right.}{7}$$

$$= \frac{8ab\left(7a^2+9b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2\left(45a^4+330a^2b^2+77b^4\right)}{}$$

Mathematica [A] time = 1.83525, size = 199, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(240\left(330a^2b^2+45a^4+77b^4\right)\sqrt{\cos(c+dx)}\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))\left(616ab\left(43a^2+36b^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4/Sec[c + d*x]^(11/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(14784*a*b*(7*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2] + 240*(45*a^4 + 330*a^2*b^2 + 77*b^4)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2] + (616*a*b*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*(159
3*a^4 + 10296*a^2*b^2 + 1848*b^4 + 72*(8*a^4 + 33*a^2*b^2)*Cos[2*(c + d*x)]
 + 616*a^3*b*Cos[3*(c + d*x)] + 63*a^4*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)])
)/(27720*d)
```

Maple [A] time = 1.592, size = 586, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*a^4*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-50400*a^4-49280*a^3*b)*sin(1/2*d
*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*a^4+98560*a^3*b+47520*a^2*b^2)*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-34920*a^4-91168*a^3*b-71280*a^2*b^2-22
176*a*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*a^4+41888*a^3*b+5
5440*a^2*b^2+22176*a*b^3+4620*b^4)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-2790*a^4-7392*a^3*b-15840*a^2*b^2-5544*a*b^3-2310*b^4)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+675*a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4950*a^2*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1155*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6468*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*a^3*b-8316*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^4 \sec(dx+c)^4 + 4\,ab^3 \sec(dx+c)^3 + 6\,a^2b^2 \sec(dx+c)^2 + 4\,a^3b \sec(dx+c) + a^4}{\sec(dx+c)^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^4*sec(d*x + c)^4 + 4*a*b^3*sec(d*x + c)^3 + 6*a^2*b^2*sec(d*x +
 c)^2 + 4*a^3*b*sec(d*x + c) + a^4)/sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx+c) + a)^4}{\sec(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4/sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^4/sec(d*x + c)^(11/2), x)
```

3.608 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=188

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd}+\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sin(c+a}{}$$

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*
d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*b*d) + (2*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) - (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(b^2*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.510955, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3851, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*
d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*b*d) + (2*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) - (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(b^2*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*d)
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

EqQ[n^2, 1/4]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}b\sec(c+dx)-\frac{3}{2}a\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{3b}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{4\int \frac{\frac{3a^2}{4}+ab\sec(c+dx)+\frac{1}{4}\left(3a^2+b^2\right)\sec^2(c+}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}}{3b^2}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{4\int \frac{\frac{3a^3}{4}+\frac{1}{4}a^2b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^2b^2} + \frac{a^2\int \frac{s}{a+}}{}$$

$$= -\frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b^2} + \frac{\int \sqrt{\sec(c+d}}{3b}$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d} - \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2d} + \frac{2\sec^{\frac{3}{2}}(c}{}$$

$$= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3bd}$$

Mathematica [A] time = 2.83434, size = 167, normalized size = 0.89

$$-\frac{\cot(c+dx)\left(-2\left(3a^2+3ab+b^2\right)\sqrt{-\tan^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)-6a^2\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{b}{a};-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]*(-(b^2*Sec[c + d*x]^(5/2)) + b^2*Cos[2*(c + d*x)]*Sec[c + d*
x]^(5/2) + 6*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*
x]^2] - 2*(3*a^2 + 3*a*b + b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[-Tan[c + d*x]^2] - 6*a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[-Tan[c + d*x]^2]))/(3*b^3*d)
```

Maple [A] time = 4.243, size = 450, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^3/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a/b^2*(-
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

3.609 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{bd}$$

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
- (2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(b*(a + b)*d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.163845, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3850, 3768, 3771, 2639, 3849, 2805}

$$-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d)
- (2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(b*(a + b)*d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 3850

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[(a*
d)/b, Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \frac{\int \sec^{\frac{3}{2}}(c+dx)\,dx}{b} - \frac{a\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos\cdots}}{b}$$

$$= -\frac{2a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a+b)d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{bd} - \frac{\left(\sqrt{\cos(c+\cdots}}{}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c\cdots}}{b(a+b)d}$$

Mathematica [A] time = 4.53838, size = 86, normalized size = 0.74

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(-(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)-a\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - a*EllipticPi[-(b/a), -ArcSin[Sqrt[S
ec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b^2*d)
```

Maple [B] time = 1.689, size = 353, normalized size = 3.

$$-2\frac{1}{(a-b)b\sqrt{-2(\sin(1/2dx+c/2))^4+(\sin(1/2dx+c/2))^2}\sin(1/2dx+c/2)\sqrt{2(\cos(1/2dx+c/2))^2-1}d}\left(-2\sqrt{-2(\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/b/(a-b)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

3.610 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/((a + b)*d)
```

Rubi [A] time = 0.0915231, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3849, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/((a + b)*d)
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a+b)d}$$

Mathematica [A] time = 0.315992, size = 63, normalized size = 1.29

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(b
/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(b*d)
```

Maple [B] time = 1.154, size = 150, normalized size = 3.1

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\text{EllipticPi}\left(\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{3}{2}}}{b\sec (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{3}{2}}}{b\sec (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

3.611 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a*(a + b)*d)
```

Rubi [A] time = 0.150151, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3848, 2803, 2641, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a*(a + b)*d)
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos}}{a}$$

$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{a(a+b)d}$$

Mathematica [A] time = 0.216921, size = 49, normalized size = 0.53

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-
Tan[c + d*x]^2])/(a*d)
```

Maple [A] time = 1.492, size = 187, normalized size = 2.

$$-2\frac{\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{a\left(a-b\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\Big(\mathrm{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.612 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=135

$$-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2\sqrt{\cos(}}{}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rubi [A] time = 0.207321, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3852, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\sqrt{\cos(c+dx)}\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```


```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```


Rule 3852

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \frac{\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{b^2 \int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2}$$

$$= \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{b\int \sqrt{\sec(c+dx)}\,dx}{a^2} + \frac{\left(b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos}}}{a^2}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+}\right.}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{a^2d}$$

Mathematica [A] time = 6.23237, size = 178, normalized size = 1.32

$$\frac{\cot(c+dx)\left(2a\sqrt{-\tan^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+2b\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+d}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (Cot[c + d*x]*(-(a*Sec[c + d*x]^(3/2)) - a*Cos[2*(c + d*x)]*Sec[c + d*x]^(3
/2) + a*Sec[c + d*x]^(7/2) + a*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*a*El
lipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*EllipticPi[-
(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*d)
```

Maple [A] time = 1.415, size = 226, normalized size = 1.7

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\,a^2\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(\text{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.613 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^3d}-\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-$$

```
[Out] (-2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a^3*d) - (2*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*Sin[c + d*x])/(3
*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.366478, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3853, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}-\frac{2b^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2b\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```


```
[Out] (-2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2
*d) + (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a^3*d) - (2*b^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a + b)*d) + (2*Sin[c + d*x])/(3
*a*d*Sqrt[Sec[c + d*x]])
```


Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}dx = \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3ab}{2}-\left(-\frac{a^2}{2}-\frac{3b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{3a^3} - \frac{b^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}dx}{a^3}$$

$$= \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2} + \frac{\left(a^2+3b^2\right)\int \sqrt{\sec(c+dx)}\,dx}{3a^3} - \frac{\left(b^3\sqrt{\cos\right.}}{}$$

$$= -\frac{2b^3\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a+b)d} + \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(b\sqrt{}}{}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{3a^3d}$$

Mathematica [A] time = 6.08485, size = 196, normalized size = 1.14

$$-\frac{\cot(c+dx)\left(-4a(a-3b)\sqrt{-\tan^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)-a^2\sqrt{\sec(c+dx)}+a^2\cos(3(c+dx))\text{se}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -(Cot[c + d*x]*(-(a^2*Sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) - 6*a*
b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + a^2*Cos[3*(c + d*x)]*Sec[c + d*x]^(
3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^
2] - 4*a*(a - 3*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c +
d*x]^2] + 12*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-
Tan[c + d*x]^2]))/(6*a^3*d)
```

Maple [B] time = 1.543, size = 516, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*a^3-4*a^2*
b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a^3+2*a^2*b)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*a^2*b+3*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+3
*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec\left(c+dx\right)\right)\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.614 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=342

$$\frac{(5a^2-2b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d(a^2-b^2)}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{(5a^2-2b^2)\sin(c+}{3b^2d(a^2}$$

```
[Out] (a*(5*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(b^3*(a^2 - b^2)*d) + ((5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d) + (a^2*(5*a^2 -
 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - (a*(5*a^2 - 4*b^2)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((5*a^2 - 2*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.944133, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}+\frac{(5a^2-2b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d(a^2-b^2)}-\frac{a(5a^2-4b^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^3d(a^2-b^2)}+\frac{(5a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (a*(5*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(b^3*(a^2 - b^2)*d) + ((5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d) + (a^2*(5*a^2 -
 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - (a*(5*a^2 - 4*b^2)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((5*a^2 - 2*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-ab\sec(c+dx)-\frac{1}{2}\left(5a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{1}{4}a(5\right.}{}}{}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{5}{2}}}{b\left(a^2-b^2\right.}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{5}{2}}}{b\left(a^2-b^2\right.}$$

$$= -\frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{5}{2}}}{b\left(a^2-b^2\right.}$$

$$= \frac{a^2\left(5a^2-7b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} - \frac{a\left(5a^2-4b^2\right)\sqrt{\sec(c+dx)}}{b^3\left(a^2-b^2\right)d}$$

$$= \frac{a\left(5a^2-4b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{3b^2\left(a^2-b^2\right.}$$

Mathematica [A] time = 6.3972, size = 319, normalized size = 0.93

$$\frac{\frac{\cot(c+dx)\left(2\left(-16a^2b^2+15a^3b+15a^4-12ab^3-2b^4\right)\sqrt{-\tan^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)-6ab\left(5a^2-4b^2\right)\sqrt{-\tan^2(c+dx)}E\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+6a\left(\right.\right.}{(a-b)(a+b)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b*(15*a^4 - 16*a^2*b^2 + 4*b^4 + 20*a*b*(a^2 - b^2)*Cos[c + d*x] + 3*(5*a
^4 - 4*a^2*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^(3/2)*Sin[c + d*x])/((-a^2 +
 b^2)*(b + a*Cos[c + d*x])) + (Cot[c + d*x]*(-6*a*b*(5*a^2 - 4*b^2)*Ellipti
cE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*(15*a^4 + 15*a
^3*b - 16*a^2*b^2 - 12*a*b^3 - 2*b^4)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[-Tan[c + d*x]^2] + 6*a*(b*(5*a^2 - 4*b^2)*Sec[c + d*x]^(3/2)*Sin[
```

```
c + d*x]^2 + a*(5*a^2 - 7*b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*(a + b)))/(6*b^4*d)
```

Maple [B] time = 6.195, size = 1002, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))-4*a^3/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*a^2/b^
2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-4/b^3*a*(-(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

3.615 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=279

$$-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(}}{b^2d\left(a^2-b^2\right)}$$

```
[Out] -(((3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(b^2*(a^2 - b^2)*d)) - (a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) - (a*(3*a^2 - 5*b^2)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a -
 b)*b^2*(a + b)^2*d) + ((3*a^2 - 2*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b
^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*
(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.643241, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|2\right)}{bd\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -(((3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(b^2*(a^2 - b^2)*d)) - (a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) - (a*(3*a^2 - 5*b^2)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a -
 b)*b^2*(a + b)^2*d) + ((3*a^2 - 2*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b
^2*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*
(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
```

```
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
```

```
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-ab\sec(c+dx)-\frac{1}{2}\left(3a^2-2b^2\right)\sec^2(c+dx)\right)}{a+b\sec(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\int \frac{\frac{1}{4}a\left(3a^2-2b^2\right)+\frac{1}{2}\cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{2\int \frac{\frac{1}{4}a^2\left(3a^2-2b^2\right)-(\cdots}{\cdots}}{\cdots}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{a\int \sqrt{\sec(c+dx)}\cdots}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} + \frac{\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\cdots}{b^2\left(a^2-b^2\right)\cdots}$$

$$= -\frac{\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} - \frac{a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\cdots}{b\left(a^2-b^2\right)d}$$

Mathematica [A] time = 5.81816, size = 355, normalized size = 1.27

$$\frac{\cot(c+dx)\left(-2\left(3a^2b+3a^3-4ab^2-2b^3\right)\sqrt{-\tan^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+2b\left(3a^2-2b^2\right)\sqrt{-\tan^2(c+dx)}E\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-3a^2b\sec^{\frac{3}{2}}(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*b*(3*a^3 - 2*a*b^2 + 2*b*(a^2 - b^2)*Sec[c + d*x])*Sin[c + d*x])/((a^2
- b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-3*a^2*b*S
ec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(3/2) + 3*a^2*b*Cos[2*(c + d*x)]*Sec
[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*b*(3*a^2 -
2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*
(3*a^3 + 3*a^2*b - 4*a*b^2 - 2*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[-Tan[c + d*x]^2] - 6*a^3*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 10*a*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2*b^3*d)
```

Maple [B] time = 4.619, size = 868, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b^2/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))-2*a/b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/
(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^
2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
```

```
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/b^2
*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{7}{2}}}{(b\sec (dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

3.616 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=214

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\left(a^2-b^2\right)}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{bd\left(a^2-b^2\right)}$$

```
[Out] (a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2
 - b^2)*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/((a^2 - b^2)*d) + ((a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b*(a + b)^2*d) - (a^2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.401769, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3845, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{bd\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2
 - b^2)*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/((a^2 - b^2)*d) + ((a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b*(a + b)^2*d) - (a^2*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
```

```
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a^2}{2}-ab\sec(c+dx)-\frac{1}{2}\left(a^2-2b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a^3}{2}-\frac{1}{2}a^2b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2b\left(a^2-b^2\right)} + \frac{\left(a^2-3b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2\left(a^2-b^2\right)} + \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2b\left(a^2-b^2\right)} + \frac{\left(\left(a^2-3b^2\right)\sqrt{\right.}{}$$

$$= \frac{\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} - \frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx)}$$

$$= \frac{a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 6.65552, size = 587, normalized size = 2.74

$$\frac{\frac{2\left(3a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)} - \frac{2\sin(c+dx)\cos(2}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((a*Sin[c + d*x])/(b*(-a^2 + b^2)) + (a*Sin[c + d*x])/(
(a^2 - b^2)*(b + a*Cos[c + d*x]))))/d + ((-8*b*Cos[c + d*x]^2*EllipticPi[-(
b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c
+ d*x]^2]*Sin[c + d*x])/((b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(3
*a^2 - 4*b^2)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + E
llipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sq
rt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d
```

```
*x]^2)) - (2*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d
*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a),
-ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 -
Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*b*(a +
 b)*d)
```

Maple [B] time = 2.751, size = 608, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a/b/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a/b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/(a^2-a*b)*a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

3.617 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=208

$$-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad\left(a^2-b^2\right)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}\right.}{d\left(a^2-b^2\right)}$$

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a^2 -
 b^2)*d)) - (b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*(a^2 - b^2)*d) + ((a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*(a + b)^2*d) + (a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.357557, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3844, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right.}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a^2 -
 b^2)*d)) - (b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a*(a^2 - b^2)*d) + ((a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*(a + b)^2*d) + (a*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
```

```
&& IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{a}{2}-b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx}{-a^2+b^2}$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2(a^2-b^2)} + \frac{(a^2+b^2)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{2a(a^2-b^2)}$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2(a^2-b^2)} - \frac{b\int \sqrt{\sec(c+dx)}\,dx}{2a(a^2-b^2)} + \frac{((a^2+b^2)\sqrt{\cos(c}}{}$$

$$= \frac{(a^2+b^2)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} + \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} -$$

$$= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a^2-b^2)\,d} - \frac{b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a^2-b^2)\,d}$$

Mathematica [B] time = 6.65143, size = 633, normalized size = 3.04

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)^2\left(-\frac{2\sin(c+dx)\cos(2(c+dx))(a+b\sec(c+dx))\left(a(a-2b)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+a\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(-(Sin[c + d*x]/(-a^2 + b^2)) +
(b*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*
x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-8*b*Cos[c + d*x]^2*Ellip
ticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2
)) - (2*a*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ellip
ticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1
 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^
```

```
2)) - (2*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^
2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] -
2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]
]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b*(b + a*Cos[c + d*x])*(1 - Co
s[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))))/(4*(-a + b)*(a + b
)*d*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 3.605, size = 707, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/(a^2-a*b)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))-2*b/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*
a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

3.618 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c}{ad\left(a^2\right.}$$

```
[Out] (b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2
 - b^2)*d) + ((2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - (b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)
*(a + b)^2*d) - (b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*S
ec[c + d*x]))
```

Rubi [A] time = 0.369199, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3843, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}I}{ad\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2
 - b^2)*d) + ((2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - (b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)
*(a + b)^2*d) - (b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*S
ec[c + d*x]))
```


Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
```

```
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{b}{2}-a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx}{-a^2+b^2}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-\frac{ab}{2}-\left(a^2-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2\left(a^2-b^2\right)} - \frac{\left(b\left(3-\frac{b^2}{a^2}\right)\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a\left(a^2-b^2\right)} + \frac{\left(2a^2-b^2\right)\int \sqrt{\sec(c+dx)}\,dx}{2a^2\left(a^2-b^2\right)} - \frac{\left(b\left(3\right.\right.}{}$$

$$= -\frac{b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} - \frac{b\sqrt{\sec(c+dx)}\sin(c+d}{\left(a^2-b^2\right)d(a+b\sec(c+}$$

$$= \frac{b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} + \frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{a^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 4.38511, size = 255, normalized size = 1.12

$$-\frac{\cos(2(c+dx))\csc(c+dx)\sqrt{\sec(c+dx)}\left(-a(a-b)\sqrt{-\tan^2(c+dx)}\sqrt{\sec(c+dx)}(a\cos(c+dx)+b)\textbf{EllipticF}\left(\sin^{-1}\left(\sqrt{}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((Cos[2*(c + d*x)]*Csc[c + d*x]*Sqrt[Sec[c + d*x]]*(-(a*(a - b)*(b + a*Cos
[c + d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[-Tan[c + d*x]^2]) - (3*a^2 - b^2)*(b + a*Cos[c + d*x])*EllipticPi[-(b/a),
 -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2]
+ a*b*(b*Tan[c + d*x]^2 - (b + a*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/(a^2*(a - b)*(a +
```

```
b)*d*(b + a*Cos[c + d*x])*(-2 + Sec[c + d*x]^2)))
```

Maple [B] time = 3.836, size = 788, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*b/a
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2))+2/a^2*b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a
-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^
2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.619 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=244

$$-\frac{b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^3d\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(2a^2-3b^2\right)\sqrt{\cos(\ldots}}{}$$

```
[Out] ((2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^2*(a^2 - b^2)*d) - (b*(4*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + (b^2*(5*a^2 - 3
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.439661, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3847, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{b\left(4a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\ldots}}{a^2d\left(a^2\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```


```
[Out] ((2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^2*(a^2 - b^2)*d) - (b*(4*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d) + (b^2*(5*a^2 - 3
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
```

```
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+\frac{3b^2}{2}+ab\sec(c+dx)-\frac{1}{2}b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{a\left(-a^2+\frac{3b^2}{2}\right)-\left(-a^2b+b\left(-a^2+\frac{3b^2}{2}\right)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^3\left(a^2-b^2\right)} + \frac{(b}{}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2a^2-3b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a^2\left(a^2-b^2\right)} - \frac{\left(b\left(4a^2-3b^2\right)\right)}{2a^3(}$$

$$= \frac{b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a-b)(a+b)^2d} + \frac{b^2\sqrt{\sec(c+}}{a\left(a^2-b^2\right)d(}$$

$$= \frac{\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} - \frac{b\left(4a^2-3b^2\right)\sqrt{\cos(c}}{}$$

Mathematica [A] time = 6.40575, size = 323, normalized size = 1.32

$$\frac{2\cot(c+dx)\left(2a\left(2a^2+ab-3b^2\right)\sqrt{-\tan^2(c+dx)}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)-2a\left(2a^2-3b^2\right)\sqrt{-\tan^2(c+dx)}E\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+10a^2b\sqrt{-\tan^2(c+dx)}\Pi}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]
]) + (2*Cot[c + d*x]*(2*a^3*Sec[c + d*x]^(3/2) - 3*a*b^2*Sec[c + d*x]^(3/2)
 - 2*a^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 3*a*b^2*Cos[2*(c + d*x)]*Sec
[c + d*x]^(3/2) - 2*a*(2*a^2 - 3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*(2*a^2 + a*b - 3*b^2)*EllipticF[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 10*a^2*b*EllipticPi[-(b/a), -
```

```
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*b^3*EllipticPi[-(
b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^3*(a - b)
*(a + b)))/(4*d)
```

Maple [B] time = 4.456, size = 809, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-6/a^2*b^2/(a^2-a*b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
-2/a^3*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2
*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\sec(c + dx))^2 \sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx + c) + a)^2 \sqrt{\sec(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.620 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=304

$$\frac{\left(16a^2b^2+2a^4-15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d\left(a^2-b^2\right)}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx)}$$

```
[Out] -((b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a^3*(a^2 - b^2)*d)) + ((2*a^4 + 16*a^2*b^2 - 15*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d
) - (b^3*(7*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a - b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)
*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (b^2*Sin[c + d*x]
)/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.680384, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3847, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{\left(16a^2b^2+2a^4-15b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec}}{3a^4d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```


```
[Out] -((b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(a^3*(a^2 - b^2)*d)) + ((2*a^4 + 16*a^2*b^2 - 15*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d
) - (b^3*(7*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a - b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)
*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]) + (b^2*Sin[c + d*x]
)/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
```

```
- b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} - \frac{\int \frac{-a^2+\frac{5b^2}{2}+ab\sec(c+dx)-\frac{3}{2}b^2\sec^2(c+}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} +$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} +$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))} -$$

$$= -\frac{b^3\left(7a^2-5b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^4(a-b)(a+b)^2d} + \frac{\left(2a^2-5\right.}{3a^2\left(a^2-b\right.}$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^4+16a^2b^2-\right.}{}$$

Mathematica [A] time = 6.59456, size = 281, normalized size = 0.92

$$\frac{\frac{\cot(c+dx)\Big(a\big(-12a^2b+2a^3-5ab^2+15b^3\big)\sqrt{-\tan^2(c+dx)}\text{EllipticF}\big(\sin^{-1}\big(\sqrt{\sec(c+dx)}\big),-1\big)+3ab\big(4a^2-5b^2\big)\sqrt{-\tan^2(c+dx)}E\big(\sin^{-1}\big(\sqrt{\sec(c+dx)}\big)\big|-1\big)+3b\Big(a\big(5b^2-4a}{(a-b)(a+b)}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((a^2*(2*a^2*b - 5*b^3 + 2*a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2
- b^2)*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(3*a*b*(4*a
^2 - 5*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
 + a*(2*a^3 - 12*a^2*b - 5*a*b^2 + 15*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 3*b*(a*(-4*a^2 + 5*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x]^2 + b*(-7*a^2 + 5*b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*(a + b)))/(3*a^4*d)
```

Maple [B] time = 4.523, size = 1064, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^2*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4/a^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*(a^2+2*a*b+3*b^2)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8/a^3*b^3/(a^2-a*b)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
))+2/a^4*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(
```

```
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**2*sec(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.621 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=388

$$-\frac{a\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{a^2\left(5a^2-11b^2 \right.}{4b^2d\left(a^2-\right.}$$

```
[Out] -((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - (a*(5*a^2 - 11*b^2)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2
)^2*d) - (a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^3*(a + b)^3
*d) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^
3*(a^2 - b^2)^2*d) - (a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) - (a^2*(5*a^2 - 11*b^2)*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.961193, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3845, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{a^2\left(5a^2-11b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\left(-29a^2b^2+15a^4+8b^4\right)\sin(c+dx)\sqrt{s}}{4b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d) - (a*(5*a^2 - 11*b^2)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2
)^2*d) - (a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^3*(a + b)^3
*d) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^
3*(a^2 - b^2)^2*d) - (a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) - (a^2*(5*a^2 - 11*b^2)*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}\left(5a^2-4b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(5a^2-11b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\sqrt{\sec(c+dx)}}{}}{}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(5\right.}{4b}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(5\right.}{4b}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(5\right.}{4b}$$

$$= -\frac{a\left(15a^4-38a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d} + \frac{\left(15a^4-29a^2\right.}{}$$

$$= -\frac{\left(15a^4-29a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a\left(5a^2-11b^2\right)\sqrt{\cos(c}}{4}$$

Mathematica [A] time = 6.7834, size = 726, normalized size = 1.87

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(-29a^2b^2+15a^4+8b^4\right)\sin(c+dx)}{4b^3\left(b^2-a^2\right)^2}+\frac{a^2\sin(c+dx)}{2b\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}+\frac{11a^2b^2\sin(c+dx)-5a^4\sin(c+dx)}{4b^2\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d} - \frac{2\left(-95a^3b^2+45a^5+56ab^4\right)\sin(c+d}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -((-2*(40*a^4*b - 80*a^2*b^3 + 16*b^5)*Cos[c + d*x]^2*EllipticPi[-(b/a), -A
rcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2
]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(45*a^5
- 95*a^3*b^2 + 56*a*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c
```

```
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1
- Cos[c + d*x]^2)) - (2*(15*a^5 - 29*a^3*b^2 + 8*a*b^4)*Cos[2*(c + d*x)]*(a
 + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a
- 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*
Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c +
 d*x]]*(2 - Sec[c + d*x]^2)))/(16*(a - b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c
+ d*x]]*(((15*a^4 - 29*a^2*b^2 + 8*b^4)*Sin[c + d*x])/(4*b^3*(-a^2 + b^2)^2
) + (a^2*Sin[c + d*x])/(2*b*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (-5*a^4*
Sin[c + d*x] + 11*a^2*b^2*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2*(b + a*Cos[c
+ d*x]))))/d
```

Maple [B] time = 7.598, size = 2014, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a/b*(1/2*a^2
/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(
a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)
/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2)))+2*a^2/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a/b^2*(a^2/b/(a^2-b^2)*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a
^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2*a/(a-b),2^(1/2)))+2/b^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c
)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^3, x)
```

3.622 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=315

$$\frac{(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4bd\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3a^2\left(a^2-3b^2\right)\sin(c}{4b^2d\left(a^2-b^2\right)^2(a}$$

```
[Out] (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b*(a^2 - b^2)^2*d) + (3*(a^4 - 2
*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^2*(a + b)^3*d) - (a^2*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (3*a^2*(a^2
 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Se
c[c + d*x]))
```

Rubi [A] time = 0.703575, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3845, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3a^2\left(a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{(a^2-7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\right.}{4bd\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b*(a^2 - b^2)^2*d) + (3*(a^4 - 2
*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^2*(a + b)^3*d) - (a^2*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (3*a^2*(a^2
 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Se
c[c + d*x]))
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}(3a^2-4b^2)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^2}dx}{2b(a^2-b^2)}$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{3a^2(a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{3}{4}a^2(a^2-3}{}}{}$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{3a^2(a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{3}{4}a^3(a^2-3}{}}{}$$

$$= -\frac{a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b(a^2-b^2)d(a+b\sec(c+dx))^2} - \frac{3a^2(a^2-3b^2)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b^2(a^2-b^2)^2d(a+b\sec(c+dx))} + \frac{(a^2-7b^2)}{8b}$$

$$= \frac{3(a^4-2a^2b^2+5b^4)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} - \frac{a^2\sec^{\frac{3}{2}}(c+dx)}{2b(a^2-b^2)d(a+}$$

$$= \frac{3a(a^2-3b^2)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2(a^2-b^2)^2d} + \frac{(a^2-7b^2)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{4b(a^2-b^2}$$

Mathematica [B] time = 6.74898, size = 697, normalized size = 2.21

$$\frac{2(-19a^2b^2+9a^4+16b^4)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b(1-\cos^2(c+dx))(a\cos(c+dx)+b)} - \frac{2(3a^4-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(8*a^3*b - 32*a*b^3)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Se
c[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x
])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(9*a^4 - 19*a^2*b^2 +
 16*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ellipt
icPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1
- Sec[c + d*x]^2]*Sin[c + d*x]))/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2
)) - (2*(3*a^4 - 9*a^2*b^2)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b -
2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt
```

```
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*Elli
pticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
 Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*
Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2))
)/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((3*a*(-a^2 + 3*b^2)
*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2) - (a*Sin[c + d*x])/(2*(-a^2 + b^2)*(b
 + a*Cos[c + d*x])^2) + (a^3*Sin[c + d*x] - 7*a*b^2*Sin[c + d*x])/(4*b*(-a^
2 + b^2)^2*(b + a*Cos[c + d*x]))))/d
```

Maple [B] time = 3.678, size = 1203, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^2/b/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*a-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b^2)/b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*a/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4/(a-b)/(a+b)/(a^2-
b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a
```

```
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))-15/4/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^{\frac{7}{2}}}{(b \sec (dx + c) + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^3, x)
```

3.623 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=313

$$\frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4ad\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2-7b^2\right)\sin(c}{4bd\left(a^2-b^2\right)^2(a}$$

```
[Out] ((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(4*b*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^
2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*(a - b)^2*b*(a + b)^3*d) - (a^2*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2 - 7*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.682791, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3845, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\right.}{4ad\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(4*b*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^
2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*(a - b)^2*b*(a + b)^3*d) - (a^2*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2 - 7*b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
```

```
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-\frac{a^2}{2}-2ab\sec(c+dx)-\frac{1}{2}\left(a^2-4b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{1}{4}a^2\left(a^2+5b^2\right)+}{}}{}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{1}{4}a^3\left(a^2+5b^2\right)-}{}}{}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(3\left(a^2+b^2\right)\right)}{8a\,(}$$

$$= \frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b(a+b)^3d} - \frac{a^2\sqrt{\sec(c+dx)}\,\mathrm{si}}{2b\left(a^2-b^2\right)d(a+b\,\mathrm{s}}$$

$$= \frac{\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{4a\left(a^2-b^2\right)^2 d}$$

Mathematica [B] time = 6.72035, size = 733, normalized size = 2.34

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)^3\left(\frac{2\left(3a^3-9ab^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin\right.\right.}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(8*a^2*b + 16*b^3)*Cos[c + d*x]
^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])
*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c
+ d*x]^2)) + (2*(3*a^3 - 9*a*b^2)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a
+ b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)) - (2*(a^3 + 5*a*b^2)*Cos[2*(c + d*x)]*(a + b*Se
c[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec
[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*
```

```
EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sq
rt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c +
 d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*
(2 - Sec[c + d*x]^2))))/(16*(a - b)^2*b*(a + b)^2*d*(a + b*Sec[c + d*x])^3)
 + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*(-((a^2 + 5*b^2)*Sin[c + d*x]
)/(4*b*(-a^2 + b^2)^2) + (b*Sin[c + d*x])/(2*(-a^2 + b^2)*(b + a*Cos[c + d*
x])^2) + (3*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(4*(-a^2 + b^2)^2*(b + a
*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 5.928, size = 1760, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/a*(1/2*a^2
/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(
a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)
/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
```

```
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2)))+2/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^3, x)
```

3.624 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=306

$$-\frac{b\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^2d\left(a^2-b^2\right)^2}+\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a}$$

```
[Out] -((5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(4*a*(a^2 - b^2)^2*d) - (b*(7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4 + 10
*a^2*b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*(a + b)^3*d) + (a*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.645643, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3844, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{b\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{4a^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] -((5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(4*a*(a^2 - b^2)^2*d) - (b*(7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4 + 10
*a^2*b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*(a + b)^3*d) + (a*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
```

```
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
```

```
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \frac{-\frac{a}{2}-2b\sec(c+dx)+\frac{3}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{4}a\left(5a^2+b^2\right)+3a^2b\,\mathrm{s}\cdots}{\sqrt{\sec(c\cdots}}}{2\cdots}$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{4}a^2\left(5a^2+b^2\right)-\left(-3a^3\cdots\right.}{\sqrt{\cdots}}}{2a^3\cdots}$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} - \frac{\left(b\left(7a^2-b^2\right)\right)\int \sqrt{\cdots}}{8a^2\left(a^2-\cdots\right.}$$

$$= \frac{\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3d} + \frac{a\sqrt{\sec(c+dx)}\sin\cdots}{2\left(a^2-b^2\right)d(a+b\,\mathrm{se}\cdots}$$

$$= -\frac{\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2 d} - \frac{b\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\cdots\right.}{4a^2\left(a^2-b^2\right)^2\cdots}$$

Mathematica [B] time = 6.69966, size = 724, normalized size = 2.37

$$\frac{\sec^{\frac{7}{2}}(c+dx)(a\cos(c+dx)+b)^3\left(\frac{(5a^2+b^2)\sin(c+dx)}{4a(a^2-b^2)^2}+\frac{b^2\sin(c+dx)}{2a(a^2-b^2)(a\cos(c+dx)+b)^2}+\frac{b^3\sin(c+dx)-7a^2b\sin(c+dx)}{4a(a^2-b^2)^2(a\cos(c+dx)+b)}\right)}{d(a+b\sec(c+dx))^3}-\frac{\sec^3(c+dx)(a\cos}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-48*b*Cos[c + d*x]^2*EllipticPi[-
(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c
 + d*x]^2]*Sin[c + d*x])/((b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(
-a^2 - 5*b^2)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + E
llipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sq
rt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d
*x]^2)) - (2*(5*a^2 + b^2)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(2*a*b - 2
*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a^2*Ellip
ticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*C
os[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))
)/(16*(a - b)^2*(a + b)^2*d*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])
^3*Sec[c + d*x]^(7/2)*(((5*a^2 + b^2)*Sin[c + d*x])/(4*a*(a^2 - b^2)^2) + (
b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (-7*a^2*b*Sin[
c + d*x] + b^3*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*
(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 6.233, size = 1858, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*b^2*(1/2*
a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-
```

b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2
c)^2)^(1/2)(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b
)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8
a/(a^2-b^2)^2(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a
-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b
^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a
/(a-b),2^(1/2)))-2/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-4*b/a^2*(a^2/b/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-
a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

3.625 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=323

$$\frac{\left(-5a^2b^2+8a^4+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b}{2d\left(\right.}$$

```
[Out] (3*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((8*a^4 - 5*a^2*b^2 + 3*b^4)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*
d) - (3*b*(5*a^4 - 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3*d) - (
b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
 - (b*(7*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.667242, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3843, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\left(-5a^2b^2+8a^4+3b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c}{4a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (3*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((8*a^4 - 5*a^2*b^2 + 3*b^4)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*
d) - (3*b*(5*a^4 - 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3*d) - (
b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
 - (b*(7*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(
a + b*Sec[c + d*x]))
```


Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-\frac{b}{2}-2a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{3}{4}b\left(3a^2-b^2\right)+a\left(2\right.}{\sqrt{}}}{}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{3}{4}ab\left(3a^2-b^2\right)-\left(\frac{3}{4}\right.}{}}{}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(3b\left(3a^2-b^2\right)\right)}{8a^2\left(a^2\right.}$$

$$= -\frac{3b\left(5a^4-2a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3d} - \frac{b\sqrt{\sec(c+dx)}}{2\left(a^2-b^2\right)d(a+}$$

$$= \frac{3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-5a^2b^2+3b^4\right)\sqrt{\cos(c+d}}{4a^3\left(a^2-\right.}$$

Mathematica [B] time = 6.72921, size = 749, normalized size = 2.32

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)^3\left(\frac{2\left(-5a^2b-b^3\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin\right.\right.}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(16*a^3 + 8*a*b^2)*Cos[c + d*x]
^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])
*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c
+ d*x]^2)) + (2*(-5*a^2*b - b^3)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[
c + d*x]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a +
 b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d
*x])*(1 - Cos[c + d*x]^2)) - (2*(9*a^2*b - 3*b^3)*Cos[2*(c + d*x)]*(a + b*S
ec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Se
c[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)
```

```
*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c
+ d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]
*(2 - Sec[c + d*x]^2))))/(16*a*(a - b)^2*(a + b)^2*d*(a + b*Sec[c + d*x])^3
) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)*((3*b*(-3*a^2 + b^2)*Sin[c +
 d*x])/(4*a^2*(-a^2 + b^2)^2) - (b^3*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*(b +
a*Cos[c + d*x])^2) + (11*a^2*b^2*Sin[c + d*x] - 5*b^4*Sin[c + d*x])/(4*a^2*
(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^3)
```

Maple [B] time = 6.461, size = 1936, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/a^3
*b^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)
/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
```

```
2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a
+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(
a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2)))+6/a^2*b/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+6/a^3*b^2*(a^2/
b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b
/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

3.626 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=342

$$-\frac{3b\left(-11a^2b^2+8a^4+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}$$

```
[Out] ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - (3*b*(8*a^4 - 11*a^2*b^2 + 5
*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
^4*(a^2 - b^2)^2*d) + (b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a -
 b)^2*(a + b)^3*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.754374, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3847, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{3b\left(-11a^2b^2+8a^4+5b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^4d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - (3*b*(8*a^4 - 11*a^2*b^2 + 5
*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a
^4*(a^2 - b^2)^2*d) + (b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]
]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a -
 b)^2*(a + b)^3*d) + (b^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
```

```
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2a^2+\frac{5b^2}{2}+2ab\sec(c+dx)-\frac{3}{2}b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} +$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} +$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} -$$

$$= \frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4(a-b)^2(a+b)^3d} + \frac{}{2}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3\left(a^2-b^2\right)^2 d} - \frac{3b\left(8a^4-\right.}{}$$

Mathematica [B] time = 6.78187, size = 712, normalized size = 2.08

$$\frac{\frac{2\left(-7a^2b^2+8a^4+5b^4\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)}-\frac{2\left(-29a^2b\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((-2*(-32*a^3*b + 8*a*b^3)*Cos[c + d*x]^2*EllipticPi[-(b/a), -ArcSin[Sqrt[S
ec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*
x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a^4 - 7*a^2*b^2 +
 5*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ellipti
cPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 -
 Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)
) - (2*(8*a^4 - 29*a^2*b^2 + 15*b^4)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*
(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + a*(a - 2*b)*EllipticF[Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
 a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*
b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c +
 d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-(b^2*(-13
*a^2 + 7*b^2)*Sin[c + d*x])/(4*a^3*(-a^2 + b^2)^2) + (b^4*Sin[c + d*x])/(2*
a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (3*(-5*a^2*b^3*Sin[c + d*x] + 3*b
^5*Sin[c + d*x]))/(4*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/d
```

Maple [B] time = 7.382, size = 1957, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/a^4*b^4*(1/2*a^2/b/(a^2-b^2)*cos
```

(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos (1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+ 1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+ 1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^ (1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1 /2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a +7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+ 1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos (1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/ 2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1 /2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin (1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+ 1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))- 3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c) ^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)* (-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2* c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2) /b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1) ^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d *x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1 /2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-12 *b^2/a^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1) ^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-8/a^4*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/ 2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/ 2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2* c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic F(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/ 2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1 /2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1 /2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1 /2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(si n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x +1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a- b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

---

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b\sec\left(c + dx\right)\right)^3 \sqrt{\sec\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**3*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx + c\right) + a\right)^3 \sqrt{\sec\left(dx + c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.627 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=406

$$\frac{\left(128a^4b^2-223a^2b^4+8a^6+105b^6\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12a^5d\left(a^2-b^2\right)^2}+\frac{b^2\left(13a^2-7b^2\right)\sin(}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(}$$

```
[Out] -(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^6 + 128*a^4*b^2 -
223*a^2*b^4 + 105*b^6)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(12*a^5*(a^2 - b^2)^2*d) - (b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sin[c
 + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]) + (b^2*Sin[c + d*x])/(
2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2) + (b^2*(13*a^2
 - 7*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(a + b*Se
c[c + d*x]))
```

Rubi [A] time = 1.0206, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3847, 4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}+\frac{\left(-61a^2b^2+8a^4+3\right.}{12a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] -(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^6 + 128*a^4*b^2 -
223*a^2*b^4 + 105*b^6)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(12*a^5*(a^2 - b^2)^2*d) - (b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)
*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sin[c
 + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]) + (b^2*Sin[c + d*x])/(
2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2) + (b^2*(13*a^2
 - 7*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(a + b*Se
```

```
c[c + d*x]))
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2} - \frac{\int \frac{-2a^2+\frac{7b^2}{2}+2ab\sec(c+dx)-\frac{5}{2}b^2\sec^2}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2} + \frac{b^2\left(13a^2-7b^2\right)\sin(}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}(}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+d}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+d}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\sqrt{\sec(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+d}$$

$$= -\frac{b^3\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^5(a-b)^2(a+b)^3d} +$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^6\right.}{}$$

Mathematica [A] time = 6.87905, size = 736, normalized size = 1.81

$$\frac{2\left(73a^2b^3-56a^4b-35b^5\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a+b\sec(c+dx))\left(\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right),-1\right)+\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{b\left(1-\cos^2(c+dx)\right)(a\cos(c+dx)+b)} - \frac{2\left(195a^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((-2*(16*a^5 + 112*a^3*b^2 - 56*a*b^4)*Cos[c + d*x]^2*EllipticPi[-(b/a), -A
rcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2
]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-56*a^4
*b + 73*a^2*b^3 - 35*b^5)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x
```

```
]]], -1] + EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[
c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1
 - Cos[c + d*x]^2)) - (2*(-72*a^4*b + 195*a^2*b^3 - 105*b^5)*Cos[2*(c + d*x
)]*(a + b*Sec[c + d*x])*(2*a*b - 2*a*b*Sec[c + d*x]^2 + 2*a*b*EllipticE[Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2] + a^2*EllipticPi[-(b/a), -ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(b/a)
, -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]
^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Se
c[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[S
ec[c + d*x]]*((b^3*(-17*a^2 + 11*b^2)*Sin[c + d*x])/(4*a^4*(-a^2 + b^2)^2)
- (b^5*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (19*a^2*b
^4*Sin[c + d*x] - 13*b^6*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(b + a*Cos[c +
d*x])) + Sin[2*(c + d*x)]/(3*a^3)))/d
```

Maple [B] time = 7.682, size = 2216, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^3*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(2*a+3*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(a^2+3*a*b+6*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/a^5*b^5*(1/2*a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
```

```
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^
2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)
*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
)))+10/a^5*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+20/a^4*b^3/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.628 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{\sqrt{a+b\sec(c+d}}{d\sqrt{\sec(c+d}}$$

```
[Out] (b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (a*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(d*Sqrt[a + b*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]) + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.652906, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3855, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}+\frac{b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (a*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]]
)/(d*Sqrt[a + b*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d
*x]]) + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 3855

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*
Csc[e + f*x])^(n - 1))/(f*(2*n - 1)), x] + Dist[d^2/(2*n - 1), Int[((d*Csc[
e + f*x])^(n - 2)*Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e + f
```

```
*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\int \frac{-a+a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{2}b$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+d}\right.}{2\sqrt{a+b\sec(c}}$$

$$= \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c}}{d}$$

$$= \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\right.}{d\sqrt{a+b\sec(c+}}$$

Mathematica [C] time = 5.13251, size = 321, normalized size = 1.35

$$\frac{\sqrt{a+b\sec(c+dx)}\left(-\frac{2i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2b\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{a\cos(c+dx)+b}\right),\frac{b-a}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b}\right.\right.\right.\right.}{ab\sqrt{\frac{1}{a-b}}\sqrt{a\cos(c+dx)+b}}\right.}{4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*a*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - ((2*I)*Sqrt[-((a*(-1 + Cos[c
 + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b
*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]],
(-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b +
a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a -
b)^(-1)]*b*Sqrt[b + a*Cos[c + d*x]]) + 4*Tan[c + d*x]))/(4*d*Sqrt[Sec[c + d
*x]])
```

Maple [C] time = 0.353, size = 788, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/d/((a-b)/(a+b))^(1/2)*(cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-cos(d*x+c)^2*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b-2*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+cos(d*x+c)*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*b-2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((
a-b)/(a+b))^(1/2)*b+((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*
((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(dx + c) + a} \sec(dx + c)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.629 $\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.354135, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3854, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx = a\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\left(a\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\sqrt{\right.}{\sqrt{a+}}$$

$$= \frac{\left(a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+a}\right.}{\sqrt{a+b\sec}}$$

$$= \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+d\right.}{d\sqrt{a+b\sec(c}}$$

Mathematica [A] time = 2.27053, size = 96, normalized size = 0.7

$$\frac{2\sqrt{a+b\sec(c+dx)}\left(a\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+b\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{d(a+b)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2, (c + d*x)/2,
(2*a)/(a + b)])*Sqrt[a + b*Sec[c + d*x]])/((a + b)*d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Maple [C] time = 0.266, size = 283, normalized size = 2.1

$$2\,\frac{\cos(dx+c)(\sin(dx+c))^2\sqrt{(\cos(dx+c))^{-1}}\sqrt{(\cos(dx+c)+1)^{-1}}}{d(-1+\cos(dx+c))(b+a\cos(dx+c))}\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right)a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
```

```
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*cos(d*x+c)*sin(
d*x+c)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)/(-1+cos(d*x
+c))/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(c+dx)}\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a}\sqrt{\sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.630 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0971235, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3856, 2655, 2653}

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx &= \frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{b+a\cos(c+dx)}\,dx}{\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\\ &= \frac{\sqrt{a+b\sec(c+dx)}\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\\ &= \frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.101777, size = 67, normalized size = 1.

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.299, size = 925, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)*(-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a+cos(d*x+c)*sin(d*x+c)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*b+cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*sin(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)/(
a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/(b+
a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sec{(c + dx)}}}{\sqrt{\sec{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec{(dx + c)} + a}}{\sqrt{\sec{(dx + c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.631 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+}}{3ad\sqrt{\sec(c+}}$$

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.377392, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3857, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}\right.}{3ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
```

```
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\int \frac{b+a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{3a} + \frac{\left(a^2-b^2\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\left(a^2-b^2\right)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+}}}{3a\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}}{3a\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+}}$$

Mathematica [A] time = 0.575689, size = 156, normalized size = 0.81

$$\frac{2\sqrt{a+b\sec(c+dx)}\left(\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+a\sin(c+dx)(a\cos(c+dx)+b)+b(a+b)\sqrt{\frac{a}{}}\right.}{3ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*(b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*S
in[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.336, size = 1021, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)/a*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-cos(d*x+c)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a*b+cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+((a-b)/(
a+b))^(1/2)*cos(d*x+c)^3*a^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-a^2*((a-b)/(a+
b))^(1/2)*cos(d*x+c)-((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+((a-b)/(a+b))^(1/2)
*cos(d*x+c)*b^2-a*b*((a-b)/(a+b))^(1/2)-b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a
*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+b\sec\left(c+dx\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.632 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=244

$$-\frac{4b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (
2*(9*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (
2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.657504, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3857, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{4b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```


```
[Out] (-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (
2*(9*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (
2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*b*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*Sqrt[Sec[c + d*x]])
```


Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
```

```
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\int \frac{b+3a\sec(c+dx)+2b\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-9a^2+2b^2\right)-\frac{7}{2}ab\sec(\cdots}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(\cdots}}}{15a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{\left(2b\left(a^2-b^2\right)\right)\int \frac{\sqrt{\sec\cdots}}{\sqrt{a+b\,s\cdots}}}{15a^2}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{\left(2b\left(a^2-b^2\right)\sqrt{b+a\,c\cdots}\right.}{1\cdots}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad\sqrt{\sec(c+dx)}} - \frac{\left(2b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos\cdots}{a+\cdots}}\right.}{15a\cdots}$$

$$= -\frac{4b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9-\frac{2b^2}{a^2}\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\cdots}}{15d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec\cdots}}$$

Mathematica [A] time = 0.854819, size = 203, normalized size = 0.83

$$\frac{\sqrt{a+b\sec(c+dx)}\left(8b\left(b^2-a^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(3a^2\cos(2(c+dx))+3a^2+8\right.}{30a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*b*(-a^2
+ b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)] + 2*a*(3*a^2 + 2*b^2 + 8*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*Si
n[c + d*x]))/(30*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.324, size = 1736, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d/((a-b)/(a+b))^(1/2)/a^2*(3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+2*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)+4*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*a*b^2+5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+2*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a*b^2-9*a^2*b*((a-b)/(a+b))^(1/2)-a*b^2*((a-b)/(a+b))^(1/
2)-9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+6*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-2*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*b^3-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*b^3*((a-b
)/(a+b))^(1/2)+7*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
```

```
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*cos(d*x+c)*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^2-9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2+2*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c)*sin(d*
x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*b^3+7*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*
x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec (dx + c) + a}}{\sec (dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec (dx + c) + a}}{\sec (dx + c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.633 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=305

$$\frac{2\left(-17a^2b^2+25a^4-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c}}{105a^2d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^3*d*Sqrt[a + b*Se
c[c + d*x]]) + (2*b*(19*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d*
x]^(3/2)) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105
*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.846879, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3857, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2\left(-17a^2b^2+25a^4-8b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{a+b\sec(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^3*d*Sqrt[a + b*Se
c[c + d*x]]) + (2*b*(19*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d
*x]^(5/2)) + (2*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d*
x]^(3/2)) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105
*a^2*d*Sqrt[Sec[c + d*x]])
```


Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\int \frac{b+5a\sec(c+dx)+4b\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{1}{2}\left(-25a^2+4b^2\right)-\frac{23}{2}ab}{\sec^{\frac{3}{2}}(c+dx)}}{3}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2-4b^2\right)\sqrt{a+}}{105a^2d}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2-4b^2\right)\sqrt{a+}}{105a^2d}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2-4b^2\right)\sqrt{a+}}{105a^2d}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35ad\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2-4b^2\right)\sqrt{a+}}{105a^2d}$$

$$= \frac{2\left(25a^4-17a^2b^2-8b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^3d\sqrt{a+b\sec(c+dx)}} + \frac{2b\left(19a^2+8b^2\right)E\left(\right.}{105a^3d\sqrt{}}$$

Mathematica [A] time = 1.18824, size = 237, normalized size = 0.78

$$\frac{\sqrt{a+b\sec(c+dx)}\left(8\left(-17a^2b^2+25a^4-8b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(a\left(145a^2-4b^2\right)\right.\right.}{420a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*(8*b*(19*a^3 + 19*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(25*
a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)] + 2*a*(136*a^2*b - 16*b^3 + a*(145*a^2 - 4*b^2)*Cos
[c + d*x] + 36*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*Sin[c + d*
x]))/(420*a^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.396, size = 2050, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d/((a-b)/(a+b))^(1/2)/a^3*(-25*a^3*b*((a-b)/(a+b))^(1/2)-19*a^2*b^2*
((a-b)/(a+b))^(1/2)+4*a*b^3*((a-b)/(a+b))^(1/2)+25*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^4+19*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-19*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-19*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+25*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^4+8*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*b^4+15*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a^4-25*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-8*b^4*((a-b)/(
a+b))^(1/2)-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+18*cos(d*x+c)^4*((a-b)/(
a+b))^(1/2)*a^3*b-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+26*cos(d*x+c)^2*
((a-b)/(a+b))^(1/2)*a^3*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-19*cos(d
*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+20*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+1
9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*a^3*b-19*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
```

```
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2+8*cos(d*
x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*a*b^3-19*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+2*cos(d*x+c)*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b^2-8*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

3.634 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{7ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (7*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) - (5*a*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (5*a*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(4*d) + (b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.994374, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3866, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}+\frac{5a\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (7*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) - (5*a*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (5*a*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(4*d) + (b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```

Rule 3866

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[d/(m + n - 1), Int[(
a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (b^
2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || IntegersQ[
2*m, 2*n])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}\left(\frac{ab}{2}+\left(2a^2\right.\right.}{\sqrt{a}}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{2d}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{2d}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{2d}$$

$$= \frac{5a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{2d}$$

$$= \frac{\left(3a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \frac{5a\sqrt{\sec(c+dx)}}{}$$

$$= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Gamma}{4d\sqrt{a+}}$$

Mathematica [C] time = 6.17171, size = 411, normalized size = 1.37

$$(a+b\sec(c+dx))^{3/2}\left(\frac{4ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a\cos(c+dx)+b)^2}-\frac{5i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2b\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{a\,c\right.\right.\right.\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((4*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + ((a^2 + 8*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(b + a*Cos[c + d*x])^2 - ((5*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]
*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[
I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] +
 a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (
-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[
b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(a - b)^(-1)]*b*(b + a*Cos
[c + d*x])^(3/2)) + (2*(5*a + 2*b*Sec[c + d*x])*Tan[c + d*x])/(b + a*Cos[c
+ d*x])))/(8*d*Sec[c + d*x]^(3/2))
```

Maple [C] time = 0.294, size = 1744, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/((a-b)/(a+b))^(1/2)*(-5*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+5*cos(d*x+c)^3
*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a*b+6*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*cos(d*x+c
)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b
```

```
)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+2*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+2*cos(d*
x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a*b-4*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^2-5*cos(d*x+c)^2*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2+5*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+6*cos(d*x+c)^2*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*a^2+8*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+2*cos(d*x+c)^
2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*a^2+2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b-4*cos(d*x+c)^2*sin(d*x+c
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*b^2+5*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2+2*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^3*a*b-5*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2+5*((a-b)/(a+b))^(1/2)*cos
(d*x+c)^2*a*b+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^2-7*((a-b)/(a+b))^(1/2)*
cos(d*x+c)*a*b-2*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/
2)*(1/cos(d*x+c))^(3/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.635 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=249

$$\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b\sqrt{a+}}{d}$$

```
[Out] ((2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (3*a*b*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (b*EllipticE[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*Sqrt[Sec[c + d*x]]) + (b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/d
```

Rubi [A] time = 0.726712, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3866, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(2a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{b\sqrt{a+b\sec(c}}{d\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```



```
[Out] ((2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (3*a*b*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (b*EllipticE[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*Sqrt[Sec[c + d*x]]) + (b*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/d
```


Rule 3866

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[d/(m + n - 1), Int[(
```

```
a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (b^
2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || IntegersQ[
2*m, 2*n])
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \frac{b\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{-\frac{ab}{2}+a^2\sec(c+dx)+\frac{3}{2}a}{\sqrt{\sec(c+dx)}\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{b\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}(3ab)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx}}$$

$$= \frac{b\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} - \frac{1}{2}b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{b\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{\left(\left(2a^2+b^2\right)\sqrt{b+a\cos(c+a}\right.}{2\sqrt{a+}}$$

$$= \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{b\sqrt{\sec(c+dx)}\sqrt{a+b}}{}$$

$$= \frac{\left(2a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{d}$$

Mathematica [C] time = 7.7084, size = 394, normalized size = 1.58

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{8a^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a\cos(c+dx)+b)^2} - \frac{2i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2b\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{a\,\mathrm{c}}\right.\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((8*a^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + (10*a*b*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b +
 a*Cos[c + d*x])^2 - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b
*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)
/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*C
os[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*(b + a*Cos[c + d*
```

```
x])^(3/2)) + (4*b*Tan[c + d*x])/(b + a*Cos[c + d*x])))/(4*d*Sec[c + d*x]^(3
/2))
```

Maple [C] time = 0.342, size = 1207, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)*(2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-2*cos(d*x+c)^2*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a*b-cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+cos(d*x+c)^2*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+
6*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+2*cos(d*x+c)*sin(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2
-2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*a*b-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+cos(d*x+c)*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*b^2+6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+((a-b)/(a+b))^(1/2)
*cos(d*x+c)^2*a*b-((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+((a-b)/(a+b))^(1/2)*co
s(d*x+c)*b^2-b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(
1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.636 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=209

$$\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a}}{}$$

```
[Out] (2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]])
```

Rubi [A] time = 0.498619, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {3868, 3856, 2655, 2653, 3854, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2b^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2ab\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(}}{d\sqrt{\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]])
```


Rule 3868

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e +
 f*x]], x], x] + Dist[b/d, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + b\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

$$= (ab)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + b^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \frac{\left(a\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{b}}{\sqrt{b+a\cos(c+dx)}\sqrt{}}$$

$$= \frac{\left(ab\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}} + \frac{\left(b^2\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+}\right.}{\sqrt{a+b\sec(c+}}$$

$$= \frac{2aE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{\left(ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{}}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 2.46034, size = 129, normalized size = 0.62

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}(a+b\sec(c+dx))^{3/2}\left(b\left(a\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+b\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)+a(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right.}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a*(a + b)*EllipticE[(c + d*x)/2, (2*
a)/(a + b)] + b*(a*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + b*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)]))*(a + b*Sec[c + d*x])^(3/2))/(d*(b + a*Cos[c +
 d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [C] time = 0.302, size = 1367, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)*(-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x
+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*a*b-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-cos(d*x+c
)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b+2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2-a^2*(
(a-b)/(a+b))^(1/2)*cos(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-a*b*((a-b)
/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/(b+
a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.637 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c}{3d\sqrt{\sec(c+}}$$

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.405912, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3864, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{8b\sqrt{a+b\sec(c+dx)}E\left(\right.}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\,c}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
```

```
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\int \frac{-4ab-\left(a^2+3b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(4b)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{3}\left(-a^2+b^2\right)\int \frac{}{\sqrt{a}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\left(\left(-a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{b+a}}}{3\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\left(\left(-a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(}{a+}}}}{3\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\,\mathrm{se}}}{3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+}}$$

Mathematica [A] time = 0.653467, size = 156, normalized size = 0.83

$$\frac{(a+b\sec(c+dx))^{3/2}\left(2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)(a\cos(c+dx)+b)+8b(a+b\right.}{3d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(8*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c + d
*x])*Sin[c + d*x]))/(3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.311, size = 1219, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-4*cos(d*x+c)*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a*b+3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d
*x+c)+4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)-4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)^
3*a^2+5*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-a^2*((a-b)/(a+b))^(1/2)*cos(d*
x+c)-4*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+4*((a-b)/(a+b))^(1/2)*cos(d*x+c)*
b^2-a*b*((a-b)/(a+b))^(1/2)-4*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/co
s(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.638 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=240

$$\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{5ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

```
[Out] (2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(5*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(5*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*b*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.614831, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3864, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(5*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(5*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*b*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
```


Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
```

```
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{1}{5}\int \frac{-6ab-\left(3a^2+5b^2\right)\sec(c+dx)-2ab\sec^2(c+d}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{\frac{3}{2}a\left(3a^2+b^2\right)+6a^2}{\sqrt{\sec(c+dx)}\sqrt{a+}}}{15a}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{\left(b\left(a^2-b^2\right)\right)\int \frac{}{\sqrt{}}}{5a}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{\left(b\left(a^2-b^2\right)\sqrt{b+}\right.}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{\left(b\left(a^2-b^2\right)\sqrt{\frac{b+}{}}\right.}{}$$

$$= \frac{2b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5ad\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^2+b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{5ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{}}$$

Mathematica [A] time = 1.05033, size = 197, normalized size = 0.82

$$\frac{(a+b\sec(c+dx))^{3/2}\left(4b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(a^2\cos(2(c+dx))+a^2+6ab\right.}{10ad\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(4*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 4*b*(a^2 - b
^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
] + 2*a*(a^2 + 4*b^2 + 6*a*b*Cos[c + d*x] + a^2*Cos[2*(c + d*x)])*Sin[c + d
*x]))/(10*a*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.336, size = 1707, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/5/d/a/((a-b)/(a+b))^(1/2)*(cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-3*cos(d*
x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3+4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b^2+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3-3*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+cos(
d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a*b^2-cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
```

```
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3+3*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a^2*b-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+3*cos(d*x+c)^2*((a-b)/(a+b))^(1
/2)*a*b^2-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-cos(d*x+c)*((a-b)/(a+b))^(1/
2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-3*a^2*b*((a-b)/(a+b))^(1/2)-2*a
*b^2*((a-b)/(a+b))^(1/2)-b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2), x)
```

3.639 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{2\left(-31a^2b^2+25a^4+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec}}{105ad\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^2*d*Sqrt[a + b*Se
c[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c +
 d*x]^(5/2)) + (16*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d
*x]^(3/2)) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(10
5*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.871319, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3864, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105ad\sqrt{\sec(c+dx)}}+\frac{2\left(-31a^2b^2+25a^4+6b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^2*d*Sqrt[a + b*Se
c[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c +
 d*x]^(5/2)) + (16*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d
*x]^(3/2)) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(10
5*a*d*Sqrt[Sec[c + d*x]])
```


Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{1}{7}\int \frac{-8ab-\left(5a^2+7b^2\right)\sec(c+dx)-4ab\sec^2(c+dx}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\frac{1}{2}a\left(25a^2+3b^2\right)+2}{\sec^{\frac{3}{2}}(}}{}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2+3b^2\right)\sqrt{}}{105a}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2+3b^2\right)\sqrt{}}{105a}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2+3b^2\right)\sqrt{}}{105a}$$

$$= \frac{2a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{16b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(25a^2+3b^2\right)\sqrt{}}{105a}$$

$$= \frac{2\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^2d\sqrt{a+b\sec(c+dx)}} + \frac{4b\left(41-\frac{3b^2}{a^2}\right)E\left(\frac{1}{2}\right.}{105d\sqrt{\frac{b+}{}}}$$

Mathematica [A] time = 1.68817, size = 237, normalized size = 0.78

$$\frac{(a+b\sec(c+dx))^{3/2}\left(8\left(-31a^2b^2+25a^4+6b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(a\left(145a^2+108\right.\right.\right.}{420a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(16*b*(41*a^3 + 41*a^2*b - 3*a*b^2 - 3*b^3)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(
25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + 2*a*(178*a^2*b + 12*b^3 + a*(145*a^2 + 108*b^2
)*Cos[c + d*x] + 78*a^2*b*Cos[2*(c + d*x)] + 15*a^3*Cos[3*(c + d*x)])*Sin[c
```

```
 + d*x]))/(420*a^2*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] time = 0.389, size = 2050, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/105/d/a^2/((a-b)/(a+b))^(1/2)*(-25*a^3*b*((a-b)/(a+b))^(1/2)-82*a^2*b^2*
((a-b)/(a+b))^(1/2)-3*a*b^3*((a-b)/(a+b))^(1/2)+25*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^4+82*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-82*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-82*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+51*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+25*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+6*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^4-6*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*b^4+15*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*cos(d*x+c)^3*
((a-b)/(a+b))^(1/2)*a^4-25*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+6*b^4*((a-b)/
(a+b))^(1/2)+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+39*cos(d*x+c)^4*((a-b)/
(a+b))^(1/2)*a^3*b+27*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+68*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-82*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+55*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b
^2+82*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
```

```
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b-82*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2-6*co
s(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a*b^3-82*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+51*cos(d*x+c)*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b^2+6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(d*x+c)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2), x)
```

3.640 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=369

$$\frac{b\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}+\frac{\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24d}$$

```
[Out] (b*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sqrt[a + b*Sec[c + d*x]]) + (5
*a*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]) - ((3
3*a^2 + 16*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((33*a^
2 + 16*b^2)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d
) + (13*a*b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d
) + (b^2*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.34757, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3842, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24d}+\frac{b\left(59a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (b*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sqrt[a + b*Sec[c + d*x]]) + (5
*a*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]) - ((3
3*a^2 + 16*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((33*a^
2 + 16*b^2)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d
) + (13*a*b*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d
) + (b^2*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a\left(2a^2\right.\right.}{}$$

$$= \frac{13ab\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec}}{3d}$$

$$= \frac{\left(33a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d} + \frac{13ab\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{\left(33a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d} + \frac{13ab\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{\left(33a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d} + \frac{13ab\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{\left(33a^2+16b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{24d} + \frac{13ab\sec^{\frac{3}{2}}(c+d}{}$$

$$= \frac{5a\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{8d\sqrt{a+b\sec(c+dx)}} + \frac{\left(33a^2+16b\right.}{}$$

$$= \frac{b\left(59a^2+16b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{24d\sqrt{a+b\sec(c+dx)}} + \frac{5a\left(a^2+4b^2\right)}{}$$

Mathematica [C] time = 6.56889, size = 602, normalized size = 1.63

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{24}\sec(c+dx)\left(33a^2\sin(c+dx)+16b^2\sin(c+dx)\right)+\frac{13}{12}ab\tan(c+dx)\sec(c+dx)+\frac{1}{3}b^2\tan(c+\right.}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] -(a*(a + b*Sec[c + d*x])^(5/2)*((-104*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(3*a
^2 - 104*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(33*a^2 + 16*b^2)*Sqrt[(
a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c +
d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
 + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)
]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*Ar
cSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin
[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*C
os[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos
[c + d*x])^2))))/(96*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a
 + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]*(33*a^2*Sin[c + d*x] + 16*b^2*Sin[c
 + d*x]))/24 + (13*a*b*Sec[c + d*x]*Tan[c + d*x])/12 + (b^2*Sec[c + d*x]^2*
Tan[c + d*x])/3))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

Maple [C] time = 0.364, size = 2295, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/24/d/((a-b)/(a+b))^(1/2)*(33*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+33*cos
(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-34*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2
-44*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+33*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
```

cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-16*c
os(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b^2+120*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
a*b^2+26*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-44*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2
+33*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-16*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+120*
cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+26*cos(d*x+c)^4*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2*b+18*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-33*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-33*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+16*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^
3-8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3+26*cos(d*x+c)^4*((a-b)/(a+b))^(1/2
)*a^2*b+16*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2+18*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a*b^2-59*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-8*b^3*((a-b)/(a+
b))^(1/2)+16*cos(d*x+c)^4*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+30*cos(d*x+c)^4*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*a^3+18*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-33*cos(d*x+c)^3*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3+16*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+30*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^

```
(1/2))*a^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(b+a*c
os(d*x+c))/cos(d*x+c)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}}\sec (dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

3.641 $\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=314

$$\frac{a\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b\left(15a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (a*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(
15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) - (9*a
*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (9*a*b*Sqrt[Sec[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (b^2*Sec[c + d*x]^(3/2)
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.07094, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3842, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{a\left(8a^2+11b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b\left(15a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (a*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + (b*(
15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) - (9*a
*b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (9*a*b*Sqrt[Sec[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (b^2*Sec[c + d*x]^(3/2)
*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(4a^2 \cdots\right.\right.}{\cdots}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec \cdots}}{2d}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec \cdots}}{2d}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec \cdots}}{2d}$$

$$= \frac{9ab\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec \cdots}}{2d}$$

$$= \frac{b\left(15a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \frac{9ab\sqrt{\sec(c \cdots}}{\cdots}$$

$$= \frac{a\left(8a^2+11b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} + \frac{b\left(15a^2+4b^2\right) \cdots}{\cdots}$$

Mathematica [C] time = 6.50423, size = 560, normalized size = 1.78

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{9}{4}ab\tan(c+dx)+\frac{1}{2}b^2\tan(c+dx)\sec(c+dx)\right)}{d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^2}+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2\left(16a^3+4ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{Elli}}{\sqrt{a\cos(c+dx)+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(16*a^3 + 4*a*b^2)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]]
+ (2*(21*a^2*b + 8*b^3)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] - ((18*I)*a^2*Sqrt[(a -
 a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x
)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c
+ d*x])/(Sqrt[(a - b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c
+ d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c +
d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*
Sec[c + d*x])^(5/2)*((9*a*b*Tan[c + d*x])/4 + (b^2*Sec[c + d*x]*Tan[c + d*x
])/2))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

Maple [C] time = 0.275, size = 1982, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/4/d/((a-b)/(a+b))^(1/2)*(30*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*
b+8*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+8*cos(d*x+c)^3*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
```

ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)) *a^3-6*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1 /2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/ sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((- 1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-4* cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/ (cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x +c),(-(a+b)/(a-b))^(1/2))*b^3-9*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d *x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x +c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+9*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+ c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+ b)/(a-b))^(1/2))*a*b^2+30*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)) /(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))* ((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+8*c os(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x +c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+8*cos(d*x+c)^2*sin(d*x+c)*(1/(a+ b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3- 6*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d *x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*c os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos (d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-4*cos(d *x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-9*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))* ((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+9*cos(d*x+c)^2*s in(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a -b))^(1/2))*a*b^2+9*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2-9*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+9*cos(d*x +c)^2*((a-b)/(a+b))^(1/2)*a*b^2+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3-11*c os(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-2*b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d* x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)/si n(d*x+c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

3.642 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=263

$$\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2}{}$$

```
[Out] (b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (5*a*b^2*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*a^2 - b^2)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[Sec[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.78028, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3842, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{b\left(4a^2+b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (5*a*b^2*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*a^2 - b^2)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[Sec[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
```

```
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \frac{b^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \int \frac{\frac{1}{2}a\left(2a^2-b^2\right)+3a^2b\sec(c+dx)+\frac{5}{2}\cdots}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c\cdots}}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\left(5ab^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \cdots$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\left(2a^2-b^2\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \cdots$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d} + \frac{\left(b\left(4a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c\cdots}}{2\sqrt{a+b\sec(c+\cdots}}$$

$$= \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx\cdots}}{d}$$

$$= \frac{b\left(4a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c\cdots\right.}{d\sqrt{a+b\sec\cdots}}$$

Mathematica [C] time = 6.51705, size = 538, normalized size = 2.05

$$\frac{b^2\sin(c+dx)(a+b\sec(c+dx))^{5/2}}{d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2} + \frac{a(a+b\sec(c+dx))^{5/2}\left(\frac{2i\left(2a^2-b^2\right)\sin(c+dx)\cos(2(c+dx))\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a\cos(c+dx)+a}{a-b}}\left(a\left(2b\text{Ellipt}\cdots\right.\right.}{b\sqrt{\frac{1}{a-b}}\sqrt{1-\cdots}}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b^2*(a + b*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])^2*Sec
[c + d*x]^(3/2)) + (a*(a + b*Sec[c + d*x])^(5/2)*((24*a*b*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c +
 d*x]] + (2*(2*a^2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2
, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(2*a^2 - b
^2)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*C
os[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b
 + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a
 - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 -
```

```
 a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a
^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*
(b + a*Cos[c + d*x])^2))))/(4*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/
2))
```

Maple [C] time = 0.291, size = 1949, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)*(2*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3-2*cos(d*x+c)^2*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b-cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+cos(d*x+c)^2*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
b^3-2*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*cos
(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a*b^2+10*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b
^2+2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3-2*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-cos(d*x+c)
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*a*b^2+cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/
```

```
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3-2*cos(d*x+c)*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3+6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+10*cos(d
*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a
^3-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a^2*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-2*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1
/2)*b^3-b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos
(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.643 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=262

$$\frac{2a\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b^3\sqrt{\sec(c+ \cdots}{}$$

```
[Out] (2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^
3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.780261, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3841, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b^3\sqrt{\sec(c+dx)}\sqrt{\frac{a\cdots}{}}}{d\sqrt{a \cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b^
3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
```

```
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{7a^2b}{2} + \frac{1}{2}a\left(a^2+9b^2\right)\sec(c+dx) + \frac{3}{2}b^3\sec^2(c+}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{7a^2b}{2} + \frac{1}{2}a\left(a^2+9b^2\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx + b^3\int \frac{\mathrm{se}}{\sqrt{a+}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(7ab)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}\left(a\left(a^2+2b^2\right)\right)\int$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(a\left(a^2+2b^2\right)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{b+a}}}{3\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} +$$

$$= \frac{2a\left(a^2+2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{d\sqrt{a+b\sec(}}$$

<u>Mathematica</u> [C] time = 5.69436, size = 409, normalized size = 1.56

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2a\left(a^2+9b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a\cos(c+dx)+b)^3} + \frac{7i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2b\,\mathrm{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{}{a}}\right.\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*a*(a^2 + 9*b^2)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + (b*
(7*a^2 + 6*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^3 + ((7*I)*Sqrt[-((a*(-1 + Cos[c +
d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a
 + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a
 + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*C
os[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a
- b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sqrt[(a - b)^(-
```

```
1)]*(b + a*Cos[c + d*x])^(5/2)) + (2*a^2*Sin[c + d*x])/(b + a*Cos[c + d*x])
^2))/(3*d*Sec[c + d*x]^(5/2))
```

Maple [C] time = 0.262, size = 1661, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)*(cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-7*cos(d*x+c)*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b+9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*cos(d*x+c)*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3+
7*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-7*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2+6*cos(d*x+c
)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*b^3+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-7*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+7*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)-7*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
```

```
/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+8*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-7
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^
2-a^2*b*((a-b)/(a+b))^(1/2)-7*a*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/
cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x
+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.644 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=239

$$\frac{16b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (16*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9
*a^2 + 23*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (22*a*b*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.689852, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{16b\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+23b^2\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```


```
[Out] (16*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9
*a^2 + 23*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (22*a*b*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]])
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
```

```
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{11a^2b}{2}+\frac{3}{2}a\left(a^2+5b^2\right)\sec(c+dx)+\frac{1}{2}b\left(2a^2+5b\right.}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}a^2\left(9a^2+23b\right.}{\sqrt{\sec}}}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{1}{15}\left(-9a^2-23b\right.$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{\left(8b\left(a^2-b^2\right)\sqrt{\right.}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{\left(8b\left(a^2-b^2\right)\sqrt{\right.}{}$$

$$= \frac{16b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2+23b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{15d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{}}$$

Mathematica [A] time = 1.45886, size = 200, normalized size = 0.84

$$\frac{(a+b\sec(c+dx))^{5/2}\left(32b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(3a^2\cos(2(c+dx))+3a^2\right.}{30d\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(4*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 32*b*(
a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/
(a + b)] + 2*a*(3*a^2 + 22*b^2 + 28*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x
)])*Sin[c + d*x]))/(30*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.309, size = 1931, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/15/d/((a-b)/(a+b))^(1/2)*(3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-23*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+14*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+34*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a*b^2-5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-23*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*a*b^2-9*a^2*b*((a-b)/(a+b))^(1/2)-11*a*b^2*((a-b)/(a+b)
)^(1/2)-9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+15*cos(
d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*b^3+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-9*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a^3+23*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-9*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
```

```
d*x+c)+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-23*b^3*((a-b)/(a+b))^(1/2)+17*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*b-23*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*cos(d*x+c)*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b
+23*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2+15*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-23*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)+23*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-23*cos(d*x+c)*sin(d*x+c)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3+
17*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/
cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2 \sec(dx+c)^2 + 2\,ab\sec(dx+c) + a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2), x)
```

3.645 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=303

$$\frac{2\left(-2a^2b^2+5a^4-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{21ad\sqrt{a+b\sec(c+dx)}}+\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+a}}{21d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(21*a*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b*(29*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(21*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
+ d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^
(5/2)) + (6*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(3
/2)) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt
[Sec[c + d*x]])
```

Rubi [A] time = 0.935625, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(-2a^2b^2+5a^4-3b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{a+b\sec(c+dx)}}+\frac{2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(21*a*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b*(29*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(21*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
+ d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^
(5/2)) + (6*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(3
/2)) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt
[Sec[c + d*x]])
```


Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{15a^2b}{2} + \frac{1}{2}a\left(5a^2+21b^2\right)\sec(c+dx) + \frac{1}{2}b\left(4a^2\right.}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+d}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} - \frac{4\int \frac{-\frac{5}{4}a^2\left(5a^2+9b\right.}{}}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+9b^2\right)}{2}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+9b^2\right)}{2}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+9b^2\right)}{2}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+9b^2\right)}{2}$$

$$= \frac{2\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{21ad\sqrt{a+b\sec(c+dx)}} + \frac{2b\left(29a^2+3b^2\right)E\left(\right.}{21ad\sqrt{\frac{b}{}}}$$

Mathematica [A] time = 2.2453, size = 237, normalized size = 0.78

$$\frac{(a+b\sec(c+dx))^{5/2}\left(8\left(-2a^2b^2+5a^4-3b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(a\left(29a^2+72b^2\right)\right.\right.}{84ad\sec}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(8*b*(29*a^3 + 29*a^2*b + 3*a*b^2 + 3*b^3)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + 8*(5
*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*a)/(a + b)] + 2*a*(44*a^2*b + 36*b^3 + a*(29*a^2 + 72*b^2)*Cos[
c + d*x] + 24*a^2*b*Cos[2*(c + d*x)] + 3*a^3*Cos[3*(c + d*x)])*Sin[c + d*x]
```

```
))/(84*a*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.352, size = 2050, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -2/21/d/a/((a-b)/(a+b))^(1/2)*(-5*a^3*b*((a-b)/(a+b))^(1/2)-29*a^2*b^2*((a-
b)/(a+b))^(1/2)-9*a*b^3*((a-b)/(a+b))^(1/2)+5*cos(d*x+c)*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+2
9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-29*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-29*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+27*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+5*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^4+3*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^4+3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+2*cos(d*x+c)^3*((a-b)/(
a+b))^(1/2)*a^4-5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-3*b^4*((a-b)/(a+b))^(1
/2)-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+12*cos(d*x+c)^4*((a-b)/(a+b))^(1
/2)*a^3*b+18*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+22*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*a^3*b+12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-29*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^3*b+11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+29*co
s(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
```

```
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a^3*b-29*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*cos(d*x+c)
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*a*b^3-29*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+27*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*b^2-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2), x)
```

3.646 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{4b\left(-62a^2b^2+57a^4+5b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\,\mathrm{s}}}{315d\sec^{\frac{3}{2}}(c+dx)}$$

```
[Out] (4*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(315*a^2*d*Sqrt[a + b*
Sec[c + d*x]]) + (2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(9*d*Sec[c + d*x]^(7/2)) + (38*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*b*(163*a^2 + 5*b^2)*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.25392, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad\sqrt{\sec(c+dx)}}+\frac{4b\left(-62a^2b^2+57a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(315*a^2*d*Sqrt[a + b*
Sec[c + d*x]]) + (2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(9*d*Sec[c + d*x]^(7/2)) + (38*a*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(63*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*b*(163*a^2 + 5*b^2)*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sqrt[Sec[c + d*x]])
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{19a^2b}{2}+\frac{1}{2}a\left(7a^2+27b^2\right)\sec(c+dx)+\frac{3}{2}b\left(2a^2+\right.}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} - \frac{4\int \frac{-\frac{1}{4}a^2\left(49a^2+75\right.}{}}{}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2+75b^2\right.}{3}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2+75b^2\right.}{3}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2+75b^2\right.}{3}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2+75b^2\right.}{3}$$

$$= \frac{2a^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(49a^2+75b^2\right.}{3}$$

$$= \frac{4b\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{315a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(147a^4+279a^2\right.}{31}$$

Mathematica [A] time = 2.57798, size = 286, normalized size = 0.79

$$\frac{(a+b\sec(c+dx))^{5/2}\left(32b\left(-62a^2b^2+57a^4+5b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(4ab\left(619a^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(9/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(16*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^
2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c
```

```
+ d*x)/2, (2*a)/(a + b)] + 32*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(301*a^4
+ 1984*a^2*b^2 + 40*b^4 + 4*a*b*(619*a^2 + 160*b^2)*Cos[c + d*x] + 8*(42*a^
4 + 85*a^2*b^2)*Cos[2*(c + d*x)] + 260*a^3*b*Cos[3*(c + d*x)] + 35*a^4*Cos[
4*(c + d*x)])*Sin[c + d*x]))/(2520*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x
]^(5/2))
```

Maple [B] time = 0.465, size = 2788, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -2/315/d/a^2/((a-b)/(a+b))^(1/2)*(-147*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-147*a^4*b*((a-b)/
(a+b))^(1/2)-163*a^3*b^2*((a-b)/(a+b))^(1/2)-279*a^2*b^3*((a-b)/(a+b))^(1/2
)-5*a*b^4*((a-b)/(a+b))^(1/2)+147*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5+10*cos(d*x+c)
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*b^5-147*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+279*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
279*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)-10*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+261*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-279*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+155*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+10*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos
```

(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-147*cos(
d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a^4*b+279*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2-279*cos(d*x+c
)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*a^2*b^3-10*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^4+261*cos(d*x+c)*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^4*b-279*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+155*cos(d*x+c)*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^3+10*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+10*b^5*((a-b)/(a+b))^(1/2)+147
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+10*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*b^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-147*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+35*cos(d*x+c)^6*
((a-b)/(a+b))^(1/2)*a^5+14*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5+98*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a^5-147*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5-10*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b^5+170*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^2
+82*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+80*cos(d*x+c)^3*((a-b)/(a+b))^(1
/2)*a^2*b^3+272*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2-5*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a*b^4-65*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-279*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a^3*b^2+199*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+1
0*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4+130*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)
*a^4*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^5*(1/cos(d*x+c))^(9/
2)/sin(d*x+c)/(b+a*cos(d*x+c))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)
```

3.647 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=312

$$-\frac{a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4bd\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] -(a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (3*a*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d) + (Sec[c + d*x]^(3/2)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.888068, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3860, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(3a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4b^2d}+\frac{3a\sqrt{a+}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] -(a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (3*a*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d) + (Sec[c + d*x]^(3/2)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(2*b*d)
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(a+2b\sec(c+dx)-3a\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{4b}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd}$$

$$= -\frac{3a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd}$$

$$= \frac{\left(4+\frac{3a^2}{b^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+d}}{4b^2d}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}} + \frac{\left(4+\frac{3a^2}{b^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 5.83731, size = 397, normalized size = 1.27

$$\frac{\sqrt{\sec(c+dx)}\left(4ab^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\frac{3i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\sqrt{a\cos(c+dx)+b}\left(a\left(2b\text{EllipticF}\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*a*b^2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)] + b*(9*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)] + ((3*I)*Sqrt[-((a*(-1 + C
os[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos
[c + d*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)
]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Ellipt
icPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)])))/Sqrt[(a - b)^(-1)] + 2*b*(2*b - 3*a*Cos[c + d*x])*(b + a*Co
s[c + d*x])*Sec[c + d*x]*Tan[c + d*x]))/(8*b^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.321, size = 1755, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/((a-b)/(a+b))^(1/2)/b^2*(6*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-2*cos(d*x+c)
^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*a*b+4*cos(d*x+c)^3*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^2-3*cos(d*x+c)^3*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2+3*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-6*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))
^(1/2))*a^2-8*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+6*cos(d*x+c)^2*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
```

```
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a^2-2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b+4*cos(d*x+c)^2*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
b^2-3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-6*cos(d
*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*b^2+3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2-2*((a-b)/(a+b))^(1/2)*cos(
d*x+c)^3*a*b-3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2+3*((a-b)/(a+b))^(1/2)*c
os(d*x+c)^2*a*b-2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^2-((a-b)/(a+b))^(1/2)*
cos(d*x+c)*a*b+2*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/
2)*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{7}{2}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.648 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}-\frac{\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\sec(c+dx}}$$

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (a*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(b*d*Sqrt[a + b*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rubi [A] time = 0.61728, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3860, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}+\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{bd\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c}{a+}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (a*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/
(b*d*Sqrt[a + b*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d)
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
```

```
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\int \frac{-a-a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} - \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}dx}{2b} - \frac{a\int \frac{\sec^{\frac{3}{2}}(c+}{\sqrt{a+b\sec(c}}}{2b}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{1}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}dx - \frac{\int \frac{\sqrt{a+b\sec(c+dx}}{\sqrt{\sec(c+dx)}}}{2b}$$

$$= \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd} + \frac{\left(\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos}}}{2\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c}{bd}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c}}{bd\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 8.63388, size = 329, normalized size = 1.34

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{2i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\sqrt{a\cos(c+dx)+b}\left(a\left(2b\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{a\cos(c+dx)+b}\right),\frac{b-a}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\right.\right.\right.}{ab\sqrt{\frac{1}{a-b}}}\right.}{4ba}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)] - ((2*I)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b)
)]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c + d*
x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sq
rt[(a - b)^(-1)]*b) + 4*(b + a*Cos[c + d*x])*Tan[c + d*x]))/(4*b*d*Sqrt[a +
```

```
b*Sec[c + d*x]])
```

Maple [C] time = 0.313, size = 996, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)/b*(2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a-cos(d*x+c)^2*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a+cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-2*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*a+2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*b-2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+cos(d*x+c)^2*((a-b)/
(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1
/2)*b-((a-b)/(a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x
+c))^(5/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.649 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.181562, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3859, 2807, 2805}

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\left(\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$
$$= \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$
$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 0.110973, size = 68, normalized size = 1.

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] time = 0.252, size = 216, normalized size = 3.2

$$2\,\frac{\left((\cos(dx+c))^{-1}\right)^{3/2}(\cos(dx+c))^{2}}{d\,(b+a\cos(dx+c))\sqrt{(\cos(dx+c)+1)^{-1}}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+}{a-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)-2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2)))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(
3/2)*cos(d*x+c)^2/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(3/2)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.650 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.0953826, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3858, 2663, 2661}

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\left(\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 0.0670675, size = 67, normalized size = 1.

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.244, size = 171, normalized size = 2.6

$$2\,\frac{\cos(dx+c)\left(\sin(dx+c)\right)^2\sqrt{\left(\cos(dx+c)\right)^{-1}}\sqrt{\left(\cos(dx+c)+1\right)^{-1}}}{d\left(-1+\cos(dx+c)\right)\left(b+a\cos(dx+c)\right)}\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2)*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)/(-1+cos(d*x+c))/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec (dx + c)}}{\sqrt{b \sec (dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.651 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=142

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.24248, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{b\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}$$

$$= -\frac{\left(b\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}\int}{a\sqrt{b+a\cos(c+}}$$

$$= -\frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{}{a}}}{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{s}}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\,s}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c}}$$

Mathematica [A] time = 2.47358, size = 96, normalized size = 0.68

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-b\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*a)
/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])*Sqrt[Sec[c + d*x]])/(a
*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.28, size = 736, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)/a*(cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
```

```
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-cos(d*x+c)*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*b-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+cos(d*
x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)/(a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b\sec(dx+c)^2+a\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^2 + a*
sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sec{(c + dx)}}\sqrt{\sec{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec{(dx + c)} + a}\sqrt{\sec{(dx + c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.652 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{2\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+\cdots}{3\cdots}$$

```
[Out] (2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (4*
b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.361855, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3863, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2+2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx)\sqrt{a+\cdots}}{3ad\sqrt{\sec(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]]) - (4*
b*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
```

```
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\int \frac{2b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(2b)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{3a^2} + \frac{1}{3}\left(1+\frac{2b^2}{a^2}\right)\int \frac{}{\sqrt{a}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(1+\frac{2b^2}{a^2}\right)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{3\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(1+\frac{2b^2}{a^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int -}{3\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\left(1+\frac{2b^2}{a^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}} - \frac{4bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

Mathematica [A] time = 0.56573, size = 147, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(2\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)(a\cos(c+dx)+b)-4b(a+b)\sqrt{\frac{a\cos}{}}\right.}{3a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-4*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)] + 2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])
/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(b + a*Cos[c + d*x])*
Sin[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.33, size = 1024, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a^2/((a-b)/(a+b))^(1/2)*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+2*cos(d*x+c)*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a*b-2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*cos(d*x+c)*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+
((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-a^
2*((a-b)/(a+b))^(1/2)*cos(d*x+c)+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-2*((a
-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-a*b*((a-b)/(a+b))^(1/2)+2*b^2*((a-b)/(a+b))
^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)
^2/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b\sec\left(dx+c\right)^{3}+a\sec\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^3 + a*
sec(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{a+b\sec\left(c+dx\right)}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.653 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=249

$$-\frac{2b(7a^2+8b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]])
 + (2*(9*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
 + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (
8*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.5671, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3863, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b(7a^2+8b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2+8b^2)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{8b\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```



```
[Out] (-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]])
 + (2*(9*a^2 + 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
 + (2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (
8*b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```


Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
```

```
f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
```

```
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{\int \frac{4b-3a\sec(c+dx)-2b\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(9a^2}{\sqrt{\sec(}}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}} - \frac{\left(b\left(7a^2+\right.\right.}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}} - \frac{\left(b\left(7a^2+\right.\right.}{}$$

$$= \frac{2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{8b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d\sqrt{\sec(c+dx)}} - \frac{\left(b\left(7a^2+\right.\right.}{}$$

$$= -\frac{2b\left(7a^2+8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^3d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2+8b^2\right)E}{15a^3d}$$

Mathematica [A] time = 0.712849, size = 193, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(-4b\left(7a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(3a^2\cos(2(c+dx))+3a^2-2ab\right.\right.}{30a^3d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - 4*b*(7*a^2 + 8*b
^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
] + 2*a*(3*a^2 - 8*b^2 - 2*a*b*Cos[c + d*x] + 3*a^2*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(30*a^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.303, size = 1736, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/a^3/((a-b)/(a+b))^(1/2)*(3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-8*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+4*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*a*b^2+10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-8*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a*b^2-9*a^2*b*((a-b)/(a+b))^(1/2)+4*a*b^2*((a-b)/(a+b))^
(1/2)-9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3+6*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*a^3-9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+8*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*b^3-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*b^3*((
a-b)/(a+b))^(1/2)+2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
```

```
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-8*cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b^2-9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+8*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2-8*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)-9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*cos(d*x+c)*sin
(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*b^3+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos
(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b\sec(dx+c)^4+a\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^4 + a*
sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.654 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$\frac{\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+}}{b^2d\left(a^2-\right.}$$

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (3*a*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((3*a^2 - b^2)*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[S
ec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.997418, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3845, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+dx}}{b^2d\left(a^2-b^2\right)\sqrt{\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (3*a*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) - ((3*a^2 - b^2)*EllipticE[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[S
ec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{b^2\left(a^2-b^2\right)d}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{b^2\left(a^2-b^2\right)d}$$

$$= -\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} +$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{sec}}}{b^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 4.08892, size = 478, normalized size = 1.39

$$\sec^{\frac{3}{2}}(c+dx)\left(\frac{4\tan(c+dx)(a\cos(c+dx)+b)\left(\left(ab^2-3a^3\right)\cos(c+dx)-a^2b+b^3\right)}{b^4-a^2b^2}-\frac{a(a\cos(c+dx)+b)^{3/2}\left(\frac{2i\left(3a^2-b^2\right)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\left(2b\right.\right.}{}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-((a*(b + a*Cos[c + d*x])^(3/2)*((8*a*b*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[
c + d*x]] + (2*(9*a^2 - 7*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticP
i[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(3*a^2
- b^2)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))
/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[
Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Ellipti
cPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
b)/(a + b)])))/(a^2*Sqrt[(a - b)^(-1)]*b)))/((a - b)*b^2*(a + b))) + (4*(b
+ a*Cos[c + d*x])*(-(a^2*b) + b^3 + (-3*a^3 + a*b^2)*Cos[c + d*x])*Tan[c +
d*x])/(-(a^2*b^2) + b^4)))/(4*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.341, size = 1501, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)/(a+b)/b^2*(6*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+4*cos(d
*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a*b-6*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
```

```
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-6*co
s(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+c
os(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*b^2+6*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+4*cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*a*b-6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-6*cos(d*x+c)*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*a*b-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+cos(d*x+c)*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^2+3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2+((a-b)/(a+b))^(1/2)*cos(d*
x+c)^2*a*b-3*a^2*((a-b)/(a+b))^(1/2)*cos(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x
+c)*b^2-a*b*((a-b)/(a+b))^(1/2)-b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/
cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(7/2)/(b+a*cos(d*x+c))/sin(d*
x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.655 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+d\right.}{bd\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(a^2 - b^2)*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.52017, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3845, 4108, 3859, 2807, 2805, 21, 3856, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+d\right.}{bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(a^2 - b^2)*d*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Sec[c + d*x]
]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
```

```
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)-\frac{1}{2}\left(a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{b} - \frac{2\int \frac{-\frac{a^2}{2}-\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{a\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{b\left(a^2-b^2\right)} + \frac{\left(\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+\cdots}{b\sqrt{a+b\sec(c\cdots}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{b\sqrt{a+b\sec(c+dx)}} + \cdots$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{(a\cdots}{\cdots}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+\cdots}}{b\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+\cdots}}$$

Mathematica [C] time = 6.00002, size = 434, normalized size = 2.11

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{4a^2\sin(c+dx)(a\cos(c+dx)+b)}{b^2-a^2}+\frac{(a\cos(c+dx)+b)^{3/2}\left(\frac{4ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{\sqrt{a\cos(c+dx)+b}}+\frac{2i\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}\left(a\cdots\right.}{\cdots}\right.}{\cdots}\right)}{2bd(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(((b + a*Cos[c + d*x])^(3/2)*((4*a*b*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c +
d*x]] + (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*Sqrt[-((a*(
-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c +
d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*Arc
Sinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(Sq
rt[(a - b)^(-1)]*b)))/((a - b)*(a + b)) + (4*a^2*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/(-a^2 + b^2)))/(2*b*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] time = 0.281, size = 1144, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d/b/(a+b)/((a-b)/(a+b))^(1/2)*(-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*a-2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b+2*cos(
```

```
d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a+cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-2*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*a*sin(d*x+c)-2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-a*((a-b)/(a+b))^(1/2))*((b+a*cos(d*
x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/(b+a*cos(d*x+c))/
sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.656 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/((a^2 -
 b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.164182, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3844, 21, 3856, 2655, 2653}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/((a^2 -
 b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{b+a\cos(c+dx)}\,dx}{(a^2-b^2)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{a+b\sec(c+dx)}\int\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

$$= -\frac{2E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A] time = 0.294343, size = 103, normalized size = 0.82

$$-\frac{2\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left((a+b)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-a\sin(c+dx)\right)}{d(a-b)(a+b)(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((a + b)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - a*Sin[c + d*x]))/((a -
 b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.253, size = 501, normalized size = 4.

$$-2\,\frac{\left((\cos(dx+c))^{-1}\right)^{3/2}(\cos(dx+c))^2}{d(a+b)(b+a\cos(dx+c))\sin(dx+c)}\left(\cos(dx+c)\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)/(a+b)*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-cos(d*x+c)*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)-((a-b)/(a+b))
^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)
^2/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.657 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=200

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\,\mathrm{co}}{}}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.379514, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3843, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*d*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
```

& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{b}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} + \frac{b\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{(}{}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{a\sqrt{a+b\sec(c+dx)}} + \frac{(b\sqrt{}}{a}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2bE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} -$$

Mathematica [A] time = 0.581527, size = 156, normalized size = 0.78

$$\frac{2\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left(\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)-ab\sin(c+dx)+b(a+b)\sqrt{\frac{a\cos(c+d}{a+b}}\right.}{ad(a-b)(a+b)(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(b*(a + b)*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b^2)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] - a*b*Sin[c
 + d*x]))/(a*(a - b)*(a + b)*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.265, size = 510, normalized size = 2.6

$$2\frac{\cos(dx+c)\sqrt{(\cos(dx+c))^{-1}}}{da(a+b)(b+a\cos(dx+c))\sin(dx+c)}\left(-\cos(dx+c)\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d/a/(a+b)/((a-b)/(a+b))^(1/2)*(-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a-cos(d*x+c)*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*b-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-(
(a-b)/(a+b))^(1/2)*b)*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

3.658 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{4b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{a+b\sec(c}{a^2d\left(a^2-b^2\right)\sqrt{\sec(c}$$

```
[Out] (-4*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)
*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.434004, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3847, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}\right.}{a^2d\sqrt{a+b\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-4*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)
*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
```

```
-1] && IntegersQ[2*m, 2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+b^2+\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{(2b)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a^2} + \frac{\left(a^2-2b^2\right)\int \frac{\sqrt{a\text{-}}}{\sqrt{}}}{a^2\left(a^2-\right.}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2b\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{b}}}{a^2\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a}{}}}}{a^2\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-2b^2\right)E\left(\frac{1}{2}(c+dx\right.}{a^2\left(a^2-b^2\right)d\sqrt{\frac{b+a\,c}{}}}$$

Mathematica [A] time = 0.654781, size = 165, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left(b\left(ab\sin(c+dx)-2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)\right)+\left(a^2b+a^3-2ab^2-2b^3\right)\sqrt{\frac{a\cos}{}}\right.}{a^2d(a-b)(a+b)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + b*(-2*(a^2 - b^2)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*
b*Sin[c + d*x])))/(a^2*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.296, size = 999, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)/(a+b)/a^2*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+2*cos(d*x+c)
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*a*b-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*cos(d*x+c)*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2
+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+2*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-((
a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2-((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b+a^2
*((a-b)/(a+b))^(1/2)*cos(d*x+c)-2*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2+a*b*((
a-b)/(a+b))^(1/2)+2*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^2\sec\left(dx+c\right)^3+2\,ab\sec\left(dx+c\right)^2+a^2\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^3 +
2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{3}{2}}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.659 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2\left(a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2\right.}{}$$

```
[Out] (2*(a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a + b*Sec[c + d*x]]) - (2*
b*(5*a^2 - 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.671952, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3847, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{2\left(a^2+8b^2\right)\sqrt{\sec(c+dx)}\sqrt{}}{3a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (2*(a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a + b*Sec[c + d*x]]) - (2*
b*(5*a^2 - 8*b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+2b^2+\frac{1}{2}ab\sec(c+dx)-b^2\sec}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\sec(c+a}}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c}}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\sec(c+a}}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c}}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\sec(c+a}}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c}}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\sec(c+a}}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c}}$$

$$= \frac{2\left(a^2+8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(5a^2-8b^2\right)E}{3a^3\left(a^2-b\right.}$$

Mathematica [A] time = 0.877481, size = 203, normalized size = 0.7

$$\frac{2\sqrt{\sec(c+dx)}\left(\left(7a^2b^2+a^4-8b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+a\sin(c+dx)\left(a\left(a^2-b^2\right)\cos(c+dx)+b\right.\right.}{3a^3d(a-b)(a+b)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(b*(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^4 + 7*a^2*
b^2 - 8*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a
)/(a + b)] + a*(b*(a^2 - 4*b^2) + a*(a^2 - b^2)*Cos[c + d*x])*Sin[c + d*x])
)/(3*a^3*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.319, size = 1315, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/a^3/(a+b)/((a-b)/(a+b))^(1/2)*(cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*cos(d*x
+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b+8*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-5*cos(d*x+c)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*a^2*b+8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*
a^3+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+8*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)-5*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-8*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-a^2*b*((a-b)/(a+b))^(1/2)+4*a*b^2*((a-b)
/(a+b))^(1/2)+8*b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2
)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^2\sec\left(dx+c\right)^4+2\,ab\sec\left(dx+c\right)^3+a^2\sec\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^4 +
2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.660 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=360

$$-\frac{8b\left(a^2+4b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{5a^4d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+$$

```
[Out] (-8*b*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(5*a^4*d*Sqrt[a + b*Sec[c + d*x]]) +
(2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^
(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 6*b^2)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) - (2*b*(3*a^2 - 8*b
^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d*Sqrt[Sec[c
+ d*x]])
```

Rubi [A] time = 0.966353, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3847, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)}-\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{\ }}{5a^3d\left(a^2-b^2\right)\sqrt{\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (-8*b*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(5*a^4*d*Sqrt[a + b*Sec[c + d*x]]) +
(2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^
(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 6*b^2)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) - (2*b*(3*a^2 - 8*b
^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d*Sqrt[Sec[c
+ d*x]])
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{a^2}{2}+3b^2+\frac{1}{2}ab\sec(c+dx)-2b^2\sec^2}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\sec(c+dx}}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c-}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\sec(c+dx}}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c-}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\sec(c+dx}}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c-}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\sec(c+dx}}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c-}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\sec(c+dx}}{5a^2\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c-}$$

$$= -\frac{8b\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{5a^4d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^4+8a^2b^2\right.}{5a^4\left(a\right.}$$

Mathematica [A] time = 1.29537, size = 250, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(-16b\left(3a^2b^2+a^4-4b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+2a\sin(c+dx)\left(-4ab\left(a^2-b^2\right)\cos(c+a\right.\right.}{10a^4d(a-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*(3*a^5 + 3*a^4*b + 8*a^3*b^2 + 8*a^2*b^3 - 16*a*b^4
- 16*b^5)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(
a + b)] - 16*b*(a^4 + 3*a^2*b^2 - 4*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + 2*a*(a^4 - 7*a^2*b^2 + 16*b^4 - 4*
```

```
a*b*(a^2 - b^2)*Cos[c + d*x] + (a^4 - a^2*b^2)*Cos[2*(c + d*x)])*Sin[c + d*
x]))/(10*a^4*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.305, size = 1861, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d/a^4/(a+b)/((a-b)/(a+b))^(1/2)*(8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^
2*b^2-2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b-3*a^3*b*((a-b)/(a+b))^(1/2)-
8*a*b^3*((a-b)/(a+b))^(1/2)-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+8*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)-12*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-16*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-16
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-16*cos(d*x+c)*sin(d*x+c)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^4+16*cos(
d*x+c)*((a-b)/(a+b))^(1/2)*b^4-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+cos(d*x
+c)^4*((a-b)/(a+b))^(1/2)*a^4+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4-16*b^4
*((a-b)/(a+b))^(1/2)+cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b-2*cos(d*x+c)^3*
((a-b)/(a+b))^(1/2)*a^2*b^2+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+8*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3+2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-6
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+3*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^4+8*cos
(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
```

```
cos(d*x+c)+1))^(1/2)*a^2*b^2-4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-12*cos(d*x+c)*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b^2-16*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+3*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(
(b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*
x+c)/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\sec(dx+c)^5+2\,ab\sec(dx+c)^4+a^2\sec(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^5 +
2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.661 $\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=458

$$\frac{(5a^2-3b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3b^2d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(5a^2-9b^2)}{3b^2d(a^2-b}$$

```
[Out] ((5*a^2 - 3*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c +
d*x]]) - (5*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^3*d*Sqrt[a + b*Sec[c + d*x]]) - ((15
*a^4 - 26*a^2*b^2 + 3*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*b*(a^2 - b^
2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(5*a^2 - 9*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + ((15*a^4
 - 26*a^2*b^2 + 3*b^4)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(3*b^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 1.41321, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {3845, 4098, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2(5a^2-9b^2)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{(-26a^2b^2+15a^4+3b^4)\sin(c+dx)\sqrt{s}}{3b^3d(a^2-b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((5*a^2 - 3*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c +
d*x]]) - (5*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^3*d*Sqrt[a + b*Sec[c + d*x]]) - ((15
*a^4 - 26*a^2*b^2 + 3*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) - (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*b*(a^2 - b^
```

```
2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(5*a^2 - 9*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + ((15*a^4
 - 26*a^2*b^2 + 3*b^4)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(3*b^3*(a^2 - b^2)^2*d)
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
```

```
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}\left(5a^2-3b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \sqrt{\sec}}{}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(15a^4-\right.}{}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(15a^4-\right.}{}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(15a^4-\right.}{}$$

$$= -\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(5a^2-9b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(15a^4-\right.}{}$$

$$= -\frac{5a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^3d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}}$$

$$= \frac{\left(5a^2-3b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{5a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx\right.}{b^3d\sqrt{a+b\sec(c}}$$

Mathematica [C] time = 6.66365, size = 677, normalized size = 1.48

$$\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)^3\left(-\frac{2a^3\sin(c+dx)}{3b^2\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}-\frac{4\left(5a^3b^2\sin(c+dx)-3a^5\sin(c+dx)\right)}{3b^3\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}+\frac{\tan(c+dx)}{b^3}\right)}{d(a+b\sec(c+dx))^{5/2}} - \frac{a\sec^{\frac{5}{2}}(c+dx)(a\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(9/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] -(a*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*((2*(20*a^3*b - 36*a*b^3)
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
Sqrt[b + a*Cos[c + d*x]] + (2*(45*a^4 - 86*a^2*b^2 + 33*b^4)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*
Cos[c + d*x]] + ((2*I)*(15*a^4 - 26*a^2*b^2 + 3*b^4)*Sqrt[(a - a*Cos[c + d*
x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a +
 b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos
[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a -
b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt
[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a
^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))
/(12*(a - b)^2*b^3*(a + b)^2*d*(a + b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[c
+ d*x])^3*Sec[c + d*x]^(5/2)*((-2*a^3*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)*(b
+ a*Cos[c + d*x])^2) - (4*(-3*a^5*Sin[c + d*x] + 5*a^3*b^2*Sin[c + d*x]))/(
3*b^3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) + Tan[c + d*x]/b^3))/(d*(a + b*S
ec[c + d*x])^(5/2))
```

Maple [C] time = 0.384, size = 4591, normalized size = 10.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/3/d/b^3/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)*(15*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^3*a^5-3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^5-3*a^3*b^2*((a-b)/(a+b))^(1/2)-3*a^
2*b^3*((a-b)/(a+b))^(1/2)+3*a*b^4*((a-b)/(a+b))^(1/2)-30*cos(d*x+c)^3*sin(d
*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a^5+3*b^5*((a-b)/(a+b))^(1/2)-15*cos(d*x+c)^3*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^5+30*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-30*cos(d*x+c)^2*sin(d*x+c)*Ellip
ticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a^5-15*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
```

x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*cos(d*x+c)^2*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*b^5+30*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-15*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^5-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5-30*cos(d*x+c)^3*sin(d*x+c)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a^4*b+30*cos(d*x+c)^3*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2+30*cos(d*x
+c)^3*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3+26*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b
^2-3*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+20*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-36*
cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-18*cos(d*x+c)^3*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-60*c
os(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4*b+60*cos(d*x+c)^2*sin(d*x+c)*Ell
ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*a^2*b^3+30*cos(d*x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^4-15*co
s(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^4*b+26*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+26*cos(d
*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b^3-3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+

c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+50*cos(d*x+c
)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^4*b-16*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-15*cos(d*x+c)*s
in(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*a^4*b+26*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3+30*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^4*b+20*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-36*cos(d*x+c)*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^3-18*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-54*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3
-18*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-30*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*a^4*b-30*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2+30*cos(d*x+c)*sin(
d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*a^2*b^3-21*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*b^2+
15*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4*b-23*((a-b)/(a+b))^(1/2)*cos(d*x+c)
^2*a^2*b^3+5*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b-3*cos(d*x+c)^3*((a-b)/(
a+b))^(1/2)*a^2*b^3+29*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2-6*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a*b^4-20*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-5*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2+29*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b
^3+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4+30*cos(d*x+c)*sin(d*x+c)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a*b^4)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+
c))^(9/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)/(a+b*sec(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{9}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.662 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+\ldots}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+\ldots}}$$

```
[Out] (-2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2
)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^2*(a
^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a
^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a
^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.1031, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3845, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2a\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+\ldots\right.}{3bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2
)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^2*(a
^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a
^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])
^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a
```

```
^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]

Rule 2805

Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[c + d, 0]

Rule 4035

Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]

Rule 3856

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 2655

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a + b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && !GtQ[a + b, 0]

Rule 2653

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3858

Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{3}{2}\left(a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{\frac{1}{4}a^2(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}\cdots}{\sqrt{a+b\,\mathrm{s}\cdots}}}{\cdots}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(a\left(3a^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{\left(a\sqrt{b+\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \cdots$$

$$= -\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{s}\cdots}}{b^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] time = 5.29025, size = 487, normalized size = 1.32

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{4ab^2\left(a^2-3b^2\right)\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{5/2}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{(a-b)^2} + \frac{i\left(\frac{1}{a-b}\right)^{3/2}\left(3a^2-7b^2\right)\csc(c+dx)\sqrt{-\frac{a(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{a(\cos(c+dx)+1)}{a-b}}(a\cos(c+dx)+b)^{5/\cdots}}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*((4*a*b^2*(a^2 - 3*b^2)*((b + a*Cos[c + d*x])/(a + b))^
(5/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(a - b)^2 + (b*(9*a^4 - 19*a^2
*b^2 + 6*b^4)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*EllipticPi[2, (c + d*x)/
```

```
2, (2*a)/(a + b)])/(a - b)^2 + (I*((a - b)^(-1))^(3/2)*(3*a^2 - 7*b^2)*Sqrt
[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*(
b + a*Cos[c + d*x])^(5/2)*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sq
rt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Elli
pticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)])))/(a + b)^2 + (2*a^2*b^2*(b + a*Cos[c + d*x])*
Sin[c + d*x])/(-a^2 + b^2) + (2*a^2*b*(-3*a^2 + 7*b^2)*(b + a*Cos[c + d*x])
^2*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^3*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [C] time = 0.292, size = 3854, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)/b^2*(-6*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*a^2*b^2-4*a^3*b*((a-b)/(a+b))^(1/2)-a^2*b^2*((a-b)/(a+b))^(1/2)+7*a*b
^3*((a-b)/(a+b))^(1/2)+6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-3*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+7
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+3*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a^4-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-7*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a*b^3+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+3*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a^3*b+7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-3*El
```

lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)*sin(d*x+c)*a^4-9*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-3*cos(d*x+c)^
2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^3+7*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-6*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*a^3*b+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b
^3+6*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-6*cos(d*
x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-6*cos(d
*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4+6*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*b^4-6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*a^2*b^2-12*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+
b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+12*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/

```
2))*cos(d*x+c)*sin(d*x+c)*a*b^3+4*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-3*cos(d*x
+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*a^3*b+7*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2+7*cos(d*x+c)*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a*b^3+10*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-5*cos(d*x+c)*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^2-12*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)
*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.663 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*(a^2 - b^2)^
2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) +
(2*a*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.664937, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3845, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)\right.}{3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*(a^2 - b^2)^
2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*a^2*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) +
(2*a*(a^2 - 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```


Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
```

```
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a^2}{2}-\frac{3}{2}ab\sec(c+dx)-\frac{1}{2}\left(a^2-3b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{a^2b^2+\frac{1}{4}\cdots}{\sqrt{\sec(\cdots}}}{3a\cdots}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{(4b)\int \frac{\sqrt{a\cdots}}{\sqrt{\cdots}}}{3\left(a^{2}\cdots\right.}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{b+a\cos\cdots}\right.}{\cdots}$$

$$= -\frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\frac{b+a\cos\cdots}{a+\cdots}}\right.}{\cdots}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{8bE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}} \cdots$$

Mathematica [A] time = 1.02627, size = 169, normalized size = 0.61

$$-\frac{2\sec^{\frac{3}{2}}(c+dx)\left(-(a-b)(a+b)^2\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+a\sin(c+dx)\left(-a^2+4ab\cos(c+dx)+5b^{2}\cdots\right.\right.}{3d(a-b)^2(a+b)^2(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sec[c + d*x]^(3/2)*(-4*b*(a + b)^2*((b + a*Cos[c + d*x])/(a + b))^(3/2)
*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(a + b)^2*((b + a*Cos[c +
d*x])/(a + b))^(3/2)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(-a^2 + 5*b^
2 + 4*a*b*Cos[c + d*x])*Sin[c + d*x]))/(3*(a - b)^2*(a + b)^2*d*(a + b*Sec[
c + d*x])^(3/2))
```

Maple [B] time = 0.31, size = 1343, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(cos(d*x+c)^2*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-3*c
os(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*a*b+4*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+4
*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b+4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
-3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b^2*sin(d*x+c)+4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2
-3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b+4*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*
b-4*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-a^2*((a-b)/(a+b))^(1/2)-a*b*((a-b)/(
a+b))^(1/2)+4*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*
cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.664 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\sec}}{3d\left(a^2-b^2\right)(a+b\sec}$$

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(
3*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^(3/2)) + (4*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^
2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.613908, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3844, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{4\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2b\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{3ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(
3*a^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^(3/2)) + (4*(a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^
2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
```

```
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{a}{2}-\frac{3}{2}b\sec(c+dx)+a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{4\int \frac{\frac{1}{4}a\left(3a^2+b^2\right.}{\sqrt{\sec(c+dx)}}}{3a\left(a\right.}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{b\int \frac{\sqrt{\sec(c+d}}{\sqrt{a+b\sec(c-}}}{3a\left(a^2-b\right.}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{\left(b\sqrt{b+a\cos}\right.}{3a}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} - \frac{\left(b\sqrt{\frac{b+a\cos(c+}{a+b}}\right.}{3a\left(\right.}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\left(3a^2+b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b}}{3a\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec}}$$

Mathematica [A] time = 1.00252, size = 178, normalized size = 0.63

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{2\sin(c+dx)(a\cos(c+dx)+b)\left(a\left(3a^2+b^2\right)\cos(c+dx)+2b\left(a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2} - \frac{2(a+b)\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{5/2}\left(b(a-b)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(3a^2+b^2\right)E\left(\frac{1}{2}(\right.\right.}{a(a-b)^2}\right.}{3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*((-2*(a + b)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*((3*a
^2 + b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a - b)*b*EllipticF[(c +
d*x)/2, (2*a)/(a + b)]))/(a*(a - b)^2) + (2*(b + a*Cos[c + d*x])*(2*b*(a^2
+ b^2) + a*(3*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*d*(
a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.253, size = 1822, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)*(3*cos(d*x+c)^2*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
-cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-3*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3-cos(d*x+c)
^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^2+3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+2*cos(d*x+c)*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*b-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3-3*cos(
d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a^2*b-cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2-cos(d*x+c)*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*b^3+3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b
^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
```

```
*sin(d*x+c)+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-cos(d*x+c)^2*((a-b)/(a+b
))^(1/2)*a^2*b-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+3*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^3-2*a^2*b*((a-b)/(a+b))^(1/2)+a*b^2*((a-b)/(a+b))^(1/2)-b^3*((a-
b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*c
os(d*x+c)^2/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.665 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{2\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin}{3d\left(a^2-\right.}$$

```
[Out] (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]]) + (4*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.644761, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3843, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(3a^2-2b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Sec[c
+ d*x]]) + (4*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2
)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{b}{2}-\frac{3}{2}a\sec(c+dx)+b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{4\int \frac{\frac{1}{2}b\left(3a^2-\right.}{\sqrt{\sec}}}{}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-2b^2\right)}{3a}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\left(3a^2-2b^2\right.\right.}{}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(\left(3a^2-2b^2\right.\right.}{}$$

$$= \frac{2\left(3a^2-2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{4b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{b+a\cos(c}{a+b}}}$$

Mathematica [A] time = 1.18405, size = 196, normalized size = 0.65

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left(\left(-3a^2b+3a^3-2ab^2+2b^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(6a^2b-2b^3\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{(a-b)^2}+\frac{ab\sin(c+dx)}{}\right.}{3a^2d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((6*a^2*b - 2*b^3)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (3*a^3 -
3*a^2*b - 2*a*b^2 + 2*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^
2 + (a*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
a^2 - b^2)^2))/(3*a^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.277, size = 2070, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^2*(-cos(d*x+c)^2*((a-b)/(a+b))^(1
/2)*a^2*b^2-5*a^2*b^2*((a-b)/(a+b))^(1/2)+a*b^3*((a-b)/(a+b))^(1/2)-3*cos(d
*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^4-6*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*b^4-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-6*cos(d*x+c)^2*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
a^3*b+2*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^3+2*b^4*((a-b)/(a+b))^(1/2)+2*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a*b^3+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b
+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+2*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
^2-3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-6*cos(
d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
```

```
os(d*x+c)+1))^(1/2)*a^3*b-6*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2+2*cos(d*x+c)*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a*b^3+5*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+2*cos(d*x+c)*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2
)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

3.666 $\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2b\left(9a^2-8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2}{3ad\,(a}$$

```
[Out] (-2*b*(9*a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c +
d*x]])
```

Rubi [A] time = 0.734148, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3847, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}-\frac{2b\left(9a^2-8b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{3a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (-2*b*(9*a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c +
d*x]])
```


Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+2b^2+\frac{3}{2}ab\sec(c+dx)-b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2b\left(9a^2-8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^4-15a\right.}{3a^3}$$

Mathematica [A] time = 1.25898, size = 208, normalized size = 0.66

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left(b\left(9a^2b-9a^3+8ab^2-8b^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(-15a^2b^2+3a^4+8b^4\right)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{(a-b)^2}+\frac{ab}{}\right.}{3a^3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((3*a^4 - 15*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
+ b*(-9*a^3 + 9*a^2*b + 8*a*b^2 - 8*b^3)*EllipticF[(c + d*x)/2, (2*a)/(a +
b)]))/(a - b)^2 + (a*b^2*(8*a^2*b - 4*b^3 + a*(9*a^2 - 5*b^2)*Cos[c + d*x])
*Sin[c + d*x])/(a^2 - b^2)^2))/(3*a^3*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.32, size = 3103, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/a^3/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(3*((a-b)/(a+b))^(1/2)*cos(d*x
+c)^3*a^5+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*cos(d*x+c)*sin(d*x+c)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*b^5-3*a^3*b^2*((a-b)/(a+b))^(1/2)-11*a^2*b^3*((a-b)/(a+b))^(1/2)+4*a*b^4*(
(a-b)/(a+b))^(1/2)+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5-3*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^5-15*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)+8*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*b^5*((a-b)/(a+b))^(1/2)+3*cos(d*
x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^5-3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a^5-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5-15*cos(d*x+c)^2*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3*b^2+8*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
```

```
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-9*cos(d*x+c)^2*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^4*b+6*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+3*cos(d*x+c)*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a
^4*b-15*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3-12*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-3*
cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^3*b^2+14*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+8*cos(d*
x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a*b^4+8*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-3*((a-b)/(a+b))
^(1/2)*cos(d*x+c)^3*a^3*b^2+3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4*b-4*((a-
b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^3+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4
*b-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3+18*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a^3*b^2-12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4-6*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^4*b-12*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2+18*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^2*b^3+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4-15*cos
(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a^3*b^2+8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^4)*((b+a*cos(d*x
+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))^2/(1/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{4}+3\,ab^{2}\sec\left(dx+c\right)^{3}+3\,a^{2}b\sec\left(dx+c\right)^{2}+a^{3}\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^4 +
3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.667 $\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{2\left(16a^2b^2+a^4-16b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(}}$$

```
[Out] (2*(a^4 + 16*a^2*b^2 - 16*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) - (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (4*b^2*(5*a
^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.022, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3847, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{2\left(-13a^2b^2+a^4+\right.}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (2*(a^4 + 16*a^2*b^2 - 16*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) - (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (4*b^2*(5*a
^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+3b^2+\frac{3}{2}ab\sec(c+dx)-2b}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\mathrm{s}}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+d}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\mathrm{s}}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+d}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\mathrm{s}}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+d}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\mathrm{s}}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+d}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\mathrm{s}}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\sec(c+d}}$$

$$= \frac{2\left(a^4+16a^2b^2-16b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^4\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{8b\left(2a^4\right.}{}$$

Mathematica [A] time = 1.63982, size = 257, normalized size = 0.66

$$\frac{2\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left(\left(16a^3b^2-16a^2b^3-a^4b+a^5-16ab^4+16b^5\right)\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)-4\left(-7a^2b^3+2a^4b+4b^5\right)E\left(\frac{1}{2}(c+d\right.\right.}{(a-b)^2}\right.}{3a^4d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*(-4*(2*a^4*b - 7*a^2*b^3 + 4*b^5)*EllipticE[(c + d*x)/2, (2*a)/(a +
b)] + (a^5 - a^4*b + 16*a^3*b^2 - 16*a^2*b^3 - 16*a*b^4 + 16*b^5)*EllipticF
```

```
[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + (a*(a^6 - 25*a^2*b^4 + 16*b^6 +
4*a*b*(a^4 - 8*a^2*b^2 + 5*b^4)*Cos[c + d*x] + (a^3 - a*b^2)^2*Cos[2*(c + d
*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*a^4*d*(a + b*Sec[c + d*x])^(5/2)
)
```

Maple [B] time = 0.337, size = 3614, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/a^4/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(((a-b)/(a+b))^(1/2)*cos(d*x+c
)^4*a^5*b-((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^4*b^2-((a-b)/(a+b))^(1/2)*cos(
d*x+c)^4*a^3*b^3-a^4*b^2*((a-b)/(a+b))^(1/2)+7*a^3*b^3*((a-b)/(a+b))^(1/2)+
20*a^2*b^4*((a-b)/(a+b))^(1/2)-8*a*b^5*((a-b)/(a+b))^(1/2)-12*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)-16*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^6-((a-b)/(
a+b))^(1/2)*cos(d*x+c)^2*a^6+16*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^6+cos(d*x+
c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^6-16*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^6+cos(d*x+c)*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^6+28*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^4-16*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^5+1
0*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^5*b+25*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+4*cos(d*
x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
```

+b)/(a-b))^(1/2))*a^3*b^3-28*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-16*cos(d*x+c)*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^5-8*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5*b+28*cos(d*x+c)^2*sin(d*
x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a^3*b^3-16*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^5+9*cos(d*x+c)^2*sin(d*x+c)
(1/(a+b)(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^5*b+16*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2-12*cos(d*x+c)^2*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^3*b^3-16*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-8*cos(d*x+c)*sin(d*x+c)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5*
b-8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*a^4*b^2+28*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^3-6*((
a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^5*b-6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^4
b^2+6((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3*b^3+6*((a-b)/(a+b))^(1/2)*cos(d
*x+c)^3*a^2*b^4+7*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^5*b-6*((a-b)/(a+b))^(1
/2)*cos(d*x+c)^2*a^4*b^2-34*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*b^3+8*((a-
b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b^4+24*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*
b^5-2*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5*b+14*((a-b)/(a+b))^(1/2)*cos(d*x+c
)*a^4*b^2+22*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*b^3-34*((a-b)/(a+b))^(1/2)*
cos(d*x+c)*a^2*b^4-16*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^5-16*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^6*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)-8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^4*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+28*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c)

```
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^4*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+16*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^3*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-16*b^6*((a-b)/(a+b))^(1/2))*((b+a
*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)
/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\sec(dx+c)^5+3\,ab^2\sec(dx+c)^4+3\,a^2b\sec(dx+c)^3+a^3\sec(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^5 +
3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.668 $\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=474

$$-\frac{2b\left(116a^2b^2+17a^4-128b^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^5d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{8b^2\left(3a^2-2b^2\right)\sin(c+}{3a^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a-}}$$

```
[Out] (-2*b*(17*a^4 + 116*a^2*b^2 - 128*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2
)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*
b^6)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^
5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) +
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d
*x])^(3/2)) + (8*b^2*(3*a^2 - 2*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*S
ec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4 - 71*a^2*b^2 + 48*b
^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)^2*d*Sec[c +
d*x]^(3/2)) - (4*b*(7*a^4 - 49*a^2*b^2 + 32*b^4)*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(15*a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 1.35331, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3847, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{8b^2\left(3a^2-2b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(-71a^2b^2+3a^4\right.}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (-2*b*(17*a^4 + 116*a^2*b^2 - 128*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2
)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*
b^6)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^
5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) +
(2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d
*x])^(3/2)) + (8*b^2*(3*a^2 - 2*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*S
ec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4 - 71*a^2*b^2 + 48*b
```

```
^4)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)^2*d*Sec[c +
d*x]^(3/2)) - (4*b*(7*a^4 - 49*a^2*b^2 + 32*b^4)*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(15*a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a^2}{2}+4b^2+\frac{3}{2}ab\sec(c+dx)-3b\cdots}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+\cdots}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3a^2-2b^2\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2 d\sec^{\frac{3}{2}}(c+d\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3a^2-2b^2\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2 d\sec^{\frac{3}{2}}(c+d\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3a^2-2b^2\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2 d\sec^{\frac{3}{2}}(c+d\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3a^2-2b^2\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2 d\sec^{\frac{3}{2}}(c+d\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3a^2-2b^2\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2 d\sec^{\frac{3}{2}}(c+d\cdots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(3a^2-2b^2\right)\mathrm{s}\cdots}{3a^2\left(a^2-b^2\right)^2 d\sec^{\frac{3}{2}}(c+d\cdots}$$

$$= -\frac{2b\left(17a^4+116a^2b^2-128b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^5\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \cdots$$

Mathematica [A] time = 1.94773, size = 292, normalized size = 0.62

$$\frac{\sec^{\frac{5}{2}}(c+dx)(a\cos(c+dx)+b)\left(\frac{2\left(\frac{a\cos(c+dx)+b}{a+b}\right)^{3/2}\left(b\left(-116a^3b^2+116a^2b^3+17a^4b-17a^5+128ab^4-128b^5\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)+\left(55a^4b^2-212a^2b^4+\cdots\right.\right.}{(a-b)^2}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((2*((b + a*Cos[c + d*x])/(a + b))
^(3/2)*((9*a^6 + 55*a^4*b^2 - 212*a^2*b^4 + 128*b^6)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)] + b*(-17*a^5 + 17*a^4*b - 116*a^3*b^2 + 116*a^2*b^3 + 128*a
*b^4 - 128*b^5)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a - b)^2 + a*((10*
b^5*Sin[c + d*x])/(-a^2 + b^2) - (10*b^4*(-15*a^2 + 11*b^2)*(b + a*Cos[c +
d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*b*(b + a*Cos[c + d*x])^2*Sin[c + d*x
] + 3*a*(b + a*Cos[c + d*x])^2*Sin[2*(c + d*x)])))/(15*a^5*d*(a + b*Sec[c +
 d*x])^(5/2))
```

Maple [B] time = 0.451, size = 4586, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d/a^5/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(-9*cos(d*x+c)^2*sin(d*x+c)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*a^7+9*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^7-9*a^5*b^2*((a-b)/(a+b))^(1/2)+5*a^4*b^
3*((a-b)/(a+b))^(1/2)-50*a^3*b^4*((a-b)/(a+b))^(1/2)-148*a^2*b^5*((a-b)/(a+
b))^(1/2)+64*a*b^6*((a-b)/(a+b))^(1/2)+128*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^7*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*cos(d*x+c)^3*
((a-b)/(a+b))^(1/2)*a^6*b+42*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*b^2+42*co
s(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b^3-48*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*
a^3*b^4-48*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^5+17*cos(d*x+c)^2*((a-b)/
(a+b))^(1/2)*a^6*b-38*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*b^2+42*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a^4*b^3+254*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^
4-64*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^5-192*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*a*b^6-18*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^6*b+16*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a^5*b^2-94*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^3-164*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a^3*b^4+260*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^5+1
28*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^6+3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*
a^6*b-3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^5*b^2-3*cos(d*x+c)^5*((a-b)/(a+b
))^(1/2)*a^4*b^3-8*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^6*b-8*cos(d*x+c)^4*((
a-b)/(a+b))^(1/2)*a^5*b^2+8*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b^3+8*cos(
d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^4+128*b^7*((a-b)/(a+b))^(1/2)-9*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a^7-128*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^7+3*cos(d
```

```
*x+c)^5*((a-b)/(a+b))^(1/2)*a^7+6*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^7-212*
cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b^4+128*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^6-26*co
s(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a^6*b-89*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5*b^2-188*cos(d*x+
c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*a^4*b^3-20*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^4+224*cos(d*x+c)*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*a^2*b^5+128*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^6+9*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^6*b+55*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b^2+55*cos(d*x+c)*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
4*b^3-212*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^4-212*cos(d*x+c)*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
^5-9*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a^7+9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^7+128*cos(d*x+
c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^7-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^6*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-17*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b^2*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-72*Ellipti
```

```
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4
*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-116*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^3*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+96*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^5*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+128*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^6*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+55*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-212*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^5*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)+128*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^6-17*cos(d*x+c)^2*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^6*b-
72*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5*b^2-116*cos(d*x+c)^2*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4*b^3+
96*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^4+128*cos(d*x+c)^2*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^5+
55*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*c
os(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}{b^{3}\sec\left(dx+c\right)^{6}+3\,ab^{2}\sec\left(dx+c\right)^{5}+3\,a^{2}b\sec\left(dx+c\right)^{4}+a^{3}\sec\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^6 +
3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

3.669 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

```
[Out] (-3*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]]) + (Sqrt[5]*EllipticE[(c + d*x)/2, 4/
5]*Sqrt[2 + 3*Sec[c + d*x]])/(d*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
)
```

Rubi [A] time = 0.179907, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3862, 3856, 2653, 3858, 2661}

$$\frac{\sqrt{5}\sqrt{3\sec(c+dx)+2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (-3*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]]
)/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]]) + (Sqrt[5]*EllipticE[(c + d*x)/2, 4/
5]*Sqrt[2 + 3*Sec[c + d*x]])/(d*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
)
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{2+3\sec(c+dx)}}\,dx = \frac{1}{2}\int \frac{\sqrt{2+3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx$$

$$= -\frac{\left(3\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3+2\cos(c+dx)}}\,dx}{2\sqrt{2+3\sec(c+dx)}} + \frac{\sqrt{2+3\sec(c+dx)}\int}{2\sqrt{3+2\cos(c-}}$$

$$= -\frac{3\sqrt{3+2\cos(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}} + \frac{\sqrt{5}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{2+}}{d\sqrt{3+2\cos(c+dx)}\sqrt{s}}$$

Mathematica [A] time = 0.13928, size = 78, normalized size = 0.64

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c +
 d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Maple [C] time = 0.274, size = 405, normalized size = 3.3

$$-\frac{1}{10\,d\sin\left(dx+c\right)\left(3+2\,\cos\left(dx+c\right)\right)}\left(2\,i\sqrt{10}\sqrt{\frac{3+2\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\sqrt{\left(\cos\left(dx+c\right)+1\right)^{-1}}\sin\left(dx+c\right)\cos\left(dx+c\right)\sqrt{5}\mathrm{El}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/10/d*(2*I*10^(1/2)*((3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*5^(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5
^(1/2)/sin(d*x+c),5^(1/2))*2^(1/2)+I*10^(1/2)*((3+2*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)*5^(1/2)*EllipticE
(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*2^(1/2)+2*I*5^(1/2)*Elli
pticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/2)*((3+2*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+I
*5^(1/2)*EllipticE(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2))*10^(1/
2)*((3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)+20*cos(d*x+c)^2+10*cos(d*x+c)-30)*((3+2*cos(d*x+c))/cos(d*x+c))
^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(3+2*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,\sec\left(dx+c\right)+2}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)^2+2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c)^2 + 2*
sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\sec(c+dx)+2}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*sec(c + d*x) + 2)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

3.670 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, -4]*Sqrt[-2 + 3*Sec
[c + d*x]])/(d*Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.175671, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3862, 3856, 2653, 3858, 2661}

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}-\frac{\sqrt{3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

```
[Out] (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]]) - (EllipticE[(c + d*x)/2, -4]*Sqrt[-2 + 3*Sec
[c + d*x]])/(d*Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-2+3\sec(c+dx)}}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{-2+3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right)+\frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx$$

$$= \frac{\left(3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}\,dx}{2\sqrt{-2+3\sec(c+dx)}} - \frac{\sqrt{-2+3\sec(c+dx)}\ \cdots}{2\sqrt{3-2\cos(c\ \cdots}}$$

$$= \frac{3\sqrt{3-2\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}} - \frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{-2\cdots}}{d\sqrt{3-2\cos(c+dx)\cdots}}$$

Mathematica [A] time = 0.117922, size = 68, normalized size = 0.62

$$-\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\right)}{d\sqrt{3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-2 + 3*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[-2 + 3*Sec[c + d*x]]))
```

Maple [B] time = 0.282, size = 374, normalized size = 3.4

$$\frac{1}{2 d \sin(dx+c)(-3+2\cos(dx+c))}\left(-2 i \cos(dx+c)\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\sqrt{5}\right)\sqrt{2}\sqrt{(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(-2*I*cos(d*x+c)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^
(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)-I*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),5^(
1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)-2*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*
EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*cos(d*x+c)^2
-10*cos(d*x+c)+6)*(-(-3+2*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2
)/sin(d*x+c)/(-3+2*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)^2-2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c)^2 - 2*
sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\sec(c+dx)-2}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

3.671 $\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] (EllipticE[(c + d*x)/2, -4]*Sqrt[2 - 3*Sec[c + d*x]])/(d*Sqrt[3 - 2*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x
)/2, -4]*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Rubi [A] time = 0.198349, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}+\frac{\sqrt{2-3\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (EllipticE[(c + d*x)/2, -4]*Sqrt[2 - 3*Sec[c + d*x]])/(d*Sqrt[3 - 2*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (3*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x
)/2, -4]*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]])
```


Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \frac{1}{2}\int \frac{\sqrt{2-3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{-3+2\cos(c+dx)}\,dx}{2\sqrt{-3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(3\sqrt{-3+2\cos(c+dx)}\sqrt{\sec(c\right.}{2\sqrt{2-3\sec}}$$

$$= \frac{\sqrt{2-3\sec(c+dx)}\int \sqrt{3-2\cos(c+dx)}\,dx}{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(3\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+\right.}{2\sqrt{2-3\sec(c}}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{2-3\sec(c+dx)}}{d\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{3\sqrt{3-2\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right.}{d\sqrt{2-3\sec(c+dx)}}$$

Mathematica [A] time = 0.0779803, size = 68, normalized size = 0.63

$$-\frac{\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\left(E\left(\frac{1}{2}(c+dx)\middle|-4\right)-3\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)\right)}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[3 - 2*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, -4] - 3*EllipticF[(c +
d*x)/2, -4])*Sqrt[Sec[c + d*x]])/(d*Sqrt[2 - 3*Sec[c + d*x]]))
```

Maple [B] time = 0.28, size = 405, normalized size = 3.8

$$-\frac{1}{10\,d\sin(dx+c)(-3+2\cos(dx+c))}\left(2\,i\cos(dx+c)\sin(dx+c)\sqrt{5}\text{EllipticF}\left(\frac{i\sqrt{5}(-1+\cos(dx+c))}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)\sqrt{-2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/10/d*(2*I*cos(d*x+c)*sin(d*x+c)*5^(1/2)*EllipticF(I*5^(1/2)*(-1+cos(d*x+
c))/sin(d*x+c),1/5*5^(1/2))*(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*2^(1/2)-5*I*cos(d*x+c)*sin(d*x+c)*5^(1/2)*EllipticE(I
*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))*(-2*(-3+2*cos(d*x+c))/(cos
```

```
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*2^(1/2)+2*I*5^(1/2)*EllipticF(I*
5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))*(-2*(-3+2*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5*I*5^(1/2)*El
lipticE(I*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))*(-2*(-3+2*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+20*c
os(d*x+c)^2-50*cos(d*x+c)+30)*((-3+2*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d
*x+c))^(1/2)/sin(d*x+c)/(-3+2*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)^2-2\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c)^2 -
2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3 \sec(dx+c)+2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))), x)
```

3.672 $\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

```
[Out] -((Sqrt[5]*EllipticE[(c + d*x)/2, 4/5]*Sqrt[-2 - 3*Sec[c + d*x]])/(d*Sqrt[3
 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) - (3*Sqrt[3 + 2*Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d
*x]])
```

Rubi [A] time = 0.204202, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{3\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}-\frac{\sqrt{5}\sqrt{-3\sec(c+dx)-2}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{d\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] -((Sqrt[5]*EllipticE[(c + d*x)/2, 4/5]*Sqrt[-2 - 3*Sec[c + d*x]])/(d*Sqrt[3
 + 2*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])) - (3*Sqrt[3 + 2*Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d
*x]])
```


Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\left(\frac{1}{2}\int \frac{\sqrt{-2-3\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right) - \frac{3}{2}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx$$

$$= -\frac{\sqrt{-2-3\sec(c+dx)}\int \sqrt{-3-2\cos(c+dx)}\,dx}{2\sqrt{-3-2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(3\sqrt{-3-2\cos(c+dx)}\sqrt{\mathrm{sec}\right.}{2\sqrt{-2-3}}$$

$$= -\frac{\left(\sqrt{5}\sqrt{-2-3\sec(c+dx)}\right)\int \sqrt{\frac{3}{5}+\frac{2}{5}\cos(c+dx)}\,dx}{2\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(3\sqrt{3+2\cos(c+dx)}\right.}{2\sqrt{5}\sqrt{-}}$$

$$= -\frac{\sqrt{5}E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{-2-3\sec(c+dx)}}{d\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{3\sqrt{3+2\cos(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right.}{\sqrt{5}d\sqrt{-2-3\sec(c+}}$$

Mathematica [A] time = 0.0886176, size = 78, normalized size = 0.63

$$\frac{\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)-3\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[3 + 2*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 4/5] - 3*EllipticF[(c +
 d*x)/2, 4/5])*Sqrt[Sec[c + d*x]])/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Maple [C] time = 0.284, size = 390, normalized size = 3.2

$$-\frac{1}{10\,d\sin(dx+c)(3+2\cos(dx+c))}\left(2\,i\sin(dx+c)\cos(dx+c)\,\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)\sqrt{(\cos(dx+c)-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/10/d*(2*I*sin(d*x+c)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1
/5*5^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*2^(1/2)-5*I*sin(d*x+c)*cos(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/si
```

```
n(d*x+c),1/5*5^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*2^(1/2)+2*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5
*5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)-5*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*
5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)-20*cos(d*x+c)^2-10*cos(d*x+c)+30)*(-(3+2*cos(d*x
+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(3+2*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3 \sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3 \sec(dx+c)-2}\sqrt{\sec(dx+c)}}{3 \sec(dx+c)^2+2 \sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c)^2 +
2*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\sec(c+dx)-2}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*sec(c + d*x) - 2)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))), x)
```

3.673 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=127

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{4\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

```
[Out] (-4*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]]) + (2*Sqrt[5]*EllipticE[(c + d*x)/2
, 6/5]*Sqrt[3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])
```

Rubi [A] time = 0.171297, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3862, 3856, 2653, 3858, 2661}

$$\frac{2\sqrt{5}\sqrt{2\sec(c+dx)+3}E\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3d\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}}-\frac{4\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```


```
[Out] (-4*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]]
)/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]]) + (2*Sqrt[5]*EllipticE[(c + d*x)/2
, 6/5]*Sqrt[3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]])
```


Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{3+2\sec(c+dx)}}\,dx &= \frac{1}{3}\int \frac{\sqrt{3+2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx - \frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx\\ &= -\frac{\left(2\sqrt{2+3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2+3\cos(c+dx)}}\,dx}{3\sqrt{3+2\sec(c+dx)}} + \frac{\sqrt{3+2\sec(c+dx)}\int}{3\sqrt{2+3\cos(c-}}\\ &= -\frac{4\sqrt{2+3\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{3+2\sec(c+dx)}} + \frac{2\sqrt{5}E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)\sqrt{3+}}{3d\sqrt{2+3\cos(c+dx)}\sqrt{}}\end{aligned}$$

Mathematica [A] time = 0.139241, size = 81, normalized size = 0.64

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)-2\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(c
 + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Maple [C] time = 0.461, size = 409, normalized size = 3.2

$$\frac{1}{15\,d\sin(dx+c)(2+3\,\cos(dx+c))}\left(3\,\sqrt{5}\sin(dx+c)\cos(dx+c)\,\mathrm{EllipticF}\left(1/5\,\frac{(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},i\sqrt{5}\right)\sqrt{2}\sqrt{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/15/d*(3*5^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF(1/5*(-1+cos(d*x+c))*5^(1/
2)/sin(d*x+c),I*5^(1/2))*2^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-5^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE(1/5
*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*2^(1/2)*10^(1/2)*((2+3*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*5^(1/2)*EllipticF(1
/5*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*10^(1/2)*((2+3*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5^(1/2)*
EllipticE(1/5*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*10^(1/2)*((2+3*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-30*cos(d*x+c)^2+10*cos(d*x+c)+20)*((2+3*cos(d*x+c))/cos(d*x+c))^(1/2)/(1
/cos(d*x+c))^(1/2)/sin(d*x+c)/(2+3*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}{2\sec(dx+c)^2+3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c)^2 + 3*
sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\sec(c+dx)+3}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*sec(c + d*x) + 3)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\sec(dx+c)+3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

3.674 $\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{4\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)}{3d\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

```
[Out] (2*EllipticE[(c + d*x)/2, 6]*Sqrt[3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2 + 3*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rubi [A] time = 0.176649, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3862, 3856, 2653, 3858, 2661}

$$\frac{4\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3-2\sec(c+dx)}}+\frac{2\sqrt{3-2\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|6\right)}{3d\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 6]*Sqrt[3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2 + 3*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]]) + (4*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx &= \frac{1}{3}\int \frac{\sqrt{3-2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx+\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx\\ &= \frac{\sqrt{3-2\sec(c+dx)}\int \sqrt{-2+3\cos(c+dx)}\,dx}{3\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{\left(2\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+\right.}{3\sqrt{3-2\sec(}\\ &= \frac{2E\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{3-2\sec(c+dx)}}{3d\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}}+\frac{4\sqrt{-2+3\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{}}{3d\sqrt{3-2\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.113527, size = 72, normalized size = 0.64

$$\frac{\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\left(4\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)+2E\left(\frac{1}{2}(c+dx)\middle|6\right)\right)}{3d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[3 - 2*Sec[c + d*x]])
```

Maple [C] time = 0.283, size = 381, normalized size = 3.4

$$\frac{2}{15d\sin\left(dx+c\right)\left(-2+3\,\cos\left(dx+c\right)\right)}\left(3\,\sin\left(dx+c\right)\cos\left(dx+c\right)\sqrt{5}\sqrt{\frac{-2+3\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\sqrt{\left(\cos\left(dx+c\right)+1\right)^{-1}}\mathrm{Ellipt}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/15/d*(3*sin(d*x+c)*cos(d*x+c)*5^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*5^(1/2)/sin(d*x+c)
,1/5*I*5^(1/2))-5*sin(d*x+c)*cos(d*x+c)*5^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*5^(1/2)/si
n(d*x+c),1/5*I*5^(1/2))+3*5^(1/2)*EllipticF((-1+cos(d*x+c))*5^(1/2)/sin(d*x
+c),1/5*I*5^(1/2))*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-5*5^(1/2)*EllipticE((-1+cos(d*x+c))*5^(1/2)/sin(d*x+c)
,1/5*I*5^(1/2))*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-15*cos(d*x+c)^2+25*cos(d*x+c)-10)*((-2+3*cos(d*x+c))/cos(
d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2+3*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,\sec\left(dx+c\right)+3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,\sec\left(dx+c\right)+3}\sqrt{\sec\left(dx+c\right)}}{2\,\sec\left(dx+c\right)^{2}-3\,\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c)^2 -
3*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3-2\sec{(c+dx)}}\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3 - 2*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\sec{(dx+c)}+3}\sqrt{\sec{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))), x)
```

3.675 $\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=129

$$\frac{4\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{6}{5}\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

```
[Out] (4*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c + d
*x]])/(3*Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]]) - (2*Sqrt[5]*EllipticE[(c + P
i + d*x)/2, 6/5]*Sqrt[-3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 - 3*Cos[c + d*x]]*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 0.174113, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3862, 3856, 2654, 3858, 2662}

$$\frac{4\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3\sqrt{5}d\sqrt{2\sec(c+dx)-3}}-\frac{2\sqrt{5}\sqrt{2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{3d\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```


```
[Out] (4*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c + d
*x]])/(3*Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]]) - (2*Sqrt[5]*EllipticE[(c + P
i + d*x)/2, 6/5]*Sqrt[-3 + 2*Sec[c + d*x]])/(3*d*Sqrt[2 - 3*Cos[c + d*x]]*S
qrt[Sec[c + d*x]])
```


Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{-3+2\sec(c+dx)}}\,dx &= -\left(\frac{1}{3}\int \frac{\sqrt{-3+2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right)+\frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx\\ &= \frac{\left(2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2-3\cos(c+dx)}}\,dx}{3\sqrt{-3+2\sec(c+dx)}}-\frac{\sqrt{-3+2\sec(c+dx)}}{3\sqrt{2-3\cos(c}\\ &= \frac{4\sqrt{2-3\cos(c+dx)}F\left(\frac{1}{2}(c+\pi+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{3\sqrt{5}d\sqrt{-3+2\sec(c+dx)}}-\frac{2\sqrt{5}E\left(\frac{1}{2}(c+\pi+dx)\right.}{3d\sqrt{2-3\cos(c}}\end{aligned}$$

Mathematica [A] time = 0.077407, size = 72, normalized size = 0.56

$$\frac{\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\left(4\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)+2E\left(\frac{1}{2}(c+dx)\middle|6\right)\right)}{3d\sqrt{2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Sec[c + d*x]]*Sqrt[-3 + 2*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[-2 + 3*Cos[c + d*x]]*(2*EllipticE[(c + d*x)/2, 6] + 4*EllipticF[(c +
d*x)/2, 6])*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Maple [C] time = 0.242, size = 370, normalized size = 2.9

$$-\frac{2}{3\,d\sin\left(dx+c\right)\left(-2+3\,\cos\left(dx+c\right)\right)}\left(3\,i\sin\left(dx+c\right)\cos\left(dx+c\right)\operatorname{EllipticF}\left(\frac{i\left(-1+\cos\left(dx+c\right)\right)}{\sin\left(dx+c\right)},i\sqrt{5}\right)\sqrt{(\cos\left(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d*(3*I*sin(d*x+c)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I*
5^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)-
I*sin(d*x+c)*cos(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*(
1/(cos(d*x+c)+1))^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)+3*I*Ellipt
icF(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*((-2+3
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*EllipticE(I*(-1+cos(d*x+c))
/sin(d*x+c),I*5^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)-3*cos(d*x+c)^2+5*cos(d*x+c)-2)*(-(-2+3*cos(d*x+c))
/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(-2+3*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,\sec\left(dx+c\right)-3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{2\,\sec\left(dx+c\right)-3}\sqrt{\sec\left(dx+c\right)}}{2\,\sec\left(dx+c\right)^{2}-3\,\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c)^2 - 3*
sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\sec{(c+dx)}-3}\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\sec\left(dx+c\right)-3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

3.676 $\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{4\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),6\right)}{3d\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}\,E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*EllipticE[(c + Pi + d*x)/2, 6]*Sqrt[-3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2
- 3*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*Sqrt[-2 - 3*Cos[c + d*x]]*Ellipt
icF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 - 2*Sec[c + d*x]]
)
```

Rubi [A] time = 0.174563, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3862, 3856, 2654, 3858, 2662}

$$-\frac{4\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-2\sec(c+dx)-3}}-\frac{2\sqrt{-2\sec(c+dx)-3}E\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{3d\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (-2*EllipticE[(c + Pi + d*x)/2, 6]*Sqrt[-3 - 2*Sec[c + d*x]])/(3*d*Sqrt[-2
- 3*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (4*Sqrt[-2 - 3*Cos[c + d*x]]*Ellipt
icF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[-3 - 2*Sec[c + d*x]]
)
```


Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2\sec(c+dx)}\sqrt{\sec(c+dx)}}\,dx = -\left(\frac{1}{3}\int \frac{\sqrt{-3-2\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\right) - \frac{2}{3}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx$$

$$= -\frac{\sqrt{-3-2\sec(c+dx)}\int \sqrt{-2-3\cos(c+dx)}\,dx}{3\sqrt{-2-3\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{-2-3\cos(c+dx)}\sqrt{\sec\right.}{3\sqrt{-3-2}}$$

$$= -\frac{2E\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{-3-2\sec(c+dx)}}{3d\sqrt{-2-3\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{4\sqrt{-2-3\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d\sqrt{-3-2\sec}}$$

Mathematica [A] time = 0.104836, size = 81, normalized size = 0.7

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\left(5E\left(\frac{1}{2}(c+dx)\middle|\frac{6}{5}\right)-2\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)\right)}{3\sqrt{5}d\sqrt{-2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 2*Sec[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*(5*EllipticE[(c + d*x)/2, 6/5] - 2*EllipticF[(c
 + d*x)/2, 6/5])*Sqrt[Sec[c + d*x]])/(3*Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x]]
)
```

Maple [C] time = 0.255, size = 394, normalized size = 3.4

$$-\frac{1}{15\,d\sin\left(dx+c\right)\left(2+3\,\cos\left(dx+c\right)\right)}\left(3\,i\sin\left(dx+c\right)\cos\left(dx+c\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(dx+c\right)\right)}{\sin\left(dx+c\right)},\frac{i}{5}\sqrt{5}\right)\sqrt{2}\sqrt{\left(\cos\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/15/d*(3*I*sin(d*x+c)*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1
/5*I*5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)-5*I*sin(d*x+c)*cos(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/
sin(d*x+c),1/5*I*5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((2+3*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)+3*I*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,1/5*I*5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5*I*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),1/5*I*5^(1/2))*2^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((2+3*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-30*cos(d*x+c)^2+10*cos(d*x+c)+20)*(-(2+3
*cos(d*x+c))/cos(d*x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(2+3*cos(d*x
+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,\sec\left(dx+c\right)-3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}{2\sec(dx+c)^2+3\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c)^2 +
3*sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\sec(c+dx)-3}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-2*sec(c + d*x) - 3)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\sec(dx+c)-3}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))), x)
```

3.677 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Rubi [A] time = 0.0568292, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3858, 2661}

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2+3\sec(c+dx)}}\,dx = \frac{\left(\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3+2\cos(c+dx)}}\,dx}{\sqrt{2+3\sec(c+dx)}}$$
$$= \frac{2\sqrt{3+2\cos(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{2+3\sec(c+dx)}}$$

Mathematica [A] time = 0.0552023, size = 61, normalized size = 1.

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{3\sec(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[2 + 3*Sec[c + d*x]])
```

Maple [C] time = 0.217, size = 142, normalized size = 2.3

$$\frac{-\frac{i}{5}\sqrt{5}\left(\sin(dx+c)\right)^2\cos(dx+c)\sqrt{10}\sqrt{2}}{d\left(2\left(\cos(dx+c)\right)^2+\cos(dx+c)-3\right)}\text{EllipticF}\left(\frac{\frac{i}{5}\left(-1+\cos(dx+c)\right)\sqrt{5}}{\sin(dx+c)},\sqrt{5}\right)\sqrt{\left(\cos(dx+c)\right)^{-1}}\sqrt{\frac{3+2\cos(dx+c}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*5^(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),5^(1/2)
)*sin(d*x+c)^2*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((3+2*cos(d*x+c))/cos(d*x+c)
)^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*2^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)/(2*cos(d*x+c)^2+cos(d*x+c)-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\,\sec\left(dx+c\right)+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{3\sec\left(c+dx\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{3\,\sec\left(dx+c\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) + 2), x)
```

3.678 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

Rubi [A] time = 0.0566, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3858, 2661}

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2+3\sec(c+dx)}}\,dx = \frac{\left(\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}\,dx}{\sqrt{-2+3\sec(c+dx)}}$$
$$= \frac{2\sqrt{3-2\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{-2+3\sec(c+dx)}}$$

Mathematica [A] time = 0.0561017, size = 54, normalized size = 1.

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 + 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-2 + 3*Sec[c + d*x]])
```

Maple [A] time = 0.234, size = 137, normalized size = 2.5

$$\frac{i(\sin(dx+c))^2\cos(dx+c)\sqrt{2}}{d\left(2(\cos(dx+c))^2-5\cos(dx+c)+3\right)}\sqrt{(\cos(dx+c))^{-1}}\sqrt{-\frac{-3+2\cos(dx+c)}{\cos(dx+c)}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\sqrt{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x)
```

```
[Out] I/d*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2)*(-(-3+2*cos(d*x+c))/cos(d*x+c))^(1/2)
*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)/(2*cos(d*x+c)
^2-5*cos(d*x+c)+3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)-2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3\sec(c+dx)-2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-2+3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3*sec(c + d*x) - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{3\sec(dx+c)-2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2+3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(3*sec(d*x + c) - 2), x)
```

3.679 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Rubi [A] time = 0.0692775, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3858, 2663, 2661}

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx &= \frac{\left(\sqrt{-3+2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{2-3\sec(c+dx)}}\\ &= \frac{\left(\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{3-2\cos(c+dx)}}\,dx}{\sqrt{2-3\sec(c+dx)}}\\ &= \frac{2\sqrt{3-2\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|-4\right)\sqrt{\sec(c+dx)}}{d\sqrt{2-3\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0352445, size = 54, normalized size = 1.

$$\frac{2\sqrt{3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),-4\right)}{d\sqrt{2-3\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 - 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, -4]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[2 - 3*Sec[c + d*x]])
```

Maple [A] time = 0.226, size = 144, normalized size = 2.7

$$\frac{-\frac{i}{5}\sqrt{5}\cos(dx+c)(\sin(dx+c))^{2}\sqrt{2}}{d\left(2(\cos(dx+c))^{2}-5\cos(dx+c)+3\right)}\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{-3+2\cos(dx+c)}{\cos(dx+c)}}\text{EllipticF}\left(\frac{i\sqrt{5}(-1+\cos(dx+c))}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*5^(1/2)*cos(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(1/2)*((-3+2*cos(d*
x+c))/cos(d*x+c))^(1/2)*EllipticF(I*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),1/5*
5^(1/2))*2^(1/2)*(-2*(-3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)/(2*cos(d*x+c)^2-5*cos(d*x+c)+3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\sec(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\sec(dx+c)+2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)-2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*sec(d*x + c) + 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c) - 2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{2-3\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(2-3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2 - 3*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\sec(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) + 2), x)
```

3.680 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}$$

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Rubi [A] time = 0.0700731, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3858, 2663, 2661}

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-2-3\sec(c+dx)}}\,dx = \frac{\left(\sqrt{-3-2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-3-2\cos(c+dx)}}\,dx}{\sqrt{-2-3\sec(c+dx)}}$$
$$= \frac{\left(\sqrt{3+2\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{3}{5}+\frac{2}{5}\cos(c+dx)}}\,dx}{\sqrt{5}\sqrt{-2-3\sec(c+dx)}}$$
$$= \frac{2\sqrt{3+2\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|\frac{4}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-2-3\sec(c+dx)}}$$

Mathematica [A] time = 0.0432494, size = 61, normalized size = 1.

$$\frac{2\sqrt{2\cos(c+dx)+3}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{4}{5}\right)}{\sqrt{5}d\sqrt{-3\sec(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-2 - 3*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[3 + 2*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 4/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-2 - 3*Sec[c + d*x]])
```

Maple [C] time = 0.232, size = 139, normalized size = 2.3

$$\frac{\frac{i}{5}(\sin(dx+c))^2\cos(dx+c)\sqrt{10}\sqrt{2}}{d\left(2(\cos(dx+c))^2+\cos(dx+c)-3\right)}\sqrt{(\cos(dx+c))^{-1}}\sqrt{-\frac{3+2\cos(dx+c)}{\cos(dx+c)}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)\sqrt{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5*I/d*(1/cos(d*x+c))^(1/2)*(-(3+2*cos(d*x+c))/cos(d*x+c))^(1/2)*sin(d*x+c
)^2*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))*(1/(cos(
d*x+c)+1))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*2^(1/2)/(
2*cos(d*x+c)^2+cos(d*x+c)-3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\sec(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\sec(dx+c)-2}\sqrt{\sec(dx+c)}}{3\sec(dx+c)+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*sec(d*x + c) - 2)*sqrt(sec(d*x + c))/(3*sec(d*x + c) + 2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3\sec(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-2-3*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(-3*sec(c + d*x) - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-3\sec(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-2-3*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-3*sec(d*x + c) - 2), x)
```

3.681 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Rubi [A] time = 0.0570815, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3858, 2661}

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3+2\sec(c+dx)}}\,dx = \frac{\left(\sqrt{2+3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2+3\cos(c+dx)}}\,dx}{\sqrt{3+2\sec(c+dx)}}$$
$$= \frac{2\sqrt{2+3\cos(c+dx)}F\left(\frac{1}{2}(c+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{3+2\sec(c+dx)}}$$

Mathematica [A] time = 0.0572409, size = 61, normalized size = 1.

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[3 + 2*Sec[c + d*x]])
```

Maple [C] time = 0.215, size = 145, normalized size = 2.4

$$\frac{\sqrt{5}(\sin(dx+c))^2\cos(dx+c)\sqrt{10}\sqrt{2}}{5d\left(3(\cos(dx+c))^2-\cos(dx+c)-2\right)}\text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{5}}{5\sin(dx+c)},i\sqrt{5}\right)\sqrt{(\cos(dx+c))^{-1}}\sqrt{\frac{2+3\cos(dx+c)}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*5^(1/2)*EllipticF(1/5*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))*s
in(d*x+c)^2*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((2+3*cos(d*x+c))/cos(d*x+c))^(
1/2)*10^(1/2)*((2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*2^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)/(3*cos(d*x+c)^2-cos(d*x+c)-2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{2\sec(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\,\sec\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{2\sec\left(c+dx\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\,\sec\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) + 3), x)
```

3.682 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rubi [A] time = 0.0570182, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3858, 2661}

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[3 - 2*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\,dx = \frac{\left(\sqrt{-2+3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-2+3\cos(c+dx)}}\,dx}{\sqrt{3-2\sec(c+dx)}}$$
$$= \frac{2\sqrt{-2+3\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{d\sqrt{3-2\sec(c+dx)}}$$

Mathematica [A] time = 0.055909, size = 54, normalized size = 1.

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)}{d\sqrt{3-2\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[3 - 2*Sec[c + d*x]])
```

Maple [C] time = 0.241, size = 138, normalized size = 2.6

$$\frac{2\sqrt{5}\left(\sin(dx+c)\right)^2\cos(dx+c)}{5d\left(3\left(\cos(dx+c)\right)^2-5\cos(dx+c)+2\right)}\sqrt{\left(\cos(dx+c)\right)^{-1}}\sqrt{\frac{-2+3\cos(dx+c)}{\cos(dx+c)}}\text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{i}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/5/d*5^(1/2)*sin(d*x+c)^2*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((-2+3*cos(d*x+c
))/cos(d*x+c))^(1/2)*EllipticF((-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(
1/2))*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)/(3*
cos(d*x+c)^2-5*cos(d*x+c)+2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-2\sec(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2 \sec(dx+c)+3}\sqrt{\sec(dx+c)}}{2 \sec(dx+c)-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*sec(d*x + c) + 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c) - 3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{3-2\sec(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(3-2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(3 - 2*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-2 \sec(dx+c)+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(3-2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) + 3), x)
```

3.683 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)-3}}$$

```
[Out] (2*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c + d
*x]])/(Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Rubi [A] time = 0.0571196, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3858, 2662}

$$\frac{2\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx+\pi)|\frac{6}{5}\right)}{\sqrt{5}d\sqrt{2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6/5]*Sqrt[Sec[c + d
*x]])/(Sqrt[5]*d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3+2\sec(c+dx)}}\,dx = \frac{\left(\sqrt{2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{2-3\cos(c+dx)}}\,dx}{\sqrt{-3+2\sec(c+dx)}}$$
$$= \frac{2\sqrt{2-3\cos(c+dx)}F\left(\frac{1}{2}(c+\pi+dx)|\frac{6}{5}\right)\sqrt{\sec(c+dx)}}{\sqrt{5}d\sqrt{-3+2\sec(c+dx)}}$$

Mathematica [A] time = 0.0441874, size = 54, normalized size = 0.87

$$\frac{2\sqrt{3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),6\right)}{d\sqrt{2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 + 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6]*Sqrt[Sec[c + d*x]])/
(d*Sqrt[-3 + 2*Sec[c + d*x]])
```

Maple [C] time = 0.241, size = 136, normalized size = 2.2

$$\frac{2\,i\cos(dx+c)(\sin(dx+c))^2}{d\left(3\,(\cos(dx+c))^2-5\,\cos(dx+c)+2\right)}\sqrt{(\cos(dx+c))^{-1}}\sqrt{-\frac{-2+3\,\cos(dx+c)}{\cos(dx+c)}}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\sqrt{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x)
```

```
[Out] 2*I/d*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*(-(-2+3*cos(d*x+c))/cos(d*x+c))^(1/2)
*sin(d*x+c)^2*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))*(1/(cos(d*x
+c)+1))^(1/2)*((-2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)/(3*cos(d*x+c)^2-5*co
s(d*x+c)+2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{2\,\sec(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\,\sec\left(dx+c\right)-3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{2\sec\left(c+dx\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-3+2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(2*sec(c + d*x) - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{2\,\sec\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3+2*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(2*sec(d*x + c) - 3), x)
```

3.684 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx$

Optimal. Leaf size=55

$$\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx+\pi),6\right)}{d\sqrt{-2\sec(c+dx)-3}}$$

```
[Out] (2*Sqrt[-2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*
x]])/(d*Sqrt[-3 - 2*Sec[c + d*x]])
```

Rubi [A] time = 0.0580804, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3858, 2662}

$$\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx+\pi)\middle|6\right)}{d\sqrt{-2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[-2 - 3*Cos[c + d*x]]*EllipticF[(c + Pi + d*x)/2, 6]*Sqrt[Sec[c + d*
x]])/(d*Sqrt[-3 - 2*Sec[c + d*x]])
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{-3-2\sec(c+dx)}}\,dx = \frac{\left(\sqrt{-2-3\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{-2-3\cos(c+dx)}}\,dx}{\sqrt{-3-2\sec(c+dx)}}$$
$$= \frac{2\sqrt{-2-3\cos(c+dx)}F\left(\frac{1}{2}(c+\pi+dx)\middle|6\right)\sqrt{\sec(c+dx)}}{d\sqrt{-3-2\sec(c+dx)}}$$

Mathematica [A] time = 0.0466407, size = 61, normalized size = 1.11

$$\frac{2\sqrt{3\cos(c+dx)+2}\sqrt{\sec(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{6}{5}\right)}{\sqrt{5}d\sqrt{-2\sec(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[-3 - 2*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[2 + 3*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 6/5]*Sqrt[Sec[c + d*x]])
/(Sqrt[5]*d*Sqrt[-3 - 2*Sec[c + d*x]])
```

Maple [C] time = 0.235, size = 142, normalized size = 2.6

$$\frac{\frac{i}{5}(\sin(dx+c))^2\cos(dx+c)\sqrt{2}\sqrt{10}}{d\left(3(\cos(dx+c))^2-\cos(dx+c)-2\right)}\text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},\frac{i}{5}\sqrt{5}\right)\sqrt{(\cos(dx+c))^{-1}}\sqrt{-\frac{2+3\cos(dx+c)}{\cos(dx+c)}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/5*I/d*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)^2*
cos(d*x+c)*(1/cos(d*x+c))^(1/2)*(-(2+3*cos(d*x+c))/cos(d*x+c))^(1/2)*2^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
/(3*cos(d*x+c)^2-cos(d*x+c)-2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{-2\sec(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\sec\left(dx+c\right)-3}\sqrt{\sec\left(dx+c\right)}}{2\sec\left(dx+c\right)+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*sec(d*x + c) - 3)*sqrt(sec(d*x + c))/(2*sec(d*x + c) + 3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(c+dx\right)}}{\sqrt{-2\sec\left(c+dx\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(-3-2*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(-2*sec(c + d*x) - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec\left(dx+c\right)}}{\sqrt{-2\sec\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(-3-2*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sec(d*x + c))/sqrt(-2*sec(d*x + c) - 3), x)
```

3.685 $\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[
c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rubi [A] time = 0.0965397, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[
c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx &= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} \\
&= -\frac{\left(\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}} \\
&= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}
\end{aligned}$$

Mathematica [B] time = 26.0036, size = 7160, normalized size = 68.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.135, size = 0, normalized size = 0.

$$\int \sec \left(dx+c\right) \sqrt[3]{a+b\sec \left(dx+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(dx+c\right)+a\right)^{\frac{1}{3}} \sec \left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(dx+c\right)+a\right)^{\frac{1}{3}} \sec \left(dx+c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec \left(c+dx\right)} \sec \left(c+dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(1/3)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c), x)
```

3.686 $\int \sqrt[3]{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\sqrt[3]{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(1/3), x]
```

Rubi [A] time = 0.0108199, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(1/3), x]
```

Rubi steps

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx=\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 1.89551, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(1/3), x]
```

Maple [A] time = 0.174, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(1/3), x)
```

3.687 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=362

$$\frac{a\left(18a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{110\sqrt{2}b^3d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\left(23a^2b^2+9a^4-32b^4\right.}{}$$

```
[Out] (3*(9*a^2 + 32*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(220*b^2*d) -
(9*a*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(44*b^2*d) + (3*Sec[c + d*x]*
(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(11*b*d) + (a*(18*a^2 + 49*b^2)*Ap
pellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
 + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(110*Sqrt[2]*b^3*d*Sqrt[1 +
 Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - ((9*a^4 + 23*a^2*b^2
 - 32*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(55*S
qrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.66823, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3865, 4082, 4002, 4007, 3834, 139, 138}

$$\frac{a\left(18a^2+49b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{110\sqrt{2}b^3d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\left(23a^2b^2+9a^4-32b^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
```


```
[Out] (3*(9*a^2 + 32*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(220*b^2*d) -
(9*a*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(44*b^2*d) + (3*Sec[c + d*x]*
(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(11*b*d) + (a*(18*a^2 + 49*b^2)*Ap
pellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
 + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(110*Sqrt[2]*b^3*d*Sqrt[1 +
 Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - ((9*a^4 + 23*a^2*b^2
 - 32*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(55*S
qrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{11bd} + \frac{3\int \sec(c+dx)(a+b\sec(c\cdots}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{44b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{5/3}\,\mathrm{ta}\cdots}{11bd}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx))^{5/3}\,\mathrm{ta}\cdots}{44b^2d}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx))^{5/3}\,\mathrm{ta}\cdots}{44b^2d}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx))^{5/3}\,\mathrm{ta}\cdots}{44b^2d}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx))^{5/3}\,\mathrm{ta}\cdots}{44b^2d}$$

$$= \frac{3(9a^2+32b^2)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{220b^2d} - \frac{9a(a+b\sec(c+dx))^{5/3}\,\mathrm{ta}\cdots}{44b^2d}$$

Mathematica [B] time = 26.6812, size = 21877, normalized size = 60.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^4 (a+b\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{2}{3}}\sec (dx+c)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec (dx+c)+a)^{\frac{2}{3}}\sec (dx+c)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^4, x)
```

3.688 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=305

$$-\frac{(6a^2-25b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{20\sqrt{2}b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3a\left(a^2-b^2\right)\tan(c+dx)}{10}$$

```
[Out] (-9*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(40*b*d) + (3*(a + b*Sec[c +
 d*x])^(5/3)*Tan[c + d*x])/(8*b*d) - ((6*a^2 - 25*b^2)*AppellF1[1/2, 1/2, -
2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[
c + d*x])^(2/3)*Tan[c + d*x])/(20*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x]]*((a
+ b*Sec[c + d*x])/(a + b))^(2/3)) + (3*a*(a^2 - b^2)*AppellF1[1/2, 1/2, 1/3
, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c
+ d*x])/(a + b))^(1/3)*Tan[c + d*x])/(10*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x
]]*(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.47028, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3840, 4002, 4007, 3834, 139, 138}

$$-\frac{(6a^2-25b^2)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{20\sqrt{2}b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{3a\left(a^2-b^2\right)\tan(c+dx)}{10}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```


```
[Out] (-9*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(40*b*d) + (3*(a + b*Sec[c +
 d*x])^(5/3)*Tan[c + d*x])/(8*b*d) - ((6*a^2 - 25*b^2)*AppellF1[1/2, 1/2, -
2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[
c + d*x])^(2/3)*Tan[c + d*x])/(20*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x]]*((a
+ b*Sec[c + d*x])/(a + b))^(2/3)) + (3*a*(a^2 - b^2)*AppellF1[1/2, 1/2, 1/3
, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c
+ d*x])/(a + b))^(1/3)*Tan[c + d*x])/(10*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x
]]*(a + b*Sec[c + d*x])^(1/3))
```


Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
```

```
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + \frac{3\int \sec(c+dx)\left(\frac{5b}{3}-a\sec(c+dx)\right)(a+}{8b}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + $$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + $$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + $$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + $$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{40bd} + \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8bd} + $$

Mathematica [B] time = 26.3891, size = 18991, normalized size = 62.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.115, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{3} (a+b\sec (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec (dx+c)+a)^{\frac{2}{3}} \sec (dx+c)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec (c+dx))^{\frac{2}{3}} \sec^{3}(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec{(dx+c)}+a)^{\frac{2}{3}}\sec{(dx+c)}^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^3, x)
```

3.689 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=260

$$-\frac{2\sqrt{2}\left(a^2-b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{2\sqrt{2}a\tan(c+dx)(a+b\sec(}{5bd}$$

```
[Out] (3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (2*Sqrt[2]*a*AppellF1[1
/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(
a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d*Sqrt[1 + Sec[c + d*x]]*((a +
 b*Sec[c + d*x])/(a + b))^(2/3)) - (2*Sqrt[2]*(a^2 - b^2)*AppellF1[1/2, 1/2
, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*S
ec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(5*b*d*Sqrt[1 + Sec[c + d*x]]*(a
+ b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.338258, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3835, 4007, 3834, 139, 138}

$$-\frac{2\sqrt{2}\left(a^2-b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5bd\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{2\sqrt{2}a\tan(c+dx)(a+b\sec(}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*d) + (2*Sqrt[2]*a*AppellF1[1
/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(
a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d*Sqrt[1 + Sec[c + d*x]]*((a +
 b*Sec[c + d*x])/(a + b))^(2/3)) - (2*Sqrt[2]*(a^2 - b^2)*AppellF1[1/2, 1/2
, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*S
ec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(5*b*d*Sqrt[1 + Sec[c + d*x]]*(a
+ b*Sec[c + d*x])^(1/3))
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
```

```
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{2}{5}\int \frac{\sec(c+dx)(b+a\sec(c+dx))}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{(2a)\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}}{5b}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}-\frac{(2a\tan(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,\right.}{5bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}-\frac{\left(2a(a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{S}}{5bd\sqrt{1-\sec(c+dx)}\sqrt{1+\text{s}}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5d}+\frac{2\sqrt{2}aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\right.}{5bd\sqrt{1+\sec(}}$$

Mathematica [B] time = 18.3005, size = 2505, normalized size = 9.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(2/3)*((3*a*Sin[c + d*x])/(5*b) + (3*Tan[c + d*x])/5)
)/d - ((-2*b + 3*a*Cos[c + d*x])*(a + b*Sec[c + d*x])^(2/3)*(3*a*(b + a*Cos
[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(2/3) - (3*(b + a*Co
s[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)
])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 -
 b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-
2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2,
1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x
])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x])))/(5*b*Sqrt[1 - Cos[c + d*x]^
2]*Sec[c + d*x]^(1/3))))/(5*b*d*((3*a*(b + a*Cos[c + d*x])^(2/3)*Sin[c + d*
x])/(Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) - (2*a^2*Sqrt[1 - Cos[c +
 d*x]^2]*Sec[c + d*x]^(2/3)*Sin[c + d*x])/(b + a*Cos[c + d*x])^(1/3) + 2*a*
(b + a*Cos[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(5/3)*Sin[
c + d*x] - (3*Sqrt[b^(-2)]*(b + a*Cos[c + d*x])^(2/3)*Sec[c + d*x]^(5/3)*Sq
rt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*
```

AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 - a*Sqrt[b^(-2)])*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]) + (3*Sqrt[b^(-2)]*(b + a*Cos[c + d*x])^(2/3)*Sec[c + d*x]^(5/3)*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 + a*Sqrt[b^(-2)])*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]) + (3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(1 - Cos[c + d*x]^2)^(3/2)*Sec[c + d*x]^(4/3)) + (2*a*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) + ((b + a*Cos[c + d*x])^(2/3)*Sec[c + d*x]^(2/3)*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*(a^2 - b^2)*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*a*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*Sqrt[1 - Cos[c + d*x]^2]) - (3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(2*a*b*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x] - 5*(a^2 - b^2)*((b*AppellF1[5/3, 1/2, 3/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(5*(a + 1/Sqrt[b^(-2)])) - (b*AppellF1[5/3, 3/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(5*(-a + 1/Sqrt[b^(-2)]))) + 2*a*(a + b*Sec[c + d*x])*((5*b*AppellF1[8/3, 1/2, 3/2, 11/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(16*(a + 1/Sqrt[b^(-2)])) - (5*b*AppellF1[8/3, 3/2, 1/2, 11/3, -((a + b*Sec[c + d*x])/(-a + 1

```
/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Ta
n[c + d*x])/(16*(-a + 1/Sqrt[b^(-2)])))))/(5*b*Sqrt[1 - Cos[c + d*x]^2]*Sec
[c + d*x]^(1/3))))
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{2}\left(a+b\sec \left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(dx+c\right)+a\right)^{\frac{2}{3}}\sec \left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec \left(dx+c\right)+a\right)^{\frac{2}{3}}\sec \left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec{(c+dx)})^{\frac{2}{3}}\sec^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec{(dx+c)}+a)^{\frac{2}{3}}\sec{(dx+c)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)^2, x)
```

3.690 $\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[
c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rubi [A] time = 0.0829725, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
```


```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Sec[
c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```


Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left((a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [B] time = 25.9442, size = 7142, normalized size = 68.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \sec (dx + c) (a + b \sec (dx + c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{2}{3}} \sec (dx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec (dx + c) + a)^{\frac{2}{3}} \sec (dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sec (c + dx))^{\frac{2}{3}} \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(2/3)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c), x)
```

3.691 $\int (a+b\sec(c+dx))^{2/3}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left((a+b\sec(c+dx))^{2/3},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(2/3), x]
```

Rubi [A] time = 0.0117092, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(2/3),x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^(2/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{2/3}\,dx=\int (a+b\sec(c+dx))^{2/3}\,dx$$

Mathematica [A] time = 2.02654, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(2/3),x]

[Out] Integrate[(a + b*Sec[c + d*x])^(2/3), x]
```

Maple [A] time = 0.129, size = 0, normalized size = 0.

$$\int (a+b\sec (dx+c))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec (c+dx))^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(2/3), x)
```

3.692 $\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{2}(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
- Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rubi [A] time = 0.0883491, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}(a+b)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
- Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{4/3}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{\left((-a-b)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}}$$

$$= \frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\,\text{t}}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}$$

Mathematica [B] time = 26.4505, size = 7313, normalized size = 67.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec(dx+c)^2+a\sec(dx+c)\right)(b\sec(dx+c)+a)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{4}{3}}\sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(4/3)*sec(d*x + c), x)
```

3.693 $\int (a+b\sec(c+dx))^{4/3}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left((a+b\sec(c+dx))^{4/3},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(4/3), x]
```

Rubi [A] time = 0.0114037, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^{4/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(4/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{4/3}\,dx=\int (a+b\sec(c+dx))^{4/3}\,dx$$

Mathematica [A] time = 18.6317, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{4/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(4/3), x]
```

Maple [A] time = 0.133, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(4/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(4/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(4/3), x)
```

3.694 $\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=412

$$\frac{\left(164a^2b^2+36a^4+605b^4\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{616\sqrt{2}b^3d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a\left(79a^2b^2\right.}{}$$

```
[Out] (3*a*(18*a^2 + 97*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(1232*b^2*d
) + (3*(18*a^2 + 121*b^2)*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(1232*b^
2*d) - (9*a*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(77*b^2*d) + (3*Sec[c
+ d*x]*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(14*b*d) + ((36*a^4 + 164*a
^2*b^2 + 605*b^4)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(616*Sq
rt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) -
(a*(18*a^4 + 79*a^2*b^2 - 97*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c +
 d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1
/3)*Tan[c + d*x])/(308*Sqrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c +
d*x])^(1/3))
```

Rubi [A] time = 0.825241, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3865, 4082, 4002, 4007, 3834, 139, 138}

$$\frac{\left(164a^2b^2+36a^4+605b^4\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{616\sqrt{2}b^3d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a\left(79a^2b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```



```
[Out] (3*a*(18*a^2 + 97*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(1232*b^2*d
) + (3*(18*a^2 + 121*b^2)*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(1232*b^
2*d) - (9*a*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(77*b^2*d) + (3*Sec[c
+ d*x]*(a + b*Sec[c + d*x])^(8/3)*Tan[c + d*x])/(14*b*d) + ((36*a^4 + 164*a
^2*b^2 + 605*b^4)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(616*Sq
rt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) -
(a*(18*a^4 + 79*a^2*b^2 - 97*b^4)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c +
```

d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(308*Sqrt[2]*b^3*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))

Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
```

```
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{14bd} + \frac{3\int \sec(c+dx)(a+b\sec(c+\ldots}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{77b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{8/3}\tan\ldots}{14bd}$$

$$= \frac{3\left(18a^2+121b^2\right)(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{1232b^2d} - \frac{9a(a+b\sec(c+dx))^{8/3}\,\mathrm{t}\ldots}{77b^2d}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)(a+b\,\mathrm{s}\ldots}{123\ldots}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)(a+b\,\mathrm{s}\ldots}{123\ldots}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)(a+b\,\mathrm{s}\ldots}{123\ldots}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)(a+b\,\mathrm{s}\ldots}{123\ldots}$$

$$= \frac{3a\left(18a^2+97b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{1232b^2d} + \frac{3\left(18a^2+121b^2\right)(a+b\,\mathrm{s}\ldots}{123\ldots}$$

Mathematica [B] time = 26.9911, size = 28057, normalized size = 68.1

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.122, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^4 (a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(dx+c\right)^{5}+a\sec\left(dx+c\right)^{4}\right)\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^5 + a*sec(d*x + c)^4)*(b*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^4, x)
```

3.695 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=356

$$-\frac{a\left(30a^2-373b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{220\sqrt{2}b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{\left(-79a^2b^2+15a^4\right.}{}$$

```
[Out] (-3*(15*a^2 - 64*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(440*b*d) -
(9*a*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(88*b*d) + (3*(a + b*Sec[c +
d*x])^(8/3)*Tan[c + d*x])/(11*b*d) - (a*(30*a^2 - 373*b^2)*AppellF1[1/2, 1/
2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
Sec[c + d*x])^(2/3)*Tan[c + d*x])/(220*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x]]
*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + ((15*a^4 - 79*a^2*b^2 + 64*b^4)*Ap
pellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
+ b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(110*Sqrt[2]*b^2*d
*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.652839, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3840, 4002, 4007, 3834, 139, 138}

$$-\frac{a\left(30a^2-373b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{220\sqrt{2}b^2d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+\frac{\left(-79a^2b^2+15a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```


```
[Out] (-3*(15*a^2 - 64*b^2)*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(440*b*d) -
(9*a*(a + b*Sec[c + d*x])^(5/3)*Tan[c + d*x])/(88*b*d) + (3*(a + b*Sec[c +
d*x])^(8/3)*Tan[c + d*x])/(11*b*d) - (a*(30*a^2 - 373*b^2)*AppellF1[1/2, 1/
2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
Sec[c + d*x])^(2/3)*Tan[c + d*x])/(220*Sqrt[2]*b^2*d*Sqrt[1 + Sec[c + d*x]]
*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + ((15*a^4 - 79*a^2*b^2 + 64*b^4)*Ap
pellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a
+ b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(110*Sqrt[2]*b^2*d
*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{11bd} + \frac{3\int \sec(c+dx)\left(\frac{8b}{3}-a\sec(c+dx)\right)(a-}{11b}$$

$$= -\frac{9a(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{88bd} + \frac{3(a+b\sec(c+dx))^{8/3}\tan(c+dx)}{11bd} +$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+dx))^{5/3}}{88bd}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+dx))^{5/3}}{88bd}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+dx))^{5/3}}{88bd}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+dx))^{5/3}}{88bd}$$

$$= -\frac{3\left(15a^2-64b^2\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{440bd} - \frac{9a(a+b\sec(c+dx))^{5/3}}{88bd}$$

Mathematica [B] time = 26.5618, size = 21890, normalized size = 61.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^3 (a + b \sec (dx + c))^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{5}{3}} \sec (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec (dx + c)^4 + a \sec (dx + c)^3\right)(b \sec (dx + c) + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^4 + a*sec(d*x + c)^3)*(b*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^3, x)
```

3.696 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=299

$$\frac{\left(2a^2+5b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{4\sqrt{2}bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a\left(a^2-b^2\right)\tan(c+dx)\sqrt[3]{}}{2\sqrt{2}b}$$

```
[Out] (3*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*d) + (3*(a + b*Sec[c + d*x
])^(5/3)*Tan[c + d*x])/(8*d) + ((2*a^2 + 5*b^2)*AppellF1[1/2, 1/2, -2/3, 3/
2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(4*Sqrt[2]*b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
+ d*x])/(a + b))^(2/3)) - (a*(a^2 - b^2)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 -
Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a +
 b))^(1/3)*Tan[c + d*x])/(2*Sqrt[2]*b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c
 + d*x])^(1/3))
```

Rubi [A] time = 0.456605, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3835, 4002, 4007, 3834, 139, 138}

$$\frac{\left(2a^2+5b^2\right)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{4\sqrt{2}bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{a\left(a^2-b^2\right)\tan(c+dx)\sqrt[3]{}}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (3*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*d) + (3*(a + b*Sec[c + d*x
])^(5/3)*Tan[c + d*x])/(8*d) + ((2*a^2 + 5*b^2)*AppellF1[1/2, 1/2, -2/3, 3/
2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x
])^(2/3)*Tan[c + d*x])/(4*Sqrt[2]*b*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
+ d*x])/(a + b))^(2/3)) - (a*(a^2 - b^2)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 -
Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a +
 b))^(1/3)*Tan[c + d*x])/(2*Sqrt[2]*b*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c
 + d*x])^(1/3))
```

Rule 3835

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*(m + 1)), x] +
Dist[m/(m + 1), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*(b + a*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4002

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1))*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B,
0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
```

```
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = \frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d}+\frac{5}{8}\int \sec(c+dx)(b+a\sec(c+dx))(a+b$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d}+\frac{3}{8}$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d}-\frac{(a}{}$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d}+\frac{(a}{}$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d}-\frac{((}{}$$

$$= \frac{3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8d}+\frac{3(a+b\sec(c+dx))^{5/3}\tan(c+dx)}{8d}+\frac{(2}{}$$

Mathematica [B] time = 26.6131, size = 19016, normalized size = 63.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (a+b\sec(dx+c))^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec(dx+c)^3+a\sec(dx+c)^2\right)(b\sec(dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*(b*sec(d*x + c) + a)^(2/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c)^2, x)
```

3.697 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
- Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rubi [A] time = 0.0887916, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}(a+b)\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1
- Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3))
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/3}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{\left((-a-b)(a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}}$$

$$= \frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [B] time = 26.4901, size = 7321, normalized size = 67.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \sec \left(dx+c\right) \left(a+b \sec \left(dx+c\right)\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec \left(dx+c\right)+a)^{\frac{5}{3}} \sec \left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(dx+c\right)^{2}+a \sec \left(dx+c\right)\right)\left(b \sec \left(dx+c\right)+a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*(b*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3)*sec(d*x + c), x)
```

3.698 $\int (a+b\sec(c+dx))^{5/3}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left((a+b\sec(c+dx))^{5/3},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(5/3), x]
```

Rubi [A] time = 0.0113138, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^{5/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/3}\,dx=\int (a+b\sec(c+dx))^{5/3}\,dx$$

Mathematica [A] time = 24.2715, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{5/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(5/3), x]
```

Maple [A] time = 0.088, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/3), x)
```

3.699 $\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=313

$$-\frac{a\left(9a^2+11b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{10\sqrt{2}b^3d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{\left(18a^2+25b^2\right)\tan(c+dx)(a+}{20\surd}$$

```
[Out] (-9*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(20*b^2*d) + (3*Sec[c + d*x]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*b*d) + ((18*a^2 + 25*b^2)*Appe
llF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(20*Sqrt[2]*b^3*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (a*(9*a^2 + 11*b^2)*App
ellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(10*Sqrt[2]*b^3*d*S
qrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.488216, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3865, 4082, 4007, 3834, 139, 138}

$$-\frac{a\left(9a^2+11b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{10\sqrt{2}b^3d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}+\frac{\left(18a^2+25b^2\right)\tan(c+dx)(a+}{20\surd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```



```
[Out] (-9*a*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(20*b^2*d) + (3*Sec[c + d*x]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(8*b*d) + ((18*a^2 + 25*b^2)*Appe
llF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(20*Sqrt[2]*b^3*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (a*(9*a^2 + 11*b^2)*App
ellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(10*Sqrt[2]*b^3*d*S
qrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```


Rule 3865

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + n - 1)), x] + Dist[d^3/(b*(m + n - 1)
), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(
m + n - 2)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3] && (IntegerQ[n] || In
tegersQ[2*m, 2*n]) &&  !IGtQ[m, 2]
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} + \frac{3\int \frac{\sec(c+dx)\left(a+\frac{5}{3}b\sec(c+dx)-2a\sec^2(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{8b}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} + \frac{9\int}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} - \frac{(a}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} + \frac{(a}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} - \frac{((1}{}$$

$$= -\frac{9a(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{20b^2d} + \frac{3\sec(c+dx)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{8bd} + \frac{(18}{}$$

Mathematica [B] time = 26.4094, size = 19015, normalized size = 60.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^4 \frac{1}{\sqrt[3]{a+b\sec (dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^4}{(b\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (dx+c)^4}{(b\sec (dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(1/3), x)
```

3.700 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=265

$$\frac{\sqrt{2}\left(3a^2+2b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5b^2d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{3\sqrt{2}a\tan(c+dx)(a+b\sec(c\ldots}{5b^2d\ldots}$$

```
[Out] (3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d) - (3*Sqrt[2]*a*AppellF1
[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b^2*d*Sqrt[1 + Sec[c + d*x]]*(
(a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2 + 2*b^2)*AppellF1[1/
2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
 + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(5*b^2*d*Sqrt[1 + Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.332234, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3840, 4007, 3834, 139, 138}

$$\frac{\sqrt{2}\left(3a^2+2b^2\right)\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5b^2d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}-\frac{3\sqrt{2}a\tan(c+dx)(a+b\sec(c\ldots}{5b^2d\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b*d) - (3*Sqrt[2]*a*AppellF1
[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]
*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(5*b^2*d*Sqrt[1 + Sec[c + d*x]]*(
(a + b*Sec[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(3*a^2 + 2*b^2)*AppellF1[1/
2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
 + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(5*b^2*d*Sqrt[1 + Sec[c + d
*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
```

```
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] &&  !LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{3\int \frac{\sec(c+dx)\left(\frac{2b}{3}-a\sec(c+dx)\right)}{\sqrt[3]{a+b\sec(c+dx)}}dx}{5b}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{1}{5}\left(2+\frac{3a^2}{b^2}\right)\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}dx - \frac{(3a)\int \sec(c}{}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{\left(\left(-2-\frac{3a^2}{b^2}\right)\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}dx\right.}{5d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} + \frac{\left(3a(a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}\right.}{\sqrt{1-}}\right.}{5b^2d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(-\right.}$$

$$= \frac{3(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{5bd} - \frac{3\sqrt{2}aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{5b^2d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec}{a+}\right.}$$

Mathematica [B] time = 25.971, size = 7195, normalized size = 27.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^3 \frac{1}{\sqrt[3]{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^3}{(b\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (dx+c)^3}{(b\sec (dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3 (c+dx)}{\sqrt[3]{a+b\sec (c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(1/3), x)
```

3.701 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=219

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\right.}{bd\sqrt{\sec(c+dx}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*AppellF1[1/2
, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
+ b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*
(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.228642, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3838, 3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)(a+b\sec(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{bd\sqrt{\sec(c+dx)+1}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}-\frac{\sqrt{2}a\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\right.}{bd\sqrt{\sec(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
+ d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) - (Sqrt[2]*a*AppellF1[1/2
, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a
+ b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(b*d*Sqrt[1 + Sec[c + d*x]]*
(a + b*Sec[c + d*x])^(1/3))
```

Rule 3838

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Dist[a/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + D
ist[1/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
```

```
b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \frac{\int \sec(c+dx)(a+b\sec(c+dx))^{2/3}\,dx}{b} - \frac{a\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx}{b}$$

$$= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} + \frac{(a\tan(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+}}\right.}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\text{s}}}$$

$$= -\frac{\left((a+b\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right)}{bd\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}} + \frac{\left(a\sqrt[3]{-\frac{a+b}{}}\right.}{}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{bd\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}} - \frac{\sqrt{2}al}{}$$

Mathematica [B] time = 19.0141, size = 2759, normalized size = 12.6

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*(b + a*Cos[c + d*x])*Tan[c + d*x])/(2*b*d*(a + b*Sec[c + d*x])^(1/3)) -
((b + 3*a*Cos[c + d*x])*(3*(b + a*Cos[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]
^2]*Sec[c + d*x]^(2/3) - (3*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*S
ec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(
1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*
x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*
AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])),
 (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x])))/(5*b*(b
+ a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(4/3))))/(2*b
*d*(a + b*Sec[c + d*x])^(1/3)*((3*(b + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/
(Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(1/3)) - (2*a*Sqrt[1 - Cos[c + d*x]^
2]*Sec[c + d*x]^(2/3)*Sin[c + d*x])/(b + a*Cos[c + d*x])^(1/3) + 2*(b + a*C
os[c + d*x])^(2/3)*Sqrt[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(5/3)*Sin[c + d*x]
 - (3*Sqrt[b^(-2)]*Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b
^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2,
5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a
 + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])
```

/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*
Sec[c + d*x]))*Sin[c + d*x])/(10*(1 - a*Sqrt[b^(-2)])*(b + a*Cos[c + d*x])^
(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 -
a*Sqrt[b^(-2)])]) + (3*Sqrt[b^(-2)]*Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])
*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF
1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b
*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a
+ b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[
b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(10*(1 + a*Sqrt[b^(-2)])*(b +
a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(1 - Sqrt[b^(-2)]*b*Se
c[c + d*x])/(1 + a*Sqrt[b^(-2)])]) - (3*Sec[c + d*x]^(2/3)*Sqrt[(1 - Sqrt[b
^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c
+ d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*
Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-
2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[
b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))
*Sin[c + d*x])/(5*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2]) + (3
*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^
(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*
AppellF1[2/3, 1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])),
(a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3
, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a +
1/Sqrt[b^(-2)])]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*
x])^(1/3)*(1 - Cos[c + d*x]^2)^(3/2)*Sec[c + d*x]^(7/3)) - (a*(a + b*Sec[c
+ d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(
1 + Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3,
1/2, 1/2, 5/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c
+ d*x])/(a + 1/Sqrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec
[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)]
)]*(a + b*Sec[c + d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x])^(4/3)*Sqrt
[1 - Cos[c + d*x]^2]*Sec[c + d*x]^(4/3)) + (4*(a + b*Sec[c + d*x])*Sqrt[(1
- Sqrt[b^(-2)]*b*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]
*b*Sec[c + d*x])/(1 - a*Sqrt[b^(-2)])]*(-5*a*AppellF1[2/3, 1/2, 1/2, 5/3, -
((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/S
qrt[b^(-2)])] + 2*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a +
1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*(a + b*Sec[c
+ d*x]))*Sin[c + d*x])/(5*b*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x
]^2]*Sec[c + d*x]^(1/3)) - (3*(a + b*Sec[c + d*x])*Sqrt[(1 - Sqrt[b^(-2)]*b
*Sec[c + d*x])/(1 + a*Sqrt[b^(-2)])]*Sqrt[(1 + Sqrt[b^(-2)]*b*Sec[c + d*x])
/(1 - a*Sqrt[b^(-2)])]*(2*b*AppellF1[5/3, 1/2, 1/2, 8/3, -((a + b*Sec[c + d
*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec
[c + d*x]*Tan[c + d*x] - 5*a*((b*AppellF1[5/3, 1/2, 3/2, 8/3, -((a + b*Sec[
c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])
]*Sec[c + d*x]*Tan[c + d*x])/(5*(a + 1/Sqrt[b^(-2)])) - (b*AppellF1[5/3, 3/
2, 1/2, 8/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c +

```
d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(5*(-a + 1/Sqrt[b^(-
2)]))) + 2*(a + b*Sec[c + d*x])*((5*b*AppellF1[8/3, 1/2, 3/2, 11/3, -((a +
b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a + b*Sec[c + d*x])/(a + 1/Sqrt[b^
(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(16*(a + 1/Sqrt[b^(-2)])) - (5*b*AppellF
1[8/3, 3/2, 1/2, 11/3, -((a + b*Sec[c + d*x])/(-a + 1/Sqrt[b^(-2)])), (a +
b*Sec[c + d*x])/(a + 1/Sqrt[b^(-2)])]*Sec[c + d*x]*Tan[c + d*x])/(16*(-a +
1/Sqrt[b^(-2)])))))/(5*b*(b + a*Cos[c + d*x])^(1/3)*Sqrt[1 - Cos[c + d*x]^2
]*Sec[c + d*x]^(4/3)))
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^2 \frac{1}{\sqrt[3]{a + b \sec (dx + c)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^2}{(b \sec (dx + c) + a)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (dx + c)^2}{(b \sec (dx + c) + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(1/3), x)
```

3.702 $\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sqrt
[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.0757841, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sqrt
[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{a+bx}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= -\frac{\left(\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}$$

Mathematica [B] time = 1.74085, size = 310, normalized size = 2.95

$$\frac{15(a-b)^2(a+b)\cos(c+dx)\cot^3(c+dx)(\sec(c+dx)+1)(b-b\sec(c+dx))(a+b}{b^2d(b-a)\left(3(a-b)(a\cos(c+dx)+b)F_1\left(\frac{5}{3};\frac{1}{2},\frac{3}{2};\frac{8}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)+(a+b)\left(10(a-b)\cos(c+dx)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2},\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (15*(a - b)^2*(a + b)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[c
+ d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(2/3))/(b^2*(-a + b)*d*(3
*(a - b)*AppellF1[5/3, 1/2, 3/2, 8/3, (a + b*Sec[c + d*x])/(a - b), (a + b*
```

```
Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(10*(a - b)*AppellF1[
2/3, 1/2, 1/2, 5/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])/(a +
 b)]*Cos[c + d*x] + 3*AppellF1[5/3, 3/2, 1/2, 8/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int \sec (dx+c) \frac{1}{\sqrt [3]{a+b\sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)}{(b\sec (dx+c)+a)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec (dx+c)}{(b\sec (dx+c)+a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (c+dx)}{\sqrt[3]{a+b\sec (c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)}{(b\sec (dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(1/3), x)
```

3.703 $\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{\sqrt[3]{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(-1/3), x]
```

Rubi [A] time = 0.0107156, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-1/3),x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^(-1/3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 0.9116, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-1/3),x]

[Out] Integrate[(a + b*Sec[c + d*x])^(-1/3), x]
```

Maple [A] time = 0.114, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-1/3), x)
```

3.704 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=105

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sqrt
[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.0837088, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sqrt
[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx &= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{2/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\\
&= -\frac{\left(\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}\\
&= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}
\end{aligned}$$

Mathematica [B] time = 1.70522, size = 310, normalized size = 2.95

$$\frac{24(a-b)^2(a+b)\cos(c+dx)\cot^3(c+dx)(\sec(c+dx)+1)(b-b\sec(c+dx))\sqrt[3]{a+b}}{b^2d(b-a)\left(3(a-b)(a\cos(c+dx)+b)F_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};\frac{a+b\sec(c+dx)}{a-b},\frac{a+b\sec(c+dx)}{a+b}\right)+(a+b)\left(8(a-b)\cos(c+dx)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (24*(a - b)^2*(a + b)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a
- b), (a + b*Sec[c + d*x])/(a + b)]*Cos[c + d*x]*Cot[c + d*x]^3*(1 + Sec[c
+ d*x])*(b - b*Sec[c + d*x])*(a + b*Sec[c + d*x])^(1/3))/(b^2*(-a + b)*d*(3
*(a - b)*AppellF1[4/3, 1/2, 3/2, 7/3, (a + b*Sec[c + d*x])/(a - b), (a + b*
```

```
Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]) + (a + b)*(8*(a - b)*AppellF1[1
/3, 1/2, 1/2, 4/3, (a + b*Sec[c + d*x])/(a - b), (a + b*Sec[c + d*x])/(a +
b)]*Cos[c + d*x] + 3*AppellF1[4/3, 3/2, 1/2, 7/3, (a + b*Sec[c + d*x])/(a -
 b), (a + b*Sec[c + d*x])/(a + b)]*(b + a*Cos[c + d*x]))))
```

Maple [F] time = 0.126, size = 0, normalized size = 0.

$$\int \sec (dx + c) (a + b \sec (dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)}{(b \sec (dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (dx + c)}{(b \sec (dx + c) + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(2/3), x)
```

3.705 $\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{(a+b\sec(c+dx))^{2/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(-2/3), x]
```

Rubi [A] time = 0.0108436, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-2/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-2/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Mathematica [A] time = 0.904801, size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-2/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(-2/3), x]
```

Maple [A] time = 0.114, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-2/3), x)
```

3.706 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/((a + b
)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rubi [A] time = 0.0876113, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}\sqrt[3]{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 4/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/((a + b
)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3))
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{4/3}}\,dx &= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{4/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\\
&= -\frac{\left(\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}\,dx, x, \sec(c+dx)\right)}{(a+b)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}\\
&= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\tan(c+dx)}{(a+b)d\sqrt{1+\sec(c+dx)}\sqrt[3]{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 26.7753, size = 10343, normalized size = 94.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(4/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{2}{3}}\sec(dx+c)}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(2/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)}{(b \sec (dx + c) + a)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(4/3), x)
```

3.707 $\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{(a+b\sec(c+dx))^{4/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(-4/3), x]
```

Rubi [A] time = 0.0111019, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-4/3),x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^(-4/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx=\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Mathematica [A] time = 23.6927, size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^{4/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-4/3),x]

[Out] Integrate[(a + b*Sec[c + d*x])^(-4/3), x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int (a + b\sec(dx + c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(4/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx + c) + a)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\sec(c + dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-4/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-4/3), x)
```

3.708 $\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=378

$$-\frac{a\left(9a^2-7b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{2\sqrt{2}b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}+\frac{\left(-10a^2b^2+9a^4-b^4\right)\mathrm{t}}{2\sqrt{}}$$

```
[Out] (-3*a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(2/3)) + (3*(3*a^2 - b^2)*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*b^2*(
a^2 - b^2)*d) - (a*(9*a^2 - 7*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c
 + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[
c + d*x])/(2*Sqrt[2]*b^3*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^(1/3)) + ((9*a^4 - 10*a^2*b^2 - b^4)*AppellF1[1/2, 1/2, 2
/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[
c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(2*Sqrt[2]*b^3*(a^2 - b^2)*d*Sqrt[1
+ Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.560408, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3845, 4082, 4007, 3834, 139, 138}

$$-\frac{a\left(9a^2-7b^2\right)\tan(c+dx)\sqrt[3]{a+b\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{2\sqrt{2}b^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}+\frac{\left(-10a^2b^2+9a^4-b^4\right)\mathrm{t}}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```


```
[Out] (-3*a^2*Sec[c + d*x]*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(2/3)) + (3*(3*a^2 - b^2)*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*b^2*(
a^2 - b^2)*d) - (a*(9*a^2 - 7*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c
 + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[
c + d*x])/(2*Sqrt[2]*b^3*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c
 + d*x])/(a + b))^(1/3)) + ((9*a^4 - 10*a^2*b^2 - b^4)*AppellF1[1/2, 1/2, 2
/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[
c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(2*Sqrt[2]*b^3*(a^2 - b^2)*d*Sqrt[1
+ Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4082

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2))
, x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A
*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; Fr
eeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{3\int \frac{\sec(c+dx)\left(a^2-\frac{2}{3}ab\sec(c+dx)-\frac{2}{3}\left(3a^2-b^2\right)\sec^2(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{4b^2\left(a^2-b^2\right)d} - \frac{9\int}{}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{4b^2\left(a^2-b^2\right)d} - \frac{(a(}{}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{4b^2\left(a^2-b^2\right)d} + \frac{(a(}{}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{4b^2\left(a^2-b^2\right)d} + \frac{(a(}{}$$

$$= -\frac{3a^2\sec(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\left(3a^2-b^2\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)}{4b^2\left(a^2-b^2\right)d} - \frac{a(9}{}$$

Mathematica [B] time = 26.4083, size = 21987, normalized size = 58.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^4 (a + b \sec (dx + c))^{-\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec (dx + c) + a)^{\frac{1}{3}} \sec (dx + c)^4}{b^2 \sec (dx + c)^2 + 2 \, ab \sec (dx + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^4/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/3), x)
```

3.709 $\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=307

$$-\frac{a\left(3a^2-4b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}+\frac{\left(3a^2-2b^2\right)\tan(c+dx)\sqrt[3]{a+}}{\sqrt{2}b^2a}$$

```
[Out] (-3*a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) + ((3*
a^2 - 2*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Se
c[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(Sqrt[2]*b^2
*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
 - (a*(3*a^2 - 4*b^2)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b
*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c +
d*x])/(Sqrt[2]*b^2*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^(2/3))
```

Rubi [A] time = 0.380614, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3839, 4007, 3834, 139, 138}

$$-\frac{a\left(3a^2-4b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}+\frac{\left(3a^2-2b^2\right)\tan(c+dx)\sqrt[3]{a+}}{\sqrt{2}b^2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```


```
[Out] (-3*a^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) + ((3*
a^2 - 2*b^2)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Se
c[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(Sqrt[2]*b^2
*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3))
 - (a*(3*a^2 - 4*b^2)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b
*(1 - Sec[c + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c +
d*x])/(Sqrt[2]*b^2*(a^2 - b^2)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x]
)^(2/3))
```


Rule 3839

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m
+ 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a
 + b*Csc[e + f*x])^(m + 1)*Simp[a*b*(m + 1) - (a^2 + b^2*(m + 1))*Csc[e + f
*x], x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1
]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{3\int \frac{\sec(c+dx)\left(-\frac{2ab}{3}-\frac{1}{3}\left(3a^2-2b^2\right)\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{\left(a\left(3a^2-4b^2\right)\right)\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2b^2\left(a^2-b^2\right)} + \frac{\left(3a^2-2b^2\right)\int}{}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{\left(a\left(3a^2-4b^2\right)\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx}\right.}{2b^2\left(a^2-b^2\right)d\sqrt{1-\sec(c+dx)}\sqrt{1+\text{se}}}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{\left(\left(3a^2-2b^2\right)\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\text{Subst}}{2b^2\left(a^2-b^2\right)d\sqrt{1-\sec(c+dx)}\sqrt{1+\text{sec}}}$$

$$= -\frac{3a^2\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{\left(3a^2-2b^2\right)F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\text{s}}{}\right.}{\sqrt{2}b^2\left(a^2-b^2\right)d\sqrt{1+\sec(c+}}$$

Mathematica [B] time = 26.2113, size = 19126, normalized size = 62.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.111, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^3 (a+b\sec(dx+c))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^3}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^3/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^3(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/3), x)
```

3.710 $\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=289

$$\frac{\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}-\frac{a\tan(c+dx)\sqrt[3]{a+b\sec(c+dx}}{\sqrt{2}bd\left(a^2-b^2\right.}$$

```
[Out] (3*a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) - (a*Appell
F1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b
)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(Sqrt[2]*b*(a^2 - b^2)*d*Sqrt[1
 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + ((a^2 - 2*b^2)*App
ellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(Sqrt[2]*b*(a^2 - b
^2)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.35933, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3836, 4007, 3834, 139, 138}

$$\frac{\left(a^2-2b^2\right)\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{\sqrt{2}bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}-\frac{a\tan(c+dx)\sqrt[3]{a+b\sec(c+dx}}{\sqrt{2}bd\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```


```
[Out] (3*a*Tan[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(2/3)) - (a*Appell
F1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b
)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(Sqrt[2]*b*(a^2 - b^2)*d*Sqrt[1
 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + ((a^2 - 2*b^2)*App
ellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a +
 b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(Sqrt[2]*b*(a^2 - b
^2)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```


Rule 3836

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(a*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*
```

```
(a^2 - b^2)), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx = \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{3\int \frac{\sec(c+dx)\left(-\frac{2b}{3}-\frac{1}{3}a\sec(c+dx)\right)}{(a+b\sec(c+dx))^{2/3}}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{a\int \sec(c+dx)\sqrt[3]{a+b\sec(c+dx)}\,dx}{2b\left(a^2-b^2\right)} + \frac{\left(a^2-2b^2\right)\int}{2b}$$

$$= \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{(a\tan(c+dx))\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(c+dx)\right.}{2b\left(a^2-b^2\right)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}$$

$$= \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} + \frac{\left(a\sqrt[3]{a+b\sec(c+dx)}\tan(c+dx)\right)\operatorname{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-}}{\sqrt{1-x}\sqrt{}}\right.}{2b\left(a^2-b^2\right)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+d}}$$

$$= \frac{3a\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^{2/3}} - \frac{aF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a}}{\sqrt{2}b\left(a^2-b^2\right)d\sqrt{1+\sec(c+dx)}\sqrt[3]{}}$$

Mathematica [B] time = 26.2453, size = 7325, normalized size = 25.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^2 (a+b\sec(dx+c))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)^2}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)^2/(b^2*sec(d*x + c)^2 + 2*
a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/3), x)
```

3.711 $\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/((a + b
)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```

Rubi [A] time = 0.0865168, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(c+dx)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}(a+b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
```


```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 5/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c +
 d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/((a + b
)*d*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3))
```


Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{5/3}}\,dx &= -\frac{\tan(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}(a+bx)^{5/3}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}} \\
&= -\frac{\left(\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\sqrt{1+x}\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/3}}\,dx, x, \sec(c+dx)\right)}{(a+b)d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}} \\
&= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{(a+b)d\sqrt{1+\sec(c+dx)}(a+b\sec(c+dx))^{2/3}}
\end{aligned}
$$

Mathematica [B] time = 26.7381, size = 10363, normalized size = 94.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sec[c + d*x])^(5/3),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(a+b\sec(dx+c))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{1}{3}}\sec(dx+c)}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(1/3)*sec(d*x + c)/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sec(c + d*x))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*sec(d*x + c) + a)^(5/3), x)
```

3.712 $\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{(a+b\sec(c+dx))^{5/3}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(-5/3), x]
```

Rubi [A] time = 0.0114356, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(-5/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(-5/3), x]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx = \int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Mathematica [A] time = 19.2178, size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^{5/3}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(-5/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(-5/3), x]
```

Maple [A] time = 0.086, size = 0, normalized size = 0.

$$\int (a + b \sec (dx + c))^{-\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^(5/3),x)
```

```
[Out] int(1/(a+b*sec(d*x+c))^(5/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec (dx + c) + a)^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \sec (c + dx))^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**(5/3),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**(-5/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(-5/3), x)
```

3.713 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+a\right.}{d\left(a^2-b^2\right)}$$

```
[Out] (a*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3
)) - (b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b
^2)*d)
```

Rubi [A] time = 0.257051, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+a\right.}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]
```



```
[Out] (a*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3
)) - (b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b
^2)*d)
```


Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx &= \left(\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\sqrt[3]{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx \\
&= -\left(\left(a\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right)+\left(b\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \\
&= -\frac{a\,\text{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}}+\frac{\left(b\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{1-x^2}(-}\right.}{d} \\
&= \frac{aF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}}-\frac{bF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right.}{\left(a^2-\right.}
\end{aligned}
$$

Mathematica [B] time = 21.3229, size = 4543, normalized size = 26.11

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x]),x]

[Out] (9*(a^2 - b^2)*Sec[c + d*x]^(5/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/6, 1, 3/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])
/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -T
an[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*Appe
llF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/
(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2
, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(d*
(Sec[c + d*x]^2)^(2/3)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c + d*x]^2)*((9
(a^2 - b^2)(Sec[c + d*x]^2)^(1/3)*((b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 -
b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d
*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]
^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2
/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 -
b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c +
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Se
c[c + d*x]^2) - (18*b^2*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*Tan[c + d*x]^2*(
(b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[
c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2
, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*Ap
pellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan
[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[
c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3,
 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2
)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b
^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Sec[c + d*x]^2)^2 - (12*(a^2 - b^2)*Tan
[c + d*x]^2*((b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6,
1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF
1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-
a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)
/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x
]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3,

1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*Appel
lF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] +
2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x
]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^(2/3)*(-a^2 + b^2*Se
c[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((b*AppellF1[1/2, 1/6, 1, 3/2,
 -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2]*Ta
n[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2
*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c +
d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6,
 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2
) + (b*Sqrt[Sec[c + d*x]^2]*((2*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x
]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2
 - b^2)) - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2
)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2
, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*
AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2
)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*((2*b^2*AppellF1[3/2, 2/3, 2, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c
+ d*x])/(3*(a^2 - b^2)) - (4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b
^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 -
 b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) - (b*AppellF1[1
/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Se
c[c + d*x]^2]*(2*(6*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Ta
n[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c
 + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Sec[c + d*x]^2*Tan[c + d*x] +
 9*(a^2 - b^2)*((2*b^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan
[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (A
ppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)
]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(6*b^2*((12*b^2*AppellF1
[5/2, 1/6, 3, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c
 + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (AppellF1[5/2, 7/6, 2, 7/2, -Tan[
c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/
5) + (-a^2 + b^2)*((6*b^2*AppellF1[5/2, 7/6, 2, 7/2, -Tan[c + d*x]^2, (b^2*
Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) -
 (7*AppellF1[5/2, 13/6, 1, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 - b^2)*AppellF1[1/2, 1/6
, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (6*b^2*Appel
lF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] +
(-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^
2)/(a^2 - b^2)])*Tan[c + d*x]^2)^2 - (a*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c +
 d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*(-4*(3*b^2*AppellF1[3/2, 2/3, 2,

```
 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*A
ppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)
])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2 - b^2)*((2*b^2*AppellF1[3/2, 2/3, 2
, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Ta
n[c + d*x])/(3*(a^2 - b^2)) - (4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2
, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9) - 2*Tan
[c + d*x]^2*(3*b^2*((12*b^2*AppellF1[5/2, 2/3, 3, 7/2, -Tan[c + d*x]^2, (b^
2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2))
 - (4*AppellF1[5/2, 5/3, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/5) + 2*(-a^2 + b^2)*((6*b^2*AppellF1[
5/2, 5/3, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c
+ d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - 2*AppellF1[5/2, 8/3, 1, 7/2, -Tan[
c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x]))
))/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)] - 2*(3*b^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2
, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(-a^2 + b^2)*AppellF1[3/2, 5/3, 1,
5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)^2)
)/((Sec[c + d*x]^2)^(2/3)*(-a^2 + b^2*Sec[c + d*x]^2))))
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sec(dx+c)}(\sec(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(2/3)/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a), x)
```

3.714 $\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)}$$

```
[Out] (a*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3
)) - (b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b
^2)*d)
```

Rubi [A] time = 0.262347, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3
)) - (b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b
^2)*d)
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx = \left(\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{2}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\right)\int$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)} + \frac{\left(b\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{}{\sqrt[6]{1-x^2}(-}\right.}{d}$$

$$= \frac{aF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)} - \frac{bF_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right.}{\left(a^2\right.}$$

Mathematica [B] time = 21.3342, size = 4544, normalized size = 26.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sec[c + d*x]^(4/3)*Sin[c + d*x]*((b*AppellF1[1/2, 1/3, 1, 3/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])
/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (
b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2,
-Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*Ap
pellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
)/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2,
(b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(d
*(Sec[c + d*x]^2)^(5/6)*(a + b*Sec[c + d*x])*(-a^2 + b^2*Sec[c + d*x]^2)*((
9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 -
 b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
+ d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d
*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2
, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2
 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a
^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c
 + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*
Sec[c + d*x]^2) - (18*b^2*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*Tan[c + d*x]^2
*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Ta
n[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/
3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2
)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b
^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2
*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -
Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5
/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b
^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2
- b^2)])*Tan[c + d*x]^2)))/(-a^2 + b^2*Sec[c + d*x]^2)^2 - (15*(a^2 - b^2)*
Tan[c + d*x]^2*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c +
 d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/
3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*Ap
pellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]
 + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c
+ d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])/(-9*(a^2 - b^2)*AppellF1[1/2,
```

5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^(5/6)*(-a^2 + b^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (b*Sqrt[Sec[c + d*x]^2]*((2*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) + (a*((2*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (5*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2) - (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sec[c + d*x]^2]*(4*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Sec[c + d*x]^2*Tan[c + d*x] + 9*(a^2 - b^2)*((2*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/9) + 2*Tan[c + d*x]^2*(3*b^2*((12*b^2*AppellF1[5/2, 1/3, 3, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/5) + (-a^2 + b^2)*((6*b^2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (8*AppellF1[5/2, 7/3, 1, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 2*(3*b^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*Tan[c + d*x]^2)^2 - (a*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*(2*(-6*b^2*Appe

```
llF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] +
 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*
x]^2)/(a^2 - b^2)])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2 - b^2)*((2*b^2*App
ellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*
Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (5*AppellF1[3/2, 11/6, 1, 5/
2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c
+ d*x])/9) + Tan[c + d*x]^2*(-6*b^2*((12*b^2*AppellF1[5/2, 5/6, 3, 7/2, -Ta
n[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x]
)/(5*(a^2 - b^2)) - AppellF1[5/2, 11/6, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c
 + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2*Tan[c + d*x]) + 5*(a^2 - b^2)*((6*b^
2*AppellF1[5/2, 11/6, 2, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 -
b^2)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (11*AppellF1[5/2, 17/6
, 1, 7/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)]*Sec[c + d*x]^2
*Tan[c + d*x])/5))))/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*
x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + (-6*b^2*AppellF1[3/2, 5/6, 2, 5/2
, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)] + 5*(a^2 - b^2)*Appell
F1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, (b^2*Tan[c + d*x]^2)/(a^2 - b^2)])*T
an[c + d*x]^2)^2))/((Sec[c + d*x]^2)^(5/6)*(-a^2 + b^2*Sec[c + d*x]^2))))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sec\left(dx+c\right)}\sqrt[3]{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{b\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a), x)
```

3.715 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=174

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{s}}$$

```
[Out] -((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1
/3))) + (a*AppellF1[1/2, -2/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3)*Sin[c + d*x])/((a^2
 - b^2)*d)
```

Rubi [A] time = 0.240486, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{s}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1
/3))) + (a*AppellF1[1/2, -2/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2/3)*Sin[c + d*x])/((a^2
 - b^2)*d)
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))}\,dx = \left(\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{4}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\cos^{\frac{2}{3}}(c+dx)\sec^{\frac{2}{3}}(c+dx)\right)\int \frac{\cos^{\frac{7}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\cos^{\frac{2}{3}}(c+dx)\,\text{s}\right.$$

$$= \frac{b\,\text{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}} - \frac{\left(a\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)\right)\text{Sub}}{a}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}} + \frac{aF_1\left(\frac{1}{2};-\frac{2}{3},1;\frac{3}{2};\sin^2(c\right.}{}$$

Mathematica [B] time = 28.4726, size = 7542, normalized size = 43.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sec\left(dx+c\right)}\frac{1}{\sqrt[3]{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

3.716 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=174

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}F_1\left(\frac{1}{2};-\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}\sec}$$

```
[Out] -((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2
/3))) + (a*AppellF1[1/2, -5/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3)*Sin[c + d*x])/((a^2
 - b^2)*d)
```

Rubi [A] time = 0.240166, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}F_1\left(\frac{1}{2};-\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}\sec}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```


```
[Out] -((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)]*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)^(1/3)*Sec[c + d*x]^(2
/3))) + (a*AppellF1[1/2, -5/6, 1, 3/2, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*(Cos[c + d*x]^2)^(1/6)*Sec[c + d*x]^(1/3)*Sin[c + d*x])/((a^2
 - b^2)*d)
```


Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{3}}(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\left(\left(a\sqrt[3]{\cos(c+dx)}\sqrt[3]{\sec(c+dx)}\right)\int \frac{\cos^{\frac{8}{3}}(c+dx)}{b^2-a^2\cos^2(c+dx)}\,dx\right) + \left(b\sqrt[3]{\cos(c+dx)}\sqrt[3]{\right.$$

$$= \frac{b\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{-a^2+b^2+a^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)} - \frac{\left(a\sqrt[6]{\cos^2(c+dx)}\sqrt[3]{\sec(c+dx)}\right)\mathrm{Subs}}{d}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[3]{\cos^2(c+dx)}\sec^{\frac{2}{3}}(c+dx)} + \frac{aF_1\left(\frac{1}{2};-\frac{5}{6},1;\frac{3}{2};\sin^2(c\right.}{}$$

Mathematica [B] time = 28.2712, size = 7588, normalized size = 43.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \frac{1}{a + b\sec(dx + c)} (\sec(dx + c))^{-\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx + c) + a)\sec(dx + c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

3.717 $\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0521502, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 32.2302, size = 0, normalized size = 0.

$$\int \sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.225, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{7}{3}} \sqrt{a+b\sec (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{7}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3), x)
```

3.718 $\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0521657, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 42.9295, size = 0, normalized size = 0.

$$\int \sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.251, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{5}{3}} \sqrt{a+b\sec (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{5}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3), x)
```

3.719 $\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0516693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 27.3747, size = 0, normalized size = 0.

$$\int \sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.219, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{\frac{4}{3}} \sqrt{a + b \sec (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3), x)
```

3.720 $\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0522901, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx=\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 33.285, size = 0, normalized size = 0.

$$\int \sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.254, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{2}{3}} \sqrt{a+b\sec (dx+c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)

[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec (c+dx)}\sec^{\frac{2}{3}}(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

3.721 $\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0525696, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx=\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

Mathematica [A] time = 2.24048, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.38, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec(dx+c)}\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(c+dx)}\sqrt[3]{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{1}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

3.722 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

Rubi [A] time = 0.052302, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]

[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A] time = 7.81607, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3),x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(1/3), x]
```

Maple [A] time = 0.308, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec\left(dx+c\right)}\frac{1}{\sqrt[3]{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sec(c + dx)}}{\sqrt[3]{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/3),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(1/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx + c) + a}}{\sec(dx + c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(1/3), x)
```

3.723 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

Rubi [A] time = 0.0520735, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
```

```
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 15.3567, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3),x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(2/3), x]
```

Maple [A] time = 0.342, size = 0, normalized size = 0.

$$\int \sqrt{a + b\sec(dx + c)}\,(\sec(dx + c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx + c) + a}}{\sec(dx + c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx + c) + a}}{\sec(dx + c)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(2/3),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(2/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(2/3), x)
```

3.724 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

Rubi [A] time = 0.0505842, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
```

```
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 20.7091, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3),x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(4/3), x]
```

Maple [A] time = 0.296, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec\left(dx+c\right)}\,\left(\sec\left(dx+c\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b\sec(c + dx)}}{\sec^{\frac{4}{3}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(4/3),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(4/3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx + c) + a}}{\sec(dx + c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(4/3), x)
```

3.725 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

Rubi [A] time = 0.0505002, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
```

```
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 29.2934, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3),x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(5/3), x]
```

Maple [A] time = 0.204, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec\left(dx+c\right)}\left(\sec\left(dx+c\right)\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{5}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/3), x)
```

3.726 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

Rubi [A] time = 0.0506645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]

[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 35.562, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3),x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sec[c + d*x]^(7/3), x]
```

Maple [A] time = 0.21, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(dx+c)}\,(\sec(dx+c))^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx+c)+a}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/3), x)
```

3.727 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0591549, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 34.3412, size = 0, normalized size = 0.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.202, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{\frac{7}{3}} \left(a+b\sec \left(dx+c\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(dx+c\right)+a\right)^{\frac{3}{2}} \sec \left(dx+c\right)^{\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(dx+c\right)^{3}+a\sec \left(dx+c\right)^{2}\right)\sqrt{b\sec \left(dx+c\right)+a}\sec \left(dx+c\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^3 + a*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*sec
(d*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3), x)
```

3.728 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0598057, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]

[Out] Defer[Int][Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 37.2151, size = 0, normalized size = 0.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2),x]

[Out] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.215, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{5}{3}} (a+b\sec (dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec (dx+c)^{2}+a\sec (dx+c)\right)\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(d
*x + c)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3), x)
```

3.729 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.05696, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx=\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 31.3189, size = 0, normalized size = 0.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.207, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{\frac{4}{3}} (a + b \sec (dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{3}{2}} \sec (dx + c)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sec (dx + c)^2 + a \sec (dx + c)\right) \sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c)^2 + a*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sec(d
*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{3}{2}} \sec (dx + c)^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3), x)
```

3.730 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0567228, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 36.4581, size = 0, normalized size = 0.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.203, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{\frac{2}{3}} (a + b \sec (dx + c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{3}{2}} \sec (dx + c)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec (dx + c) + a)^{\frac{3}{2}} \sec (dx + c)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3), x)
```

3.731 $\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0573267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx=\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

Mathematica [A] time = 27.7259, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.193, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec (dx+c)}\,(a+b\sec (dx+c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{3}{2}}\sec (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec (dx+c)+a)^{\frac{3}{2}}\sec (dx+c)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{3}{2}}\sec (dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3), x)
```

3.732 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

Rubi [A] time = 0.0579205, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A] time = 37.1165, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(1/3), x]
```

Maple [A] time = 0.192, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{3}{2}} \frac{1}{\sqrt[3]{\sec(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(1/3), x)
```

3.733 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

Rubi [A] time = 0.0572684, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 21.1343, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(2/3), x]
```

Maple [A] time = 0.3, size = 0, normalized size = 0.

$$\int (a + b \sec(dx + c))^{\frac{3}{2}} (\sec(dx + c))^{-\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b \sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(2/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(2/3), x)
```

3.734 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

Rubi [A] time = 0.0574033, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 25.8837, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(4/3), x]
```

Maple [A] time = 0.232, size = 0, normalized size = 0.

$$\int (a + b\sec(dx + c))^{\frac{3}{2}} (\sec(dx + c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\sec(dx + c) + a)^{\frac{3}{2}}}{\sec(dx + c)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(4/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(4/3), x)
```

3.735 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

Rubi [A] time = 0.0565043, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 27.6315, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(5/3), x]
```

Maple [A] time = 0.195, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(\sec\left(dx+c\right)\right)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sec\left(dx+c\right)^{\frac{5}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(5/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec (dx + c) + a)^{\frac{3}{2}}}{\sec (dx + c)^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/3), x)
```

3.736 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3), x]
```

Rubi [A] time = 0.05757, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 34.1727, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sec[c + d*x]^(7/3), x]
```

Maple [A] time = 0.205, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{3}{2}} (\sec(dx+c))^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(7/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/3), x)
```

3.737 $\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.0560519, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 37.8797, size = 0, normalized size = 0.

$$\int \sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.205, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{\frac{7}{3}} \left(a+b\sec \left(dx+c\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int {\left(b \sec \left(d x+c\right)+a\right)}^{\frac{5}{2}} \sec \left(d x+c\right)^{\frac{7}{3}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(b^{2} \sec \left(d x+c\right)^{4}+2 \, a b \sec \left(d x+c\right)^{3}+a^{2} \sec \left(d x+c\right)^{2}\right)} \sqrt{b \sec \left(d x+c\right)+a} \sec \left(d x+c\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^4 + 2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2)*s
qrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3), x)
```

3.738 $\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.0563213, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]

[Out] Defer[Int][Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 42.9066, size = 0, normalized size = 0.

$$\int \sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2),x]

[Out] Integrate[Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.222, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{\frac{5}{3}} (a + b \sec (dx + c))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{5}{2}} \sec (dx + c)^{\frac{5}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \sec (dx + c)^3 + 2\, ab \sec (dx + c)^2 + a^2 \sec (dx + c)\right)\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3), x)
```

3.739 $\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.0565535, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 37.1492, size = 0, normalized size = 0.

$$\int \sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.201, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{4}{3}} (a+b\sec (dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\sec (dx+c)^3+2\,ab\sec (dx+c)^2+a^2\sec (dx+c)\right)\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^3 + 2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c))*sqr
t(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{4}{3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3), x)
```

3.740 $\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.0561414, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx=\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 41.2232, size = 0, normalized size = 0.

$$\int \sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.222, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{2}{3}} (a+b\sec (dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec (dx+c)^2+2\,ab\sec (dx+c)+a^2\right)\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)*sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3), x)
```

3.741 $\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.0553584, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx=\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

Mathematica [A] time = 30.6608, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.209, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec (dx+c)}\, (a+b\sec (dx+c))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}}\sec (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec (dx+c)^2+2\,ab\sec (dx+c)+a^2\right)\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)*sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3), x)
```

3.742 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3), x]
```

Rubi [A] time = 0.0576654, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx=\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Mathematica [A] time = 41.3904, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3),x]

[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(1/3), x]
```

Maple [A] time = 0.203, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{5}{2}} \frac{1}{\sqrt[3]{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(1/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}}{\sec(dx+c)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(1/3), x)
```

3.743 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

Rubi [A] time = 0.0574173, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 31.7981, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(2/3), x]
```

Maple [A] time = 0.199, size = 0, normalized size = 0.

$$\int (a + b \sec(dx + c))^{\frac{5}{2}} (\sec(dx + c))^{-\frac{2}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(2/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{2} \sec(dx + c)^{2} + 2\,ab \sec(dx + c) + a^{2}\right)\sqrt{b \sec(dx + c) + a}}{\sec(dx + c)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(2/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(2/3), x)
```

3.744 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

Rubi [A] time = 0.0561252, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 37.2591, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{4}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(4/3), x]
```

Maple [A] time = 0.195, size = 0, normalized size = 0.

$$\int (a + b\sec(dx + c))^{\frac{5}{2}} (\sec(dx + c))^{-\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx + c) + a)^{\frac{5}{2}}}{\sec(dx + c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(4/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\sec(dx + c)^2 + 2\,ab\sec(dx + c) + a^2\right)\sqrt{b\sec(dx + c) + a}}{\sec(dx + c)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(4/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(4/3), x)
```

3.745 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3), x]
```

Rubi [A] time = 0.0558461, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 31.17, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(5/3), x]
```

Maple [A] time = 0.194, size = 0, normalized size = 0.

$$\int (a+b\sec (dx+c))^{\frac{5}{2}} (\sec (dx+c))^{-\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec (dx+c)+a)^{\frac{5}{2}}}{\sec (dx+c)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{2} \sec (dx+c)^{2}+2\, ab \sec (dx+c)+a^{2}\right)\sqrt{b\sec (dx+c)+a}}{\sec (dx+c)^{\frac{5}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(5/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(5/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/3), x)
```

3.746 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3), x]
```

Rubi [A] time = 0.0576089, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx = \int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Mathematica [A] time = 37.6343, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{3}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sec[c + d*x]^(7/3), x]
```

Maple [A] time = 0.19, size = 0, normalized size = 0.

$$\int \left(a+b\sec \left(dx+c\right)\right)^{\frac{5}{2}} \left(\sec \left(dx+c\right)\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

```
[Out] int((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\sec \left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec \left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2\sec \left(dx+c\right)^2+2\,ab\sec \left(dx+c\right)+a^2\right)\sqrt{b\sec \left(dx+c\right)+a}}{\sec \left(dx+c\right)^{\frac{7}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)/sec(d*x + c)^(7/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(7/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/3), x)
```

3.747 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0520441, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 27.2099, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(7/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.201, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{\frac{7}{3}} \frac{1}{\sqrt{a + b \sec (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^{\frac{7}{3}}}{\sqrt{b \sec (dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec (dx + c)^{\frac{7}{3}}}{\sqrt{b \sec (dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.748 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0522877, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 30.992, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(5/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.211, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{\frac{5}{3}} \frac{1}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{5}{3}}}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^{\frac{5}{3}}}{\sqrt{b \sec (dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.749 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0518766, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 1.56019, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(4/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.266, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{4}{3}} \frac{1}{\sqrt{a+b\sec (dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{4}{3}}}{\sqrt{b\sec (dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec (dx+c)^{\frac{4}{3}}}{\sqrt{b\sec (dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{4}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.750 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0523929, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 1.77153, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(2/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.277, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{\frac{2}{3}} \frac{1}{\sqrt{a + b \sec (dx + c)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^{\frac{2}{3}}}{\sqrt{b \sec (dx + c) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec (dx + c)^{\frac{2}{3}}}{\sqrt{b \sec (dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(2/3)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.751 $\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0520848, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 1.21617, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[Sec[c + d*x]^(1/3)/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.278, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec\left(dx+c\right)}\frac{1}{\sqrt{a+b\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{1}{3}}}{\sqrt{b\sec\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**(1/3)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/sqrt(b*sec(d*x + c) + a), x)
```

3.752 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [A] time = 0.0531695, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 2.80306, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(1/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(dx+c)}}\frac{1}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}}{b\sec(dx+c)^{2}+a\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^2 + a*
sec(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sec(c + dx)}\sqrt[3]{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(1/3)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx + c) + a}\sec(dx + c)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)), x)
```

3.753 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [A] time = 0.0522012, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]

[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 21.9496, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A] time = 0.316, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{-\frac{2}{3}} \frac{1}{\sqrt{a+b\sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{1}{3}}}{b\sec (dx+c)^2+a\sec (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^2 + a*
sec(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sec(c+dx)}\sec^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(2/3)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)), x)
```

3.754 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [A] time = 0.0528158, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 27.0593, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A] time = 0.255, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{-\frac{4}{3}} \frac{1}{\sqrt{a + b \sec (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{2}{3}}}{b \sec (dx + c)^{3} + a \sec (dx + c)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^3 + a*
sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)), x)
```

3.755 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [A] time = 0.051882, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 30.2649, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A] time = 0.204, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{-\frac{5}{3}} \frac{1}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{1}{3}}}{b\sec(dx+c)^3+a\sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b*sec(d*x + c)^3 + a*
sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)), x)
```

3.756 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi [A] time = 0.0535342, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 36.8311, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(7/3)*Sqrt[a + b*Sec[c + d*x]]), x]
```

Maple [A] time = 0.196, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{-\frac{7}{3}} \frac{1}{\sqrt{a + b \sec (dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{2}{3}}}{b \sec (dx + c)^{4} + a \sec (dx + c)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b*sec(d*x + c)^4 + a*
sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)), x)
```

3.757 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0592837, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 29.5773, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.203, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{7}{3}} (a+b\sec (dx+c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{7}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{7}{3}}}{b^2\sec (dx+c)^2+2\,ab\sec (dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.758 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0588555, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 35.9752, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.191, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{5}{3}} (a+b\sec (dx+c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{5}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\,\sec (dx+c)^{\frac{5}{3}}}{b^2\sec (dx+c)^2+2\,ab\sec (dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.759 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0586029, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 30.631, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.207, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{4}{3}} (a+b\sec (dx+c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{4}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{4}{3}}}{b^2\sec (dx+c)^2+2\,ab\sec (dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{4}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.760 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0582848, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]

[Out] Defer[Int][Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 37.6154, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.193, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{\frac{2}{3}} (a+b\sec(dx+c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,\sec(dx+c)^{\frac{2}{3}}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{2}{3}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.761 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0591886, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 31.3629, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.204, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec (dx+c)}\, (a+b\sec (dx+c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{1}{3}}}{(b\sec (dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\,\sec (dx+c)^{\frac{1}{3}}}{b^2\sec (dx+c)^2+2\,ab\sec (dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**(1/3)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.762 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [A] time = 0.0579604, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 37.2718, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A] time = 0.194, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec \left(dx+c\right)}}\left(a+b\sec \left(dx+c\right)\right)^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec \left(dx+c\right)+a\right)^{\frac{3}{2}}\sec \left(dx+c\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec \left(dx+c\right)+a}\sec \left(dx+c\right)^{\frac{2}{3}}}{b^{2}\sec \left(dx+c\right)^{3}+2\,ab\sec \left(dx+c\right)^{2}+a^{2}\sec \left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^3 +
2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt[3]{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(1/3)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(1/3)), x)
```

3.763 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [A] time = 0.0597639, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 35.0197, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A] time = 0.26, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{-\frac{2}{3}} (a+b\sec (dx+c))^{-\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{1}{3}}}{b^2\sec (dx+c)^3+2\,ab\sec (dx+c)^2+a^2\sec (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^3 +
2*a*b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(2/3)), x)
```

3.764 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [A] time = 0.0596298, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 41.6404, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A] time = 0.195, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{-\frac{4}{3}} (a+b\sec (dx+c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec (dx+c)+a)^{\frac{3}{2}}\sec (dx+c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}}{b^2\sec (dx+c)^4+2\,ab\sec (dx+c)^3+a^2\sec (dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^4 +
2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(4/3)), x)
```

3.765 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [A] time = 0.0592022, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 15.4513, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A] time = 0.212, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{-\frac{5}{3}} (a + b \sec (dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec (dx + c) + a)^{\frac{3}{2}} \sec (dx + c)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{1}{3}}}{b^2 \sec (dx + c)^4 + 2\, ab \sec (dx + c)^3 + a^2 \sec (dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^2*sec(d*x + c)^4 +
2*a*b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/3)), x)
```

3.766 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi [A] time = 0.0594766, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx=\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 43.2596, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(3/2)), x]
```

Maple [A] time = 0.198, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{-\frac{7}{3}} (a+b\sec (dx+c))^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec (dx+c)+a)^{\frac{3}{2}} \sec (dx+c)^{\frac{7}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}}{b^2\sec (dx+c)^5+2\,ab\sec (dx+c)^4+a^2\sec (dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^2*sec(d*x + c)^5 +
2*a*b*sec(d*x + c)^4 + a^2*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/3)), x)
```

3.767 $\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

[Out] Unintegrable[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]

Rubi [A] time = 0.0590322, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]

[Out] Defer[Int][Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]

Rubi steps

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 34.6178, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{7}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2),x]

```
[Out] Integrate[Sec[c + d*x]^(7/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.202, size = 0, normalized size = 0.

$$\int \left(\sec \left(dx+c\right)\right)^{\frac{7}{3}} \left(a+b\sec \left(dx+c\right)\right)^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec \left(dx+c\right)^{\frac{7}{3}}}{\left(b\sec \left(dx+c\right)+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec \left(dx+c\right)+a}\sec \left(dx+c\right)^{\frac{7}{3}}}{b^{3}\sec \left(dx+c\right)^{3}+3\,ab^{2}\sec \left(dx+c\right)^{2}+3\,a^{2}b\sec \left(dx+c\right)+a^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(7/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.768 $\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.0587356, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 41.9273, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{5}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(5/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.23, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{5}{3}} (a+b\sec (dx+c))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{5}{3}}}{(b\sec (dx+c)+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{5}{3}}}{b^3\sec (dx+c)^3+3\,ab^2\sec (dx+c)^2+3\,a^2b\sec (dx+c)+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{5}{3}}}{(b\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.769 $\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.059279, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 33.9085, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{4}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(4/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.202, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{\frac{4}{3}} (a+b\sec (dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{4}{3}}}{(b\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\,\sec (dx+c)^{\frac{4}{3}}}{b^3\sec (dx+c)^3+3\,ab^2\sec (dx+c)^2+3\,a^2b\sec (dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(4/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{4}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(4/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.770 $\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.059908, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 38.8191, size = 0, normalized size = 0.

$$\int \frac{\sec^{\frac{2}{3}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(2/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.219, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{\frac{2}{3}} (a + b\sec (dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^{\frac{2}{3}}}{(b\sec (dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx + c) + a}\sec (dx + c)^{\frac{2}{3}}}{b^3\sec (dx + c)^3 + 3\,ab^2\sec (dx + c)^2 + 3\,a^2b\sec (dx + c) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{2}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(2/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.771 $\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi [A] time = 0.058759, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 38.014, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Integrate[Sec[c + d*x]^(1/3)/(a + b*Sec[c + d*x])^(5/2), x]
```

Maple [A] time = 0.209, size = 0, normalized size = 0.

$$\int \sqrt[3]{\sec (dx+c)}\,(a+b\sec (dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{1}{3}}}{(b\sec (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\,\sec (dx+c)^{\frac{1}{3}}}{b^3\sec (dx+c)^3+3\,ab^2\sec (dx+c)^2+3\,a^2b\sec (dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{1}{3}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(1/3)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.772 $\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [A] time = 0.0600129, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 43.5595, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(1/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A] time = 0.251, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\sec(dx+c)}}(a+b\sec(dx+c))^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{2}{3}}}{b^3\sec(dx+c)^4+3ab^2\sec(dx+c)^3+3a^2b\sec(dx+c)^2+a^3\sec(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^4 +
3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(1/3)), x)
```

3.773 $\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [A] time = 0.0593236, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 42.7313, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{2}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(2/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A] time = 0.204, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{-\frac{2}{3}} (a+b\sec (dx+c))^{-\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec (dx+c)+a)^{\frac{5}{2}}\sec (dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{1}{3}}}{b^3\sec (dx+c)^4+3\,ab^2\sec (dx+c)^3+3\,a^2b\sec (dx+c)^2+a^3\sec (dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^4 +
3*a*b^2*sec(d*x + c)^3 + 3*a^2*b*sec(d*x + c)^2 + a^3*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(2/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(2/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(2/3)), x)
```

3.774 $\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [A] time = 0.0598054, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 50.09, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{4}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(4/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A] time = 0.195, size = 0, normalized size = 0.

$$\int (\sec (dx + c))^{-\frac{4}{3}} (a + b \sec (dx + c))^{-\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec (dx + c) + a)^{\frac{5}{2}} \sec (dx + c)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec (dx + c) + a} \sec (dx + c)^{\frac{2}{3}}}{b^3 \sec (dx + c)^5 + 3 \, ab^2 \sec (dx + c)^4 + 3 \, a^2 b \sec (dx + c)^3 + a^3 \sec (dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^5 +
3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(4/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(4/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(4/3)), x)
```

3.775 $\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [A] time = 0.0592679, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]

[Out] Defer[Int][1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 19.5992, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{5}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(5/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A] time = 0.204, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^{-\frac{5}{3}} (a+b\sec(dx+c))^{-\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}} \sec(dx+c)^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sec(dx+c)^{\frac{1}{3}}}{b^3\sec(dx+c)^5+3\,ab^2\sec(dx+c)^4+3\,a^2b\sec(dx+c)^3+a^3\sec(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(1/3)/(b^3*sec(d*x + c)^5 +
3*a*b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/3)), x)
```

3.776 $\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}},x\right)$$

```
[Out] Unintegrable[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi [A] time = 0.0596412, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Defer[Int][1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Rubi steps

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx=\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Mathematica [A] time = 56.813, size = 0, normalized size = 0.

$$\int \frac{1}{\sec^{\frac{7}{3}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Integrate[1/(Sec[c + d*x]^(7/3)*(a + b*Sec[c + d*x])^(5/2)), x]
```

Maple [A] time = 0.206, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^{-\frac{7}{3}} (a+b\sec (dx+c))^{-\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] int(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec (dx+c)+a)^{\frac{5}{2}} \sec (dx+c)^{\frac{7}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec (dx+c)+a}\sec (dx+c)^{\frac{2}{3}}}{b^3\sec (dx+c)^6+3\,ab^2\sec (dx+c)^5+3\,a^2b\sec (dx+c)^4+a^3\sec (dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(2/3)/(b^3*sec(d*x + c)^6 +
3*a*b^2*sec(d*x + c)^5 + 3*a^2*b*sec(d*x + c)^4 + a^3*sec(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(7/3)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(7/3)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/3)), x)
```

3.777 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx$

Optimal. Leaf size=251

$$-\frac{ad\left(a^2(n+1)+3b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{b\left(3a^2(n+2)\right.}{}$$

```
[Out] -((a*d*(3*b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2,
 Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[
Sin[e + f*x]^2])) + (b*(b^2*(1 + n) + 3*a^2*(2 + n))*Hypergeometric2F1[1/2,
 -n/2, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*(2
+ n)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 + 2*n)*(d*Sec[e + f*x])^n*Tan[e + f*
x])/(f*(1 + n)*(2 + n)) + (b^2*(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])*Tan[
e + f*x])/(f*(2 + n))
```

Rubi [A] time = 0.349022, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3842, 4047, 3772, 2643, 4046}

$$-\frac{ad\left(a^2(n+1)+3b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{b\left(3a^2(n+2)+b^2(n+1)\right)\sin(e}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```



```
[Out] -((a*d*(3*b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2,
 Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[
Sin[e + f*x]^2])) + (b*(b^2*(1 + n) + 3*a^2*(2 + n))*Hypergeometric2F1[1/2,
 -n/2, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*(2
+ n)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 + 2*n)*(d*Sec[e + f*x])^n*Tan[e + f*
x])/(f*(1 + n)*(2 + n)) + (b^2*(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])*Tan[
e + f*x])/(f*(2 + n))
```


Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
```

```
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^3\,dx = \frac{b^2(d\sec(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+n)} + \frac{\int (d\sec(e+fx))^n \left(ad\right.}{}$$

$$= \frac{b^2(d\sec(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2+n)} + \frac{\int (d\sec(e+fx))^n \left(ad\right.}{}$$

$$= \frac{ab^2(5+2n)(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)(2+n)} + \frac{b^2(d\sec(e+fx))^n(a+b\sec(e+}{f(2+n)}$$

$$= \frac{b\left(b^2(1+n)+3a^2(2+n)\right)\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin}{fn(2+n)\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{a\left(a^2+\frac{3b^2n}{1+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.832583, size = 231, normalized size = 0.92

$$-\frac{\left(-\tan^2(e+fx)\right)^{3/2}\csc^3(e+fx)(d\sec(e+fx))^n\left(bn\left(3a^2\left(n^2+5n+6\right)\cos^2(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+}{}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((Csc[e + f*x]^3*(a^3*(6 + 11*n + 6*n^2 + n^3)*Cos[e + f*x]^3*Hypergeometr
ic2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(3*a^2*(6 + 5*n + n^2)*Cos
[e + f*x]^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2] +
b*(1 + n)*(3*a*(3 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sec[e + f*x]^2] + b*(2 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)
/2, Sec[e + f*x]^2])))*(d*Sec[e + f*x])^n*(-Tan[e + f*x]^2)^(3/2))/(f*n*(1
+ n)*(2 + n)*(3 + n)))
```

Maple [F] time = 3.113, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^3\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\sec\left(fx+e\right)^3+3\,ab^2\sec\left(fx+e\right)^2+3\,a^2b\sec\left(fx+e\right)+a^3\right)\left(d\sec\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(e+fx\right)\right)^n\left(a+b\sec\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{3}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*sec(f*x + e))^n, x)
```

3.778 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx$

Optimal. Leaf size=181

$$-\frac{d\left(a^2(n+1)+b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e+fx)(d$$

```
[Out] -((d*(b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos
[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[
e + f*x]^2])) + (2*a*b*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]
^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) + (b^2*(d*S
ec[e + f*x])^n*Tan[e + f*x])/(f*(1 + n))
```

Rubi [A] time = 0.14996, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3788, 3772, 2643, 4046}

$$-\frac{d\left(a^2(n+1)+b^2n\right)\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f\left(1-n^2\right)\sqrt{\sin^2(e+fx)}}+\frac{2ab\sin(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\right.}{fn\sqrt{\sin^2(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] -((d*(b^2*n + a^2*(1 + n))*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos
[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n^2)*Sqrt[Sin[
e + f*x]^2])) + (2*a*b*Hypergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]
^2]*(d*Sec[e + f*x])^n*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) + (b^2*(d*S
ec[e + f*x])^n*Tan[e + f*x])/(f*(1 + n))
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^2\,dx = \frac{(2ab)\int (d\sec(e+fx))^{1+n}\,dx}{d} + \int (d\sec(e+fx))^n\left(a^2+b^2\sec^2(e+fx)\right)dx$$

$$= \frac{b^2(d\sec(e+fx))^n\tan(e+fx)}{f(1+n)} + \left(a^2+\frac{b^2n}{1+n}\right)\int (d\sec(e+fx))^n\,dx + \frac{(2ab$$

$$= \frac{2ab\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} + \frac{b^2(d\sec(e$$

$$= -\frac{\left(a^2+\frac{b^2n}{1+n}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.310979, size = 171, normalized size = 0.94

$$\frac{\sqrt{-\tan^2(e+fx)}\csc(e+fx)\sec(e+fx)(d\sec(e+fx))^n\left(a^2\left(n^2+3n+2\right)\cos^2(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Csc[e + f*x]*(a^2*(2 + 3*n + n^2)*Cos[e + f*x]^2*Hypergeometric2F1[1/2, n/
2, (2 + n)/2, Sec[e + f*x]^2] + b*n*(2*a*(2 + n)*Cos[e + f*x]*Hypergeometri
c2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2] + b*(1 + n)*Hypergeometric2
F1[1/2, (2 + n)/2, (4 + n)/2, Sec[e + f*x]^2]))*Sec[e + f*x]*(d*Sec[e + f*x
])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n)*(2 + n))
```

Maple [F] time = 1.867, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{2}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2}\sec\left(fx+e\right)^{2}+2\,ab\sec\left(fx+e\right)+a^{2}\right)\left(d\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*sec(f*x + e))^n
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(e+fx\right)\right)^n\left(a+b\sec\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^2\left(d\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*sec(f*x + e))^n, x)
```

3.779 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx$

Optimal. Leaf size=137

$$\frac{b\sin(e+fx)(d\sec(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{n}{2},\frac{2-n}{2},\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{n-1}\text{Hyperg}}{f(1-n)\sqrt{}}$$

```
[Out] -((a*d*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(d*Sec[
e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (b*Hyp
ergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^n*Sin
[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.094441, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3787, 3772, 2643}

$$\frac{b\sin(e+fx)(d\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(e+fx)\right)}{fn\sqrt{\sin^2(e+fx)}}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+\right.}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] -((a*d*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(d*Sec[
e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2])) + (b*Hyp
ergeometric2F1[1/2, -n/2, (2 - n)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^n*Sin
[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))\,dx = a\int (d\sec(e+fx))^n\,dx + \frac{b\int (d\sec(e+fx))^{1+n}\,dx}{d}$$

$$= \left(a\left(\frac{\cos(e+fx)}{d}\right)^n (d\sec(e+fx))^n\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{-n} dx + \frac{\left(b\left(\frac{\cos(e+fx)}{d}\right)^n\right.}{}$$

$$= -\frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(d\sec(e+fx))^n\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}} +$$

Mathematica [A] time = 0.148842, size = 107, normalized size = 0.78

$$\frac{\sqrt{-\tan^2(e+fx)}\csc(e+fx)(d\sec(e+fx))^n\left(a(n+1)\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\sec^2(e+fx)\right)+bn\right.}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] (Csc[e + f*x]*(a*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2
, Sec[e + f*x]^2] + b*n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[e
+ f*x]^2])*(d*Sec[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n*(1 + n))
```

Maple [F] time = 0.549, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(fx+e)+a)(d\sec(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec(fx+e)+a)(d\sec(fx+e))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (d\sec(e+fx))^n (a+b\sec(e+fx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)+a\right)\left(d\sec \left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.780 $\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=192

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^n F_1\left(\frac{1}{2};\frac{n-1}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e+fx)\cos^2(e$$

```
[Out] (a*AppellF1[1/2, (-1 + n)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(
a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n*S
in[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, n/2, 1, 3/2, Sin[e + f*x]^2
, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*(d*Sec[e + f*x])
^n*Sin[e + f*x])/((a^2 - b^2)*f)
```

Rubi [A] time = 0.302176, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2823, 3189, 429}

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^n F_1\left(\frac{1}{2};\frac{n-1}{2},1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e+fx)\cos^2(e$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```


```
[Out] (a*AppellF1[1/2, (-1 + n)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(
a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n*S
in[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, n/2, 1, 3/2, Sin[e + f*x]^2
, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*(d*Sec[e + f*x])
^n*Sin[e + f*x])/((a^2 - b^2)*f)
```


Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d\sec(e+fx))^n}{a+b\sec(e+fx)}\,dx &= \left(\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{1-n}(e+fx)}{b+a\cos(e+fx)}\,dx\\
&= -\left(\left(a\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{2-n}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right)+\left(b\cos^n(e+fx)(d\sec(e+f\right.\\
&= -\frac{\left(a\cos^{2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}(d\sec(e+fx))^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1-n}{2}}}{-a^2+b^2+a^2x^2}\,dx,x,\sin(e\right.}{f}\\
&= \frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1+n),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+f}{\left(a^2-b^2\right)f}
\end{aligned}$$

Mathematica [B] time = 25.4514, size = 5280, normalized size = 27.5

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.73, size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{a+b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^n}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.781 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=299

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^n F_1\left(\frac{1}{2};\frac{n-3}{2},2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin(e+fx)\cos(}{}$$

```
[Out] (a^2*AppellF1[1/2, (-3 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n
*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 + n)/2, 2, 3/2, S
in[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]
^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n*Sin[e + f*x])/((a^2 - b^2)^2*f) - (2*a*
b*AppellF1[1/2, (-2 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*(d*Sec[e + f*x])^n*Sin[e + f*x])/((a^2 -
b^2)^2*f)
```

Rubi [A] time = 0.443625, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3869, 2824, 3189, 429}

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{n-1}{2}}(d\sec(e+fx))^n F_1\left(\frac{1}{2};\frac{n-3}{2},2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin(e+fx)\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*AppellF1[1/2, (-3 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n
*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 + n)/2, 2, 3/2, S
in[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(Cos[e + f*x]
^2)^((-1 + n)/2)*(d*Sec[e + f*x])^n*Sin[e + f*x])/((a^2 - b^2)^2*f) - (2*a*
b*AppellF1[1/2, (-2 + n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(Cos[e + f*x]^2)^(n/2)*(d*Sec[e + f*x])^n*Sin[e + f*x])/((a^2 -
b^2)^2*f)
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \left(\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{2-n}(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \left(\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \left(\frac{b^2\cos^{2-n}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} - \frac{2ab\cos^{3-n}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} + \frac{a}{\left(-b^2\right.}\right.$$

$$= \left(a^2\cos^n(e+fx)(d\sec(e+fx))^n\right)\int \frac{\cos^{4-n}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx - \left(2ab\cos^n(e+fx)(d\sec(\right.$$

$$= \frac{\left(a^2\cos^{2\left(\frac{1}{2}-\frac{n}{2}\right)+n}(e+fx)\cos^2(e+fx)^{-\frac{1}{2}+\frac{n}{2}}(d\sec(e+fx))^n\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{3-n}{2}}}{\left(a^2-b^2-a^2x^2\right)^2}\,dx, x, \sin(\right.}{f}$$

$$= \frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(-3+n),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-1+n)}(d\sec(e+}{\left(a^2-b^2\right)^2 f}$$

Mathematica [B] time = 32.7371, size = 10428, normalized size = 34.88

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{\left(a+b\sec\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^n}{\left(b\sec\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^n}{b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^n}{\left(a+b\sec\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(b\sec\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

3.782 $\int (d\sec(e+fx))^n (a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left((a+b\sec(e+fx))^{3/2}(d\sec(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi [A] time = 0.0686343, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (d\sec(e+fx))^n (a+b\sec(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]

[Out] Defer[Int][(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi steps

$$\int (d\sec(e+fx))^n (a+b\sec(e+fx))^{3/2}\,dx = \int (d\sec(e+fx))^n (a+b\sec(e+fx))^{3/2}\,dx$$

Mathematica [A] time = 13.5194, size = 0, normalized size = 0.

$$\int (d\sec(e+fx))^n (a+b\sec(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2),x]

[Out] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^(3/2), x]
```

Maple [A] time = 0.204, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n} \left(a+b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}} \left(d\sec\left(fx+e\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}} \left(d\sec\left(fx+e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*(d*sec(f*x + e))^n, x)
```

3.783 $\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sqrt{a+b\sec(e+fx)}(d\sec(e+fx))^n,x\right)$$

```
[Out] Unintegrable[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi [A] time = 0.0615866, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi steps

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx=\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx$$

Mathematica [A] time = 0.419903, size = 0, normalized size = 0.

$$\int (d\sec(e+fx))^n\sqrt{a+b\sec(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] Integrate[(d*Sec[e + f*x])^n*Sqrt[a + b*Sec[e + f*x]], x]
```

Maple [A] time = 0.212, size = 0, normalized size = 0.

$$\int \left(d\sec \left(fx+e\right)\right)^{n}\sqrt{a+b\sec \left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec \left(fx+e\right)+a}\left(d\sec \left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec \left(fx+e\right)+a}\left(d\sec \left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec \left(e+fx\right)\right)^{n}\sqrt{a+b\sec \left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n*sqrt(a + b*sec(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx + e\right) + a}\left(d \sec\left(fx + e\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n, x)
```

3.784 $\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}},x\right)$$

```
[Out] Unintegrable[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi [A] time = 0.0701936, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx=\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Mathematica [A] time = 2.45017, size = 0, normalized size = 0.

$$\int \frac{(d\sec(e+fx))^n}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] Integrate[(d*Sec[e + f*x])^n/Sqrt[a + b*Sec[e + f*x]], x]
```

Maple [A] time = 0.193, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n} \frac{1}{\sqrt{a+b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{b\sec\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{n}}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/sqrt(a + b*sec(e + f*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/sqrt(b*sec(f*x + e) + a), x)
```

3.785 $\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi [A] time = 0.0712641, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx = \int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 2.22915, size = 0, normalized size = 0.

$$\int \frac{(d\sec(e+fx))^n}{(a+b\sec(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(d*Sec[e + f*x])^n/(a + b*Sec[e + f*x])^(3/2), x]
```

Maple [A] time = 0.181, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] int((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\left(d\sec\left(fx+e\right)\right)^{n}}{b^{2}\sec\left(fx+e\right)^{2}+2\,ab\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e))^n/(b^2*sec(f*x + e)^2 +
2*a*b*sec(f*x + e) + a^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{n}}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n/(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*sec(e + f*x))**n/(a + b*sec(e + f*x))**(3/2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{n}}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^n/(b*sec(f*x + e) + a)^(3/2), x)
```

3.786 $\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\sec^n(e+fx)(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi [A] time = 0.041781, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx=\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 2.17769, size = 0, normalized size = 0.

$$\int \sec^n(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[Sec[e + f*x]^n*(a + b*Sec[e + f*x])^m, x]
```

Maple [A] time = 0.816, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^{n} \left(a+b\sec\left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m} \sec\left(fx+e\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^{m} \sec\left(fx+e\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m} \sec^{n}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**n, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^n, x)
```

3.787 $\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left((d\sec(e+fx))^n(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi [A] time = 0.0465105, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx=\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 0.495998, size = 0, normalized size = 0.

$$\int (d\sec(e+fx))^n(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(d*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m, x]
```

Maple [A] time = 0.835, size = 0, normalized size = 0.

$$\int \left(d \sec \left(f x + e\right)\right)^{n} \left(a + b \sec \left(f x + e\right)\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x + e\right) + a\right)^{m} \left(d \sec \left(f x + e\right)\right)^{n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(f x + e\right) + a\right)^{m} \left(d \sec \left(f x + e\right)\right)^{n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sec \left(e + f x\right)\right)^{n} \left(a + b \sec \left(e + f x\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**n*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((d*sec(e + f*x))**n*(a + b*sec(e + f*x))**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\left(d\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^n*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*(d*sec(f*x + e))^n, x)
```

3.788 $\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=273

$$\frac{\sqrt{2}\left(a^2+b^2(m+1)\right)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\sec(e+fx)+1}}$$

```
[Out] ((a + b*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*a*(a +
 b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f
*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 +
 Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2 + b^2*(1 +
 m))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]
))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Se
c[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Rubi [A] time = 0.348737, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3840, 4007, 3834, 139, 138}

$$\frac{\sqrt{2}\left(a^2+b^2(m+1)\right)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{b^2f(m+2)\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] ((a + b*Sec[e + f*x])^(1 + m)*Tan[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*a*(a +
 b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f
*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 +
 Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2 + b^2*(1 +
 m))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]
))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Se
c[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Rule 3840

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
```

```
, 0] &&  !LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^3(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} + \frac{\int \sec(e+fx)(b(1+m)-a\sec(e+fx))}{b(2+m)}$$

$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{a\int \sec(e+fx)(a+b\sec(e+fx))^{1+m}\,dx}{b^2(2+m)}$$

$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} + \frac{(a\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x\right.}{b^2f(2+m)\sqrt{1-\sec(e+fx)}\sqrt{1+\sec}}$$

$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{\left(a(-a-b)(a+b\sec(e+fx))^m\left(-\frac{a+b\sec}{-a-}\right.\right.}{b^2f(2+m}$$

$$= \frac{(a+b\sec(e+fx))^{1+m}\tan(e+fx)}{bf(2+m)} - \frac{\sqrt{2}a(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sec\right.}{}$$

Mathematica [B] time = 26.2263, size = 8908, normalized size = 32.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int \left(\sec\left(fx+e\right)\right)^3\left(a+b\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m}\sec^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

3.789 $\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{2}(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{bf\sqrt{\sec(e+fx)+1}}-\frac{\sqrt{2}a\,\mathrm{t}}{}$$

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(
1 - Sec[e + f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[
1 + Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]))/(a + b)]*(a
+ b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec[e + f*x]]*((a + b*Sec[e
 + f*x])/(a + b))^m)
```

Rubi [A] time = 0.221205, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3838, 3834, 139, 138}

$$\frac{\sqrt{2}(a+b)\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m-1;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{bf\sqrt{\sec(e+fx)+1}}-\frac{\sqrt{2}a\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*(a + b)*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Sec[e + f*x])/2, (b*(
1 - Sec[e + f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[
1 + Sec[e + f*x]]*((a + b*Sec[e + f*x])/(a + b))^m) - (Sqrt[2]*a*AppellF1[1
/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e + f*x]))/(a + b)]*(a
+ b*Sec[e + f*x])^m*Tan[e + f*x])/(b*f*Sqrt[1 + Sec[e + f*x]]*((a + b*Sec[e
 + f*x])/(a + b))^m)
```

Rule 3838

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> -Dist[a/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + D
ist[1/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
 b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec^2(e+fx)(a+b\sec(e+fx))^m\,dx = \frac{\int \sec(e+fx)(a+b\sec(e+fx))^{1+m}\,dx}{b} - \frac{a\int \sec(e+fx)(a+b\sec(e+fx))^m}{b}$$

$$= -\frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(e+fx)\right)}{bf\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}} + \frac{(a\tan(e+fx))\,\text{Subst}}{bf\sqrt{1-\sec(e}}$$

$$= \frac{\left(a(a+b\sec(e+fx))^m\left(-\frac{a+b\sec(e+fx)}{-a-b}\right)^{-m}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^m}{\sqrt{1-x}\sqrt{1+x}}\right.}{bf\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}(a+b)F_1\left(\frac{1}{2};\frac{1}{2},-1-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+}{bf\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 22.7531, size = 5564, normalized size = 25.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int \left(\sec \left(fx+e\right)\right)^{2}\left(a+b\sec \left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)+a\right)^{m}\sec \left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec \left(fx+e\right)+a\right)^{m}\sec \left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \sec \left(e + fx\right)\right)^{m} \sec^{2}\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx + e\right) + a\right)^{m} \sec \left(fx + e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

3.790 $\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{2}\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{f\sqrt{\sec(e+fx)+1}}$$

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e +
f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*
x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Rubi [A] time = 0.073033, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2}\tan(e+fx)(a+b\sec(e+fx))^m\left(\frac{a+b\sec(e+fx)}{a+b}\right)^{-m}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)}{f\sqrt{\sec(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Sec[e + f*x])/2, (b*(1 - Sec[e +
f*x]))/(a + b)]*(a + b*Sec[e + f*x])^m*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*
x]]*((a + b*Sec[e + f*x])/(a + b))^m)
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))^m\,dx = -\frac{\tan(e+fx)\,\text{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= -\frac{\left((a+b\sec(e+fx))^m\left(-\frac{a+b\sec(e+fx)}{-a-b}\right)^{-m}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^m}{\sqrt{1-x}\sqrt{1+x}}\,d \cdots\right.}{f\sqrt{1-\sec(e+fx)}\sqrt{1+\sec(e+fx)}}$$

$$= \frac{\sqrt{2}F_1\left(\frac{1}{2};\frac{1}{2},-m;\frac{3}{2};\frac{1}{2}(1-\sec(e+fx)),\frac{b(1-\sec(e+fx))}{a+b}\right)(a+b\sec(e+fx))^m\left(\frac{a+b\,\text{se}\cdots}{a\cdots}\right.}{f\sqrt{1+\sec(e+fx)}}$$

Mathematica [B] time = 14.5709, size = 2828, normalized size = 27.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] (-6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[
(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^(1 + m)*(a + b
*Sec[e + f*x])^m*Tan[(e + f*x)/2])/(f*(-1 + Tan[(e + f*x)/2]^2)*(3*(a + b)*
AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]
^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x
)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1 + m)*AppellF1[3
/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b
```

```
)])*Tan[(e + f*x)/2]^2)*((6*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e +
f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*Cos[e + f*x])^m*Sec
[(e + f*x)/2]^2*Sec[e + f*x]^m*Tan[(e + f*x)/2]^2)/((-1 + Tan[(e + f*x)/2]^
2)^2*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*
Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5
/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1
 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f
*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) - (3*(a + b)*AppellF1[1/2, 1 + m,
-m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*C
os[e + f*x])^m*Sec[(e + f*x)/2]^2*Sec[e + f*x]^m)/((-1 + Tan[(e + f*x)/2]^2
)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan
[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2,
 Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a + b)*(1 +
m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)
/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) + (6*a*(a + b)*m*AppellF1[1/2, 1 + m,
 -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b + a*
Cos[e + f*x])^(-1 + m)*Sec[e + f*x]^m*Sin[e + f*x]*Tan[(e + f*x)/2])/((-1 +
 Tan[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)
/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2,
 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b
)]) + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a
 - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) - (6*(a + b)*m*App
ellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)
/(a + b)]*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^(1 + m)*Sin[e + f*x]*Tan[(e +
 f*x)/2])/((-1 + Tan[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/
2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)
*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f
*x)/2]^2)/(a + b)]) + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e
+ f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) -
(6*(a + b)*(b + a*Cos[e + f*x])^m*Sec[e + f*x]^m*Tan[(e + f*x)/2]*(-((a - b
)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e +
f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3*(a + b)) + ((1
+ m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*
x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/((-1 + Tan[(e +
f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a
 - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1 + m, 1
- m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]) + (a +
 b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[
(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)) + (6*(a + b)*AppellF1[1/2, 1
 + m, -m, 3/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*(b
 + a*Cos[e + f*x])^m*Sec[e + f*x]^m*Tan[(e + f*x)/2]*(2*(-((a - b)*m*Appell
F1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)
/(a + b)]) + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]
^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2] + 3*(a + b)*(-((a - b)*m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)
```

```
/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/(3*(a + b)) + ((1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2
]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)
/2])/3) + 2*Tan[(e + f*x)/2]^2*(-((a - b)*m*((3*(a - b)*(1 - m)*AppellF1[5/
2, 1 + m, 2 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a +
 b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(5*(a + b)) + (3*(1 + m)*AppellF1
[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(
a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)) + (a + b)*(1 + m)*((-3*(a
- b)*m*AppellF1[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e
 + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(5*(a + b)) + (
3*(2 + m)*AppellF1[5/2, 3 + m, -m, 7/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e
 + f*x)/2]^2)/(a + b)]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/((-1 + Ta
n[(e + f*x)/2]^2)*(3*(a + b)*AppellF1[1/2, 1 + m, -m, 3/2, Tan[(e + f*x)/2]
^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)] + 2*(-((a - b)*m*AppellF1[3/2, 1
+ m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, ((a - b)*Tan[(e + f*x)/2]^2)/(a + b)])
 + (a + b)*(1 + m)*AppellF1[3/2, 2 + m, -m, 5/2, Tan[(e + f*x)/2]^2, ((a -
b)*Tan[(e + f*x)/2]^2)/(a + b)])*Tan[(e + f*x)/2]^2)^2)))
```

Maple [F] time = 0.238, size = 0, normalized size = 0.

$$\int \sec \left(fx+e\right) \left(a+b\sec \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(sec(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec \left(fx+e\right)+a\right)^{m} \sec \left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(e+fx\right)\right)^{m}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m*sec(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

3.791 $\int (a + b\sec(e + fx))^m \, dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left((a + b\sec(e + fx))^m, x\right)$$

```
[Out] Unintegrable[(a + b*Sec[e + f*x])^m, x]
```

Rubi [A] time = 0.0098666, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a + b\sec(e + fx))^m \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int (a + b\sec(e + fx))^m \, dx = \int (a + b\sec(e + fx))^m \, dx$$

Mathematica [A] time = 1.75919, size = 0, normalized size = 0.

$$\int (a + b\sec(e + fx))^m \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[(a + b*Sec[e + f*x])^m, x]
```

Maple [A] time = 0.193, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^m,x)
```

```
[Out] int((a+b*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m, x)
```

3.792 $\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\cos(e+fx)(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

Rubi [A] time = 0.0328224, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx=\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 5.55849, size = 0, normalized size = 0.

$$\int \cos(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[Cos[e + f*x]*(a + b*Sec[e + f*x])^m, x]
```

Maple [A] time = 0.335, size = 0, normalized size = 0.

$$\int \cos\left(fx+e\right)\left(a+b\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)*(a+b*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(e+fx\right)\right)^{m}\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Integral((a + b*sec(e + f*x))**m*cos(e + f*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e), x)
```

3.793 $\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\cos^2(e+fx)(a+b\sec(e+fx))^m,x\right)$$

```
[Out] Unintegrable[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

Rubi [A] time = 0.0406279, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx=\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Mathematica [A] time = 4.67782, size = 0, normalized size = 0.

$$\int \cos^2(e+fx)(a+b\sec(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m,x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sec[e + f*x])^m, x]
```

Maple [A] time = 0.459, size = 0, normalized size = 0.

$$\int \left(\cos \left(fx+e\right)\right)^{2} \left(a+b \sec \left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(fx+e\right)+a\right)^{m} \cos \left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sec \left(fx+e\right)+a\right)^{m} \cos \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sec(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{m}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sec(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

3.794 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{10b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{14a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2b\sin\cdots}{\cdots}$$

```
[Out] (14*a*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*b*EllipticF[(c + d*x)/2, 2])/
(21*d) + (10*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (14*a*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(45*d) + (2*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) +
 (2*a*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.104106, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2641, 2639}

$$\frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{14a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{10bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\sin(c+dx\cdots}{7\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (14*a*EllipticE[(c + d*x)/2, 2])/(15*d) + (10*b*EllipticF[(c + d*x)/2, 2])/
(21*d) + (10*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (14*a*Cos[c + d*x]
^(3/2)*Sin[c + d*x])/(45*d) + (2*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) +
 (2*a*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \int \cos^{\frac{7}{2}}(c+dx)(b+a\cos(c+dx))\,dx \\
&= a\int \cos^{\frac{9}{2}}(c+dx)\,dx + b\int \cos^{\frac{7}{2}}(c+dx)\,dx \\
&= \frac{2b\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}(7a)\int \cos^{\frac{5}{2}}(c+ \\
&= \frac{10b\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{14a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{2b\cos^{\frac{5}{2}}(c+dx)}{7d} \\
&= \frac{14aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{10bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10b\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{14
\end{aligned}$$

Mathematica [A] time = 0.341606, size = 90, normalized size = 0.67

$$\frac{600b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(266a\sin(2(c+dx))+35a\sin(4(c+dx))+690b\sin(c+dx)+90b\sin(3(c}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (1176*a*EllipticE[(c + d*x)/2, 2] + 600*b*EllipticF[(c + d*x)/2, 2] + Sqrt[
Cos[c + d*x]]*(690*b*Sin[c + d*x] + 266*a*Sin[2*(c + d*x)] + 90*b*Sin[3*(c
+ d*x)] + 35*a*Sin[4*(c + d*x)]))/(1260*d)
```

Maple [A] time = 1.557, size = 318, normalized size = 2.4

$$-\frac{2}{315\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-1120\,a\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}+\left(2240\,a+720\,b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a+720*b)*sin(1/2*d*x+1/2*c)^8*c
os(1/2*d*x+1/2*c)+(-2072*a-1080*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+
(952*a+840*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a-240*b)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)\cos\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\cos\left(dx+c\right)^{4}\sec\left(dx+c\right)+a\cos\left(dx+c\right)^{4}\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4*sec(d*x + c) + a*cos(d*x + c)^4)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a) \cos (dx + c)^{\frac{9}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(9/2), x)
```

3.795 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{10a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10a\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin$$

```
[Out] (6*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.088547, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2639, 2641}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10a\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \int \cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))\,dx\\ &= a\int \cos^{\frac{7}{2}}(c+dx)\,dx+b\int \cos^{\frac{5}{2}}(c+dx)\,dx\\ &= \frac{2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{1}{7}(5a)\int \cos^{\frac{3}{2}}(c+\\ &= \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2b\cos^{\frac{3}{2}}(c+dx)\sin(c+d\\ &= \frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2b\end{aligned}$$

Mathematica [A] time = 0.489563, size = 77, normalized size = 0.69

$$\frac{50a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(15a\cos(2(c+dx))+65a+42b\cos(c+dx))+126bE\left(\frac{1}{2}(c+dx\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (126*b*EllipticE[(c + d*x)/2, 2] + 50*a*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(65*a + 42*b*Cos[c + d*x] + 15*a*Cos[2*(c + d*x)])*Sin[c + d*x]
```

```
)/(105*d)
```

Maple [A] time = 1.383, size = 290, normalized size = 2.6

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,a\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-360\,a-168\,b\right)\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a-168*b)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(280*a+168*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*a
-42*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
a-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos(dx+c)^{3}\sec(dx+c)+a\cos(dx+c)^{3}\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

3.796 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{2b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.07644, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2635, 2641, 2639}

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```


```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```


Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \int \cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))\,dx\\ &= a\int \cos^{\frac{5}{2}}(c+dx)\,dx+b\int \cos^{\frac{3}{2}}(c+dx)\,dx\\ &= \frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}(3a)\int \sqrt{\cos(c+\\ &= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2a\,\mathrm{co}\end{aligned}$$

Mathematica [A] time = 0.224216, size = 66, normalized size = 0.76

$$\frac{2\left(5b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(3a\cos(c+dx)+5b)+9aE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(9*a*EllipticE[(c + d*x)/2, 2] + 5*b*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*b + 3*a*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 1.607, size = 262, normalized size = 3.

$$-\frac{2}{15d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,a\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(24\,a+20\,b\right)\left(\sin\left(\frac{d}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a+20*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-6*a-10*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos(dx+c)^{2}\sec(dx+c)+a\cos(dx+c)^{2}\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(cos(d*x +
c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a) \cos (dx + c)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.797 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.066688, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2639, 2635, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\,dx &= \int \sqrt{\cos(c+dx)}(b+a\cos(c+dx))\,dx\\ &= a\int \cos^{\frac{3}{2}}(c+dx)\,dx+b\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.111108, size = 53, normalized size = 0.87

$$\frac{2\left(a\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)+3bE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(3*b*EllipticE[(c + d*x)/2, 2] + a*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos
[c + d*x]]*Sin[c + d*x])))/(3*d)
```

Maple [B] time = 1.472, size = 228, normalized size = 3.7

$$-\frac{2}{3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,a\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a-3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*b-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c)\sec(dx+c)+a\cos(dx+c))\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(cos(d*x + c)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.798 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$\frac{2b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/d
```

Rubi [A] time = 0.0573418, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4225, 2748, 2641, 2639}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/d
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))\,dx = \int \frac{b+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= a\int \sqrt{\cos(c+dx)}\,dx + b\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [A] time = 0.0674208, size = 32, normalized size = 0.91

$$\frac{2\left(b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+aE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a*EllipticE[(c + d*x)/2, 2] + b*EllipticF[(c + d*x)/2, 2]))/d
```

Maple [A] time = 1.513, size = 152, normalized size = 4.3

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(b\text{EllipticF}\left(\right.\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(b*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
```

```
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx + c) + a)\sqrt{\cos(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec(dx + c) + a)\sqrt{\cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))\sqrt{\cos(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx + c) + a)\sqrt{\cos(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.799 $\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*b*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.066769, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*b*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + b\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - b\int \sqrt{\cos(c+dx)}\,dx \\
&= -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.140197, size = 51, normalized size = 0.89

$$\frac{2\left(a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-bE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(-(b*EllipticE[(c + d*x)/2, 2]) + a*EllipticF[(c + d*x)/2, 2] + (b*Sin[c
 + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [A] time = 1.556, size = 148, normalized size = 2.6

$$-2\,\frac{\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}a+\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\mathrm{Elliptic}}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,d\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a+(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b-2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\sec\left(dx+c\right)+a}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.800 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 0.0749682, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2641, 2639}

$$-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*b*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```


Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{b+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + b\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}b\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.402604, size = 65, normalized size = 0.78

$$\frac{2b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + \frac{2\sin(c+dx)(3a\cos(c+dx)+b)}{\cos^{\frac{3}{2}}(c+dx)} - 6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2] + 2*b*EllipticF[(c + d*x)/2, 2] + (2*(b + 3
*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Maple [B] time = 3.502, size = 397, normalized size = 4.8

$$\frac{2}{3d}\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b*sin(1/2*d*x+1/2*c)^2+6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*sin(1/2*d*x+
1/2*c)^2-12*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2*a+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.801 $\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*b*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.087413, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {4225, 2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*b*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{b+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + b\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3b)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(3b)\int \sqrt{\cos
\end{aligned}$$

$$= -\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\sin(c}{5d\sqrt{\cos(c}}$$

Mathematica [A] time = 0.29844, size = 95, normalized size = 0.86

$$\frac{10a\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 10a\sin(c+dx) + 9b\sin(2(c+dx)) + 6b\tan(c+dx) - 18b\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-18*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*Cos[c + d*x]^(3/
2)*EllipticF[(c + d*x)/2, 2] + 10*a*Sin[c + d*x] + 9*b*Sin[2*(c + d*x)] + 6
*b*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 3.946, size = 502, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+
1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-1
2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\sec\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.802 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{20ab\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2\left(7a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(7a^2+9b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a^2\sin(c+dx}{9}$$

```
[Out] (2*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a*b*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (20*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*(7*a^2 + 9*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a*b*Cos[c +
d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9
*d)
```

Rubi [A] time = 0.187469, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(7a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2\left(7a^2+9b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{20abF\left(\frac{1}{2}(c+\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (2*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a*b*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (20*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*(7*a^2 + 9*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a*b*Cos[c +
d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9
*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}\,dx + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.$$

$$= \frac{4ab\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{7}\left(10ab\sqrt{\cos(c-\right.$$

$$= \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2\left(7a^2+9b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4ab\,\mathrm{c}}{}$$

$$= \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2\left(7a^2+9b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4ab\,\mathrm{c}}{}$$

$$= \frac{2\left(7a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20ab\sqrt{\cos(c+dx)}\sin(}{21d}$$

Mathematica [A] time = 0.766843, size = 113, normalized size = 0.71

$$\frac{600ab\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+84\left(7a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7\left(43a^2+36b^2\right)\cos(c+dx)\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (84*(7*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(7*(43*a^2 + 36*b^2)*Cos[c + d*x] + 5*a*(156*b
+ 36*b*Cos[2*(c + d*x)] + 7*a*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 1.828, size = 398, normalized size = 2.5

$$-\frac{2}{315\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-1120\,a^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{10}+\left(2240\,a^2+144\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^2+1440*a*b)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^2-2160*a*b-504*b^2)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(952*a^2+1680*a*b+504*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-168*a^2-480*a*b-126*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+150*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-147*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^4\sec\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)^4\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)^4\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^4*sec(d*x
+ c) + a^2*cos(d*x + c)^4)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x)
```

3.803 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{2(5a^2+7b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2(5a^2+7b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{12ab}{}$$

```
[Out] (12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a^2 + 7*b^2)*EllipticF[(c
+ d*x)/2, 2])/(21*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (4*a*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.172011, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{2(5a^2+7b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5a^2+7b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{12abE\left(\frac{1}{2}(c+\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a^2 + 7*b^2)*EllipticF[(c
+ d*x)/2, 2])/(21*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (4*a*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\right.$$

$$= \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}\left(6ab\sqrt{\cos(c\right.$$

$$= \frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2}{}$$

$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{4ab\cos^{\frac{3}{2}}(c}{}$$

$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(5a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c}}{21d}$$

Mathematica [A] time = 0.583926, size = 98, normalized size = 0.73

$$\frac{10\left(5a^2+7b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(15a^2\cos(2(c+dx))+65a^2+84ab\cos(c+dx)+70\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(5*a^2 + 7*b^2)*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(65*a^2 + 70*b^2 + 84*a*b*Cos[c + d*x] + 15*a^2
*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 1.443, size = 362, normalized size = 2.7

$$-\frac{2}{105\,d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,a^2\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8+\left(-360\,a^2-336\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^2-336*a*b)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(280*a^2+336*a*b+140*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-80*a^2-84*a*b-70*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+25*a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+35*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^3\sec(dx+c)^2+2\,ab\cos(dx+c)^3\sec(dx+c)+a^2\cos(dx+c)^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^3*sec(d*x
+ c) + a^2*cos(d*x + c)^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x)
```

3.804 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{4ab\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+}}{3d}$$

```
[Out] (2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (4*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.152798, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (4*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\text{se}\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}\left(2ab\sqrt{\cos(c+\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(2ab)\int \frac{}{\sqrt{\cos}}$$
$$= \frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4ab\sqrt{\cos(c+dx)}\sin(c-}{3d}$$

Mathematica [A] time = 0.295482, size = 79, normalized size = 0.78

$$\frac{20ab\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+6\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2a\sin(c+dx)\sqrt{\cos(c+dx)}(3a\cos(c+dx)+10b)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (6*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*EllipticF[(c + d*x)/2
, 2] + 2*a*Sqrt[Cos[c + d*x]]*(10*b + 3*a*Cos[c + d*x])*Sin[c + d*x])/(15*d
)
```

Maple [B] time = 1.498, size = 321, normalized size = 3.2

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,a^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6+\left(24\,a^2+40\,ab\right)\left(\text{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^2+40*a*b)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-6*a^2-20*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-15*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2\sec(dx+c)^2+2\,ab\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^2\cos (dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.805 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=72

$$\frac{2\left(a^2+3b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2 + 3*b^2)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.139091, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3788, 3771, 2639, 4045, 2641}

$$\frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2 + 3*b^2)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + (2ab)\int \sqrt{\cos(c+dx)}\,dx - \frac{1}{3}\left(\left(-a^2-3b^2\right)\sqrt{\cos\right.$$

$$= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}\left(-a^2-3b^2\right)\int \frac{1}{\sqrt{\cos(c}}$$

$$= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+}{3d}$$

Mathematica [A] time = 0.155578, size = 64, normalized size = 0.89

$$\frac{2\left(\left(a^2+3b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+a^2\sin(c+dx)\sqrt{\cos(c+dx)}+6abE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2,
2] + a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 1.731, size = 283, normalized size = 3.9

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(4\,a^{2}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+a^{2}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*b^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*a^2*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{2}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{2}\cos(dx+c)\sec(dx+c)^{2}+2\,ab\cos(dx+c)\sec(dx+c)+a^{2}\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c)
 + a^2*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

3.806 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{4ab\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*(a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (4*a*b*EllipticF[(c + d*x)/2,
 2])/d + (2*b^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.136169, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3788, 3771, 2641, 4046, 2639}

$$\frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (4*a*b*EllipticF[(c + d*x)/2,
 2])/d + (2*b^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx\\ &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a^2+b^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.\\ &= \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (2ab)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.\right.\\ &= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(a^2-b^2\right)\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.291093, size = 62, normalized size = 0.91

$$\frac{2\left(b\left(2a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)+\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*((a^2 - b^2)*EllipticE[(c + d*x)/2, 2] + b*(2*a*EllipticF[(c + d*x)/2, 2
] + (b*Sin[c + d*x])/Sqrt[Cos[c + d*x]])))/d
```

Maple [A] time = 1.653, size = 202, normalized size = 3.

$$-2\,\frac{2\,\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)ab-\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(cos(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))^2 \sqrt{\cos(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx + c) + a)^2 \sqrt{\cos(dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

3.807 $\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(3a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a*b*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.144892, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a*b*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\left(a^2+b^2\sec^2(c+dx)\right)dx + \left(2ab\sqrt{\cos(c+dx\right.$$
$$= \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (2ab)\int \sqrt{\cos(c+dx)}\,dx + \frac{1}{3}\left(3a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+}}$$
$$= -\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.5771, size = 73, normalized size = 0.77

$$\frac{2\left(\left(3a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - 6abE\left(\frac{1}{2}(c+dx)\middle|2\right) + \frac{b\sin(c+dx)(6a\cos(c+dx)+b)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2,
 2] + (b*(b + 6*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 3.614, size = 514, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(12*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2+6*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
```

```
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*sin(1/2
*d*x+1/2*c)^2+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*sin(1/2*d*x+1/2*c)^2-24*a*b*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-3*a^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*a*b*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/sqrt(cos(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.808 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{4ab\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}-\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2+3b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b^2\sin(c}{5d\cos^{\frac{5}{2}}(c}$$

```
[Out] (-2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*b^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*a^2 + 3*b^2)*Sin[c + d*x])
/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.164317, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2641, 4046, 2639}

$$-\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(5a^2+3b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*b^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*a^2 + 3*b^2)*Sin[c + d*x])
/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$

$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx + \left(2ab\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+3b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{1}{3}(2ab)\int \frac{1}{\sqrt{\cos(c+dx)}}\,d$$

$$= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(5a^2+3b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}\left(\right.$$

$$= -\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.355176, size = 124, normalized size = 0.92

$$\frac{20ab\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - 6\left(5a^2+3b^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 15a^2\sin(2(c+dx)) + 20ab\sin(}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-6*(5*a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*C
os[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 15*a^2*
Sin[2*(c + d*x)] + 9*b^2*Sin[2*(c + d*x)] + 6*b^2*Tan[c + d*x])/(15*d*Cos[c
 + d*x]^(3/2))
```

Maple [B] time = 4.947, size = 660, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*
d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a^2*(
-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \sec(dx+c)^2 + 2\,ab\sec(dx+c) + a^2}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/cos(d*x + c)^(3/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.809 $\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=160

$$\frac{2\left(7a^2+5b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{12ab\,\text{si}}{5d\sqrt{\text{co}}}$$

```
[Out] (-12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2 + 5*b^2)*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (2*b^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(7*a^2 + 5*b^2)*Sin[c + d*x
])/(21*d*Cos[c + d*x]^(3/2)) + (12*a*b*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]
])
```

Rubi [A] time = 0.182658, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}-\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{12ab\sin(c+dx}{5d\sqrt{\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2 + 5*b^2)*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (2*b^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*(7*a^2 + 5*b^2)*Sin[c + d*x
])/(21*d*Cos[c + d*x]^(3/2)) + (12*a*b*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]
])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\left(a^2+b^2\sec^2(c+dx)\right)dx + \left(2ab\sqrt{\cos(c+dx)}\right.$$
$$= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{5}\left(6ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\,d$$
$$= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{12ab\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}\left(6\right.$$
$$= \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{12ab\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(6a$$
$$= -\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2b^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+}{5d\cos^{\frac{5}{2}}(c+}$$

Mathematica [A] time = 0.535485, size = 142, normalized size = 0.89

$$\frac{10\left(7a^2+5b^2\right)\cos^{\frac{5}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+35a^2\sin(2(c+dx))+84ab\sin(c+dx)+252ab\sin(c+dx)\cos^2(}{105d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-252*a*b*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2)
*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 84*a*b*Sin[c + d*x] + 252*a
*b*Cos[c + d*x]^2*Sin[c + d*x] + 35*a^2*Sin[2*(c + d*x)] + 25*b^2*Sin[2*(c
+ d*x)] + 30*b^2*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 5.016, size = 689, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))-4/5*a*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*
d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*b^2*(
-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)/cos(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.810 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=194

$$\frac{2b\left(15a^2+7b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2a\left(7a^2+27b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}$$

```
[Out] (2*a*(7*a^2 + 27*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*b*(15*a^2 + 7*
b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*(7*a^2 + 27*b^2)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(45*d) + (40*a^2*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*a
^2*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.286489, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3841, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2b\left(15a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\left(7a^2+27b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\left(7a^2+27b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2b\left(15\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (2*a*(7*a^2 + 27*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*b*(15*a^2 + 7*
b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(15*a^2 + 7*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*a*(7*a^2 + 27*b^2)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(45*d) + (40*a^2*b*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*a
^2*Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(9*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
```

```
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{2a\left(7a^2+27b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{40a^2b\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2a}{}$$

$$= \frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a\left(7a^2+27b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c}{45d}$$

$$= \frac{2a\left(7a^2+27b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2b\left(15a^2+7b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

$$= \frac{2a\left(7a^2+27b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{2b\left(15a^2+7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2b\left(15a^2\right.}{}$$

Mathematica [A] time = 0.942154, size = 137, normalized size = 0.71

$$\frac{60\left(15a^2b+7b^3\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+84\left(7a^3+27ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7a\left(43a^2+108\right.\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (84*(7*a^3 + 27*a*b^2)*EllipticE[(c + d*x)/2, 2] + 60*(15*a^2*b + 7*b^3)*El
lipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*a*(43*a^2 + 108*b^2)*Cos[c
+ d*x] + 5*(234*a^2*b + 84*b^3 + 54*a^2*b*Cos[2*(c + d*x)] + 7*a^3*Cos[3*(c
 + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 1.598, size = 470, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*a^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2240*a^3+2160*a^2*b)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*a^3-3240*a^2*b-1512*a*b^2)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(952*a^3+2520*a^2*b+1512*a*b^2+420*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*a^3-720*a^2*b-378*a*b^2-210*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+225*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+105*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*b^3-147*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\cos(dx+c)^4\sec(dx+c)^3+3\,ab^2\cos(dx+c)^4\sec(dx+c)^2+3\,a^2b\cos(dx+c)^4\sec(dx+c)+a^3\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^4*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^4*sec(d*
x + c)^2 + 3*a^2*b*cos(d*x + c)^4*sec(d*x + c) + a^3*cos(d*x + c)^4)*sqrt(c
```

```
os(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2), x)
```

3.811 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=159

$$\frac{2a(5a^2+21b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}+\frac{2b(9a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5a^2+21b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}$$

```
[Out] (2*b*(9*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*a^2 + 21*b^
2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*d) + (32*a^2*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) + (2*a^2*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.260216, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3841, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a(5a^2+21b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(9a^2+5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5a^2+21b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{32a^2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (2*b*(9*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*a^2 + 21*b^
2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*d) + (32*a^2*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) + (2*a^2*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
```

```
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{32a^2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} +$$

$$= \frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{32a^2b\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} +$$

$$= \frac{2b\left(9a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\left(5a^2+21b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a\left(5a^2\right.}{}$$

Mathematica [A] time = 0.714118, size = 110, normalized size = 0.69

$$\frac{10\left(5a^3+21ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+42\left(9a^2b+5b^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(c+dx)\sqrt{\cos(c+dx)}\left(15a^2\cos(2(c\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (42*(9*a^2*b + 5*b^3)*EllipticE[(c + d*x)/2, 2] + 10*(5*a^3 + 21*a*b^2)*Ell
ipticF[(c + d*x)/2, 2] + a*Sqrt[Cos[c + d*x]]*(65*a^2 + 210*b^2 + 126*a*b*C
os[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 1.764, size = 421, normalized size = 2.7

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,a^3\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-360\,a^3-504\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-360*a^3-504*a^2*b)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(280*a^3+504*a^2*b+420*a*b^2)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-80*a^3-126*a^2*b-210*a*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+25*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*a*b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-189*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \cos(dx+c)^3 \sec(dx+c)^3 + 3\,ab^2 \cos(dx+c)^3 \sec(dx+c)^2 + 3\,a^2 b \cos(dx+c)^3 \sec(dx+c) + a^3 \cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^3*sec(d*
x + c)^2 + 3*a^2*b*cos(d*x + c)^3*sec(d*x + c) + a^3*cos(d*x + c)^3)*sqrt(c
os(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^3\cos (dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2), x)
```

3.812 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=116

$$\frac{2b\left(a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}+\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2b\sin(c+dx)\sqrt{\cos(c+dx)}}{5d}+\frac{2a^2\sin(c+dx)}{}$$

```
[Out] (6*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(a^2 + b^2)*Elli
pticF[(c + d*x)/2, 2])/d + (8*a^2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d)
+ (2*a^2*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.227948, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3841, 4047, 3771, 2639, 4045, 2641}

$$\frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^2b\sin(c+dx)\sqrt{\cos(c+dx)}}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (6*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(a^2 + b^2)*Elli
pticF[(c + d*x)/2, 2])/d + (8*a^2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d)
+ (2*a^2*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
```

```
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx\right.$$

$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx\right.$$

$$= \frac{8a^2b\sqrt{\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{5d}$$

$$= \frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2b\sqrt{\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{3}{2}}(c+}{}$$

$$= \frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2b\sqrt{\cos(c+}}{5}$$

Mathematica [A] time = 0.364365, size = 84, normalized size = 0.72

$$\frac{2\left(5b\left(a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\left(a^3+5ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a^2\sin(c+dx)\sqrt{\cos(c+dx)}(a\cos(c+dx)+5b\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(3*(a^3 + 5*a*b^2)*EllipticE[(c + d*x)/2, 2] + 5*b*(a^2 + b^2)*EllipticF
[(c + d*x)/2, 2] + a^2*Sqrt[Cos[c + d*x]]*(5*b + a*Cos[c + d*x])*Sin[c + d*
x]))/(5*d)
```

Maple [B] time = 1.634, size = 376, normalized size = 3.2

$$-\frac{2}{5\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\,a^3\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6+\left(8\,a^3+20\,a^2b\right)\left(\sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*a^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(8*a^3+20*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-2*a^3-10*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3-3*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*a^3-15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\cos(dx+c)^2\sec(dx+c)^3+3\,ab^2\cos(dx+c)^2\sec(dx+c)^2+3\,a^2b\cos(dx+c)^2\sec(dx+c)+a^3\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^2*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2*sec(d*
x + c)^2 + 3*a^2*b*cos(d*x + c)^2*sec(d*x + c) + a^3*cos(d*x + c)^2)*sqrt(c
os(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2), x)
```

3.813 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=126

$$\frac{2a(a^2+9b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b(a^2-3b^2)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{c}}{}$$

```
[Out] (2*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) - (2*b*(a^2 - 3*b^2)*Sin[c + d*x])/(3*d*Sqrt[Cos
[c + d*x]]) + (2*a^2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/
(3*d)
```

Rubi [A] time = 0.23136, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3841, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a(a^2+9b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b(3a^2-b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b(a^2-3b^2)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) - (2*b*(a^2 - 3*b^2)*Sin[c + d*x])/(3*d*Sqrt[Cos
[c + d*x]]) + (2*a^2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*Sin[c + d*x])/
(3*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
```

```
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= -\frac{2b\left(a^2-3b^2\right)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}(a+b\sec(c+dx))\sin(c+dx)}{3d}$$

$$= \frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2b\left(a^2-3b^2\right)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}(}{}$$

$$= \frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2b\left(a^2-3b^2\right.}{3d\sqrt{\cos}}$$

Mathematica [A] time = 0.522461, size = 87, normalized size = 0.69

$$\frac{2\left(\left(a^3+9ab^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\left(9a^2b-3b^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(a^3\cos(c+dx)+3b^3\right)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*((9*a^2*b - 3*b^3)*EllipticE[(c + d*x)/2, 2] + (a^3 + 9*a*b^2)*EllipticF
[(c + d*x)/2, 2] + ((3*b^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c + d
*x]]))/(3*d)
```

Maple [A] time = 1.954, size = 303, normalized size = 2.4

$$-\frac{2}{3\,d}\left(4\,a^3\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^4+a^3\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\text{EllipticF}\left(\cos\left(\frac{dx}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/3*(4*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+a^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+9*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*b^3-2*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-6
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\cos(dx+c)\sec(dx+c)^3+3\,ab^2\cos(dx+c)\sec(dx+c)^2+3\,a^2b\cos(dx+c)\sec(dx+c)+a^3\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)*sec(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sec(d*x +
c)^2 + 3*a^2*b*cos(d*x + c)*sec(d*x + c) + a^3*cos(d*x + c))*sqrt(cos(d*x +
 c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.814 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=118

$$\frac{2b\left(9a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d}+\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (16*a*b^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.229916, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3842, 4047, 3771, 2641, 4046, 2639}

$$\frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)(a+b\sec(c+dx))}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (2*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (16*a*b^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*b^2*(a + b*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
```

& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) && !(IGtQ[n, 2] && !IntegerQ[m])

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a\left(3a^{2}\right.}{}$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a\left(3a^{2}\right.}{}$$

$$= \frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{1}{3}\left(b\left(9a^2+b^2\right)\right)\int \frac{}{\sqrt{}}$$

$$= \frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{16ab^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{16ab^2\sin(c+}{3d\sqrt{\cos(c+}}$$

Mathematica [A] time = 1.11281, size = 84, normalized size = 0.71

$$\frac{2\left(b\left(\left(9a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\frac{b\sin(c+dx)(9a\cos(c+dx)+b)}{\cos^{\frac{3}{2}}(c+dx)}\right)+3\left(a^3-3ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (2*(3*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, 2] + b*((9*a^2 + b^2)*Elliptic
F[(c + d*x)/2, 2] + (b*(b + 9*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3
/2))))/(3*d)
```

Maple [B] time = 3.619, size = 631, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(18*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*a^2*b*sin(1/2*d*x+1/2*c)^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3*sin(1
/2*d*x+1/2*c)^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3*sin(1/2*d*x+1/2*c)^2+18*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*a*b^2*sin(1/2*d*x+1/2*c)^2-36*a*b^2*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*b^3+3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+18
*a*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+2*b^3*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

3.815 $\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=149

$$\frac{2a\left(a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d}-\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6b\left(5a^2+b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+$$

```
[Out] (-6*b*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(a^2 + b^2)*Ell
ipticF[(c + d*x)/2, 2])/d + (8*a*b^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
 + (6*b*(5*a^2 + b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*(a +
b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.245708, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3842, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6b\left(5a^2+b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-6*b*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(a^2 + b^2)*Ell
ipticF[(c + d*x)/2, 2])/d + (8*a*b^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
 + (6*b*(5*a^2 + b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*(a +
b*Sec[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
```

```
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3\,dx$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}a\right.$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}a\right.$$

$$= \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\left(5a^2+b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \left(a\left(a^2\right.\right.$$

$$= \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\left(5a^2+b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \left(a\left(a^2\right.\right.$$

$$= -\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8ab^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6b\left(5\right.}{}$$

Mathematica [A] time = 0.8762, size = 125, normalized size = 0.84

$$\frac{10a\left(a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+3\left(5a^2b+b^3\right)\sin(2(c+dx))-6b\left(5a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+\right.}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-6*b*(5*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*a*(a^
2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a*b^2*Sin[c + d*
x] + 3*(5*a^2*b + b^3)*Sin[2*(c + d*x)] + 2*b^3*Tan[c + d*x])/(5*d*Cos[c +
d*x]^(3/2))
```

Maple [B] time = 4.534, size = 738, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a*b
^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*b^3/(8*sin(1/2*d*x+1/2*c)^6
-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)+6*a^2*b*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+
1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\sec(c + dx))^3}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx + c) + a)^3}{\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.816 $\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{2b(21a^2+5b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d}-\frac{2a(5a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5a^2+9b^2)}{5d\sqrt{\cos}}$$

```
[Out] (-2*a*(5*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(21*a^2 + 5*b
^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (32*a*b^2*Sin[c + d*x])/(35*d*Cos[c
 + d*x]^(5/2)) + (2*b*(21*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/
2)) + (2*a*(5*a^2 + 9*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*
(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.285038, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3842, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2b(21a^2+5b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2a(5a^2+9b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5a^2+9b^2)\sin(c+}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*(5*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(21*a^2 + 5*b
^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (32*a*b^2*Sin[c + d*x])/(35*d*Cos[c
 + d*x]^(5/2)) + (2*b*(21*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/
2)) + (2*a*(5*a^2 + 9*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*
(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(5/2))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3\,dx$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a\,(7$$

$$= \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a\,(7$$

$$= \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b^2(a+b\sec(c+dx))\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{21}\,($$

$$= \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\left(5a^2+9b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2(a+b\,\mathrm{se}}{7d}$$

$$= \frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\,(5}{5}$$

$$= -\frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{32ab^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \cdots$$

Mathematica [A] time = 0.763865, size = 177, normalized size = 0.91

$$\frac{10b\left(21a^2+5b^2\right)\cos^{\frac{5}{2}}(c+dx)\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right) - 42a\left(5a^2+9b^2\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 105a^2b\sin(2(c+}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-42*a*(5*a^2 + 9*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*b*
(21*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a*b^2*S
in[c + d*x] + 210*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 378*a*b^2*Cos[c + d*x]^
2*Sin[c + d*x] + 105*a^2*b*Sin[2*(c + d*x)] + 25*b^3*Sin[2*(c + d*x)] + 30*
b^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 5.513, size = 847, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(6*a^2*b*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/
42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/5*a*b^2/(8*sin(1/2*d*x+1/2*c)^6-12
*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2
*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x
+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-8*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)+2*a^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c
)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\sec(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.817 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=152

$$-\frac{2b\left(a^2+3b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d}+\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}+\frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a+b)}-\frac{2b\sin(c+dx)\sqrt{}}{3a^2d}$$

```
[Out] (2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*b*(a^2 + 3*b^2
)*EllipticF[(c + d*x)/2, 2])/(3*a^4*d) + (2*b^4*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
a^2*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.602274, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3853, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{2b\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d}+\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}+\frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a+b)}-\frac{2b\sin(c+dx)\sqrt{\cos(c+}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```


```
[Out] (2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*b*(a^2 + 3*b^2
)*EllipticF[(c + d*x)/2, 2])/(3*a^4*d) + (2*b^4*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
a^2*d) + (2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
```

```
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5b}{2}+\frac{3}{2}a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx}{5a}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{b^4\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a^4} -$$

$$= \frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4(a+b)d} - \frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\left(b\left(a^2\right.\right.}{}$$

$$= \frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d} - \frac{2b\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d} + \frac{2b^4\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4(a+b)d} -$$

Mathematica [A] time = 1.58215, size = 228, normalized size = 1.5

$$-\frac{\frac{6\left(3a^2+5b^2\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}} + 8b\left(2\text{E}\right.}{30a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*(9*a^2 + 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*
b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b)) + 4*Sqrt[Cos[c + d*x]]*(-5*b + 3*a*Cos[c + d*x])*Sin[c + d*x
] - (6*(3*a^2 + 5*b^2)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2
*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Ellipt
icPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Si
n[c + d*x]^2]))/(30*a^2*d)
```

Maple [B] time = 1.995, size = 668, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*a^4+24*
a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*a^4-44*a^3*b+20*a^2*b^2)
*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*a^4+16*a^3*b-10*a^2*b^2)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+5*a^2*b
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*b^4*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+9*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*a^3*b-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b
^3-15*b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^4/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

3.818 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=112

$$\frac{2\left(a^2+3b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^3d}-\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(a^2 + 3*b^2)*EllipticF[(c +
d*x)/2, 2])/(3*a^3*d) - (2*b^3*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
a^3*(a + b)*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.390238, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4264, 3853, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}-\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(a^2 + 3*b^2)*EllipticF[(c +
d*x)/2, 2])/(3*a^3*d) - (2*b^3*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
a^3*(a + b)*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3b}{2}+\frac{1}{2}a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx}{3a}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3ab}{2}-\left(-\frac{a^2}{2}-\frac{3b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^3} - \frac{\left(b^3\right.}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{b^3\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a^3} - \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{a^2}$$

$$= -\frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a+b)d} + \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{b\int\sqrt{\cos(c+dx)}\,dx}{a^2} + \frac{\left(a^2+3b^2\right)}{}$$

$$= -\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d} - \frac{2b^3\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a+b)d} + \frac{2\sqrt{\cos(c+}}{}$$

Mathematica [A] time = 1.6599, size = 160, normalized size = 1.43

$$\frac{\frac{6\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2\sqrt{\sin^2(c+dx)}} + 4\text{EllipticF}\left(\frac{1}{2}(c\right.}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (4*EllipticF[(c + d*x)/2, 2] - (6*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (6*(2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/(6*a*d)
```

Maple [B] time = 1.714, size = 516, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*a^3-4*a^2*
b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a^3+2*a^2*b)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*a^2*b+3*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+3
*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+3*b^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(dx+c)}^{\frac{3}{2}}}{b\sec{(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

3.819 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{2b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d}+\frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticF[(c + d*x)/2, 2])/(a^2*
d) + (2*b^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d)
```

Rubi [A] time = 0.242028, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4264, 3852, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticF[(c + d*x)/2, 2])/(a^2*
d) + (2*b^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3852

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-b\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} + \frac{\left(b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,d}{a^2}$$

$$= \frac{b^2\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \left(b\sqrt{\cos(c+dx)}\right.$$

$$= \frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d} + \frac{\int \sqrt{\cos(c+dx)}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2b^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d}$$

Mathematica [A] time = 0.291558, size = 84, normalized size = 1.12

$$-\frac{2\sin(c+dx)\left(-(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-b\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+aE\left(\sin^{-1}\left(\sqrt{\cos\right.\right.}{a^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(a*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin
[Sqrt[Cos[c + d*x]]], -1] - b*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]
], -1])*Sin[c + d*x])/(a^2*d*Sqrt[Sin[c + d*x]^2])
```

Maple [A] time = 1.507, size = 226, normalized size = 3.

$$2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}}{(a-b)\,a^2\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1}d}\left(\text{EllipticF}\left(\text{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a*b-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a^2-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+b^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/a^2/(a-b)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.820 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=53

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad}-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

```
[Out] (2*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2])/(a*(a + b)*d)
```

Rubi [A] time = 0.188849, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3848, 2803, 2641, 2805}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2])/(a*(a + b)*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
```

```
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx \\
&= \int \frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx \\
&= \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{a} \\
&= \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}
\end{aligned}
$$

Mathematica [A] time = 0.0684706, size = 48, normalized size = 0.91

$$
\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b))/(a*d)
```

Maple [A] time = 1.849, size = 187, normalized size = 3.5

$$-2\,{\frac {\sqrt {\left(2\, \left(\cos \left(1/2\,dx+c/2 \right) \right) ^{2}-1 \right) \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}\sqrt { \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}\sqrt {-2\, \left(\cos \left(1/2\,dx+c/2 \right) \right) ^{2}+1}}{a \left(a-b \right) \sqrt {-2\, \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{4}+ \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}\sin \left(1/2\,dx+c/2 \right) \sqrt {2\, \left(\cos \left(1/2\,dx+c/2 \right) \right) ^{2}-1}d}} \Bigl({\rm EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2)))/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sec \left(dx + c\right) + a\right)\sqrt{\cos \left(dx + c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \sec{(c + dx)}) \sqrt{\cos{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sec{(dx + c)} + a)\sqrt{\cos{(dx + c)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.821 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=29

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] (2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rubi [A] time = 0.128332, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4264, 3849, 2805}

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx\\ &= \int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx\\ &= \frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.0777983, size = 29, normalized size = 1.

$$\frac{2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Maple [B] time = 1.392, size = 150, normalized size = 5.2

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\mathrm{EllipticPi}\left(\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
```

```
,2*a/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.822 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=77

$$-\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(b*(a + b)*d) + (2*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.202038, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4264, 3850, 3768, 3771, 2639, 3849, 2805}

$$-\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*a)/(a + b), (c +
d*x)/2, 2])/(b*(a + b)*d) + (2*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3850

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[(a*
d)/b, Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{b}$$

$$= \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{a\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{b} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{b}$$

$$= -\frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d} + \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{b}$$

$$= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2a\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d} + \frac{2\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

Mathematica [B] time = 1.89428, size = 197, normalized size = 2.56

$$\frac{\frac{2\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}} - \frac{2b\left(2\text{EllipticF}\left(\frac{1}{2}(c+d\right.\right.}{}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((-6*a*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (2*b*(2*Ellipti
cF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b
)))/a + (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(2*a*b*EllipticE[ArcSin[Sq
rt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]],
-1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Si
n[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*b*d)
```

Maple [B] time = 1.821, size = 353, normalized size = 4.6

$$-2\,\frac{1}{(a-b)\,b\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(-2\sqrt{-2\,(\mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/b/(a-b)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.823 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=128

$$\frac{2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd}+\frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*
b*d) + (2*a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d) +
(2*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/2)) - (2*a*Sin[c + d*x])/(b^2*d*Sqr
t[Cos[c + d*x]])
```

Rubi [A] time = 0.540456, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3851, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}+\frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*
b*d) + (2*a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d) +
(2*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/2)) - (2*a*Sin[c + d*x])/(b^2*d*Sqr
t[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
```

```
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}b\sec(c+dx)-\frac{3}{2}a}{a+b\sec(c+dx)}}{3b}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{3a^2}{4}+ab}{}}{3b^2}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{3a^3}{4}+\frac{1}{4}}{\sqrt{s}}}{3a^2b^2}$$

$$= \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{a^2\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{b^2} + \frac{\left(a\sqrt{c}\right.}{}$$

$$= \frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d} + \frac{2\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} - \frac{2a\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{a\int\sqrt{\cos(}}{b^2}$$

$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd} + \frac{2a^2\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d} + \frac{2\sin(}{3bd\cos^{\frac{3}{2}}}$$

Mathematica [A] time = 4.66451, size = 213, normalized size = 1.66

$$\frac{\frac{6\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b\sqrt{\sin^2(c+dx)}} + 8b\left(2\text{EllipticF}\left(\frac{1}{2}\right.\right.}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((2*(9*a^2 + 2*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*
b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b)) + (4*(b - 3*a*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)
 + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Ellip
ticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*EllipticPi[-(a/b), -Ar
cSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/(6*b
```

```
^2*d)
```

Maple [B] time = 4.096, size = 450, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^3/b^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a/b^2*(-
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.824 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=244

$$\frac{(16a^2b^2+2a^4-15b^4)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^4d\,(a^2-b^2)}-\frac{b\,(4a^2-5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\,(a^2-b^2)}-\frac{b^3\,(7a^2-5b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a-b)(a+b)^2}+$$

```
[Out] -((b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d)) + ((2*
a^4 + 16*a^2*b^2 - 15*b^4)*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d)
 - (b^3*(7*a^2 - 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a
- b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^
2*(a^2 - b^2)*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(
a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.726656, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3847, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{(16a^2b^2+2a^4-15b^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d\,(a^2-b^2)}-\frac{b\,(4a^2-5b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\,(a^2-b^2)}-\frac{b^3\,(7a^2-5b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a-b)(a+b)^2}+\frac{(2a^2-5}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d)) + ((2*
a^4 + 16*a^2*b^2 - 15*b^4)*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d)
 - (b^3*(7*a^2 - 5*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a
- b)*(a + b)^2*d) + ((2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^
2*(a^2 - b^2)*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(
a + b*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-a^2+\frac{5b^2}{2}+ab\sec(c+dx)-\frac{3}{2}b^2\sec^2}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx)}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\left(2\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(b^3\left(7a^2-5b^2\right)\right)}{2}$$

$$= -\frac{b^3\left(7a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4(a-b)(a+b)^2d} + \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos}}{a\left(a^2-b^2\right.}$$

$$= -\frac{b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^4+16a^2b^2-15b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4\left(a^2-b^2\right)d} - \frac{b^3\left(7a^2-5b^2\right)}{a^4(a-}$$

Mathematica [A] time = 1.81213, size = 268, normalized size = 1.1

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3b^3}{\left(b^2-a^2\right)(a\cos(c+dx)+b)}+2\right) - \frac{\frac{8\left(a^2+2b^2\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6\left(4a^2-5b^2\right)\sin(c+dx)\left(-2b(a+b)\text{Ellip}\right.}{}}{}}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2 + (3*b^3)/((-a^2 + b^2)*(b + a*Cos[c + d*x])))*Sin
[c + d*x] - ((2*(-8*a^2*b + 5*b^3)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a + b) + (8*(a^2 + 2*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*Ellipt
icPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(4*a^2 - 5*b^2)*(2*a*b*E
llipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqr
t[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c
```

```
 + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)
))/(12*a^2*d)
```

Maple [B] time = 4.984, size = 1064, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^2*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4/a^3*(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*(a^2+2*a*b+3*b^2)/a^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8/a^3*b^3/(a^2-a*b)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
))+2/a^4*b^4*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

3.825 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=184

$$-\frac{b\left(4a^2-3b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^3d\left(a^2-b^2\right)}+\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{b^2\left(5a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}+\frac{\cdots}{ad\left(a^2\cdots\right.}$$

```
[Out] ((2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - (b*(4*a^2
 - 3*b^2)*EllipticF[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + (b^2*(5*a^2 - 3*
b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) +
 (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]
))
```

Rubi [A] time = 0.478678, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4264, 3847, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\left(4a^2-3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\left(a^2-b^2\right)}+\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{b^2\left(5a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}+\frac{\cdots}{ad\left(a^2-b^2\right)\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - (b*(4*a^2
 - 3*b^2)*EllipticF[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + (b^2*(5*a^2 - 3*
b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) +
 (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]
))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
```

```
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-a^2+\frac{3b^2}{2}+ab\,\mathrm{se}\cdots}{\sqrt{\sec(c+\cdots}}}{a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a\left(-a^2+\frac{3b^2}{2}\right)-(\cdots}{\cdots}}{a^3\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} + \frac{\left(b^2\left(5a^2-3b^2\right)\right)\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}\,dx}{2a^3\left(a^2-b^2\right)}$$

$$= \frac{b^2\left(5a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3(a-b)(a+b)^2d} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} + \frac{\left(2a^{2}\cdots\right.}{\cdots}$$

$$= \frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} - \frac{b\left(4a^2-3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{b^2\left(5a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c\cdots\right.}{a^3(a-b)(a+b)^2d}$$

Mathematica [A] time = 1.76664, size = 254, normalized size = 1.38

$$\frac{-\frac{\frac{2\left(2a^2-3b^2\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2b\sqrt{\sin^2(c+dx)}}+8b\left(\frac{b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}-\text{EllipticF}\cdots\right.}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x]))
 + ((2*(2*a^2 - b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 8
*b*(-EllipticF[(c + d*x)/2, 2] + (b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
2])/(a + b)) - (2*(2*a^2 - 3*b^2)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]
]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*
b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^
2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

Maple [B] time = 4.926, size = 809, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-6/a^2*b^2/(a^2-a*b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
-2/a^3*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2
*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx + c)}}{(b\sec(dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c + dx)}}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx + c)}}{(b\sec(dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.826 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=167

$$\frac{(2a^2-b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{a^2d\,(a^2-b^2)}+\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\,(a^2-b^2)}-\frac{b\,(3a^2-b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}-\frac{b\sin(c+}{d\,(a^2-b^2)\sqrt{\cos(c+dx)}}$$

```
[Out] (b*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((2*a^2 - b^2)*EllipticF[
(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - (b*(3*a^2 - b^2)*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) - (b*Sin[c + d*x])/((a^2
- b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.419403, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4264, 3843, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{(2a^2-b^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\,(a^2-b^2)}+\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\,(a^2-b^2)}-\frac{b\,(3a^2-b^2)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}-\frac{b\sin(c+dx)}{d\,(a^2-b^2)\sqrt{\cos(c+dx)}(a+b\,\text{se}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```



```
[Out] (b*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((2*a^2 - b^2)*EllipticF[
(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - (b*(3*a^2 - b^2)*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) - (b*Sin[c + d*x])/((a^2
- b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
```

```
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\ldots}{-a^2\ldots}$$

$$= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\ldots}{a^2\left(-a^2+\ldots\right.}$$

$$= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} - \frac{\left(b\left(3-\frac{b^2}{a^2}\right)\right)\int\frac{1}{\sqrt{\cos(c+dx)}(b+a\cos\ldots}}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\left(3a^2-b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a-b)(a+b)^2d} - \frac{b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+\ldots}$$

$$= \frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{\left(2a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} - \frac{b\left(3a^2-b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\ldots\right.}{a^2(a-b)(a+b)^2d}$$

Mathematica [A] time = 3.45592, size = 196, normalized size = 1.17

$$\frac{\frac{4b\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(b^2-a^2\right)(a\cos(c+dx)+b)} - \frac{-\frac{2\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a^2\sqrt{\sin^2(c+dx)}}+8\text{EllipticF}\left(\ldots\right.}{(b-a)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x]))
- (8*EllipticF[(c + d*x)/2, 2] - (10*b*EllipticPi[(2*a)/(a + b), (c + d*x)/
```

```
2, 2])/(a + b) - (2*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*
(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticP
i[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c +
 d*x]^2]))/((-a + b)*(a + b)))/(4*d)
```

Maple [B] time = 3.556, size = 788, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*b/a
/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2))+2/a^2*b^2*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a
-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^
2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\sec{(c+dx)})^2\sqrt{\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec{(dx+c)}+a)^2\sqrt{\cos{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.827 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$-\frac{b\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{ad\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+$$

```
[Out] -(EllipticE[(c + d*x)/2, 2]/((a^2 - b^2)*d)) - (b*EllipticF[(c + d*x)/2, 2]
)/(a*(a^2 - b^2)*d) + ((a^2 + b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a*(a - b)*(a + b)^2*d) + (a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.393874, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4264, 3844, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{\left(a^2+b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```


```
[Out] -(EllipticE[(c + d*x)/2, 2]/((a^2 - b^2)*d)) - (b*EllipticF[(c + d*x)/2, 2]
)/(a*(a^2 - b^2)*d) + ((a^2 + b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a*(a - b)*(a + b)^2*d) + (a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d
*x]]*(a + b*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
```

```
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -\frac{a}{2}}{-a^2+b^2}$$

$$= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{a^2\left(-a^2+b^2\right)}$$

$$= \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} + \frac{\left(a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos(c+dx))}}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2+b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a-b)(a+b)^2d} + \frac{a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} -$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} - \frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{\left(a^2+b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a-b)(a+b)^2d} + \frac{}{\left(a^2\right.}$$

Mathematica [A] time = 3.23772, size = 233, normalized size = 1.57

$$\frac{\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a\cos(c+dx)+b)} - \frac{\frac{2\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b\sqrt{\sin^2(c+dx)}}+4b\left(2\text{EllipticF}\left(\frac{1}{2}\right.\right.}{a(a-b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) -
 ((-2*a^2*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 4*b*(2*Ellip
```

```
ticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a +
 b)) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*E
llipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*EllipticPi[-(a/b),
 -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/
(a*(a - b)*(a + b)))/(4*d)
```

Maple [B] time = 3.88, size = 707, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/(a^2-a*b)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2))-2*b/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*
a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.828 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\left(a^2-b^2\right)}+\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

```
[Out] (a*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + EllipticF[(c + d*x)/2, 2]
/((a^2 - b^2)*d) + ((a^2 - 3*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/((a - b)*b*(a + b)^2*d) - (a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.445169, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4264, 3845, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}+\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```


```
[Out] (a*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + EllipticF[(c + d*x)/2, 2]
/((a^2 - b^2)*d) + ((a^2 - 3*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/((a - b)*b*(a + b)^2*d) - (a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c
+ d*x]]*(a + b*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
```

```
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{b}$$

$$= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{a^2b\left(a^2-\right.}$$

$$= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))} + \frac{\left(a^2-3b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}(b+a\cos}}{2b\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b(a+b)^2d} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx}$$

$$= \frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b(a+b)^2d} - \frac{}{b\left(\right.}$$

Mathematica [A] time = 3.26628, size = 242, normalized size = 1.57

$$\frac{\frac{\frac{2\sin(c+dx)\left(2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)-\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b\sqrt{\sin^2(c+dx)}}+4b\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx\right.}{a+b}\right.}{(a-b)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(b + a*Cos[c + d*x])
) + ((2*(3*a^2 - 4*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)
+ 4*b*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2])/(a + b)) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a^2 - 2*b^2)*Elli
pticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[
c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

Maple [B] time = 3.063, size = 608, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+a/b/(a^2-b^2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-a/b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-1/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3*b/(a^2-b^2)/(a^2-a*b)*a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.829 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$-\frac{a\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(3a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{\left(3a^2-2b^2\right)\sin(c+a}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+}}$$

```
[Out] -(((3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)) - (a*Ell
ipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) - (a*(3*a^2 - 5*b^2)*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/((a - b)*b^2*(a + b)^2*d) + ((3*a^2 - 2*b^2)
*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) - (a^2*Sin[c + d*x])/
(b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.684068, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\left(3a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```


```
[Out] -(((3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)) - (a*Ell
ipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) - (a*(3*a^2 - 5*b^2)*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/((a - b)*b^2*(a + b)^2*d) + ((3*a^2 - 2*b^2)
*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) - (a^2*Sin[c + d*x])/
(b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))} -$$

$$= \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))} -$$

$$= \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))} -$$

$$= -\frac{a\left(3a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b^2(a+b)^2d} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{}{b\left(a^2-b\right.}$$

$$= -\frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} - \frac{a\left(3a^2-5b^2\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c\right.}{(a-b)b^2(a+b)^2}$$

Mathematica [A] time = 3.07779, size = 280, normalized size = 1.28

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{a^3\sin(c+dx)}{\left(a^2-b^2\right)(a\cos(c+dx)+b)}+2\tan(c+dx)\right) - \frac{\frac{\left(8a^2b-4b^3\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a} - 2\left(3a^2-2b^2\right)\sin(c+dx)\left(-2b(a+b)\text{Ellipt}\right.}{}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] (-(((2*(9*a^3 - 10*a*b^2)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b
) + ((8*a^2*b - 4*b^3)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)
/(a + b), (c + d*x)/2, 2])/(a + b)))/a - (2*(3*a^2 - 2*b^2)*(2*a*b*Elliptic
```

```
E[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c
 + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))) + 4*S
qrt[Cos[c + d*x]]*((a^3*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) +
2*Tan[c + d*x]))/(4*b^2*d)
```

Maple [B] time = 5.325, size = 868, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2/b^2/(a^2-
a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))-2*a/b*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/
(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^
2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2/b^2
*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/
2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)), x)
```

3.830 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=346

$$\frac{\left(128a^4b^2-223a^2b^4+8a^6+105b^6\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{12a^5d\left(a^2-b^2\right)^2}-\frac{b\left(-65a^2b^2+24a^4+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}-\frac{b^3\left(-86a^2b^2\right.}{}$$

```
[Out] -(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 -
 b^2)^2*d) + ((8*a^6 + 128*a^4*b^2 - 223*a^2*b^4 + 105*b^6)*EllipticF[(c +
d*x)/2, 2])/(12*a^5*(a^2 - b^2)^2*d) - (b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) +
((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*a^3*(a^
2 - b^2)^2*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a
 + b*Sec[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 1.05687, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4264, 3847, 4100, 4104, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(128a^4b^2-223a^2b^4+8a^6+105b^6\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^5d\left(a^2-b^2\right)^2}-\frac{b\left(-65a^2b^2+24a^4+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}-\frac{b^3\left(-86a^2b^2+63a^4\right.}{4a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```



```
[Out] -(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 -
 b^2)^2*d) + ((8*a^6 + 128*a^4*b^2 - 223*a^2*b^4 + 105*b^6)*EllipticF[(c +
d*x)/2, 2])/(12*a^5*(a^2 - b^2)^2*d) - (b^3*(63*a^4 - 86*a^2*b^2 + 35*b^4)*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) +
((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*a^3*(a^
2 - b^2)^2*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a
 + b*Sec[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*
x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx \\
&= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-2a^2+\frac{7b^2}{2}+2ab\sec(c+dx)-\frac{5}{2}b}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+}}{2a\left(a^2-b^2\right)} \\
&= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2\left(13a^2-7b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(a+b\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+\right.}{} \\
&= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2(}{} \\
&= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2(}{} \\
&= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b^2(}{} \\
&= -\frac{b^3\left(63a^4-86a^2b^2+35b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^5(a-b)^2(a+b)^3d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,\mathrm{s}}{12a^3\left(a^2-b^2\right)^2 d} \\
&= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^6+128a^4b^2-223a^2b^4+105b^6\right)F\left(\frac{1}{2}(c+\right.}{12a^5\left(a^2-b^2\right)^2 d}
\end{aligned}
$$

Mathematica [A] time = 3.59901, size = 355, normalized size = 1.03

$$
\frac{\frac{16\left(14a^2b^2+2a^4-7b^4\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6\left(-65a^2b^2+24a^4+35b^4\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos\right.\right.\right.}{a^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(4*a^6 - 57*a^2*b^4 + 35*b^6 + a*b*(16*a^4 - 83*a^2*
b^2 + 49*b^4)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*
x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(56*a^4*b - 73*a^2*b^3 +
35*b^5)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^4 + 1
4*a^2*b^2 - 7*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/
(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(24*a^4 - 65*a^2*b^2 + 35*b^4)*(2*a
*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin
[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[C
os[c + d*x]]], -1])*Sin[c + d*x])/(a^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a
 + b)^2))/(48*a^3*d)
```

Maple [B] time = 7.931, size = 2216, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/a^3*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-3*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/a^4*(2*a+3*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(a^2+3*a*b+6*b^2)/a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/a^5*b^5*(1/2*a^2/b/(a^2-b^
2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/
2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^
2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)
*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
)))+20*b^3/a^4/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+10/a^5*b^4*(a^2/b/(a^2-b^2)*cos(1/2
*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2
*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b
)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x)
```

3.831 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=282

$$-\frac{3b\left(-11a^2b^2+8a^4+5b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2b^2+8a^4+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{b^2\left(-38a^2b^2+35a^4+\right.}{4a^4d(a}$$

```
[Out] ((8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2
)^2*d) - (3*b*(8*a^4 - 11*a^2*b^2 + 5*b^4)*EllipticF[(c + d*x)/2, 2])/(4*a^
4*(a^2 - b^2)^2*d) + (b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + (b^2*Sin[c + d*x])
/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2) + (b^2*(11*a
^2 - 5*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*
Sec[c + d*x]))
```

Rubi [A] time = 0.806788, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3847, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{3b\left(-11a^2b^2+8a^4+5b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2b^2+8a^4+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{b^2\left(-38a^2b^2+35a^4+15b^4\right)\text{I}}{4a^4d(a-b)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2
)^2*d) - (3*b*(8*a^4 - 11*a^2*b^2 + 5*b^4)*EllipticF[(c + d*x)/2, 2])/(4*a^
4*(a^2 - b^2)^2*d) + (b^2*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + (b^2*Sin[c + d*x])
/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2) + (b^2*(11*a
^2 - 5*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*
Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-2a^2+\frac{5b^2}{2}+2a}{\sqrt{\sec(c}}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\sec(c+}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\sec(c+}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}(a+b\sec(c+}$$

$$= \frac{b^2\left(35a^4-38a^2b^2+15b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^4(a-b)^2(a+b)^3 d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} - \frac{3b\left(8a^4-11a^2b^2+5b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{b^2\left(35a\right.}{}$$

Mathematica [A] time = 2.75075, size = 315, normalized size = 1.12

$$\frac{-\frac{\frac{16\left(4a^2b-b^3\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}-\frac{2\left(-29a^2b^2+8a^4+15b^4\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right.}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*(11*a^2*b - 5*b^3 + a*(13*a^2 - 7*b^2)*Cos[c + d
*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(8*a^4 - 7*
a^2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (16*(
4*a^2*b - b^3)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2]))/(a + b) - (2*(8*a^4 - 29*a^2*b^2 + 15*b^4)*(2*a*b*El
lipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt
```

```
[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c
+ d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a +
b)^2))/(16*a^2*d)
```

Maple [B] time = 7.339, size = 1957, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*b*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2/a^4*b^4*(1/2*a^2/b/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos
(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)
/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(
a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c
```

```
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-12
*b^2/a^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-8/a^4*b^3*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/
2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-
b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x)
```

3.832 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=263

$$\frac{\left(-5a^2b^2+8a^4+3b^4\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{3b\left(-2a^2b^2+5a^4+b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+d\right.}{4a^3d(a-b)^2(a+b)^3}$$

```
[Out] (3*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((8
*a^4 - 5*a^2*b^2 + 3*b^4)*EllipticF[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d
) - (3*b*(5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(4*a^3*(a - b)^2*(a + b)^3*d) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*d*Sqrt[C
os[c + d*x]]*(a + b*Sec[c + d*x])^2) - (b*(7*a^2 - b^2)*Sin[c + d*x])/(4*a*
(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.682805, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3843, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(-5a^2b^2+8a^4+3b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}+\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{3b\left(-2a^2b^2+5a^4+b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a-b)^2(a+b)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] (3*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((8
*a^4 - 5*a^2*b^2 + 3*b^4)*EllipticF[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d
) - (3*b*(5*a^4 - 2*a^2*b^2 + b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(4*a^3*(a - b)^2*(a + b)^3*d) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*d*Sqrt[C
os[c + d*x]]*(a + b*Sec[c + d*x])^2) - (b*(7*a^2 - b^2)*Sin[c + d*x])/(4*a*
(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{2\left(\right.}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sin}{4a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sin}{4a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sin}{4a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= -\frac{3b\left(5a^4-2a^2b^2+b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3(a-b)^2(a+b)^3d} - \frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+}$$

$$= \frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-5a^2b^2+3b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} - \frac{3b\left(5\right.}{}$$

Mathematica [A] time = 2.81952, size = 288, normalized size = 1.1

$$\frac{\frac{\frac{16\left(2a^2+b^2\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}-\frac{6\left(3a^2-b^2\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin\right.\right.}{a^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((4*b*Sqrt[Cos[c + d*x]]*(-7*a^2*b + b^3 + (-9*a^3 + 3*a*b^2)*Cos[c + d*x])
*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((-2*(5*a^2*b + b^3
)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^2 + b^2)*((
a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]))/(a + b) - (6*(3*a^2 - b^2)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]
], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b
```

```
^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2
*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

Maple [B] time = 6.48, size = 1936, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2/a^3
*b^3*(1/2*a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)
/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^
2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a
+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(
a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
```

```
1/2*c),2*a/(a-b),2^(1/2)))+6/a^2*b/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+6/a^3*b^2*(a^2/
b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b
/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.833 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=246

$$-\frac{b\left(7a^2-b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{\left(10a^2b^2+3a^4-b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a-b)^2(a+b)^3}+\frac{\ \ }{4d\left(\right.}$$

```
[Out] -((5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) - (b*(7*a^
2 - b^2)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4 + 10*
a^2*b^2 - b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)^2*
(a + b)^3*d) + (a*Sin[c + d*x])/(2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*
Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*Sqrt[Cos
[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.657735, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3844, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\left(7a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{\left(10a^2b^2+3a^4-b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a-b)^2(a+b)^3}+\frac{\ \ }{4d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -((5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) - (b*(7*a^
2 - b^2)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4 + 10*
a^2*b^2 - b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)^2*
(a + b)^3*d) + (a*Sin[c + d*x])/(2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*
Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*Sqrt[Cos
[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{2\left(a^2-\right.}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sin(c+}{4\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}(a+}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sin(c+}{4\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}(a+}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sin(c+}{4\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}(a+}$$

$$= \frac{\left(3a^4+10a^2b^2-b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2(a-b)^2(a+b)^3d} + \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec}$$

$$= -\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2d} - \frac{b\left(7a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d} + \frac{\left(3a^4+10a^2b^2-\right.}{4a^2(a}$$

Mathematica [A] time = 1.98825, size = 274, normalized size = 1.11

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(a\left(5a^2+b^2\right)\cos(c+dx)+3b\left(a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2} - \frac{-\frac{2\left(5a^2+b^2\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(3*b*(a^2 + b^2) + a*(5*a^2 + b^2)*Cos[c + d*x])*Sin
[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) - ((-2*(a^2 + 5*b^2)*Elli
pticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*b*(2*EllipticF[(c + d*x
)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) - (2*(5*
a^2 + b^2)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*E
```

```
llipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b),
 -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2
]))/((a - b)^2*(a + b)^2))/(16*d)
```

Maple [B] time = 6.548, size = 1858, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*b^2*(1/2*
a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-
b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b
)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8
*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a
-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b
^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a
/(a-b),2^(1/2)))-2/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
```

```
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-4*b/a^2*(a^2/b/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-
a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

3.834 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=253

$$\frac{3\left(a^2+b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{\left(-10a^2b^2+a^4-3b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4abd(a-b)^2(a+b)^3}-\frac{}{2bd\left(a\right.}$$

```
[Out] ((a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (3*(a^2 +
 b^2)*EllipticF[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^2
 - 3*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b*(a +
b)^3*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*S
ec[c + d*x])^2) + (a*(a^2 - 7*b^2)*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.721445, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3845, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}+\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{\left(-10a^2b^2+a^4-3b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4abd(a-b)^2(a+b)^3}-\frac{}{2bd\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] ((a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (3*(a^2 +
 b^2)*EllipticF[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^2
 - 3*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b*(a +
b)^3*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*S
ec[c + d*x])^2) + (a*(a^2 - 7*b^2)*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{\cdots}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sin\cdots}{4b\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d\cdots}}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sin\cdots}{4b\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d\cdots}}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sin\cdots}{4b\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d\cdots}}$$

$$= \frac{\left(a^4-10a^2b^2-3b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a(a-b)^2b(a+b)^3d} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cdots}$$

$$= \frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} + \frac{\left(a^4-10a^2b^2-3\cdots\right.}{4a(a-\cdots}$$

Mathematica [A] time = 3.10519, size = 291, normalized size = 1.15

$$\frac{\frac{\frac{8b\left(a^2+2b^2\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a} - \frac{2\left(a^2+5b^2\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\cdots\right.\right.}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((-4*a*Sqrt[Cos[c + d*x]]*(-(a^2*b) + 7*b^3 + a*(a^2 + 5*b^2)*Cos[c + d*x])
*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((6*(a^3 - 3*a*b^2)
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(a^2 + 2*b^2)*(2
*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
```

```
)/(a + b)))/a - (2*(a^2 + 5*b^2)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]
], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b
^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b
*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b*d)
```

Maple [B] time = 6.368, size = 1760, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/a*(1/2*a^2
/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(
a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)
/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
```

```
-b),2^(1/2)))+2/a*(a^2/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.835 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=255

$$\frac{\left(a^2-7b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{3\left(-2a^2b^2+a^4+5b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a-b)^2(a+b)^3}-\frac{\cdots}{2b\cdots}$$

```
[Out] (3*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((a
^2 - 7*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (3*(a^4 - 2*
a^2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^
2*(a + b)^3*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(
a + b*Sec[c + d*x])^2) - (3*a^2*(a^2 - 3*b^2)*Sin[c + d*x])/(4*b^2*(a^2 - b
^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rubi [A] time = 0.746259, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {4264, 3845, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{3\left(-2a^2b^2+a^4+5b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a-b)^2(a+b)^3}-\frac{\cdots}{2bd\left(a^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] (3*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((a
^2 - 7*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (3*(a^4 - 2*
a^2*b^2 + 5*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^
2*(a + b)^3*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(
a + b*Sec[c + d*x])^2) - (3*a^2*(a^2 - 3*b^2)*Sin[c + d*x])/(4*b^2*(a^2 - b
^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-3b^2\right)\sin}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-3b^2\right)\sin}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{3a^2\left(a^2-3b^2\right)\sin}{4b^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= \frac{3\left(a^4-2a^2b^2+5b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4(a-b)^2b^2(a+b)^3d} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\,\mathrm{s}}$$

$$= \frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{3\left(a^4-2a^2b^2+5\right.}{4(a-}$$

Mathematica [A] time = 2.77507, size = 299, normalized size = 1.17

$$\frac{\frac{\frac{16b\left(a^2-4b^2\right)\left((a+b)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}-\frac{6\left(a^2-3b^2\right)\sin(c+dx)\left(-2b(a+b)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right),-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-}\right.\right.}{b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((-4*a^2*Sqrt[Cos[c + d*x]]*(5*a^2*b - 11*b^3 + 3*a*(a^2 - 3*b^2)*Cos[c + d
*x])*Sin[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(9*a^4 - 19
*a^2*b^2 + 16*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16
*b*(a^2 - 4*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a
```

```
 + b), (c + d*x)/2, 2]))/(a + b) - (6*(a^2 - 3*b^2)*(2*a*b*EllipticE[ArcSin
[Sqrt[Cos[c + d*x]]], -1] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]
], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])
*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b^2*d)
```

Maple [B] time = 3.999, size = 1203, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(a^2/b/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/2*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*a-a+b)-3/4/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/2/(a+b)/(a^2-b^2)/b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*a+7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/4*a/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3/4*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+9/4*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/4/(a-b)/(a+b)/(a^2-
b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a
^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))-15/4/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))
```

```
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)), x)
```

3.836 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=328

$$-\frac{a\left(5a^2-11b^2\right)\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2b^2+15a^4+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{a\left(-38a^2b^2+15a^4+35b^4\right)\Pi\left(\frac{2a}{a+b}\right.}{4b^3d(a-b)^2(a+b)^3}$$

```
[Out] -((15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^
2)^2*d) - (a*(5*a^2 - 11*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)
^2*d) - (a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*Si
n[c + d*x])/(4*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - (a^2*Sin[c + d*x])
/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2) - (a^2*(5*a^
2 - 11*b^2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*
Sec[c + d*x]))
```

Rubi [A] time = 0.999823, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {4264, 3845, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a\left(5a^2-11b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2b^2+15a^4+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{a\left(-38a^2b^2+15a^4+35b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{4b^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```


```
[Out] -((15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^
2)^2*d) - (a*(5*a^2 - 11*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)
^2*d) - (a*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*Si
n[c + d*x])/(4*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) - (a^2*Sin[c + d*x])
/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2) - (a^2*(5*a^
2 - 11*b^2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*
Sec[c + d*x]))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2} - \frac{a^2\left(5a^2-11b^2\right)\mathrm{s}}{4b^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+d}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} - \frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+}$$

$$= -\frac{a\left(15a^4-38a^2b^2+35b^4\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4(a-b)^2b^3(a+b)^3d} + \frac{\left(15a^4-29a^2b^2+8b^4\right)\sin}{4b^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c}}$$

$$= -\frac{\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a\left(5a^2-11b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} - $$

Mathematica [A] time = 3.32688, size = 336, normalized size = 1.02

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{a^3\sin(c+dx)\left(a\left(7a^2-13b^2\right)\cos(c+dx)+9a^2b-15b^3\right)}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+8\tan(c+dx)\right) - \frac{\frac{8b\left(-10a^2b^2+5a^4+2b^4\right)\left(2\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right.}{a}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] (-(((2*(45*a^5 - 95*a^3*b^2 + 56*a*b^4)*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(a + b) + (8*b*(5*a^4 - 10*a^2*b^2 + 2*b^4)*(2*EllipticF[(c + d*x)/
2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a - (2*(1
5*a^4 - 29*a^2*b^2 + 8*b^4)*(2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1
] - 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*E
llipticPi[-(a/b), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt
[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + 4*Sqrt[Cos[c + d*x]]*((a^3*(9*a
^2*b - 15*b^3 + a*(7*a^2 - 13*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b^2)
^2*(b + a*Cos[c + d*x])^2) + 8*Tan[c + d*x]))/(16*b^3*d)
```

Maple [B] time = 7.894, size = 2014, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a/b*(1/2*a^2
/b/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2
)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(
a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)
/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)
```

```
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a
-b),2^(1/2)))+2*a^2/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*a/b^2*(a^2/b/(a^2-b^2)*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*a/b/(a^2-b^2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*a/b/(a
^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a
^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2*a/(a-b),2^(1/2)))+2/b^3*(-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c
)^2/(2*sin(1/2*d*x+1/2*c)^2-1))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^3\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)), x)
```

3.837 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=244

$$-\frac{4b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (
2*(9*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (
2*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2
*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.658907, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3857, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{4b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-4*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (
2*(9*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (
2*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2
*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{1}{5}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{5d}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{5d}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{5d}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15ad} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{5d}$$

$$= -\frac{4b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(\right.}{15a^2d\sqrt{\frac{b+}{}}}$$

Mathematica [C] time = 8.76432, size = 340, normalized size = 1.39

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\left(a\sin(c+dx)(3a\cos(c+dx)+b) - \frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(ia\left(9a^2+7ab-2b^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(a*(b + 3*a*Cos[c + d*x])*Si
n[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(9*a^3 + 9*a^2*
b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)] + I*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a
```

```
 + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2)/(a + b)] - (9*a^2 - 2*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^
(3/2)*Tan[(c + d*x)/2]))/((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2))))/(15*a^
2*d)
```

Maple [B] time = 0.327, size = 1726, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/((a-b)/(a+b))^(1/2)/a^2*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)*(-9*a^2*b*((a-b)/(a+b))^(1/2)-a*b^2*((a-b)/(a+b))^(1/2)-9*cos(d*x
+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^3+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)+9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-9*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+2
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+4*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^
2*b-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+7*cos(d*x+c)*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+
2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-2*cos(d*x+c
)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*a*b^2+2*b^3*((a-b)/(a+b))^(1/2)+2*cos(d*x+c)*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
```

```
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^
3+7*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.838 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{a}}{3}$$

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d
*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.432075, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4264, 3857, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}}{3ad\sqrt{\frac{a\cos}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sq
rt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d
*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3857

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^n)/(f*n), x] - Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[
b - 2*a*(n + 1)*Csc[e + f*x] - b*(2*n + 3)*Csc[e + f*x]^2, x])/Sqrt[a + b*C
sc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && L
eQ[n, -1] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{3a}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{\left(\left(-a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\right)}{3a\sqrt{\cos(c+dx)}\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{\left(\left(-a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int}{3a\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

$$= \frac{2\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 8.12308, size = 273, normalized size = 1.42

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\left(-ia(a+b)\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\mathrm{EllipticF}\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(I*b*(a + b)*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(a +
 b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c
 + d*x]] + a^2*Sin[c + d*x] + a*b*Tan[(c + d*x)/2] + b^2*Sec[c + d*x]*Tan[(
c + d*x)/2] + a*b*Tan[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.309, size = 1011, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)/a*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b+cos(d*x+c)*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+((a-b)/(a+b))^(1/2)*cos(d*x+c)
^3*a^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+
2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-a^2*((a-b)/(a+b))^(1/2)*cos(d*x+c)-(
(a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-a*b*((
a-b)/(a+b))^(1/2)-b^2*((a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a} \cos (dx + c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sec (dx + c) + a} \cos (dx + c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a} \cos (dx + c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

3.839 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```

Rubi [A] time = 0.145862, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3856, 2655, 2653}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx\\
&= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{b+a\cos(c+dx)}\,dx}{\sqrt{b+a\cos(c+dx)}}\\
&= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}\\
&= \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}
\end{aligned}$$

Mathematica [C] time = 3.36157, size = 198, normalized size = 2.96

$$\frac{\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\sec(c+dx)}\left(-i\text{EllipticF}\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{b-a}{a+b}\right)+\sin(c+dx)\sqrt{\frac{1}{\cos(c+d}}\right.}{d\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*Sqrt[a + b*Sec[c + d*x]]*(I*Elliptic
E[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - I*EllipticF[I*ArcSinh[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] + Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sin[c + d*x]))/(d*Sqrt[(1 +
```

```
Cos[c + d*x])^(-1)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
])
```

Maple [B] time = 0.287, size = 923, normalized size = 13.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/
2)*(-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a+cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b+cos(d*x+c)*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*b-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x
+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b*sin(d*x+c)+cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-((a-b)/(a+b))^(1/2)*b)/(b+a*co
s(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec (dx + c) + a} \sqrt{\cos (dx + c)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.840 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.406017, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4264, 3854, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
```

```
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}\,dx \\
&= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + \left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{}} \\
&= \frac{\left(a\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\right)\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{\left(a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 28.1773, size = 14885, normalized size = 107.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.25, size = 275, normalized size = 2.

$$
2\,\frac{\sqrt{\cos(dx+c)}\,(\sin(dx+c))^2\sqrt{(\cos(dx+c)+1)^{-1}}}{d\,(-1+\cos(dx+c))\,(b+a\cos(dx+c))}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right)a-\mathrm{EllipticF}\left(-\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
```

```
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b)*cos(d*x+c)^(1/2
)*sin(d*x+c)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)/(-1+cos(d*x+c))/(b+a*cos(d
*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.841 $\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+\right.}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (a*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.689847, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {4264, 3855, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (a*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3855

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*
Csc[e + f*x])^(n - 1))/(f*(2*n - 1)), x] + Dist[d^2/(2*n - 1), Int[((d*Csc[
e + f*x])^(n - 2)*Simp[2*a*(n - 2) + b*(2*n - 3)*Csc[e + f*x] + a*Csc[e + f
*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 NeQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-a+a\sec^2(c+dx}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx +$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(a\sqrt{\frac{b+a\cos(c+dx}{a+b}}\right.}{2\sqrt{\cos(c+dx}}$$

$$= \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int -}{2\sqrt{\cos(c+dx)}\sqrt{a-}}$$

$$= \frac{b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.}{}$$

Mathematica [C] time = 30.7325, size = 23549, normalized size = 99.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.289, size = 781, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)*(-cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+cos(d*x+c)^2*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b+2*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a+cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*b+2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*
((a-b)/(a+b))^(1/2)*b-((a-b)/(a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + b \sec(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b \sec(dx + c) + a}}{\cos(dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.842 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=303

$$\frac{2\left(-31a^2b^2+25a^4+6b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}{105ad}$$

```
[Out] (2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*(25*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(105*a*d) + (16*b*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(35*d) + (2*a*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.939951, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3864, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{105ad}+\frac{2\left(-31a^2b^2+25a^4+6b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]) + (4*b*(41*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*(25*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(105*a*d) + (16*b*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(35*d) + (2*a*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(7*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d} - \frac{1}{7}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= \frac{16b\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{35d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(}}{7d}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105ad} + \frac{16b\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105ad} + \frac{16b\cos^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105ad} + \frac{16b\cos^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{105ad} + \frac{16b\cos^{\frac{3}{2}}(c+dx}{}$$

$$= \frac{2\left(25a^4-31a^2b^2+6b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{4b\left(41a^2-3b^2\right)\sqrt{\mathrm{co}}}{}$$

Mathematica [C] time = 9.86517, size = 383, normalized size = 1.26

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(a\sin(c+dx)(a\cos(c+dx)+b)\left(15a^2\cos(2(c+dx))+65a^2+48ab\cos(c+dx)+6b^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(a*(b + a*Cos[c + d*x])*(65*
a^2 + 6*b^2 + 48*a*b*Cos[c + d*x] + 15*a^2*Cos[2*(c + d*x)])*Sin[c + d*x] -
```

```
(2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-41*a^3 - 41*a^2*b +
3*a*b^2 + 3*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
ec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] +
 I*a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + 2*b*(-41*a^2 + 3*b^2)*(b + a*Cos[c + d*x])*(Sec
[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(105*a^2*d*(
b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.309, size = 2040, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/105/d/a^2/((a-b)/(a+b))^(1/2)*(-25*a^3*b*((a-b)/(a+b))^(1/2)-82*a^2*b^2*
((a-b)/(a+b))^(1/2)-3*a*b^3*((a-b)/(a+b))^(1/2)+25*cos(d*x+c)*sin(d*x+c)*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^4+82*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-82*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-82*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)+51*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+25*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)+6*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^4-6*cos(d*x+c)*((a-b
```

```
)/(a+b))^(1/2)*b^4+15*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*cos(d*x+c)^3*
((a-b)/(a+b))^(1/2)*a^4-25*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+6*b^4*((a-b)/
(a+b))^(1/2)+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+39*cos(d*x+c)^4*((a-b)/
(a+b))^(1/2)*a^3*b+27*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+68*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-82*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+55*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b
^2+82*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b-82*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2-6*co
s(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a*b^3-82*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+51*cos(d*x+c)*
sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b^2+6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos(dx+c)^3\sec(dx+c)+a\cos(dx+c)^3\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^3*sec(d*x + c) + a*cos(d*x + c)^3)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^{\frac{3}{2}} \cos (dx + c)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2), x)
```

3.843 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=240

$$\frac{2b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+$$

```
[Out] (2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(5*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (4*b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d
*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.681495, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3864, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3
*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(5*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (4*b*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d
*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} - \frac{1}{5}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d$$

$$= \frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(}{5d}$$

$$= \frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(}{5d}$$

$$= \frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(}{5d}$$

$$= \frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(}{5d}$$

$$= \frac{2b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.}{5ad\sqrt{\frac{b\cdot}{}}}$$

Mathematica [C] time = 8.07437, size = 344, normalized size = 1.43

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(2\sin(c+dx)(a\cos(c+dx)+b)(a\cos(c+dx)+2b) - \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(ia\left(3a^2\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(2*(b + a*Cos[c + d*x])*(2*b
 + a*Cos[c + d*x])*Sin[c + d*x] - (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2
)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
+ d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b + b^2)*EllipticF[I*ArcSinh[Tan[(
```

```
c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2 + b^2)*(b + a*Cos[c + d*x])*(Sec[(
c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(3/2))))/(5*d*(b +
a*Cos[c + d*x])^2)
```

Maple [B] time = 0.271, size = 1697, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d/a/((a-b)/(a+b))^(1/2)*(cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3-3*cos(d*
x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3+4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c)
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b^2+3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-3*cos(d*x+c)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+cos(
d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a*b^2-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+3*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a^2*b-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
```

```
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+3*cos(d*x+c)^2*((a-b)/(a+b))^(1
/2)*a*b^2-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-cos(d*x+c)*((a-b)/(a+b))^(1/
2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-3*a^2*b*((a-b)/(a+b))^(1/2)-2*a
*b^2*((a-b)/(a+b))^(1/2)-b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos(dx+c)^2\sec(dx+c)+a\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2*sec(d*x + c) + a*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{3}{2}}\cos (dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2), x)
```

3.844 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=187

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{8b\sqrt{\cos(c+dx)}}{}$$

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.46912, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4264, 3864, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{8b\sqrt{\cos(c+dx)}\sqrt{a+b\,\mathrm{s}}}{3d\sqrt{\frac{a\,\mathrm{c}}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3864

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(3/2), x_Symbol] :> Simp[(a*Cot[e + f*x]*Sqrt[a + b*Csc[e + f*x]]*(d*C
sc[e + f*x])^n)/(f*n), x] + Dist[1/(2*d*n), Int[((d*Csc[e + f*x])^(n + 1)*S
imp[a*b*(2*n - 1) + 2*(b^2*n + a^2*(n + 1))*Csc[e + f*x] + a*b*(2*n + 3)*Cs
c[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegersQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(4b\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{\left(\left(-a^2+b^2\right)\sqrt{b+a\cos(c+\right.}{3\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} - \frac{\left(\left(-a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)}{3\sqrt{\cos(c+dx)}\sqrt{a+}}$$

$$= \frac{2\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{3d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

Mathematica [C] time = 6.34616, size = 284, normalized size = 1.52

$$\frac{2\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\left(\frac{1}{2}a\sin(2(c+dx))(a\cos(c+dx)+b)+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-i\left(a^2+4ab+3b^2\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+\right.}{}}\right.}{}\right.}{3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*((a*(b + a*Cos[c + d*x])*S
in[2*(c + d*x)])/2 + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*b*(a + b
)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)*EllipticF
[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/
2]^2]*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(3*d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.28, size = 1209, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)*(4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*cos(d*x+c)*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*b^2+cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-4*cos(d*x+c)*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b+3*co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^2+((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2+4*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)-4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d
```

```
*x+c)+5*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-a^2*((a-b)/(a+b))^(1/2)*cos(d*
x+c)-4*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b+4*((a-b)/(a+b))^(1/2)*cos(d*x+c)*
b^2-a*b*((a-b)/(a+b))^(1/2)-4*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/co
s(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c)\sec(dx+c)+a\cos(dx+c))\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)*sec(d*x + c) + a*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{3}{2}}\cos (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

3.845 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$

Optimal. Leaf size=209

$$\frac{2ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{d\sqrt{\frac{a\cos(c+dx)+}{a+b}}}$$

```
[Out] (2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```

Rubi [A] time = 0.55178, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {4264, 3868, 3856, 2655, 2653, 3854, 3858, 2663, 2661, 3859, 2807, 2805}

$$\frac{2b^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^2*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3868

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_
)]*(d_.)], x_Symbol] :> Dist[a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e +
 f*x]], x], x] + Dist[b/d, Int[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3854

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + \left(b\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.$$
$$= \left(ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + \left(b^2\sqrt{\cos(c+dx)}\sqrt{\right.$$
$$= \frac{\left(ab\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(b^2\sqrt{b+a\cos(c+dx)}\right)\int \frac{\sec(c\text{-}}{\sqrt{b+a\cos}}}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}$$
$$= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{\left(ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int -}{\sqrt{\cos(c+dx)}\sqrt{a+}}$$
$$= \frac{2ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} +$$

Mathematica [C] time = 29.4704, size = 25369, normalized size = 121.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.26, size = 1365, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/
2)*(-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
```

```
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b-cos(d*x+c)*s
in(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*b^2+cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*c
os(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b^2*sin(d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2*
sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2-a^2*((a-b)/(a+b))^(1/2)*cos
(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-a*b*((a-b)/(a+b))^(1/2))/(b+a*co
s(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.846 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=249

$$\frac{\left(2a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+a}}{d\sqrt{\frac{a\cos(c+dx)}{a+b}}}$$

```
[Out] ((2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (3*a*b*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (b*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]])
```

Rubi [A] time = 0.775235, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {4264, 3866, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(2a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c\right.}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```


```
[Out] ((2*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (3*a*b*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (b*Sqrt[Cos[c + d*x]]*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt
[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3866

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[d/(m + n - 1), Int[(
a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (b^
2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || IntegersQ[
2*m, 2*n])
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}\,dx$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{ab}{2}+a^2\sec(c+dx)+}{\sqrt{\sec(c+dx)}\sqrt{a+b}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{ab}{2}+a^2\sec(c+}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec}}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,d$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(\left(2a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(3}{}$$

$$= \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(\left(2a^2+b^2\right)\sqrt{\frac{b}{}}\right.}{2\sqrt{\cos(c}}$$

$$= \frac{\left(2a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{3ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{b\sqrt{c}}{}$$

Mathematica [C] time = 31.2735, size = 24604, normalized size = 98.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.293, size = 1204, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d/((a-b)/(a+b))^(1/2)*(cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-sin(d*x+c)*cos(d*x+c
)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^2-6*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-2*cos(d*x+c)^2*sin(
d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*a^2+2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b+cos(d*x+c)*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*b^2-6*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-2*cos
(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b-((a-b)/(a+b))^(1/2)*
cos(d*x+c)^2*a*b+((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-((a-b)/(a+b))^(1/2)*cos
(d*x+c)*b^2+b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b
+a*cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.847 $\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=299

$$\frac{7ab\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec}}{2d\cos^{\frac{3}{2}}(c+dx}$$

```
[Out] (7*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (5*a*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(2*d*Cos[c + d*x]^(3/2)) + (5*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
4*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.02438, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4264, 3866, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{(3a^2+4b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2d\cos^{\frac{3}{2}}(c+dx)}+\frac{5a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\cos(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```


```
[Out] (7*a*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (5*a*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]) + (b*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(2*d*Cos[c + d*x]^(3/2)) + (5*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(
4*d*Sqrt[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3866

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + n - 1)), x] + Dist[d/(m + n - 1), Int[(
a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n - 1)*Simp[a*b*(n - 1) + (b^
2*(m + n - 2) + a^2*(m + n - 1))*Csc[e + f*x] + a*b*(2*m + n - 2)*Csc[e + f
*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
0, m, 2] && LtQ[0, n, 3] && NeQ[m + n - 1, 0] && (IntegerQ[m] || IntegersQ[
2*m, 2*n])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\,dx$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{ab}{2}+\right.}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\text{se}\right.}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\text{se}\right.}{}$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{1}{8}\left(5a\sqrt{\cos(c+dx\right.$$

$$= \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{5a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(7ab\sqrt{b+a\cos(c\right.}{8\sqrt{\cos(c+dx}}$$

$$= \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + 5a\sqrt{}$$

$$= \frac{7ab\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{(3a^2+4b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{5a}{}$$

Mathematica [C] time = 32.0869, size = 51315, normalized size = 171.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.253, size = 1742, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d/((a-b)/(a+b))^(1/2)*(-5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2+5*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)
*cos(d*x+c)^3*a*b+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^2+2*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)
^3*a*b-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*b^2+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^3*a^2+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^2-5*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^2*a^2+5*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*cos(d*x+c)^2*sin(d*x+c)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^
2+2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b-4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*b^2+6*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c
)^2*b^2+5*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2+2*((a-b)/(a+b))^(1/2)*cos(d*
```

```
x+c)^3*a*b-5*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2+5*((a-b)/(a+b))^(1/2)*cos
(d*x+c)^2*a*b+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^2-7*((a-b)/(a+b))^(1/2)*
cos(d*x+c)*a*b-2*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/
2)/(b+a*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.848 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=363

$$\frac{4b\left(-62a^2b^2+57a^4+5b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\text{s}}}{315d}$$

```
[Out] (4*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]) + (2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(163*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(49*a^2 + 75*b^2)*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (38*a*b*
Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(63*d) + (2*a^2*C
os[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.32757, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{315d}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{315ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (4*b*(57*a^4 - 62*a^2*b^2 + 5*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]) + (2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(163*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(49*a^2 + 75*b^2)*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d) + (38*a*b*
Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(63*d) + (2*a^2*C
os[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{38ab\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{63d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\,\mathrm{se}}}{9d}$$

$$= \frac{2\left(49a^2+75b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315d} + \frac{38ab\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315ad} + \frac{2\left(49a^2+75b\right.}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315ad} + \frac{2\left(49a^2+75b\right.}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315ad} + \frac{2\left(49a^2+75b\right.}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{315ad} + \frac{2\left(49a^2+75b\right.}{}$$

$$= \frac{4b\left(57a^4-62a^2b^2+5b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(147a^4+279a^2b\right.}{}$$

Mathematica [C] time = 13.5271, size = 477, normalized size = 1.31

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{630}\left(133a^2+150b^2\right)\sin(2(c+dx))+\frac{b\left(747a^2+20b^2\right)\sin(c+dx)}{630a}+\frac{1}{36}a^2\sin(4(c+dx))+\frac{19}{126}ab\sin(3(c+dx))\right)(a-}{d(a\cos(c+dx)+b)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*((b*(747*a^2 + 20*b^2)*Sin[c
 + d*x])/(630*a) + ((133*a^2 + 150*b^2)*Sin[2*(c + d*x)])/630 + (19*a*b*Sin
[3*(c + d*x)])/126 + (a^2*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])^2)
 - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec
[c + d*x])^(5/2)*((-I)*(147*a^5 + 147*a^4*b + 279*a^3*b^2 + 279*a^2*b^3 - 1
0*a*b^4 - 10*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ I*a*(147*a^4 + 261*a^3*b + 279*a^2*b^2 + 155*a*b^3 - 10*b^4)*EllipticF[I*
ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4 + 279*a^2*b^2 - 10*
b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(31
5*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.41, size = 2778, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/315/d/a^2/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)*(-147*a^4*b*((a-b)/(a+b))^(1/2)+147*sin(d*x+c)*cos(d*x+c)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5+
35*((a-b)/(a+b))^(1/2)*cos(d*x+c)^6*a^5-163*a^3*b^2*((a-b)/(a+b))^(1/2)-279
*a^2*b^3*((a-b)/(a+b))^(1/2)-5*a*b^4*((a-b)/(a+b))^(1/2)+10*cos(d*x+c)*sin(
d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*b^5+130*((a-b)/(a+b))^(1/2)*cos(d*x+c)^5*a^4*b+170*((a-b)/(a+b))^(
1/2)*cos(d*x+c)^4*a^3*b^2+82*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+80*cos(
d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3+272*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^3*b^2-5*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4-65*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a^4*b-279*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2+199*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^2*b^3+10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4-147*sin(d*x
+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^5-147*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+279*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-279*El
```

lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)-10*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+261*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-279*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
b^2(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)+155*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+10*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+10*b^5*((a-b)/
(a+b))^(1/2)+147*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+10*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-147*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-
147*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*a^4*b-279*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3+261*c
os(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^4*b-279*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+155*cos(d*
x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b^3+10*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+279*sin(d*x+c)*c
os(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*a^3*b^2-10*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^4+98*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a^5-10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5+14*((a-b)/(a+b))^(1
/2)*cos(d*x+c)^4*a^5-147*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^5)/(b+a*cos(d*x+c
))/sin(d*x+c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^4\sec(dx+c)^2+2\,ab\cos(dx+c)^4\sec(dx+c)+a^2\cos(dx+c)^4\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^4*sec(d*x
+ c) + a^2*cos(d*x + c)^4)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.849 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=303

$$\frac{2\left(-2a^2b^2+5a^4-3b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{21d}$$

```
[Out] (2*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(21*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b*(29*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(21*a*d*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]) + (2*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(21*d) + (6*a*b*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(7*d)
```

Rubi [A] time = 1.00581, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{21d}+\frac{2\left(-2a^2b^2+5a^4-3b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2}{\ }$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(21*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]) + (2*b*(29*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(21*a*d*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]) + (2*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*S
in[c + d*x])/(21*d) + (6*a*b*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(7*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(7*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= \frac{6ab\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec}}{7d}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d} + \frac{6ab\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d} + \frac{6ab\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d} + \frac{6ab\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{21d} + \frac{6ab\cos^{\frac{3}{2}}(c+dx)}{}$$

$$= \frac{2\left(5a^4-2a^2b^2-3b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{21ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\left(29a^2+3b^2\right)\sqrt{\cos}}{}$$

Mathematica [C] time = 11.8731, size = 419, normalized size = 1.38

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{42}\left(23a^2+36b^2\right)\sin(c+dx)+\frac{1}{14}a^2\sin(3(c+dx))+\frac{3}{7}ab\sin(2(c+dx))\right)(a+b\sec(c+dx))^{5/2}}{d(a\cos(c+dx)+b)^2} + \frac{2\cos^{\frac{3}{2}}(c}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((23*a^2 + 36*b^2)*Sin[c +
d*x])/42 + (3*a*b*Sin[2*(c + d*x)])/7 + (a^2*Sin[3*(c + d*x)])/14))/(d*(b +
 a*Cos[c + d*x])^2) + (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x
```

```
])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(I*b*(29*a^3 + 29*a^2*b + 3*a*b^2 + 3*b
^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/
2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(5*a^3 +
 29*a^2*b + 27*a*b^2 + 3*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2)/(a + b)] + b*(29*a^2 + 3*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^
(3/2)*Tan[(c + d*x)/2]))/(21*a*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.304, size = 2040, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/21/d/a/((a-b)/(a+b))^(1/2)*(-5*a^3*b*((a-b)/(a+b))^(1/2)-29*a^2*b^2*((a-
b)/(a+b))^(1/2)-9*a*b^3*((a-b)/(a+b))^(1/2)+5*cos(d*x+c)*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+2
9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-29*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-29*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+27*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+5*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)-3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^4+3*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^4+3*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+2*cos(d*x+c)^3*((a-b)/(
```

```
a+b))^(1/2)*a^4-5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-3*b^4*((a-b)/(a+b))^(1
/2)-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+12*cos(d*x+c)^4*((a-b)/(a+b))^(1
/2)*a^3*b+18*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+22*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*a^3*b+12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-29*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^3*b+11*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+29*co
s(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a^3*b-29*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2+3*cos(d*x+c)
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*a*b^3-29*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+27*cos(d*x+c)*sin(d*x+
c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*b^2-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^3\sec(dx+c)^2+2\,ab\cos(dx+c)^3\sec(dx+c)+a^2\cos(dx+c)^3\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^3*sec(d*x
+ c) + a^2*cos(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2), x)
```

3.850 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=239

$$\frac{16b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+23b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (16*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9
*a^2 + 23*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (22*a*b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a^2*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.755703, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3841, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{16b\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+23b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a^2\,\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (16*b*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9
*a^2 + 23*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (22*a*b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a^2*C
os[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
```

```
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}}}{5d}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}}}{5d}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}}}{5d}$$

$$= \frac{22ab\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15d} + \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}}}{5d}$$

$$= \frac{16b\left(a^2-b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2+23b^2\right)\sqrt{\cos(c+dx)}}{15d\sqrt{}}$$

Mathematica [C] time = 11.6245, size = 391, normalized size = 1.64

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{1}{5}a^2\sin(2(c+dx))+\frac{22}{15}ab\sin(c+dx)\right)(a+b\sec(c+dx))^{5/2}}{d(a\cos(c+dx)+b)^2} - \frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*((22*a*b*Sin[c + d*x])/15 +
(a^2*Sin[2*(c + d*x)])/5))/(d*(b + a*Cos[c + d*x])^2) - (2*Cos[c + d*x]^(3/
2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*((-I)
*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]
], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/(a + b)] + I*(9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^3)*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
```

```
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^2 + 23*b^2)*(b + a*Cos
[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(15*d*(b + a*Cos[c
 + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] time = 0.312, size = 1921, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d/((a-b)/(a+b))^(1/2)*(-9*a^2*b*((a-b)/(a+b))^(1/2)-11*a*b^2*((a-b)/(
a+b))^(1/2)-9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+9*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-23*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d
*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)+15*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3+9*cos(d*x+c)*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a^3+23*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-9*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^3-5*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-23*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*a*b^2+14*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+34*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a*b^2+15*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+17*cos(d*x+c)*sin(d*x+c)*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b-23*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+23*co
s(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
```

```
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a*b^2-23*b^3*((a-b)/(a+b))^(1/2)-23*cos(d*x+c)*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*b^3+17*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-23*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
+23*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2
)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^2\sec(dx+c)^2+2\,ab\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x
+ c) + a^2*cos(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2), x)
```

3.851 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=262

$$\frac{2a\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b^3\sqrt{\frac{a\cos(c+}{a+}}}{d\sqrt{\cos(c+}}$$

```
[Out] (2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^
3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.852741, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {4264, 3841, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2a\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b^3\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(\right.}{d\sqrt{\cos(c+dx)}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*a*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b^
3*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (14*a*b*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(7ab\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d} + \frac{\left(a\left(a^2+2b^2\right)\sqrt{b+a\cos(c+\right.}{3\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$= \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,\mathrm{si}}{3d}$$

$$= \frac{2a\left(a^2+2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b^3\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+d\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}$$

Mathematica [C] time = 32.584, size = 36372, normalized size = 138.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.224, size = 1651, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)*(cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-7*cos(d*x+c)*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b+9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-3*cos(d*x+c)*sin(d*x+c)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^3+
7*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-7*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2+6*cos(d*x+c
)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*b^3+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-7*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+7*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)-7*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+8*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3-7
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b+7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^
```

```
2-a^2*b*((a-b)/(a+b))^(1/2)-7*a*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/
cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{\frac{5}{2}}\cos (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2), x)
```

3.852 $\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$

Optimal. Leaf size=263

$$\frac{b\left(4a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2}{}$$

```
[Out] (b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (5*a*b^2*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*a^2 - b^2)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.849044, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {4264, 3842, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{b\left(4a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b^2\sin(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (b*(4*a^2 + b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (5*a*b^2*
Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)
])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*a^2 - b^2)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sqrt[a + b*Sec[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
```

```
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a\left(2a^2\right.}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a\left(2a^2\right.}{\sqrt{\sec(c}}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left(b\left(4a^2+b^2\right)\sqrt{b+a\cos(c+dx)}\right)\int \frac{}{\sqrt{b+a}}}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+a}}$$

$$= \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{b\left(4a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{5ab^2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}}$$

Mathematica [C] time = 31.8775, size = 44191, normalized size = 168.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.24, size = 1947, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(10*sin(d*x+c)
*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+2*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3-2*sin(d
*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a^2*b-sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2+sin(d*x+c)*cos(d
*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*b^3-2*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+
c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*sin(d*x+c)*cos(d*x+c)^2*(1/
(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b-4*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+10*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*a*b^2+2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-2*cos(d*x+c)*sin(d*x+c)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b-co
s(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a*b^2+cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-2*cos(d*x+c)*sin(d*
x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3+6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+
```

```
2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^3+2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+cos(d*x+c)^2*((a-b)/(a+b))^(1/2
)*a*b^2-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/
2)*a*b^2+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-b^3*((a-b)/(a+b))^(1/2))/(b+a*c
os(d*x+c))/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2\right)\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*sqrt(b*sec(d*x + c
) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

3.853 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=314

$$\frac{a\left(8a^2+11b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\left(15a^2+4b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b^2\sin(c+\cdots}{2\cdots}$$

```
[Out] (a*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (b*(
15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (9*a
*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)) + (9*a*b*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.15199, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4264, 3842, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{a\left(8a^2+11b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b\left(15a^2+4b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{a+\cdots}}{2d\cos^{\frac{3}{2}}(c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (a*(8*a^2 + 11*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (b*(
15*a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (9*a
*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[
c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sqrt[a + b*Sec[c
 + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)) + (9*a*b*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
```

```
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a\left(\right.\right.}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} - \frac{1}{8}\left(9ab\sqrt{\cos(c+\right.$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{\left(a\left(8a^2+11b^2\right)\right.}{8\sqrt{\text{co}}}$$

$$= \frac{b\left(15a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{9}{}$$

$$= \frac{a\left(8a^2+11b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b\left(15a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}$$

Mathematica [C] time = 31.9623, size = 52888, normalized size = 168.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.287, size = 1972, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d/((a-b)/(a+b))^(1/2)*(-9*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+9*sin(d*x+c)
*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^2+30*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/
(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+8*si
n(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+8*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-6
*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-4*sin(d*
x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d
*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*b^3-9*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+9*sin(d*x+c)*cos(
d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a*b^2+30*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+8*sin(d*x
+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+8*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-6*sin(
d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2*b+2*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x
```

```
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-4*sin(d*x+c)*
cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*b^3+9*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+2*((a-b)/(a+b))
^(1/2)*cos(d*x+c)^3*a*b^2-9*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b+9*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^3-11*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-2*b^3*((a-b)/(a+b))^(1/2))*((b+a*cos(d
*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

3.854 $\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=369

$$\frac{b\left(59a^2+16b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\cos(c+dx)}}-\frac{\left(33a^2+16b^2\right.}{}$$

```
[Out] (b*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (5
*a*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)])/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((3
3*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + (13*a*b*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d*Cos[c + d*x]^(3/2)) + ((33*a^2 +
16*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.42062, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4264, 3842, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{b\left(59a^2+16b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{\left(33a^2+16b^2\right)\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*(59*a^2 + 16*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (5
*a*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)
/2, (2*a)/(a + b)])/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((3
3*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sq
rt[a + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b^2*Sq
rt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + (13*a*b*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d*Cos[c + d*x]^(3/2)) + ((33*a^2 +
```

16*b^2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]])

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
```

```
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a\left(2\right.\right.}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{(33a^2+16b^2)}{2}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{(33a^2+16b^2)}{2}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{(33a^2+16b^2)}{2}$$

$$= \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{12d\cos^{\frac{3}{2}}(c+dx)} + \frac{(33a^2+16b^2)}{2}$$

$$= \frac{5a\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{b^2\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + 13$$

$$= \frac{b\left(59a^2+16b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{5a\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{8d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+a}}$$

Mathematica [C] time = 32.2794, size = 61979, normalized size = 167.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

[Out] Result too large to show

Maple [C] time = 0.312, size = 2285, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)

[Out] -1/24/d/((a-b)/(a+b))^(1/2)*(-8*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^3-33*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3-59*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-33*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3+16*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*b^3+26*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^2*b+16*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a*b^2+18*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*b^3+33*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+18*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a*b^2-34*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+33*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-8*b^3*((a-b)/(a+b))^(1/2)+30*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3+18*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^3-33*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3+16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^3+30*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^3+33*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-16*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+26*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2

```
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-44*sin(d*x+c)*cos(d*x+c)^3*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+120
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a*b^2+33*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4
*a^2*b-16*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^2+120*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*co
s(d*x+c)^4*a*b^2+26*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b-44*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x
+c)^4*a*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c
)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

3.855 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=249

$$-\frac{2b\left(7a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
 + (2*(9*a^2 + 8*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
 - (8*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d
) + (2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```

Rubi [A] time = 0.651309, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3863, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\left(7a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(9a^2+8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{8b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*b*(7*a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
 + (2*(9*a^2 + 8*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
 - (8*b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d
) + (2*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5ad} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{4b-3a\sec(c+dx)-}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a-}}}{5a}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{5ad}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{5ad}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{5ad}$$

$$= -\frac{8b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+}{5ad}$$

$$= -\frac{2b\left(7a^2+8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(9a^2+8b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{15a^3d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

<u>Mathematica</u> [C] time = 8.93993, size = 340, normalized size = 1.37

$$\frac{2a\sin(c+dx)(a\cos(c+dx)+b)(3a\cos(c+dx)-4b) + \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-ia\left(9a^2+2ab+8b^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\,c}{a+b}}\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*a*(b + a*Cos[c + d*x])*(-4*b + 3*a*Cos[c + d*x])*Sin[c + d*x] + (2*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(9*a^3 + 9*a^2*b + 8*a*b^2 + 8*b^3)*E
llipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(9*a^2 + 2*a*
b + 8*b^2)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (9*a^
```

```
2 + 8*b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]
))/Sec[c + d*x]^(3/2))/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
])
```

Maple [B] time = 0.263, size = 1726, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/a^3/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)*(-9*a^2*b*((a-b)/(a+b))^(1/2)+4*a*b^2*((a-b)/(a+b))^(1/2)-9*cos(d
*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^3+9*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+9*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+3*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3+6
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-
9*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+10*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*
b-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2-cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a
^2*b+4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2+2*cos(d*x+c)*sin(d*x+c)*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2
*b-8*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+8*cos(d*
x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*a*b^2-8*b^3*((a-b)/(a+b))^(1/2)-8*cos(d*x+c)*sin(d*x+c)*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
```

```
*b^3+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-8*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.856 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{2\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+\ldots}{\ldots}$$

```
[Out] (2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (4*
b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.439267, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4264, 3863, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\left(a^2+2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{4b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sin(c+dx)\sqrt{\cos(\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a^2 + 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (4*
b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3863

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(Cos[e + f*x]*(d*Csc[e + f*x])^(n + 1)*Sqrt[a +
 b*Csc[e + f*x]])/(a*d*f*n), x] + Dist[1/(2*a*d*n), Int[((d*Csc[e + f*x])^(
n + 1)*Simp[-(b*(2*n + 1)) + 2*a*(n + 1)*Csc[e + f*x] + b*(2*n + 3)*Csc[e +
 f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x]
&& NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{2b-a\sec(c\cdot}{\sqrt{\sec(c+dx)}\sqrt{a+b}}}{3a}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left(2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+d}}{\sqrt{\sec(c+dx)}}}{3a^2}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(\left(a^2+2b^2\right)\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+}}}{3a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(\left(a^2+2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,d}{3a^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2\left(a^2+2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{4b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c}}{3a^2d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 7.22111, size = 265, normalized size = 1.36

$$\frac{2\sqrt{\cos(c+dx)}\left(-ia(a-2b)\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{b-a}{a+b}\right)+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*((-2*I)*b*(a + b)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] - I*a*(a - 2*b)*Sqrt[(b + a*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]] + a^2*Sin
[c + d*x] - 2*a*b*Tan[(c + d*x)/2] - 2*b^2*Sec[c + d*x]*Tan[(c + d*x)/2] +
a*b*Tan[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.287, size = 1014, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a^2/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c
))^(1/2)*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b-2*cos(d
*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b+2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+((a-b)/(a+b))^(1/2)*co
s(d*x+c)^3*a^2+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)-((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-a^2*((a-b)/(a+b))^(1/2)*
cos(d*x+c)+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-2*((a-b)/(a+b))^(1/2)*cos(d
*x+c)*b^2-a*b*((a-b)/(a+b))^(1/2)+2*b^2*((a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c
))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a), x)
```

3.857 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=142

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)])
```

Rubi [A] time = 0.307214, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4264, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```


```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
```

```
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}$$

$$= -\frac{\left(b\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{b+a\cos(c+dx)}\,dx}{a\sqrt{b+a\cos(c+dx)}}$$

$$= -\frac{\left(b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\right)\int \sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}\,dx}{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 3.76795, size = 216, normalized size = 1.52

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{\sec(c+dx)+1}\left(-ia\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{b-a}{a+b}\right)+\sqrt{\frac{}{\cos(c}}\right.}{ad\sqrt{a+b\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]*(I*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - I*a*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + Sqrt[(1 + Cos[c + d*x])^(-1)]*(b + a*Cos[c + d*x])*Tan[(c + d*x)/2
]))/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] time = 0.258, size = 732, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)/a*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1
/2)*(cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*a-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+cos(d*x+c)*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-cos(d*x+c)^2*((a-b)/(a
+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b))^(1/2
)*b+((a-b)/(a+b))^(1/2)*b)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{b\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

3.858 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.149271, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3858, 2663, 2661}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \frac{\sqrt{b+a\cos(c+dx)}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 0.745638, size = 102, normalized size = 1.52

$$
-\frac{2i\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\,\text{EllipticF}\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right),\frac{b-a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{a+b\sec(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] ((-2*I)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[(
1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A] time = 0.24, size = 163, normalized size = 2.4

$$2\,\frac{(\sin(dx+c))^2\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\cos(dx+c)}}{d(-1+\cos(dx+c))(b+a\cos(dx+c))}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/d/((a-b)/(a+b))^(1/2)*sin(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/c
os(d*x+c))^(1/2)/(-1+cos(d*x+c))/(b+a*cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)\sec(dx+c)+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)*sec(d*
x + c) + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sec(c + dx)}\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx + c) + a}\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.859 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.234762, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4264, 3859, 2807, 2805}

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx \\
&= \frac{\sqrt{b+a\cos(c+dx)}\int \frac{\sec(c+dx)}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} \\
&= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 28.4472, size = 14986, normalized size = 220.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.234, size = 208, normalized size = 3.1

$$-2\frac{\sqrt{\cos(dx+c)}}{d\,(b+a\cos(dx+c))\sqrt{(\cos(dx+c)+1)^{-1}}}\sqrt{\frac{b+a\cos(dx+c)}{(a+b)(\cos(dx+c)+1)}}\left(2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\frac{a+}{a-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(2
*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((
a-b)/(a+b))^(1/2))-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2)))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2
)/(b+a*cos(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.860 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\cos(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (a*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.696252, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {4264, 3860, 4109, 3859, 2807, 2805, 3862, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```


```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (a*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Sec[c + d*x]]) - (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4109

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3862

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-a-a\sec^2(c+}{\sqrt{\sec(c+dx)}\sqrt{a+b\,\mathrm{se}}}}{2b}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}\sqrt{a+b}}}{2b}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+}}{\sqrt{a+b\sec(}}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{b+a\cos(c+dx)}\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(a\sqrt{\frac{b}{}}\right.}{2b\sqrt{}}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\frac{b+a\,\mathrm{c}}{}}}{2\sqrt{\cos}}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\sqrt{\cos}}{}$$

Mathematica [C] time = 30.1463, size = 21698, normalized size = 88.2

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.271, size = 986, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)/b*(-cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+cos(d*x+c)^2*sin(d
*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*b-2*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+2*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^2*a-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-2*cos(d*x+c)*s
in(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*a+2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a+cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*a-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+cos(d*x+c)*((a-b)/(a+b))^(
1/2)*b-((a-b)/(a+b))^(1/2)*b)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(
d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.861 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=312

$$-\frac{a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2+4b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{a+b\sec}}{4b^2d\sqrt{\cos(c+dx}}$$

```
[Out] -(a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
)/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (3*a*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b
^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(2*b*d*Cos[c + d*x]^(3/2)) - (3*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.952222, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4264, 3860, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{\left(3a^2+4b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{3a\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\cos(c+dx)}}+\frac{3a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx}}{4b^2d\sqrt{\frac{a\cos(c+dx)+}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] -(a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)
])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 + 4*b^2)*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
)/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (3*a*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b
^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(2*b*d*Cos[c + d*x]^(3/2)) - (3*a*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(4*b^2*d*Sqrt[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3860

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[(-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*S
qrt[a + b*Csc[e + f*x]])/(b*f*(2*n - 3)), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)*Simp[2*a*(n - 3) + b*(2*n - 5)*Csc[e + f*x] - 2*a
*(n - 2)*Csc[e + f*x]^2, x])/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
```

```
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}(a+2b}{\sqrt{a}}}{4b}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c\right.}}{}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c\right.}}{}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} + \frac{\left(3a\sqrt{\cos\right.}}{}$$

$$= \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} - \frac{3a\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{4b^2d\sqrt{\cos(c+dx)}} - \frac{\left(a\sqrt{b+a\right.}}{8b\sqrt{c}}$$

$$= \frac{\left(3a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{a+b\sec(c+dx)}\sin(c+dx}{2bd\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}$$

Mathematica [C] time = 32.0656, size = 51323, normalized size = 164.5

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.266, size = 1745, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/4/d/((a-b)/(a+b))^(1/2)/b^2*(-3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2+3*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^3*a*b-6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*a^2-8*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b
))^(1/2))*sin(d*x+c)*cos(d*x+c)^3*b^2+6*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*a^2-2*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)*cos(d*x+c)^3*a*b+4*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^
(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*b^2-3*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*c
os(d*x+c)^2*a^2+3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-6*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^2*a^2-8*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+6*cos(d*x+
c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*a^2-2*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b+4*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*
x+c)^2*b^2+3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2-2*((a-b)/(a+b))^(1/2)*cos
```

```
(d*x+c)^3*a*b-3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2+3*((a-b)/(a+b))^(1/2)*
cos(d*x+c)^2*a*b-2*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*b^2-((a-b)/(a+b))^(1/2)
*cos(d*x+c)*a*b+2*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1
/2)/(b+a*cos(d*x+c))/cos(d*x+c)^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sec (dx + c) + a} \cos (dx + c)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.862 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=360

$$-\frac{8b\left(a^2+4b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\cos}{5a^2d\,(}$$

```
[Out] (-8*b*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
(2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) + (2*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b
^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*b*(3*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d) + (2*(a^2 - 6*b
^2)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 -
 b^2)*d)
```

Rubi [A] time = 1.05035, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3847, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d\left(a^2-b^2\right)}-\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-8*b*(a^2 + 4*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*a)/(a + b)])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
(2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(5*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) + (2*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b
^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*b*(3*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d) + (2*(a^2 - 6*b
^2)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 -
 b^2)*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a^2}{2}+3b^2+\frac{1}{2}ab\sec(c+dx)-2b^2 s}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-6b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{5a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(3a^2-8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+d}{5a^3\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(3a^2-8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+d}{5a^3\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(3a^2-8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+d}{5a^3\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(3a^2-8b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+d}{5a^3\left(a^2-b^2\right)d}$$

$$= -\frac{8b\left(a^2+4b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^4+8a^2b^2-16b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}\right.}{5a^4\left(a^2-b^2\right)d\sqrt{\frac{b+}{}}}$$

Mathematica [C] time = 12.4714, size = 419, normalized size = 1.16

$$\frac{(a\cos(c+dx)+b)\left(a\sec^{\frac{3}{2}}(c+dx)\left(6b\left(b^2-a^2\right)\sin(c+dx)(a\cos(c+dx)+b)+a\left(a^2-b^2\right)\sin(2(c+dx))(a\cos(c+dx)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(a*Sec[c + d*x]^(3/2)*(10*b^4*Sin[c + d*x] + 6*b*(-a^
2 + b^2)*(b + a*Cos[c + d*x])*Sin[c + d*x] + a*(a^2 - b^2)*(b + a*Cos[c + d
*x])*Sin[2*(c + d*x)]) + 2*(a^2 + 4*b^2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^
(3/2)*(I*(3*a^3 + 3*a^2*b - 4*a*b^2 - 4*b^3)*EllipticE[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] - I*a*(3*a^2 - a*b - 4*b^2)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (3*a^2 - 4*b^2)*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(5*a^4*(a^2 - b^2)*d*Cos[c
+ d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.275, size = 1853, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/5/d/a^4/(a+b)/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(
d*x+c))^(1/2)*(-8*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2+2*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a^3*b+3*a^3*b*((a-b)/(a+b))^(1/2)+8*a*b^3*((a-b)/(a+b))^
(1/2)+3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+12*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+16*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+16*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+16*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b^4-16*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*b^4+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-cos(d*x+c)^4*((a-b)/(a+b))^(1
```

```
/2)*a^4-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4+16*b^4*((a-b)/(a+b))^(1/2)-c
os(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+2*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^
2*b^2-2*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-8*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a*b^3-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+6*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^2*b^2-3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^4-8*cos(d*x+c)*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a
^2*b^2+4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+12*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+1
6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b^3-3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))/(b+a*cos(d*x+c))/sin(d*
x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.863 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2\left(a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{\cos(c}{3a^2d\left(a^2-\right.}$$

```
[Out] (2*(a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*
b*(5*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.752364, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3847, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2\left(a^2+8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```


```
[Out] (2*(a^2 + 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*
b*(5*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a +
 b*Sec[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a^2}{2}+2b^2+\frac{1}{2}ab\sec(c+dx)-b^2}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2\left(a^2+8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2b\left(5a^2-8b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{3a^3\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c}{a+b}}}$$

Mathematica [C] time = 8.65943, size = 382, normalized size = 1.32

$$-\frac{2(a\cos(c+dx)+b)\left(a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(3b^3-\left(a^2-b^2\right)(a\cos(c+dx)+b)\right)-\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*(a*(3*b^3 - (a^2 - b^2)*(b + a*Cos[c + d*x]))*Sec[
c + d*x]^(3/2)*Sin[c + d*x] - (Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*b*
(-5*a^3 - 5*a^2*b + 8*a*b^2 + 8*b^3)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d
```

```
*x)/2]^2)/(a + b)] - I*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*EllipticF[I*ArcS
inh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Co
s[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + b*(-5*a^2 + 8*b^2)*(b + a*Cos[c
+ d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(3*a^3*(a^2 - b^2)*d
*Cos[c + d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.3, size = 1305, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/a^3/(a+b)/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos
(d*x+c))^(1/2)*(cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+6*cos(d*x+c)*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
+8*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-5*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b+8*cos(d*x+
c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^3+cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3+cos(d*x+c)^3*((a-b
)/(a+b))^(1/2)*a^2*b+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)-5*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a^2*b-4*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2-cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^3+4*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b-8*cos(d*x+c)*((a-b)/(a+b
```

```
))^(1/2)*b^3-a^2*b*((a-b)/(a+b))^(1/2)+4*a*b^2*((a-b)/(a+b))^(1/2)+8*b^3*((
a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2), x)
```

3.864 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{4b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx}}{a^2d\left(a^2\right.}$$

```
[Out] (-4*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Si
n[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.525519, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4264, 3847, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{4b\sqrt{\cdots}}{a^2d\sqrt{c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-4*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2 - 2*b^2)
*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Si
n[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a^2}{2}+b^2+}{\sqrt{\sec(c+dx}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\left(-\frac{a^2}{2}+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\left(-\frac{a^2b}{2}+b\left(-\frac{a^2}{2}+b^2\right)\right)\right)\sqrt{b+a\cos(c+}}{a^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+}}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\left(-\frac{a^2b}{2}+b\left(-\frac{a^2}{2}+b^2\right)\right)\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int}{a^2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\,s}}$$

$$= -\frac{4b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+}}{a^2\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 8.95844, size = 330, normalized size = 1.54

$$\frac{2(a\cos(c+dx)+b)\left(ab^2\sin(c+dx)+\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-ia\left(a^2-ab-2b^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{a+b}}\text{EllipticF}\left(i\,\text{si}\right.\right.}{}\right.}{a^2d\left(a^2-b^2\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(a*b^2*Sin[c + d*x] + ((Cos[(c + d*x)/2]^2*Sec[c +
d*x])^(3/2)*(I*(a^3 + a^2*b - 2*a*b^2 - 2*b^3)*EllipticE[I*ArcSinh[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^2 - a*b - 2*b^2)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (a^2 - 2*b^2)*(b + a*Cos[c + d*x])*(
Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(a^2*(a^2
 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.27, size = 997, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)/(a+b)/a^2*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d
*x+c))^(1/2)*(-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b+co
s(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2-2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*sin(d*x+c)+(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*co
s(d*x+c)^2*a^2+((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b-a^2*((a-b)/(a+b))^(1/2)
```

```
*cos(d*x+c)+2*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-a*b*((a-b)/(a+b))^(1/2)-2*
b^2*((a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2), x)
```

3.865 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=200

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}{ad\left(a^2-b^2\right)\sqrt{}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.462187, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4264, 3843, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*Sqrt[Cos[c + d*x
]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2
- b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{a^2}$$

$$= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{a}$$

$$= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{b+a\cos(c+dx)}\int \frac{1}{\sqrt{b+a\cos}}}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}}$$

$$= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+}{a+b}}}}{a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+d}{a+b}}}$$

Mathematica [C] time = 7.38603, size = 245, normalized size = 1.22

$$\frac{2\sqrt{\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left(-ia(a+b)\sqrt{\sec(c+dx)+1}\sqrt{\frac{a\cos(c+dx)+b}{(a+b)(\cos(c+dx)+1)}}\text{EllipticF}\left(i\sinh^{-1}\left(\tan\left(\right.\right.\right.\right.}{ad\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(I*b*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] - I*a*(a + b)*
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + b*(-a + b)*S
qrt[Sec[c + d*x]]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
(3/2))
```

Maple [B] time = 0.269, size = 502, normalized size = 2.5

$$2\,\frac{\sqrt{\cos(dx+c)}}{da\,(a+b)\,(b+a\cos(dx+c))\sin(dx+c)}\left(-\cos(dx+c)\sin(dx+c)\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d/a/(a+b)/((a-b)/(a+b))^(1/2)*(-cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a-cos(d*x+c)*sin(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*b-EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)-(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-(
(a-b)/(a+b))^(1/2)*b)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/
(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^2\cos\left(dx+c\right)\sec\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)\sec\left(dx+c\right)+a^2\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)*sec(
d*x + c)^2 + 2*a*b*cos(d*x + c)*sec(d*x + c) + a^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.866 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} dx$

Optimal. Leaf size=126

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*Sin[c
 + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.233128, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4264, 3844, 21, 3856, 2655, 2653}

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*Sin[c
 + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
```

```
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{a^2-b^2}$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{a^2-b^2}$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx}\right.}{\left(a^2-b^2\right)\sqrt{b+a}}$$

$$= \frac{2a\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx}\right.}{\left(a^2-b^2\right)\sqrt{\frac{b+a}{}}}$$

$$= -\frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}} + \frac{2a\sin(}{\left(a^2-b^2\right)d\sqrt{\cos(c+a}}$$

Mathematica [C] time = 7.18075, size = 260, normalized size = 2.06

$$\frac{\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)\left(i\sqrt{\sec(c+dx)+1}(a\cos(c+dx)+b)\text{EllipticF}\left(i\sinh^{-1}\left(\right.\right.\right.}{d\left(a^2-b^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sec[c + d*x]^(3
/2)*((-I)*(b + a*Cos[c + d*x])*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + I*(b + a*Cos[c + d*x])*EllipticF[I*Ar
cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 + Sec[c + d*x]] + (a - b)
*Sqrt[Sec[c + d*x]]*Sqrt[(a + b*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))]
*Sin[c + d*x]))/((a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.245, size = 491, normalized size = 3.9

$$-2\frac{\sqrt{\cos(dx+c)}}{d(a+b)(b+a\cos(dx+c))\sin(dx+c)}\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\sqrt{\frac{a-b}{a+b}},\sqrt{-\frac{a+b}{a-b}}\right)\sin(dx+c)\cos(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/d/((a-b)/(a+b))^(1/2)/(a+b)*(EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)-((a-b)/(a+b))
^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c
))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^2\sec(dx+c)^2+2\,ab\cos(dx+c)^2\sec(dx+c)+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^2*se
c(d*x + c)^2 + 2*a*b*cos(d*x + c)^2*sec(d*x + c) + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.867 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(
a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c + d*x])/(b*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.598232, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3845, 4108, 3859, 2807, 2805, 21, 3856, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a +
 b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*Sqrt[Cos[c +
 d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(
a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c + d*x])/(b*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)\right.}}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)\right.}}{b\left(a^2\right.}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)\right.}}{b\left(a^2-b^2\right.}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{\left(\left(-a^2+b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int}{b\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+}}{b\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

Mathematica [C] time = 32.0603, size = 47811, normalized size = 232.09

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.267, size = 1134, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/d/b/(a+b)/((a-b)/(a+b))^(1/2)*(-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d
*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*cos(d*x+c)*si
n(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*a-2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b+2*cos(
d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a+cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*b-(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-2*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*a*sin(d*x+c)-2*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*(1
/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)+cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-a*((a-b)/(a+b))^(1/2))*((b+a*cos(d*
x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.868 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$\frac{\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+}}$$

```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (3*a*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^2*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b
^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.08889, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4264, 3845, 4102, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a}}{b^2d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```


```
[Out] (Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/
(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (3*a*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^2*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2*(a^2 - b
^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+a}}{b^2\left(a^2-b^2\right)d\sqrt{\cos(}}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+a}}{b^2\left(a^2-b^2\right)d\sqrt{\cos(}}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+a}}{b^2\left(a^2-b^2\right)d\sqrt{\cos(}}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\sec(c+a}}{b^2\left(a^2-b^2\right)d\sqrt{\cos(}}$$

$$= -\frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{se}}}$$

$$= \frac{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{3a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} -$$

Mathematica [C] time = 32.2082, size = 51610, normalized size = 149.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.317, size = 1491, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/d/((a-b)/(a+b))^(1/2)/(a+b)/b^2*(6*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*
(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+4*cos(d
*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*a*b-6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2-6*si
n(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2+s
in(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*b^2+6*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(
d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2+4*cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*a*b-6*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-6*sin(d*x+c)*cos(
d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*a*b-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+cos(d*x+c)*sin(d*x
+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^2+3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2+((a-b)/(a+b))^(1/2)*cos(d*
x+c)^2*a*b-3*a^2*((a-b)/(a+b))^(1/2)*cos(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x
+c)*b^2-a*b*((a-b)/(a+b))^(1/2)-b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/
cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.869 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=391

$$\frac{2\left(16a^2b^2+a^4-16b^4\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin}{3ad\left(a^2-\right.}$$

```
[Out] (2*(a^4 + 16*a^2*b^2 - 16*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^4*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) - (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2
 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*b^2*(5*a
^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
+ b*Sec[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 1.09542, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {4264, 3847, 4100, 4104, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2\left(-13a^2b^2+a^4+8b^4\right)\sin(c+dx)\sqrt{c}}{3a^3d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (2*(a^4 + 16*a^2*b^2 - 16*b^4)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^4*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) - (8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2
 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (4*b^2*(5*a
^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
+ b*Sec[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4104

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3a^2}{2}+3b^2+\frac{3}{2}ab\sec(c+dx)\cdots}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec\cdots}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{\left(4\sqrt{\cos(\cdots}\right.}{\cdots}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^4-1\cdots\right.}{\cdots}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^4-1\cdots\right.}{\cdots}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^4-1\cdots\right.}{\cdots}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}} + \frac{2\left(a^4-1\cdots\right.}{\cdots}$$

$$= \frac{2\left(a^4+16a^2b^2-16b^4\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{\cos(c+\cdots}}{3a^4\left(a^2-b^{2}\cdots\right.}$$

Mathematica [C] time = 14.4517, size = 527, normalized size = 1.35

$$\frac{(a\cos(c+dx)+b)^3\left(\frac{2b^4\sin(c+dx)}{3a^3\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{8\left(2b^5\sin(c+dx)-3a^2b^3\sin(c+dx)\right)}{3a^3\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2\sin(c+dx)}{3a^3}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(\cos\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*Sin[c + d*x])/(3*a^3) + (2*b^4*Sin[c + d*x])/(3
*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (8*(-3*a^2*b^3*Sin[c + d*x] + 2*
b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c +
d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c
 + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-4*
I)*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2 - 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*EllipticE[
I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^5 - 8*a^4*b + 7*a^3
*b^2 + 28*a^2*b^3 - 4*a*b^4 - 16*b^5)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]]
, (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2)/(a + b)] - 4*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*(b + a*Cos[c + d*x])*
(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*(a + b
*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.313, size = 3604, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/a^4/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*cos(d*x+c)^(1/2)*((b+a*cos(d*x
+c))/cos(d*x+c))^(1/2)*(((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^5*b-((a-b)/(a+b)
)^(1/2)*cos(d*x+c)^4*a^4*b^2-((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3*b^3-a^4*b
^2*((a-b)/(a+b))^(1/2)+7*a^3*b^3*((a-b)/(a+b))^(1/2)+20*a^2*b^4*((a-b)/(a+b
))^(1/2)-8*a*b^5*((a-b)/(a+b))^(1/2)-12*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-16*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^6-((a-b)/(a+b))^(1/2)*cos(d*x+c)
^2*a^6+16*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^6+cos(d*x+c)^2*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6-
16*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*b^6+cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^6+28*cos(d*x+c)*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
```

+1))^(1/2)*a^2*b^4-16*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^5+10*cos(d*x+c)*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^5*b+25*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+4*cos(d*x+c)*sin(d*x+c)*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*
b^3-28*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-16*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b
+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5-8*c
os(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*a^5*b+28*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^3-16*cos(
d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*a*b^5+9*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+16*cos(d*x+c)
^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^4*b^2-12*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-16*cos(d*x+c)^
2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b^4-8*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5*b-8*cos(d*x+c)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*a^4*b^2+28*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d
x+c)+1))^(1/2)(1/(cos(d*x+c)+1))^(1/2)*a^3*b^3-6*((a-b)/(a+b))^(1/2)*cos(
d*x+c)^3*a^5*b-6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^4*b^2+6*((a-b)/(a+b))^(
1/2)*cos(d*x+c)^3*a^3*b^3+6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b^4+7*((a-
b)/(a+b))^(1/2)*cos(d*x+c)^2*a^5*b-6*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4*b
^2-34*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*b^3+8*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^2*a^2*b^4+24*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^5-2*((a-b)/(a+b))^(1

```
/2)*cos(d*x+c)*a^5*b+14*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^4*b^2+22*((a-b)/(a
+b))^(1/2)*cos(d*x+c)*a^3*b^3-34*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b^4-16*
((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^5-16*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^6*(1/(a+b)*(b+a*cos(d*x+c))/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-8*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)+28*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)+EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+16*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)-16*b^6*((a-b)/(a+b))^(1/2))/(b+a*cos(d*x+c))^2/sin(d*x+
c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2), x)
```

3.870 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{2b\left(9a^2-8b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{}{3ad\left(a^2-\right.}$$

```
[Out] (-2*b*(9*a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^
2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*
Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]])
```

Rubi [A] time = 0.822351, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3847, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}-\frac{2b\left(9a^2-8b^2\right)\sqrt{}}{3a^3d\left(a^2-b^2\right)\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-2*b*(9*a^2 - 8*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x
)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]) + (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^
2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (8*b^2*(2*a^2 - b^2)*
Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{3a^2}{2}+2b}{\sqrt{se}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

$$= -\frac{2b\left(9a^2-8b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}E\left(\right.}{3a^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

Mathematica [C] time = 13.6198, size = 507, normalized size = 1.6

$$\frac{(a\cos(c+dx)+b)^3\left(-\frac{2b^3\sin(c+dx)}{3a^2\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}-\frac{2\left(5b^4\sin(c+dx)-9a^2b^2\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} - \frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+d\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((-2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(b + a*Co
s[c + d*x])^2) - (2*(-9*a^2*b^2*Sin[c + d*x] + 5*b^4*Sin[c + d*x]))/(3*a^2*
(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c +
d*x])^(5/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5
```

```
/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^5 + 3*a^4*b - 15*a^3
*b^2 - 15*a^2*b^3 + 8*a*b^4 + 8*b^5)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]],
 (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] + I*a*(3*a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 + 8*b^4)*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^4 - 15*a^2*b
^2 + 8*b^4)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2
]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.275, size = 3101, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/a^3/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(-3*cos(d*x+c)^3*((a-b)/(a+b))
^(1/2)*a^3*b^2+3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5-3*cos(d*x+c)^2*sin(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*a^5+3*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^5-3*a^3*b^2*((a-b)/(a+b))^(1/2)-11*a
^2*b^3*((a-b)/(a+b))^(1/2)+4*a*b^4*((a-b)/(a+b))^(1/2)+8*cos(d*x+c)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*b^5+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b-4*cos(d*x+c)^2*((a-b)/(a
+b))^(1/2)*a^2*b^3+3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos
(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+3*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^4*b-3*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3+18*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^3*b^2-12*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^4-6*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b-12*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^
2+18*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+8*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a*b^4-3*sin(d*x+c)*cos(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+3*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-15*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(
1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin
```

(d*x+c)-3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-9*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*b^5*((a-b)/(a+b))^(1/2)+8*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4*b-15*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3-12*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+14*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+8*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-15*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2+8*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^4-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5-8*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5-15*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+8*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-9*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4*b+6*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^3*b^2+8*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^3)*cos(d*x+c)^(

```
1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*sec(d*x + c)^3 +
3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\sec(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2), x)
```

3.871 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=302

$$\frac{2\left(3a^2-2b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3a^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{}{3d\left(a^2-b^2\right)}$$

```
[Out] (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + (4*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*
x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sin[c + d*x])/(3*a*(a^
2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.740032, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3843, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2b\left(5a^2-b^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{2\left(3a^2-2b^2\right)\sqrt{}}{3a^2d\left(a^2-b^2\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (2*(3*a^2 - 2*b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + (4*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*
x]]*(a + b*Sec[c + d*x])^(3/2)) - (2*b*(5*a^2 - b^2)*Sin[c + d*x])/(3*a*(a^
2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\ldots}{3\ldots}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\mathrm{s}\ldots}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d\ldots}}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\mathrm{s}\ldots}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d\ldots}}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\mathrm{s}\ldots}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d\ldots}}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{2b\left(5a^2-b^2\right)\mathrm{s}\ldots}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+d\ldots}}$$

$$= \frac{2\left(3a^2-2b^2\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{4b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\ldots}{3a^2\left(a^2-\ldots\right.}$$

Mathematica [C] time = 8.60322, size = 398, normalized size = 1.32

$$\frac{(a\cos(c+dx)+b)^2\left(\frac{2b\sin(c+dx)\left(\left(2ab^2-6a^3\right)\cos(c+dx)-5a^2b+b^3\right)}{a\left(a^2-b^2\right)^2(a\cos(c+dx)+b)} - \frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(ia\left(6a^2b+3a^3+ab^2-2b^3\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(\ldots\right.}{\ldots}}\ldots\right.}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((2*b*(-5*a^2*b + b^3 + (-6*a^3 + 2*a*b^2)*Cos[c +
d*x])*Sin[c + d*x])/(a*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) - (2*(Cos[(c + d
*x)/2]^2*Sec[c + d*x])^(3/2)*((2*I)*b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*Elli
```

```
pticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqr
t[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^3 + 6*a^2*b
 + a*b^2 - 2*b^3)*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
+ 2*b*(-3*a^2 + b^2)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c
 + d*x)/2]))/((a^3 - a*b^2)^2*Sec[c + d*x]^(3/2))))/(3*d*Cos[c + d*x]^(5/2)
*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.272, size = 2062, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/a^2*(-cos(d*x+c)^2*((a-b)/(a+b))^(1
/2)*a^2*b^2-5*a^2*b^2*((a-b)/(a+b))^(1/2)+a*b^3*((a-b)/(a+b))^(1/2)-3*cos(d
*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^4-6*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+
1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)+2*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*cos(d*x+c)*sin(d*x
+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*b^4-2*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4+2*b^4*((a-b)/(a+b))^(1/2)+2*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+6*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3
*b-3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3-6*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a^3*b+6*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2-6*sin(d*x+c)*cos(d*x+c)^2*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*a^3*b+2*sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)
```

```
+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^3+2*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
^2-3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-6*cos(
d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*a^3*b-6*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2+2*cos(d*x+c)*si
n(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*a*b^3+5*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x
+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+2*cos(d*x+c)*sin(d*x+c)*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^3)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+
c))^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)\sec(dx+c)^3+3\,ab^2\cos(dx+c)\sec(dx+c)^2+3\,a^2b\cos(dx+c)\sec(dx+c)+a^3\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)*sec(
d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)*sec
(d*x + c) + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.872 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{4\left(a^2+b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin}{3d\left(a^2-b^2\right)\sqrt{\cos(c+}}$$

```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(
3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)) + (4*(a^2 + b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.687986, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3844, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$\frac{4\left(a^2+b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}-\frac{2b\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (-2*b*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(
3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[
a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c +
d*x])^(3/2)) + (4*(a^2 + b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.}{3}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sin}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sin}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sin}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{4\left(a^2+b^2\right)\sin}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx}}$$

$$= -\frac{2b\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}}{3a\left(a^2-\right.}$$

Mathematica [C] time = 11.299, size = 447, normalized size = 1.59

$$\frac{(a\cos(c+dx)+b)^3\left(\frac{2b\sin(c+dx)}{3\left(b^2-a^2\right)(a\cos(c+dx)+b)^2}+\frac{2\left(3a^2\sin(c+dx)+b^2\sin(c+dx)\right)}{3\left(b^2-a^2\right)^2(a\cos(c+dx)+b)}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\mathrm{s}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*b*Sin[c + d*x])/(3*(-a^2 + b^2)*(b + a*Cos[c +
d*x])^2) + (2*(3*a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b
 + a*Cos[c + d*x]))))/(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (
2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*
```

```
x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*EllipticE
[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a^2 + 4*a*b + b^2)
*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^
2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2 + b^2)*(
b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a*(a^2
 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.244, size = 1812, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)/(a+b)^2/a/((a-b)/(a+b))^(1/2)*(-3*sin(d*x+c)*cos(d*x+c)^2*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^
3-sin(d*x+c)*cos(d*x+c)^2*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2+3*sin(d*x+c)*cos(d*x+c)^2*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-sin(d*x+c
)*cos(d*x+c)^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-3*cos(d*x+c)*sin(d*x+c
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)*a^2*b-cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b^2-cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+3*cos(d
*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^3+2*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-cos(d*x+c)*sin(d*x+c
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
```

```
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b^2-3*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+3*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*(1/(a+b)*(b+a*c
os(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+3*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a^2*b-3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3+3*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a^2*b-cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^2+cos(d*x+c)*((a-b)/(a+
b))^(1/2)*b^3-2*a^2*b*((a-b)/(a+b))^(1/2)+a*b^2*((a-b)/(a+b))^(1/2)-b^3*((a
-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/(b+a
*cos(d*x+c))^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^2\sec(dx+c)^3+3\,ab^2\cos(dx+c)^2\sec(dx+c)^2+3\,a^2b\cos(dx+c)^2\sec(dx+c)+a^3\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^2*se
c(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2*sec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)
```

```
^2*sec(d*x + c) + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.873 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5\right.}{3bd\left(a^2-b^2\right)^2\sqrt{\cos(\right.}}$$

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c +
 d*x])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) +
(2*a*(a^2 - 5*b^2)*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 0.744574, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4264, 3845, 4100, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}+\frac{2a\left(a^2-5b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\sqrt{\frac{a\cos(c+\right.}{a+b}}}{3d\left(a^2-b^2\right)\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (8*b*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a^2*Sin[c +
 d*x])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) +
(2*a*(a^2 - 5*b^2)*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)s}{3b\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)s}{3b\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)s}{3b\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}} + \frac{2a\left(a^2-5b^2\right)s}{3b\left(a^2-b^2\right)^2 d\sqrt{\cos(c+}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{8b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{b}{}}}$$

Mathematica [C] time = 9.08617, size = 311, normalized size = 1.12

$$\frac{2(a\cos(c+dx)+b)^2\left(\frac{a\sin(c+dx)\left(a^2-4ab\cos(c+dx)-5b^2\right)}{a\cos(c+dx)+b} + \frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-i\left(a^2+4ab+3b^2\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{a+b}}\text{EllipticF}\left(i\sin\right.\right.}{}\right.}{3d\left(a^2-b^2\right)^2\cos^{\frac{5}{2}}(c+dx)(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])^2*((a*(a^2 - 5*b^2 - 4*a*b*Cos[c + d*x])*Sin[c + d*
x])/(b + a*Cos[c + d*x]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*((4*I)*b*
```

```
(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*(a^2 + 4*a*b + 3*b^2)*Ell
ipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 4*b*(b + a*Cos[c + d*x])*Sqrt[Sec[(c +
 d*x)/2]^2]*Tan[(c + d*x)/2]))/Sqrt[Sec[c + d*x]]))/(3*(a^2 - b^2)^2*d*Cos[
c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.303, size = 1333, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)/(a+b)^2/((a-b)/(a+b))^(1/2)*(cos(d*x+c)^2*sin(d*x+c)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a
+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2-3*c
os(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)*a*b+4*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+
1))^(1/2)*a^2-2*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a*b-3*cos(d*x+c)*sin(d*x+c)*(1/(a+b
)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+4
*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b+4*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+
c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*(1/(a+b)
*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
-3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*b^2*sin(d*x+c)+4*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*b^2*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2
```

```
-3*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b+4*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*
b-4*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^2-a^2*((a-b)/(a+b))^(1/2)-a*b*((a-b)/(
a+b))^(1/2)+4*b^2*((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*
cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^3\sec(dx+c)^3+3\,ab^2\cos(dx+c)^3\sec(dx+c)^2+3\,a^2b\cos(dx+c)^3\sec(dx+c)+a^3\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^3*se
c(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^3*sec(d*x + c)^2 + 3*a^2*b*cos(d*x + c)
^3*sec(d*x + c) + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.874 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{2a\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\text{EllipticF}\left(\frac{1}{2}(c+dx),\frac{2a}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2-\right.}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos}}$$

```
[Out] (-2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
)/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2
)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a
^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])
^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rubi [A] time = 1.20468, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.56, Rules used = {4264, 3845, 4098, 4108, 3859, 2807, 2805, 4035, 3856, 2655, 2653, 3858, 2663, 2661}

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2a\sqrt{\frac{a\cos(c+}{a+b}}}{3bd\left(a^2-b^2\right)\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```


```
[Out] (-2*a*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
b)])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]
)/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*a*(3*a^2 - 7*b^2
)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c
 + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*a
^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])
^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[
```

```
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4108

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rule 3859

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[(d*Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x
]])/Sqrt[a + b*Csc[e + f*x]], Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
```

```
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 4035

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3856

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3858

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[(Sqrt[d*Csc[e + f*x]]*Sqrt[b + a*Sin[e + f*x]])/
Sqrt[a + b*Csc[e + f*x]], Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+}}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a^2\left(3a^2-7b^2\right)}{3b^2\left(a^2-b^2\right)^2d\sqrt{\cos(c+}}$$

$$= \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c}$$

$$= -\frac{2a\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+\right.}{b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec}}$$

Mathematica [C] time = 32.9697, size = 92128, normalized size = 248.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.289, size = 3844, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/((a-b)/(a+b))^(1/2)/(a+b)^2/(a-b)/b^2*(6*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*a^2*b^2+4*a^3*b*((a-b)/(a+b))^(1/2)+a^2*b^2*((a-b)/(a+b))^(1/2)-7*a*b
^3*((a-b)/(a+b))^(1/2)-6*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(
b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-7
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1
))^(1/2)*sin(d*x+c)-6*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*(1/(a+b)*(b+a*cos(d*
x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+9*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3
*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)+3*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-3*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a^4+3*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)-6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^4*(1/(a+b)*(b+a*cos(d*x
+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+7*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a*b^3-cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b-3*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a^3*b-7*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+3*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)*sin(d*x+c)*a^4+9*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c
))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+3*cos(d*x+c)^
2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)
+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^3-7*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+6*(1/(a+b)*(b+a*co
s(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos
```

(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*a^3*b-6*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b
^3-6*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*
cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+6*cos(d*
x+c)^2*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4+3*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a
*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+6*cos(d
*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1)
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^4-6*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2
)*b^4+6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*a^3*b*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*EllipticPi((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2*(1/(a+
b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)-6*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*a*b^3*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-6*cos(d*x+c)^2*sin(d*x+c)*EllipticPi(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^
(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*a^2*b^2+12*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+
b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-12*(1/(a+b)*(b+
a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b^3-4*cos(d*x+c)^2*sin(d*x+c)*(1/(a+b)*(b+a*cos
(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d
x+c))((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+3*cos(d*x
+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)*a^3*b-7*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c)
)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*a^2*b^2-7*cos(d*x+c)*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)*a*b^3-10*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c

```
)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+5*cos(d*x+c)*sin(d*x+c)*(1/(
a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^2+12*cos(d*x+c)*sin(d*x+c)*(1/(a+b)*(b+a*cos(d*x+c))/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)
*cos(d*x+c)^(1/2)/(b+a*cos(d*x+c))^2/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2)), x)
```

3.875 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx$

Optimal. Leaf size=266

$$-\frac{b\left(3a^2(2-n)+b^2(1-n)\right)\sin(e+fx)(d\cos(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a\left(a^2(1-n)-\right.}{}$$

```
[Out] -((b*(b^2*(1 - n) + 3*a^2*(2 - n))*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2
, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 - n)*n*Sqrt[Sin[e + f
*x]^2])) - (a*(a^2*(1 - n) - 3*b^2*n)*Cos[e + f*x]*(d*Cos[e + f*x])^n*Hyper
geometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1
 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 - 2*n)*(d*Cos[e + f*x])^n*T
an[e + f*x])/(f*(1 - n)*(2 - n)) + (b^2*(d*Cos[e + f*x])^n*(a + b*Sec[e + f
*x])*Tan[e + f*x])/(f*(2 - n))
```

Rubi [A] time = 0.456245, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4264, 3842, 4047, 3772, 2643, 4046}

$$-\frac{b\left(3a^2(2-n)+b^2(1-n)\right)\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+fx)\right)}{f(2-n)n\sqrt{\sin^2(e+fx)}}-\frac{a\left(a^2(1-n)-3b^2n\right)\sin(e+fx)}{f(}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((b*(b^2*(1 - n) + 3*a^2*(2 - n))*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2
, n/2, (2 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(2 - n)*n*Sqrt[Sin[e + f
*x]^2])) - (a*(a^2*(1 - n) - 3*b^2*n)*Cos[e + f*x]*(d*Cos[e + f*x])^n*Hyper
geometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1
 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (a*b^2*(5 - 2*n)*(d*Cos[e + f*x])^n*T
an[e + f*x])/(f*(1 - n)*(2 - n)) + (b^2*(d*Cos[e + f*x])^n*(a + b*Sec[e + f
*x])*Tan[e + f*x])/(f*(2 - n))
```

Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^3\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}(a+b\sec(e+fx))^3\,d$$

$$= \frac{b^2(d\cos(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2-n)} + \frac{\left((d\cos(e+fx))^n(d\sec\right.}{}$$

$$= \frac{b^2(d\cos(e+fx))^n(a+b\sec(e+fx))\tan(e+fx)}{f(2-n)} + \frac{\left((d\cos(e+fx))^n(d\sec\right.}{}$$

$$= \frac{ab^2(5-2n)(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)(2-n)} + \frac{b^2(d\cos(e+fx))^n(a+b\sec(e+}{f(2-n)}$$

$$= -\frac{b\left(b^2(1-n)+3a^2(2-n)\right)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin}{f(2-n)n\sqrt{\sin^2(e+fx)}}$$

$$= -\frac{b\left(b^2(1-n)+3a^2(2-n)\right)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin}{f(2-n)n\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.689, size = 222, normalized size = 0.83

$$-\frac{\sqrt{\sin^2(e+fx)}\csc(e+fx)\sec^2(e+fx)(d\cos(e+fx))^n\left(\frac{1}{2}a(n-2)\cos(e+fx)\left(2a(n-1)\cos(e+fx)\left(an\cos(e+fx)\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^3,x]
```

```
[Out] -((((d*Cos[e + f*x])^n*Csc[e + f*x]*(b^3*n*(-1 + n^2)*Hypergeometric2F1[1/2,
 (-2 + n)/2, n/2, Cos[e + f*x]^2] + (a*(-2 + n)*Cos[e + f*x]*(6*b^2*n*(1 +
n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + 2*a*(-1
+ n)*Cos[e + f*x]*(3*b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e
 + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2])))/2)*Sec[e + f*x]^2*Sqrt[Sin[e + f*x]^2])/(f*(-2 + n)*(-1
+ n)*n*(1 + n)))
```

Maple [F] time = 3.103, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^n\left(a+b\sec\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^3 \left(d\cos\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\sec\left(fx+e\right)^3+3\,ab^2\sec\left(fx+e\right)^2+3\,a^2b\sec\left(fx+e\right)+a^3\right)\left(d\cos\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(f*x + e)^3 + 3*a*b^2*sec(f*x + e)^2 + 3*a^2*b*sec(f*x + e
) + a^3)*(d*cos(f*x + e))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^3\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^3*(d*cos(f*x + e))^n, x)
```

3.876 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx$

Optimal. Leaf size=186

$$-\frac{\left(a^2(1-n)-b^2n\right)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2ab\sin(e$$

```
[Out] (-2*a*b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f
*x]^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) - ((a^2*(1 - n) - b^2*n)*Co
s[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (b
^2*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n))
```

Rubi [A] time = 0.223934, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3788, 3772, 2643, 4046}

$$-\frac{\left(a^2(1-n)-b^2n\right)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(1-n)(n+1)\sqrt{\sin^2(e+fx)}}-\frac{2ab\sin(e+fx)(d\cos(e+fx}{fn\sqrt{\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```


```
[Out] (-2*a*b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f
*x]^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2]) - ((a^2*(1 - n) - b^2*n)*Co
s[e + f*x]*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - n)*(1 + n)*Sqrt[Sin[e + f*x]^2]) + (b
^2*(d*Cos[e + f*x])^n*Tan[e + f*x])/(f*(1 - n))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (d\cos(e+fx))^n(a+b\sec(e+fx))^2\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}(a+b\sec(e+fx))^2\,d$$

$$= \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int (d\sec(e+fx))^{-n}\left(a^2+b^2\sec^2(e+fx)\right.$$

$$= \frac{b^2(d\cos(e+fx))^n\tan(e+fx)}{f(1-n)} + \frac{\left(2ab\left(\frac{\cos(e+fx)}{d}\right)^{-n}(d\cos(e+fx))^n\right)\int\left(\frac{\text{co}}{}\right.}{d}$$

$$= -\frac{2ab(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} + \frac{b^2(d\cos(e}{}$$

$$= -\frac{2ab(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}} - \frac{\left(a^2-\frac{b^2n}{1-n}\right)}{}$$

Mathematica [A] time = 0.396495, size = 161, normalized size = 0.87

$$-\frac{d\sqrt{\sin^2(e+fx)}\csc(e+fx)(d\cos(e+fx))^{n-1}\left(a(n-1)\cos(e+fx)\left(an\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x])^2,x]
```

```
[Out] -((d*(d*Cos[e + f*x])^(-1 + n)*Csc[e + f*x]*(b^2*n*(1 + n)*Hypergeometric2F
1[1/2, (-1 + n)/2, (1 + n)/2, Cos[e + f*x]^2] + a*(-1 + n)*Cos[e + f*x]*(2*
b*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]^2] + a*n*Cos[
e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]))*Sqr
t[Sin[e + f*x]^2])/(f*(-1 + n)*n*(1 + n)))
```

Maple [F] time = 1.933, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{2}\left(d\cos\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2\right)\left(d\cos\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)*(d*cos(f*x + e))^n
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sec\left(fx+e\right)+a\right)^2\left(d\cos\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^2*(d*cos(f*x + e))^n, x)
```

3.877 $\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx$

Optimal. Leaf size=132

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)\sqrt{\sin^2(e+fx)}}-\frac{b\sin(e+fx)(d\cos(e+fx))^n\text{Hyper}}{fn\sqrt{\text{s}}}$$

```
[Out] -((b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])
/(d*f*(1 + n)*Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.114997, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {4225, 16, 2748, 2643}

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)\sqrt{\sin^2(e+fx)}}-\frac{b\sin(e+fx)(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(e+f\right.}{fn\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] -((b*(d*Cos[e + f*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[e + f*x]
^2]*Sin[e + f*x])/(f*n*Sqrt[Sin[e + f*x]^2])) - (a*(d*Cos[e + f*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])
/(d*f*(1 + n)*Sqrt[Sin[e + f*x]^2])
```

Rule 4225

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[(Activate
Trig[u]*(B + A*Sin[a + b*x]))/Sin[a + b*x], x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (d\cos(e+fx))^n(a+b\sec(e+fx))\,dx &= \int (d\cos(e+fx))^n(b+a\cos(e+fx))\sec(e+fx)\,dx\\ &= d\int (d\cos(e+fx))^{-1+n}(b+a\cos(e+fx))\,dx\\ &= a\int (d\cos(e+fx))^n\,dx+(bd)\int (d\cos(e+fx))^{-1+n}\,dx\\ &= -\frac{b(d\cos(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(e+fx)\right)\sin(e+fx)}{fn\sqrt{\sin^2(e+fx)}}-\frac{a(d\cos(e+f}{}\end{aligned}$$

Mathematica [A] time = 0.11885, size = 106, normalized size = 0.8

$$-\frac{\sqrt{\sin^2(e+fx)}\csc(e+fx)(d\cos(e+fx))^n\left(an\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)+b(n+1\right.}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(a + b*Sec[e + f*x]),x]
```

```
[Out] -(((d*Cos[e + f*x])^n*Csc[e + f*x]*(b*(1 + n)*Hypergeometric2F1[1/2, n/2, (
2 + n)/2, Cos[e + f*x]^2] + a*n*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*n*(1 + n)))
```

Maple [F] time = 0.576, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{n}\left(a+b\sec\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(e+fx\right)\right)^{n}\left(a+b\sec\left(e+fx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((d*cos(e + f*x))**n*(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(fx+e)+a)(d\cos(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)*(d*cos(f*x + e))^n, x)
```

3.878 $\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=196

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-1),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e+\ldots}{}$$

```
[Out] (a*AppellF1[1/2, (-1 - n)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(
a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)*S
in[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, -n/2, 1, 3/2, Sin[e + f*x]^
2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(d*Cos[e + f*x])^n*Sin[e + f*x])/((a^2
 - b^2)*f*(Cos[e + f*x]^2)^(n/2))
```

Rubi [A] time = 0.365425, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3869, 2823, 3189, 429}

$$\frac{a\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-1),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)}-\frac{b\sin(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```


```
[Out] (a*AppellF1[1/2, (-1 - n)/2, 1, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(
a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)*S
in[e + f*x])/((a^2 - b^2)*f) - (b*AppellF1[1/2, -n/2, 1, 3/2, Sin[e + f*x]^
2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*(d*Cos[e + f*x])^n*Sin[e + f*x])/((a^2
 - b^2)*f*(Cos[e + f*x]^2)^(n/2))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
```

```
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{a+b\sec(e+fx)}\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}}{a+b\sec(e+fx)}\,dx$$

$$= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{1+n}(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= -\left(\left(a\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{2+n}(e+fx)}{b^2-a^2\cos^2(e+fx)}\,dx\right) + \left(b\cos^{-n}(e+fx)(d\cos(e+$$

$$= -\frac{\left(a\cos^{2\left(\frac{1}{2}+\frac{n}{2}\right)-n}(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{-\frac{1}{2}-\frac{n}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{1+n}{2}}}{-a^2+b^2+a^2x^2}\,dx,x,\sin(e}{f}$$

$$= \frac{aF_1\left(\frac{1}{2};\frac{1}{2}(-1-n),1;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}(-}}{\left(a^2-b^2\right)f}$$

Mathematica [B] time = 25.6473, size = 5216, normalized size = 26.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.782, size = 0, normalized size = 0.

$$\int \frac{\left(d \cos \left(f x + e\right)\right)^{n}}{a + b \sec \left(f x + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \cos \left(f x + e\right)\right)^{n}}{b \sec \left(f x + e\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d \cos \left(f x + e\right)\right)^{n}}{b \sec \left(f x + e\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(e+fx\right)\right)^{n}}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{n}}{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a), x)
```

3.879 $\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=309

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-3),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin(e}{}$$

```
[Out] (a^2*AppellF1[1/2, (-3 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)
*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 - n)/2, 2, 3/2, S
in[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*
x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) - (2*a*
b*AppellF1[1/2, (-2 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(d*Cos[e + f*x])^n*Sin[e + f*x])/((a^2 - b^2)^2*f*(Cos[e + f*x]^
2)^(n/2))
```

Rubi [A] time = 0.513906, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4264, 3869, 2824, 3189, 429}

$$\frac{a^2\sin(e+fx)\cos(e+fx)\cos^2(e+fx)^{\frac{1}{2}(-n-1)}(d\cos(e+fx))^n F_1\left(\frac{1}{2};\frac{1}{2}(-n-3),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)}{f\left(a^2-b^2\right)^2}+\frac{b^2\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```


```
[Out] (a^2*AppellF1[1/2, (-3 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)
/(a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)
*Sin[e + f*x])/((a^2 - b^2)^2*f) + (b^2*AppellF1[1/2, (-1 - n)/2, 2, 3/2, S
in[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a^2 - b^2)]*Cos[e + f*x]*(d*Cos[e + f*
x])^n*(Cos[e + f*x]^2)^((-1 - n)/2)*Sin[e + f*x])/((a^2 - b^2)^2*f) - (2*a*
b*AppellF1[1/2, (-2 - n)/2, 2, 3/2, Sin[e + f*x]^2, (a^2*Sin[e + f*x]^2)/(a
^2 - b^2)]*(d*Cos[e + f*x])^n*Sin[e + f*x])/((a^2 - b^2)^2*f*(Cos[e + f*x]^
2)^(n/2))
```


Rule 4264

```
Int[(u_)*((c_.)*sin[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Csc[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSecantIntegrandQ[u, x]
```

Rule 3869

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Dist[Sin[e + f*x]^n*(d*Csc[e + f*x])^n, Int[(b +
a*Sin[e + f*x])^m/Sin[e + f*x]^(m + n), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^n}{(a+b\sec(e+fx))^2}\,dx = \left((d\cos(e+fx))^n(d\sec(e+fx))^n\right)\int \frac{(d\sec(e+fx))^{-n}}{(a+b\sec(e+fx))^2}\,dx$$

$$= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{2+n}(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \left(\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \left(\frac{b^2\cos^{2+n}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} - \frac{2ab\cos^{3+n}(e+fx)}{\left(b^2-a^2\cos^2(e+fx)\right)^2} + \frac{}{(-}\right.$$

$$= \left(a^2\cos^{-n}(e+fx)(d\cos(e+fx))^n\right)\int \frac{\cos^{4+n}(e+fx)}{\left(-b^2+a^2\cos^2(e+fx)\right)^2}\,dx - \left(2ab\cos^{-n}(e+fx)(d\cos\right.$$

$$= \frac{\left(a^2\cos^{2\left(\frac{1}{2}+\frac{n}{2}\right)-n}(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{-\frac{1}{2}-\frac{n}{2}}\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{\frac{3+n}{2}}}{\left(a^2-b^2-a^2x^2\right)^2}\,dx, x, \sin(\right.}{f}$$

$$= \frac{a^2F_1\left(\frac{1}{2};\frac{1}{2}(-3-n),2;\frac{3}{2};\sin^2(e+fx),\frac{a^2\sin^2(e+fx)}{a^2-b^2}\right)\cos(e+fx)(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1}{2}}}{\left(a^2-b^2\right)^2 f}$$

Mathematica [B] time = 32.6211, size = 10296, normalized size = 33.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^n/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^n}{\left(a+b\sec\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^n}{\left(b\sec\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^n}{b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(e+fx\right)\right)^n}{\left(a+b\sec\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((d*cos(e + f*x))**n/(a + b*sec(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^n}{\left(b\sec\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n/(a+b*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n/(b*sec(f*x + e) + a)^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
    ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```